Proton Radius Measurement with AMBER
An approach complementary to PRES

Jan Friedrich
Technische Universität München

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PRES Collaboration Meeting

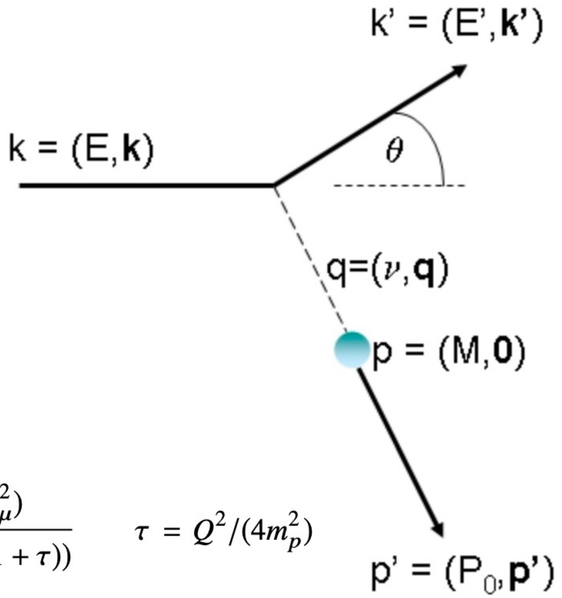
Planned, ongoing, recent scattering experiments to measure the proton form factor at low Q^2

The discrepancy between the results – the proton radius puzzle - triggered many new proposals and experiments:

- e^- scattering radiative: ISR electron scattering
- e^- scattering at medium *E* with active-target TPC at MAMI (PRES)
- e^- scattering at higher *E*: PRad at Jefferson Lab

- $\mu^{+/-}, e^{+/-}$ scattering at low energy: MUSE / PSI

- $\mu^{+/-}$ at high *E* at CERN (AMBER)
  different systematics

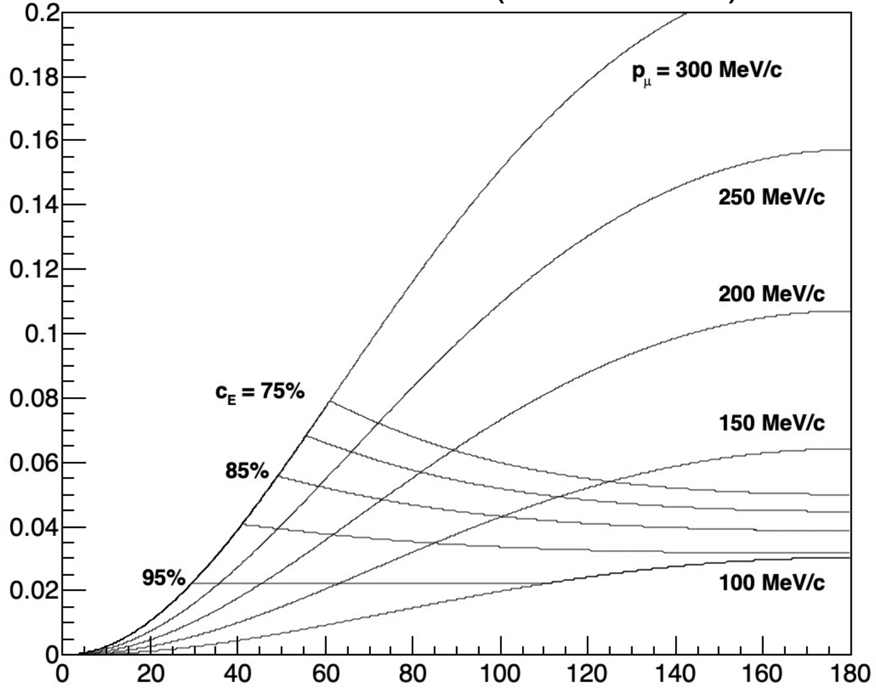


$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

$$R = \frac{\vec{p}_\mu^{\,2} - \tau(s - 2m_p^2(1+\tau))}{\vec{p}_\mu^{\,2}(1+\tau)} \qquad \varepsilon = \frac{E_\mu^2 - \tau(s - m_\mu^2)}{\vec{p}_\mu^{\,2} - \tau(s - 2m_p^2(1+\tau))} \qquad \tau = Q^2/(4m_p^2)$$

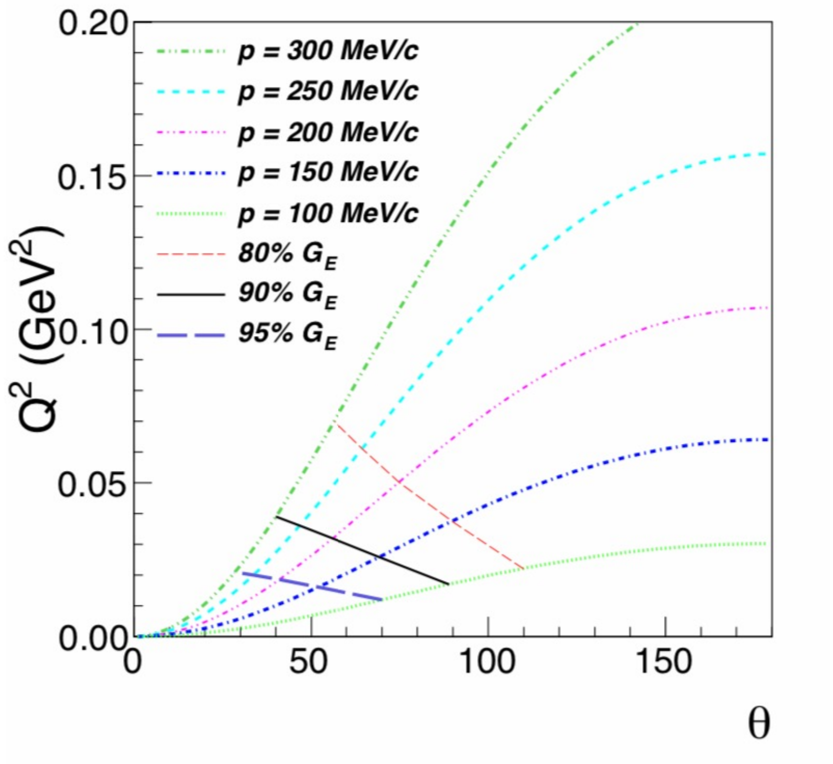
MUSE – kinematics of low-energy elastic muon scattering

our calculation (muon mass)

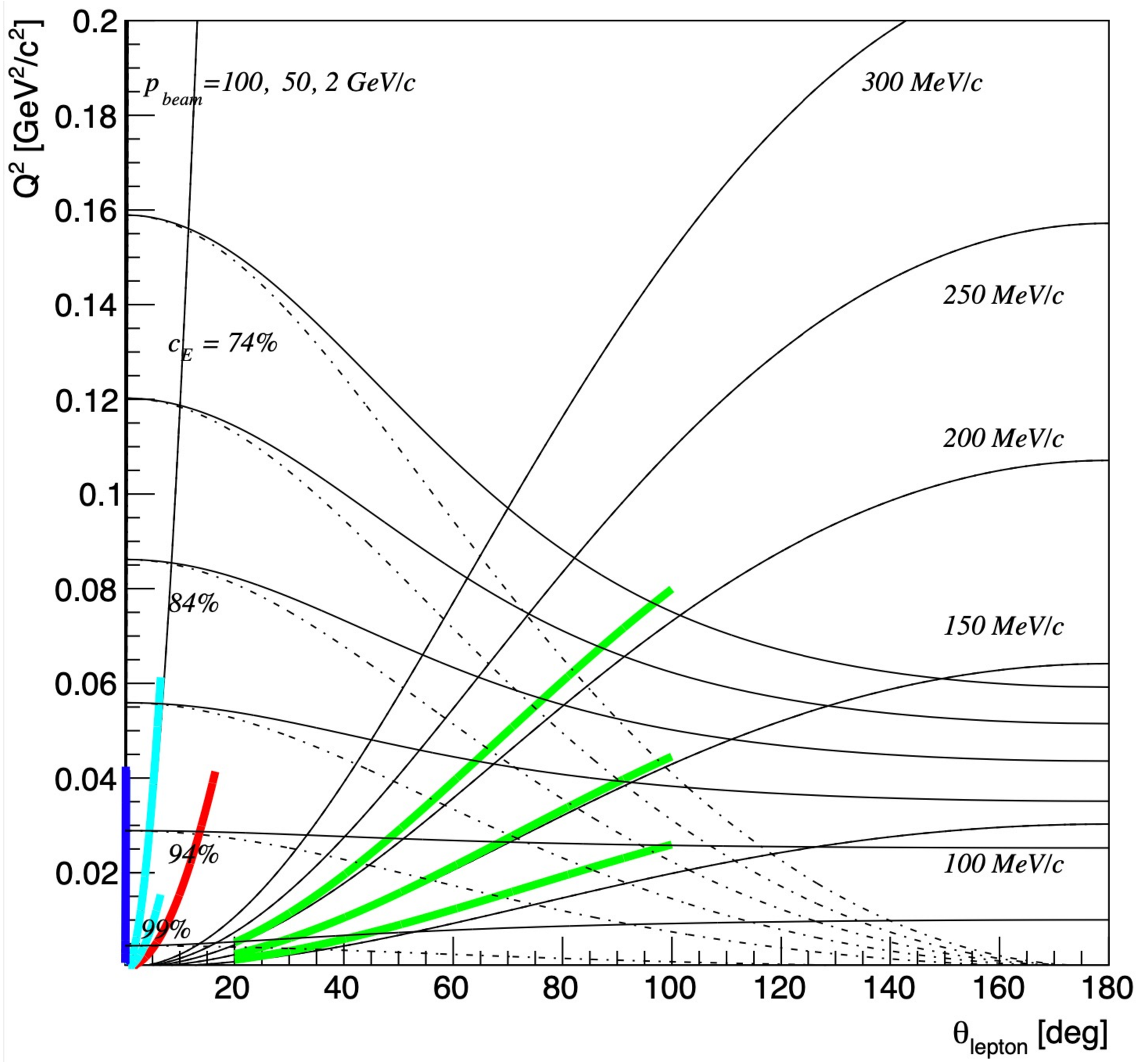


A Proposal for the Paul Scherrer Institute πM1 beam line

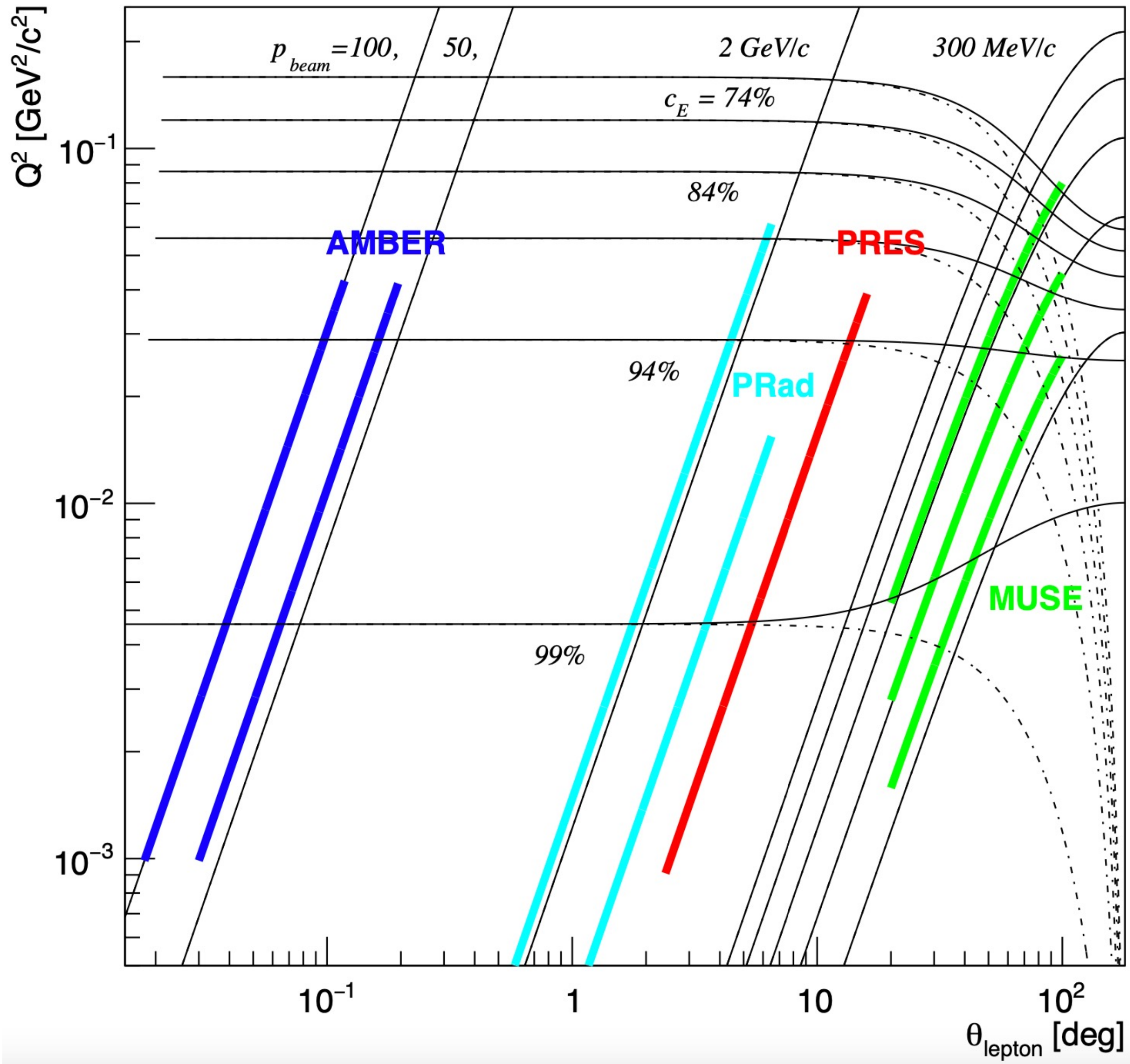
Studying the Proton "Radius" Puzzle with μp Elastic Scattering



Kinematic ranges



Kinematic ranges

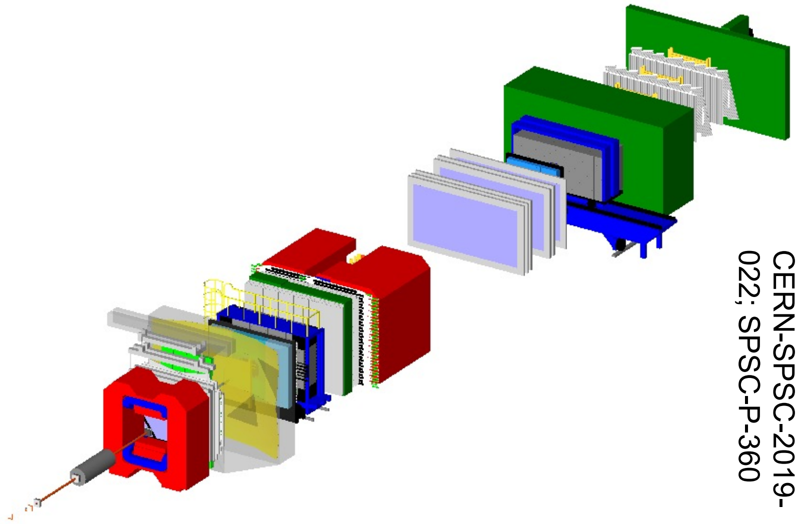


Comparison of kinematics

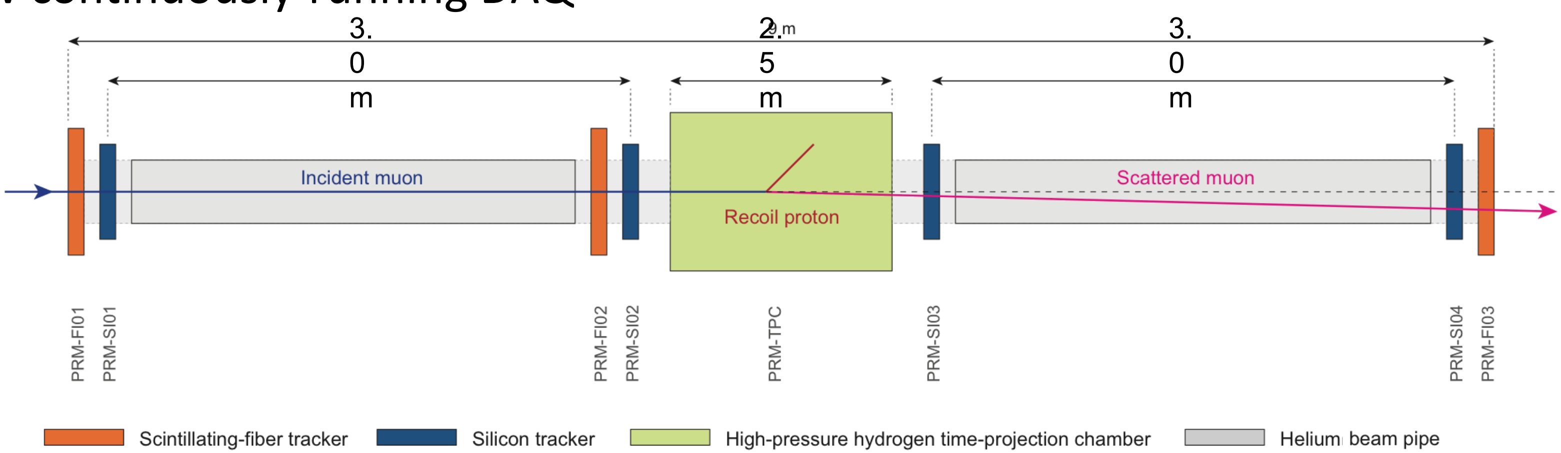
- at low Q^2 the cross section is dominated by G_E
- The cross section is practically independent on the lepton energy (above 500 MeV)

AMBER Proton Radius Measurement

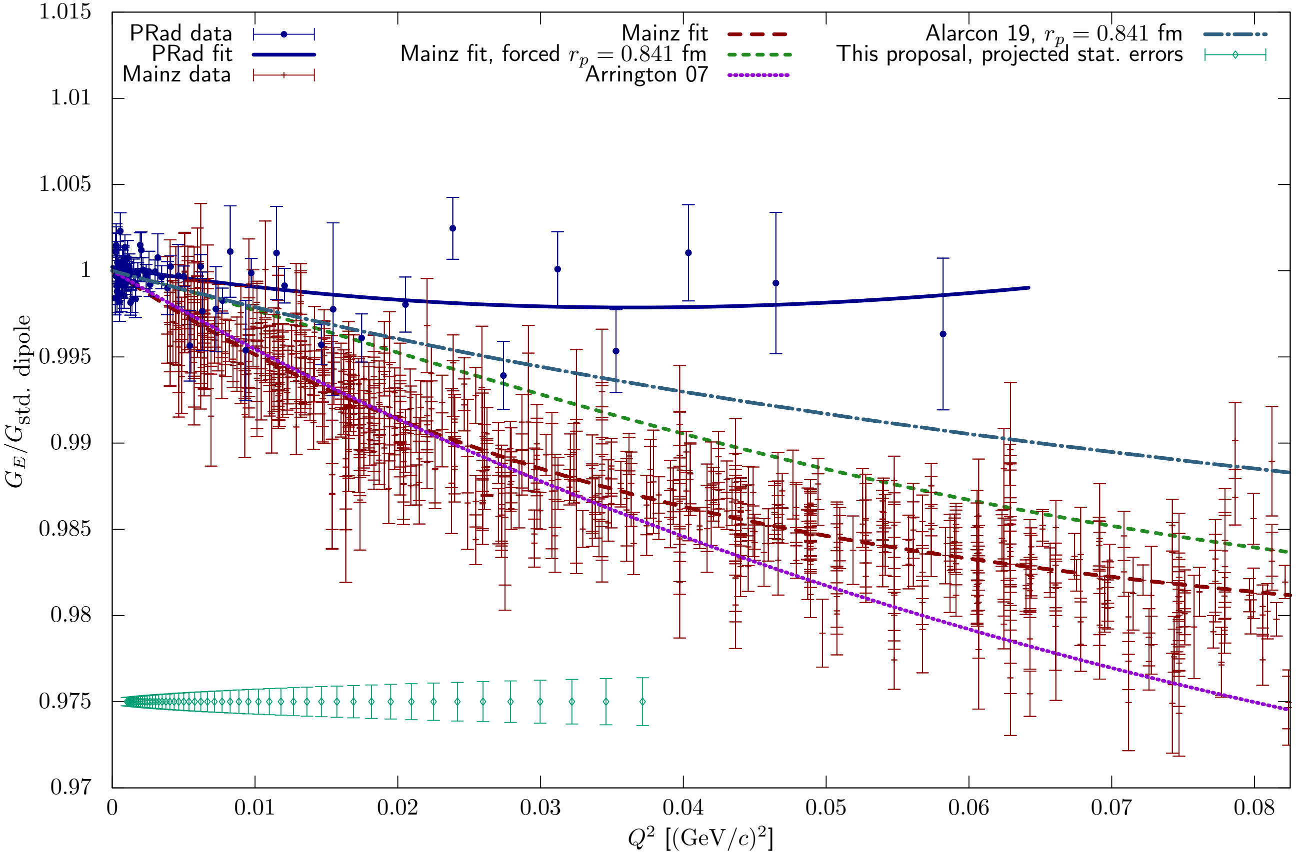
- **Measurement of low-Q^2 elastic-scattering**
- Detection of low-energetic recoil-protons and scattered muons with small scattering-angle.
- Silicon trackers along large lever arm to measure small scattering-angles
- Fiber tracker timing (and trigger)
- TPC as an active target with the ability to measure the low-energetic recoil-proton
- New continuously-running DAQ



CERN-SPSC-2019-022; SPSC-P-360



Mainz vs JLab data

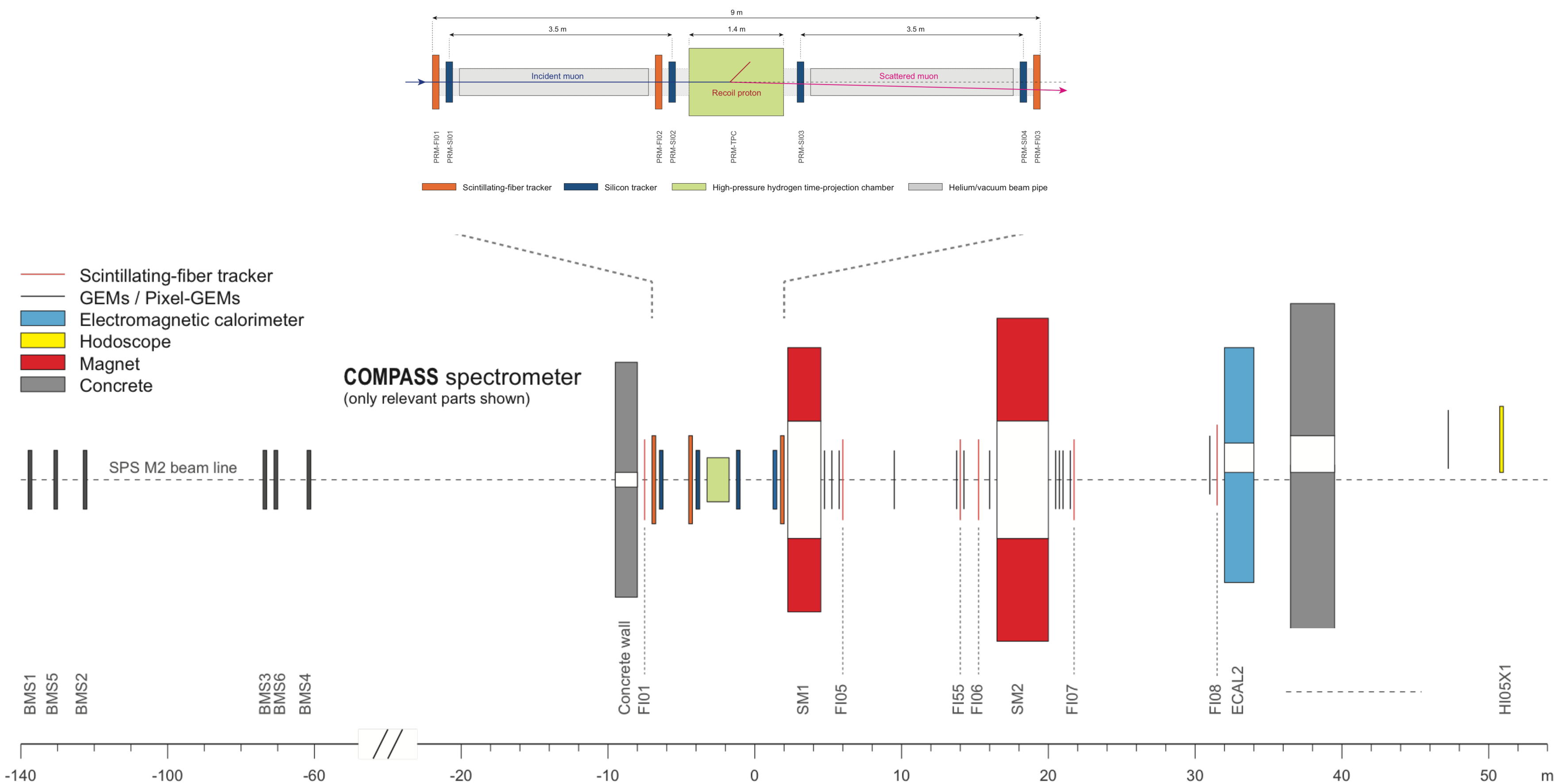


uncertainties for the COMPASS++/AMBER proposal

- program for 200 days of beam
- precision on the proton radius < 0.01 fm

Layout of the AMBER PRM

- **Advantages of using the *COMPASS* spectrometer**
- Measurement of muon momentum and understanding of background.



Layout of the AMBER PRM

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CERN-SPSC-2019-022;
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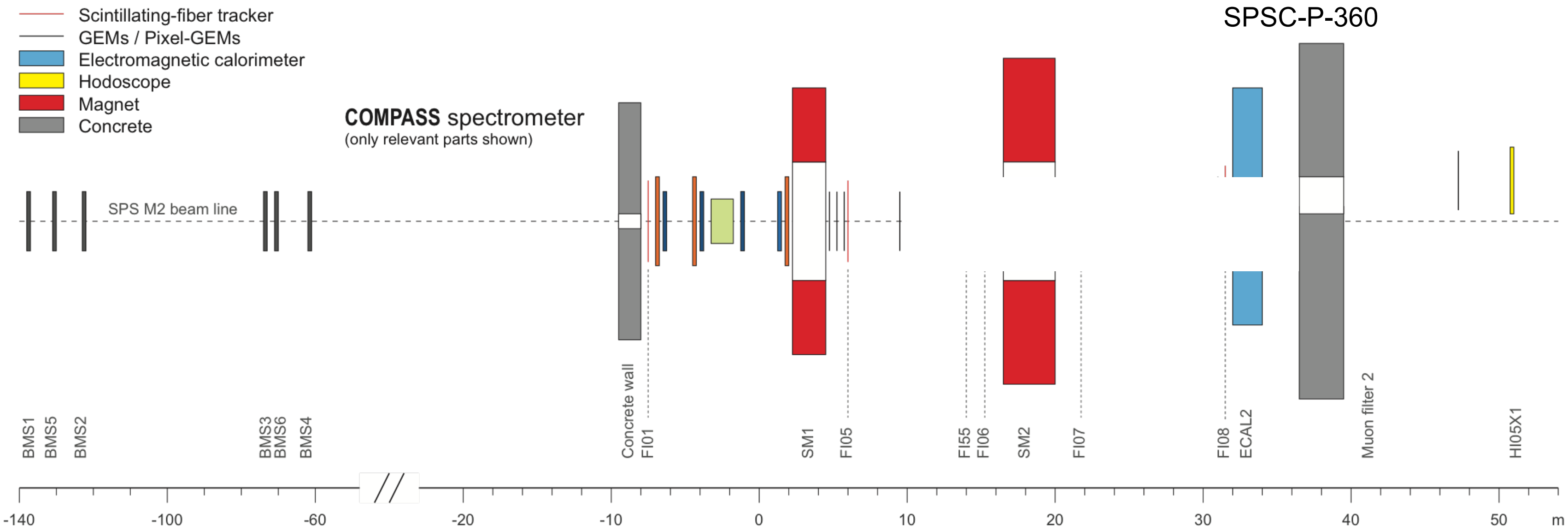
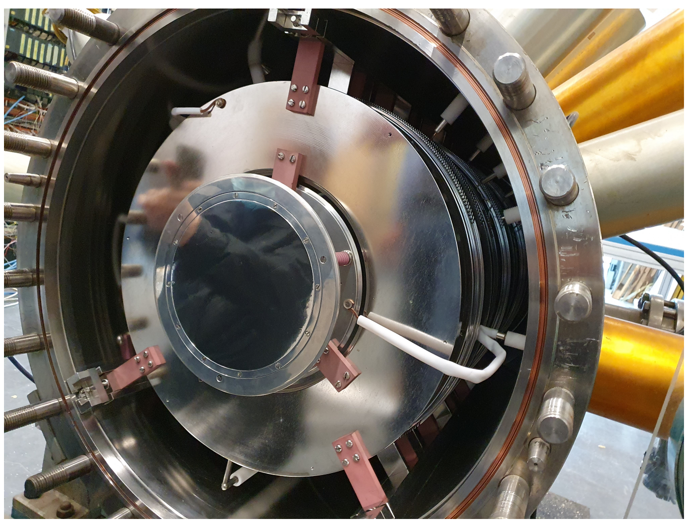


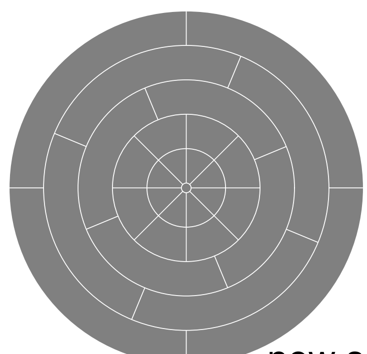
- *COMPASS* spectrometer
  → Momentum measurement of scattered muon
  → Radiative background using electromagnetic calorimeter
  → Muon identification with muon filter and hodoscope

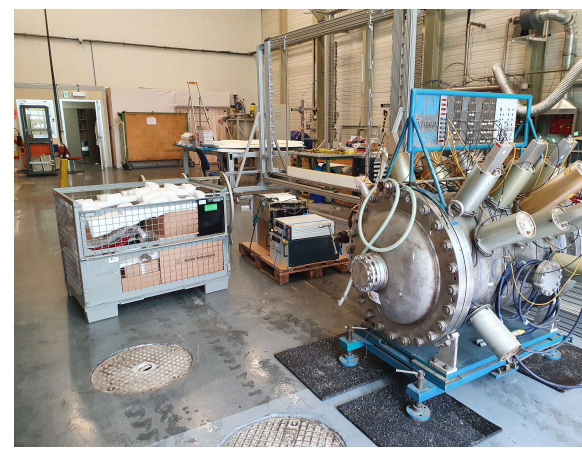
TPC for the pilot run

- IKAR TPC was transported from GSI to CERN on 22 November 2020
- Refurbishing of the inner part is ongoing
- Pressure and valve tests foreseen in April
- New readout plane has been produced, ready to be installed



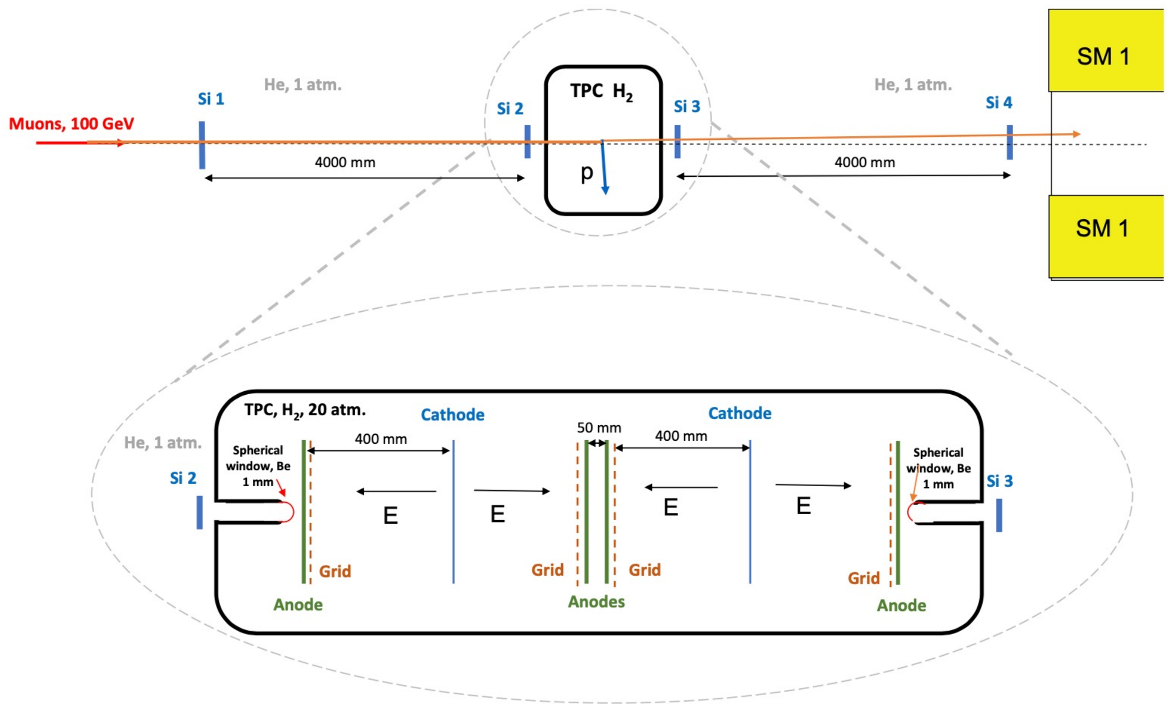
opened TPC with old electrode structure

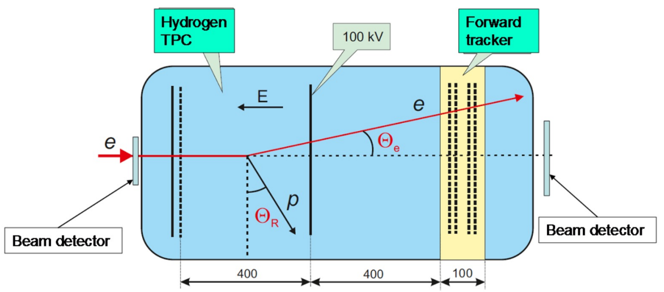




new segmented readout plane

AMBER and PRES TPCs

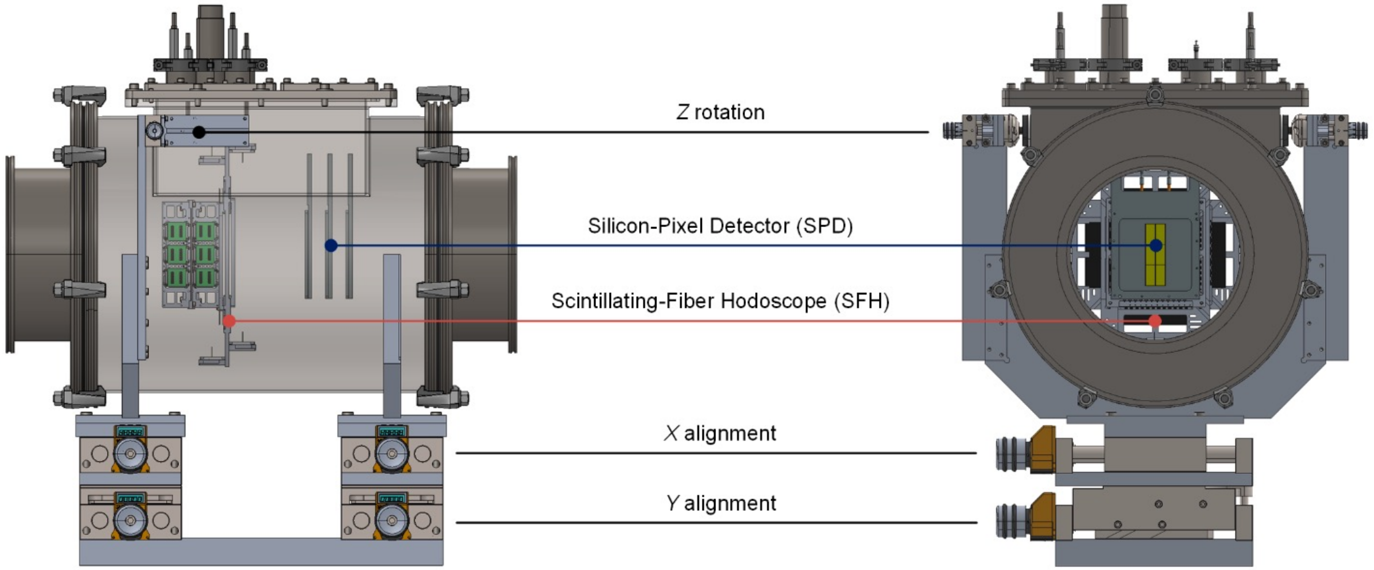




cathode–grid distance (drift zone)	400.0 mm
grid–anode distance	10.0 mm
grid wire diameter	0.1 mm
grid wire spacing	1.0 mm
grid transparency	1.8%
anode outer diameter	600 mm
hydrogen pressure	20 bar and 4 bar
electric field in drift space E/P	0.116 kV/(cm bar)
electric field in grid–anode zone E/P	0.340 kV/(cm bar)
electron drift velocity in the drift zone	0.41 cm/μs
electron drift velocity in the grid–anode space	0.70 cm/μs

- many similar parameters for the two setups
- Similar geometry allows for using calibrations (e.g. the drift velocity)
- similar technology for gas system (purification, temperature and pressure control)

Unified Tracker Station



New design of the detector holding structure to

- accommodate a small distance between the Silicon-pixel detectors (SPD) and the Scintillating-Fibre Hodoscope (SFH) (for hit-timing association)
- Allow for independent access and cooling infrastructure
- Compatible to connect to beam line elements for the He volume

Charge radius: definition and model dependence

Determination of the rms radius from a form factor measurement

- the rms radius of a charge distribution seen in lepton scattering is *defined* as the slope of the electric form factor at vanishing momentum transfer Q^2

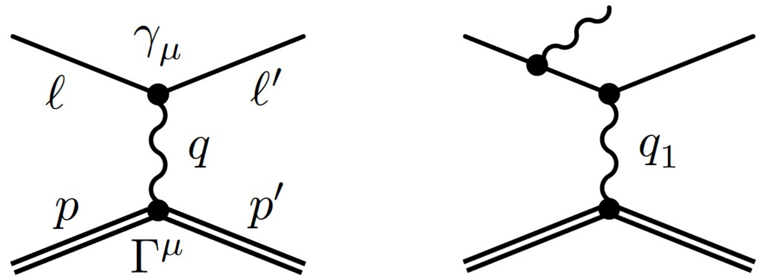
$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

- elastic scattering experiments provide data for G_E at non-vanishing Q^2 and thus require an extrapolation procedure towards zero
  $\to$ mathematical ansatz may take more or less bounds into account (physics/theory/whatever motivated)
- Any approach (Padé, CF, DI, CM,...) *must* boil down to a series expansion

$$G_E(Q^2) = 1 + c_2 Q^2 + c_4 Q^4 + \ldots$$

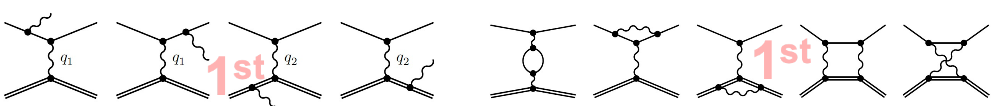
  introducing possibly very different assumptions on the coefficients c_i
- recipe for experimenters: measure a sufficiently large range of Q^2 down to values as small as possible and as precise as possible

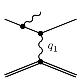
Accelerated charge radiates: correction to elastic lepton-nucleon scattering

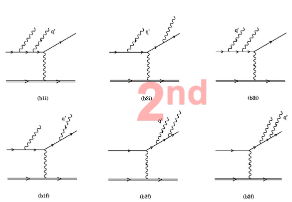


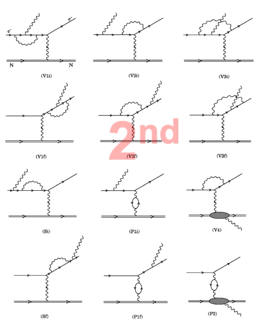
figs. from: Gramolin et al., arXiv:1401.2959

$$d\sigma_{Exp} = d\sigma_{Born}(1+\delta)$$



 includes:





real

virtual

internal corrections

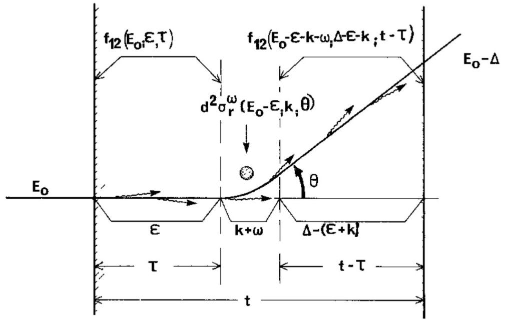
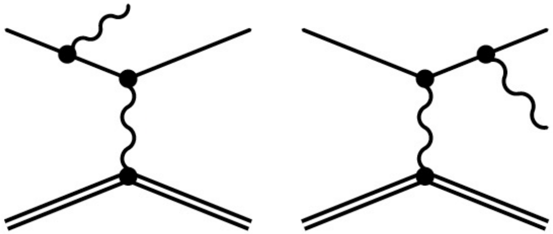


Fig. 1. The path of an electron with incident energy E_0 through a target of thickness t. Energy loss before, during and after the large-angle scattering (which occurs at target depth τ and which is shown enlarged) is ϵ, $k+\omega$ and $\Delta-(\epsilon+k+\omega)$, respectively. For further details of the nomenclature see text.

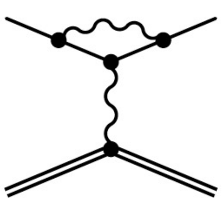
from: Pieroth et al., NIM B36 (1989)

external bremsstrahlung

1^{st} order internal corrections

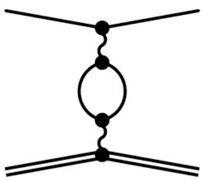


$\delta_{vertex} + \delta_R$



$Q^2 \gg m^2$

$$\delta_{vac} + \delta_{vertex} + \delta_R = \frac{\alpha_{em}}{\pi}\left\{ \ln\left(\frac{(\Delta E_s)^2}{E_e E'_e}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] + \frac{13}{6}\ln\left(\frac{Q^2}{m^2}\right) - \frac{28}{9} - \frac{1}{2}\ln^2\left(\frac{E_e}{E'_e}\right) - \frac{\pi^2}{6} + Sp\left(\cos^2\frac{\theta_e}{2}\right)\right\},$$

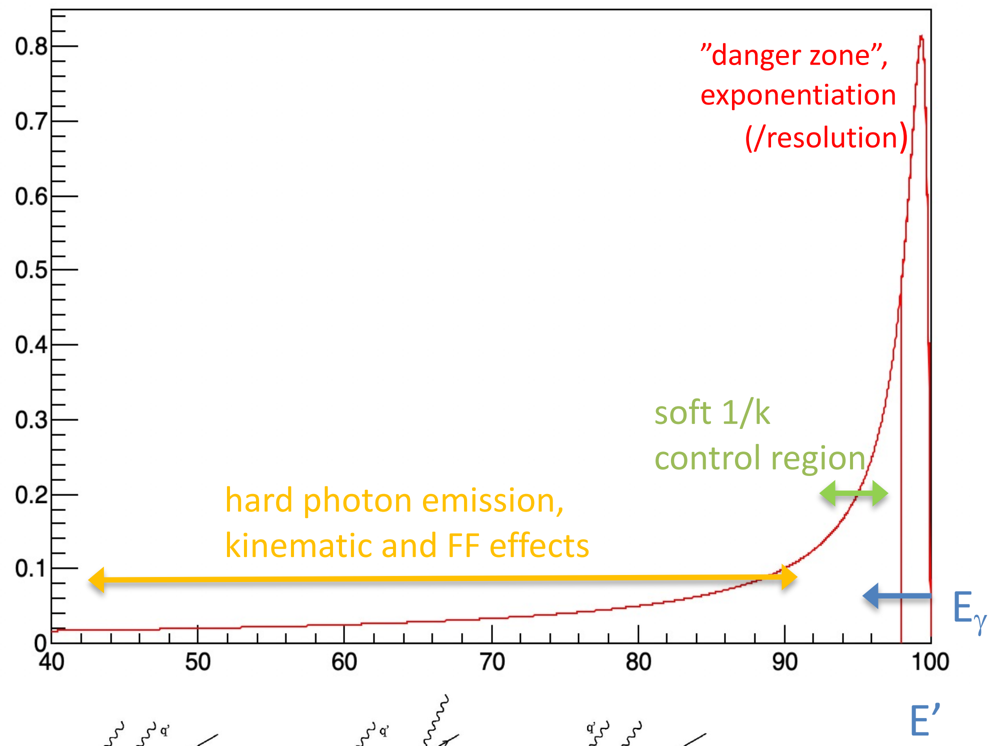


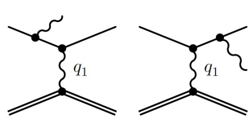
δ_{vac}

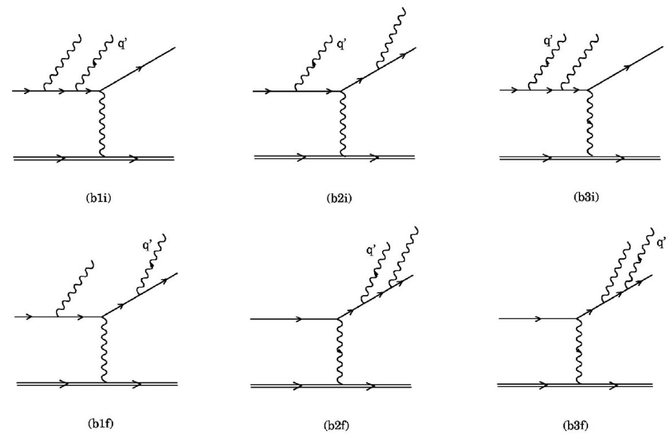
- the first-order real and virtual corrections need to be merged on cross-section level
- formally: $\delta_R = +\infty$ and $\delta_V = -\infty$
- the underlying "infrared" divergence is not related to the regularization scheme
- under certain kinematic conditions and depending on the choice of the cut-off energy ΔE, parts of the corrections may cancel (or become even zero) – this does not imply that the correction is "really small"
- uncertainty has to be estimated in any case, and can be larger than the correction

Peak shape with no experimental (resp. external) smearing

- the correction $\delta_R \xrightarrow{\Delta E \to 0} +\infty$ was originally introduced as "small correction"
- it expresses the probability to emit one real photon along the Born process
- if the emission of a photon with a certain energy is large, it is plausible that two or more photons are emitted:



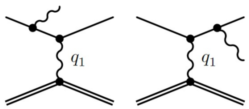


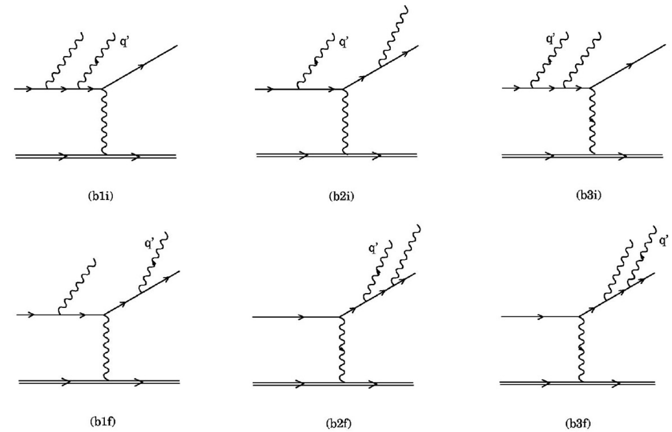


Exponentiation procedure

QED radiative corrections to virtual Compton scattering

M. Vanderhaeghen, J. M. Friedrich, D. Lhuillier, D. Marchand, L. Van Hoorebeke, and J. Van de Wiele
Phys. Rev. C **62**, 025501 – Published 25 July 2000





- if the emission of a photon with a certain energy is large, it is plausible that two (and more) photons are emitted

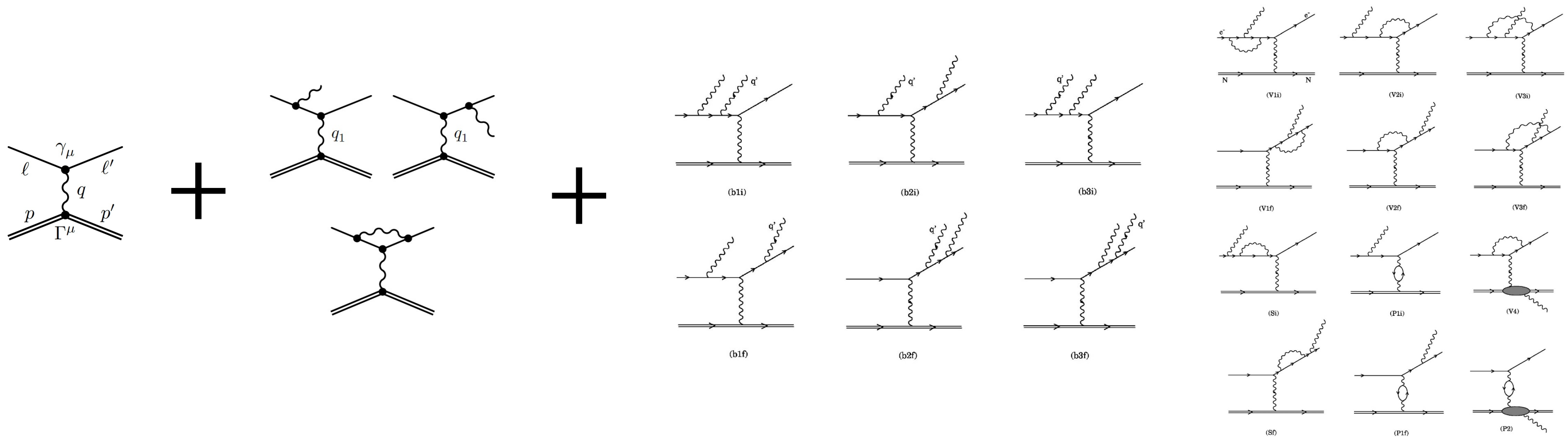
$$\left(\frac{d\sigma}{d\Omega'_e}\right)_{VIRTUAL\,\gamma} + \left(\frac{d\sigma}{d\Omega'_e}\right)_{REAL\,SOFT\,\gamma} = \left(\frac{d\sigma}{d\Omega'_e}\right)_{BORN} (1+\delta_{vac}+\delta_{vertex}+\delta_R)$$

→

$$\left(\frac{d\sigma}{d\Omega'_e}\right)_{VIRTUAL\,\gamma} + \left(\frac{d\sigma}{d\Omega'_e}\right)_{REAL\,SOFT\,\gamma} = \left(\frac{d\sigma}{d\Omega'_e}\right)_{BORN} \frac{e^{\delta_{vertex}+\delta_R}}{(1-\delta_{vac}/2)^2}.$$

- inspired by the higher-order divergence cancellation proof (Jennie, Frautschi, Suura 1961): infinitely soft photon emission / absorption becomes independent
- unclear for finite ΔE (no cheap way around calculating the higher orders)

Exponentiation procedure

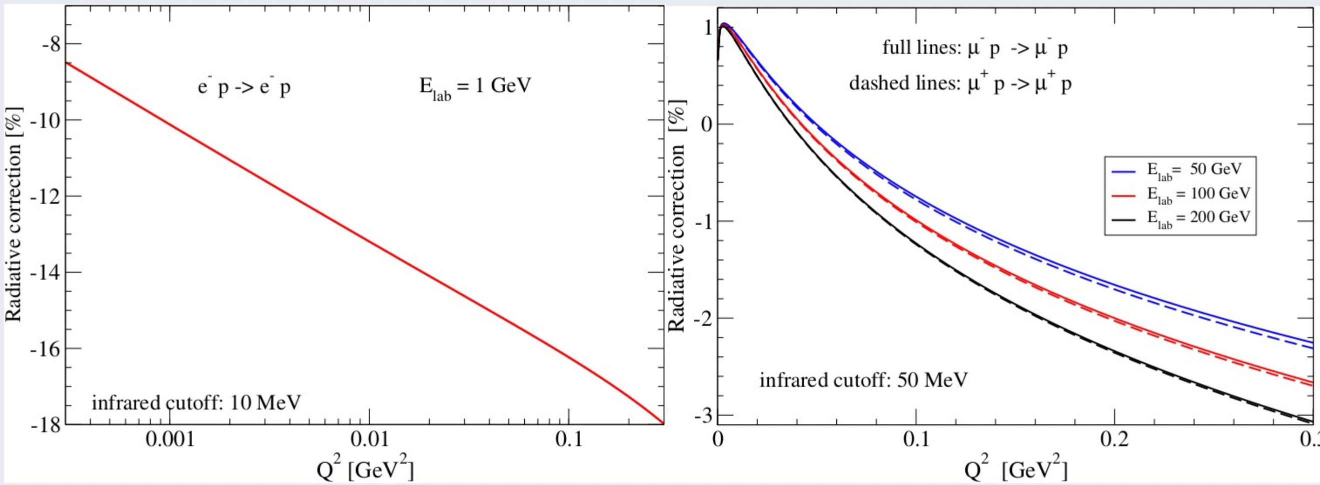


$$1 + \delta(\Delta E) \;\rightarrow\; e^{\delta(\Delta E)} = 1 + \delta(\Delta E) + \frac{\delta^2(\Delta E)}{2} + \cdots$$

- unclear for finite ΔE (no cheap way around the calculation of the higher orders)
- theory homework for 2nd order Feynman diagrams is done, check integral (over 4-particle f.s.)

Radiative corrections for electron and muon scattering

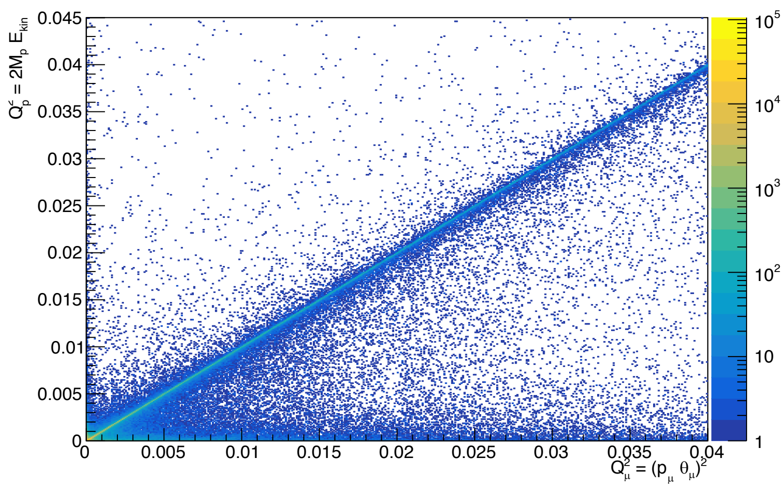
QED radiative corrections



- for soft bremsstrahlung photon energies ($E_\gamma / E_{beam} \sim 0.01$), QED radiative corrections amount to $\sim$ 15-20% for electrons, and to $\sim$ 1.5% for muons
- important contribution to the uncertainty of elastic scattering intensities: *change* of this correction over the kinematic range of interest
- check: impact of exponantiation procedure (stricty valid only for vanishing photon energies): e^-: $2 - 4\%$, μ^-: 0.1%
- integrating the radiative tail out to large fraction of beam energy: shifts the correction to smaller values, but only *increases* the uncertainty

Radiative corrections in the interpretation of lepton-proton scattering data

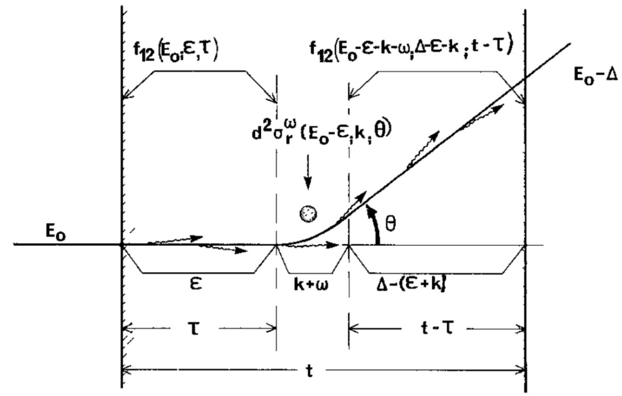
- Measuring the recoil of the proton can save to mix different Q^2 (as happens in case of single-arm measurement without constraint to the elastic peak)
- The interpretation of the Q^2 cross-section dependence still requires a precise understanding of the influence of radiative effects



“Primakoff effect” in forward kinematics (high-energy muon scattering)

External bremsstrahlung

- calculus of the total bremsstrahlung probability down to zero scattering angle
  - screening by atomic electrons
  - long-wavelength limit: contribution from different scattering centers
  - coherent bremsstrahlung in crystals
- "sublimation" of all effects into Tsai's radiation lengths X_0 may not be the full answer to describe correctly the external bremsstrahlung in a given setup
- best way: measure it



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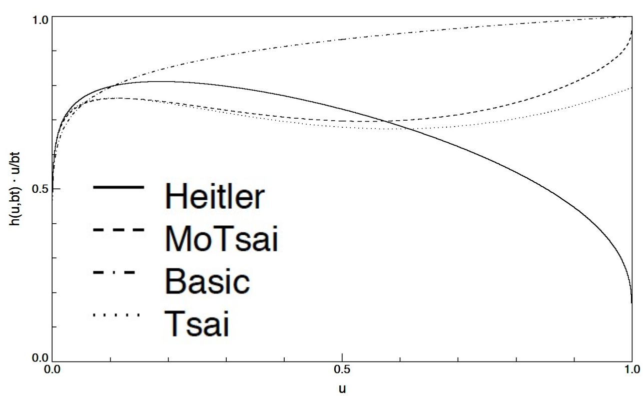


$$h_H(u,bt)\,du = \frac{1}{\Gamma(bt)}\left(-\ln[1-u]\right)^{bt-1}\,du$$

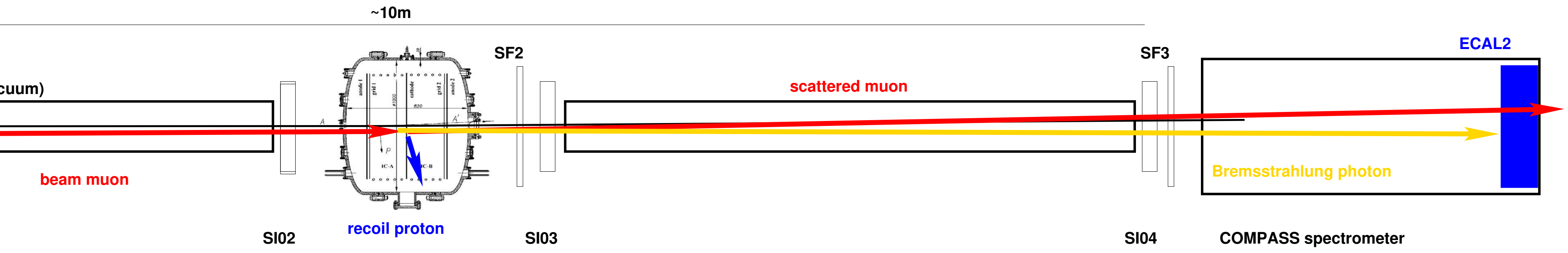
$$h_{MT}(u,bt) = n_{MT}\cdot\frac{bt}{u}\left(1-u+0.75u^2\right)\left(-\ln[1-u]\right)^{bt}$$

$$h_B(u,bt)\,du = bt\cdot u^{bt-1}\,du$$

$$h_T(u,bt) = n_T\left(1-u+0.75u^2\right)u^{bt-1}$$

from: J.F. PhD thesis 2000

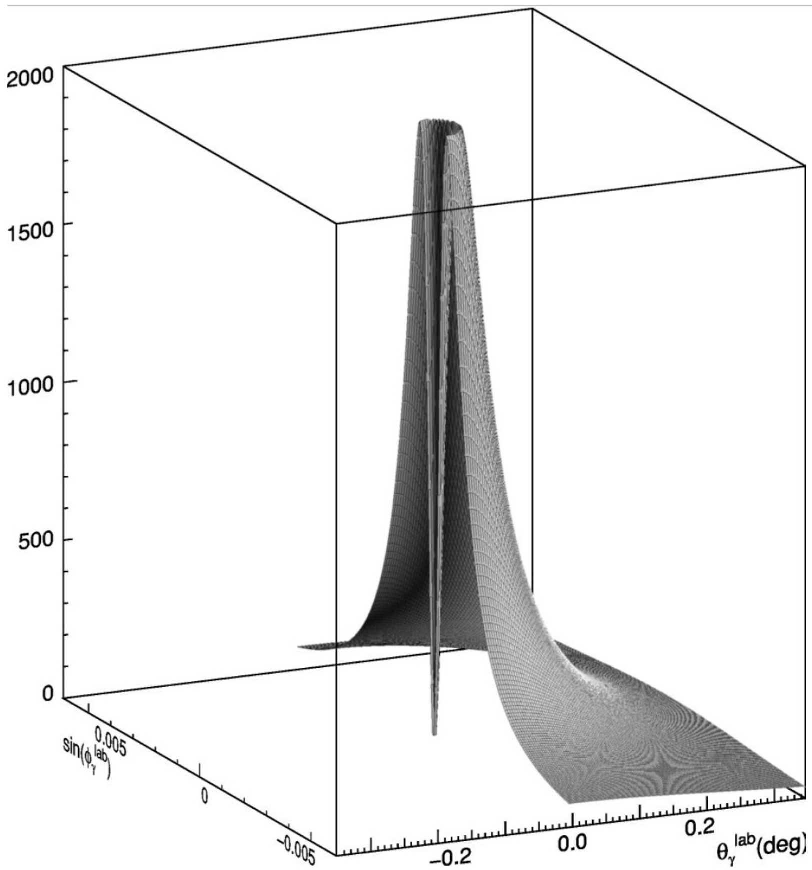
Bremsstrahlung: real-photon emission and observation along muon-proton scattering



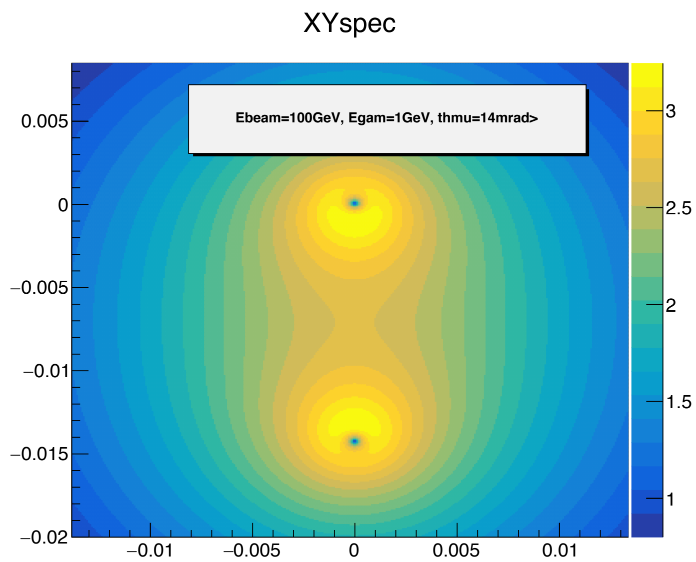
- Bremsstrahlung accompanies the elastic process
- for low-energy photons roughly $1/E_\gamma$ (‘infrared divergence‘)
- angular spectrum: peaking in the relativistic case, opening angle $1/\gamma$ [Lorentz factor]
- 100 GeV beam: E_γ between 50 MeV and 5 GeV emission probability at θ_μ =0.3mrad (Q^2=0.001): 5 x10^{-4}
- Bremsstrahlung events for 7e7 elastic events in Q^2=0.001…0.04 GeV^2/c^2 are about 38000

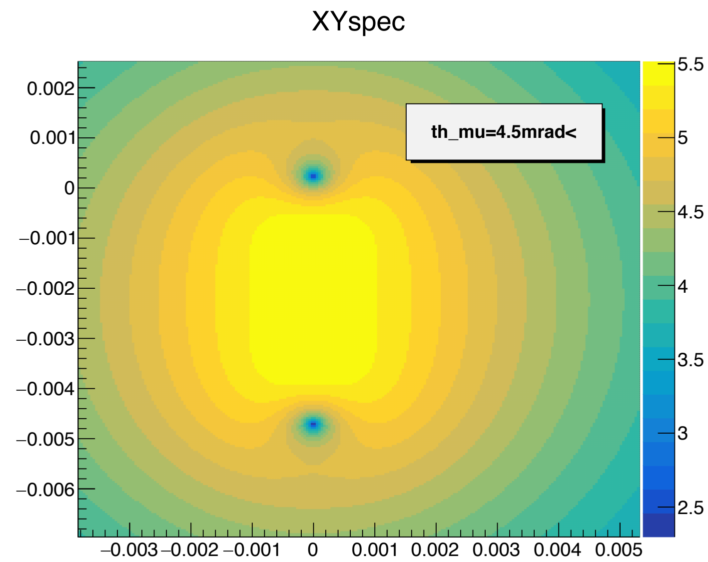
Peaking and forward effect for small Q^2

- Bremsstrahlung of ultra-relativistic moving charges is peaked with opening angle $1/\gamma$
- emission probability in exact forward direction practically vanishes
- if the lepton scattering angle is in the order of the radiation opening angle ($Q^2 \approx m^2$), interference becomes important



Bremsstrahlung emission angle, E=100GeV





- forward cancellation in case of 100 GeV muon scattering at $Q^2 < m^2 \approx 0.01 \text{ GeV }^2$ ($\vartheta_\mu \approx 1\text{mrad}$)
- similar effect discussed in Fadin & Gerasimov, PLB 795 (2019) (however “neglect m^2 compared with M^2 and Q^2”)

Generators

- for a concise description of the experimental conditions, the simulation must include all effects from
  - atomic collision energy loss (Landau straggling)
  - external bremsstrahlung
  - internal radiative corrections

- for the internal 1st order corrections, the ESEPP generator became available (arXiv-1401.2959)
  - implementation of full corrections including the real-photon distributions
  - usage of the TFoam (CERN/root) library for importance sampling

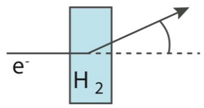
from arXiv-1401.2959 about higher orders:

$$\frac{d\sigma_{\text{meas}}}{d\Omega_\ell} = \exp(\delta) \frac{d\sigma_{\text{Born}}}{d\Omega_\ell}. \tag{2.3}$$

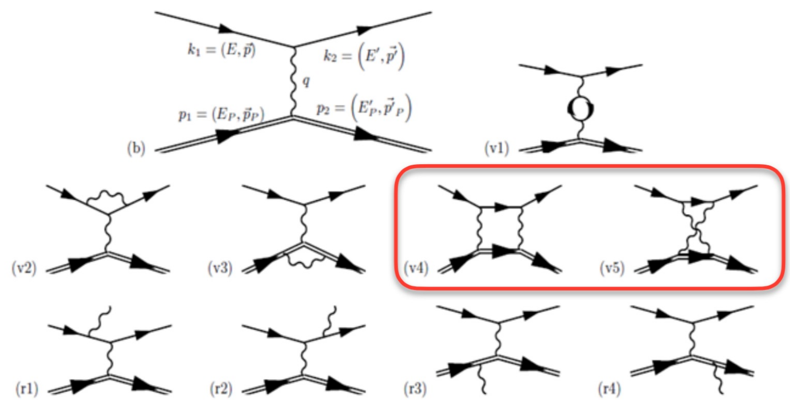
This exponentiation procedure is incompatible with our approach, but we can use the formula (2.3) to make a rough estimation of the contribution of higher-order bremsstrahlung. To do this, we choose the following numerical parameters approximately corresponding to the Novosibirsk TPE experiment: $E_\ell = 1$ GeV, $-q^2 = 1$ GeV^2, and $\Delta E = 0.1$ GeV.

do the calculation with $\Delta E = 0.01$ GeV and get the uncertainty orders of magnitude higher.

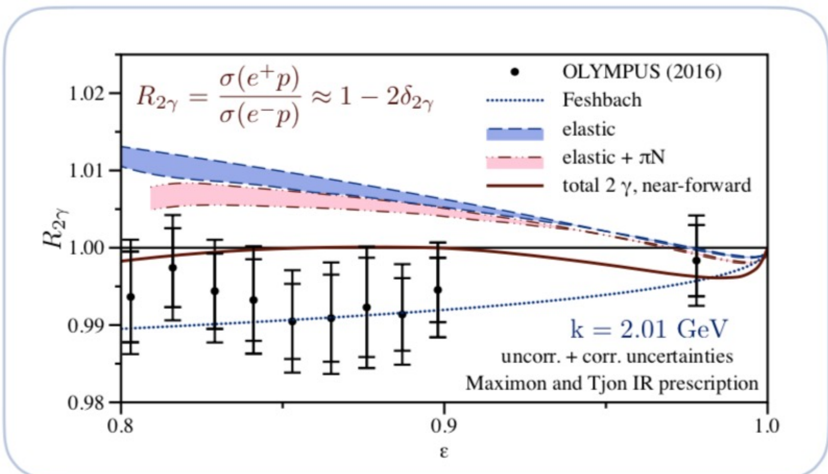
two-photon exchange
Radiative corrections



e⁻-p scattering



$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$



near-forward 2γ agree with data
multi-particle 2γ, e.g. πN, is important

Tomalak, Pasquini, Vdh (2017)
Pasquini, Vdh, Ann.Rev.Nucl.Part.Sci(2018)

- Effect of positive vs. negative muons presumably too small (unless two full experiments are made)

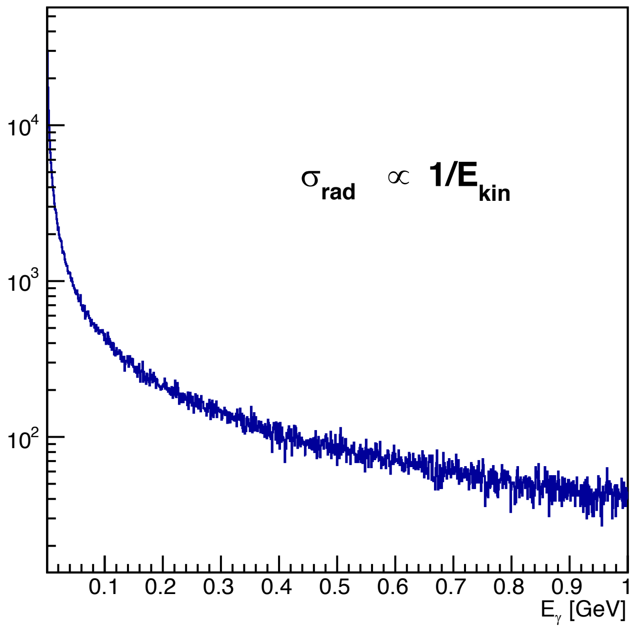
Summary

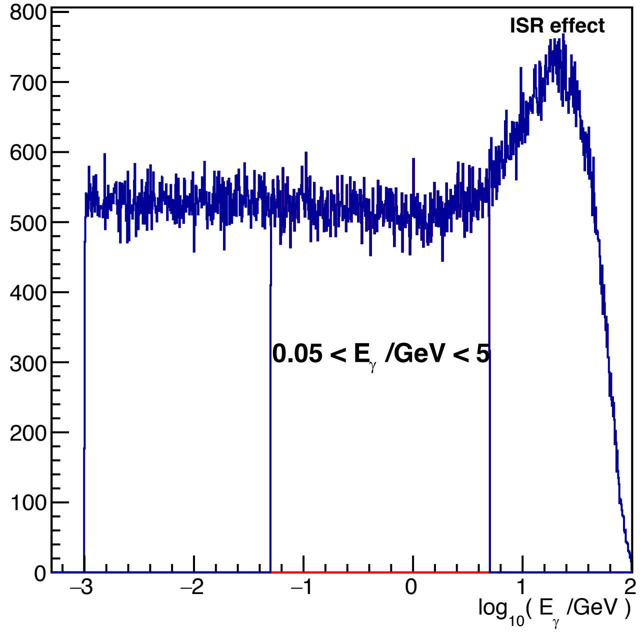
- AMBER is approved at CERN for various measurements of QCD, including the proton charge radius
  - with a 100 GeV muon beam
  - about 150 days of beam time

- The AMBER measurement has many similarities with PRES
  - a similar active-target TPC is employed
  - four 400 mm drift cells instead of two
  - Similar geometry allows for synergy effects in the construction, operation and calibration procedures

Real-photon energy spectrum

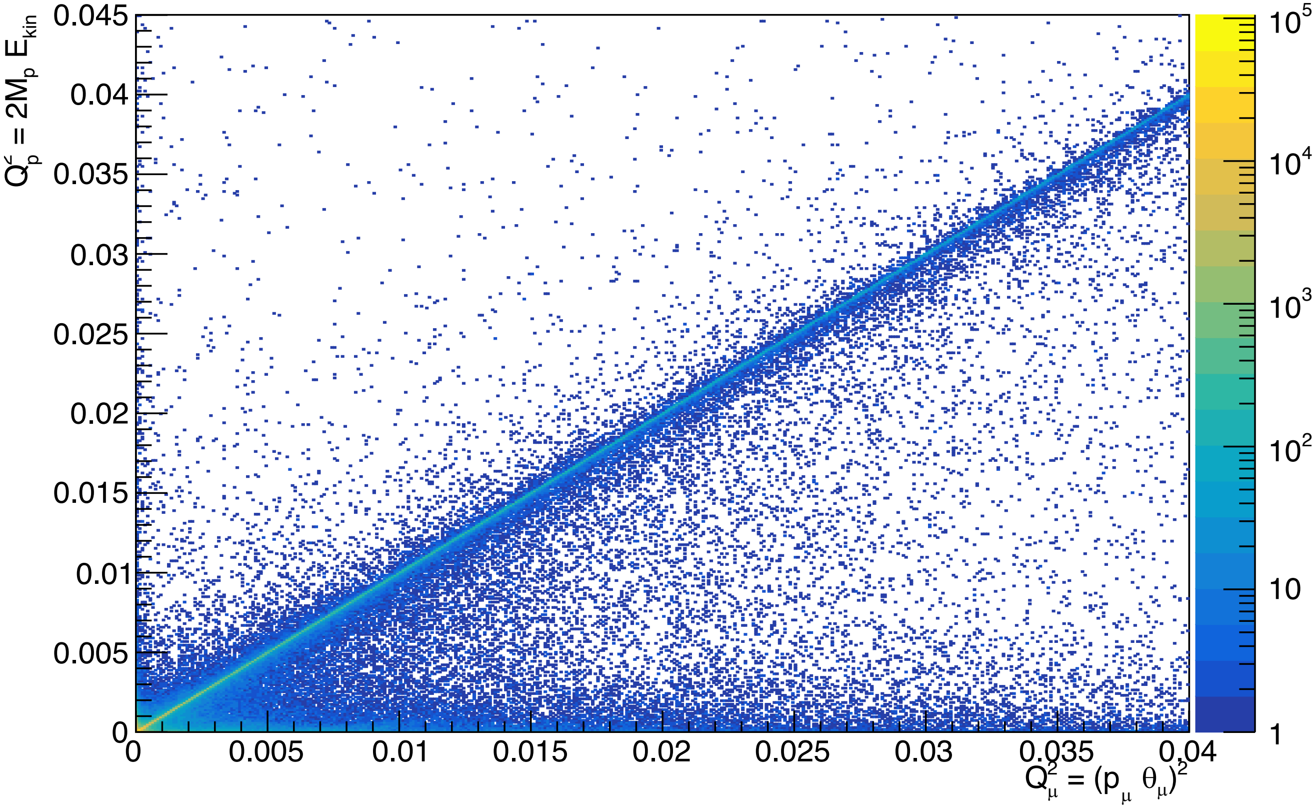
MC simulation of 500k events in θ_μ =0.3 … 2 mrad, E_γ > 1 MeV





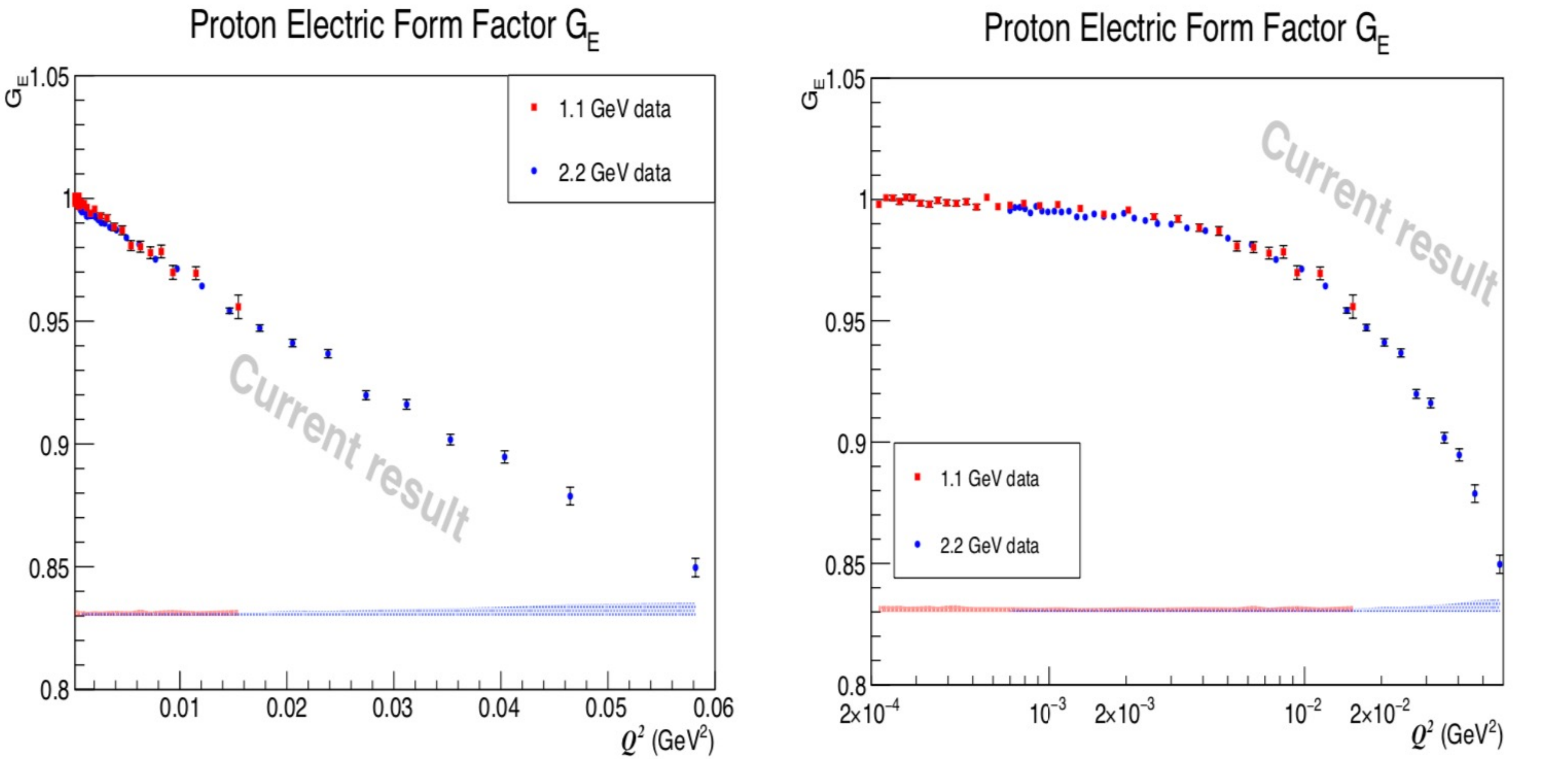
ISR effect: if incoming muon loses much of its energy, the scattering off the proton under a specific scattering angle happens at lower average Q^2 and accordingly a larger cross section

Impact on Q2 reconstruction



real-photon emission distorts the kinematics, correlation of reconstruction from muon and recoil proton becomes blurred

PRad



Lowest Q^2 ever achieved from ep elastic scattering

from: H. Gao, ICSAC2019, Losinj, Croatia

General cross-section behavior

- steep increase towards smaller Q^2 with $1/Q^4$
- forever rising?
- not for scattering off atoms / molecules: