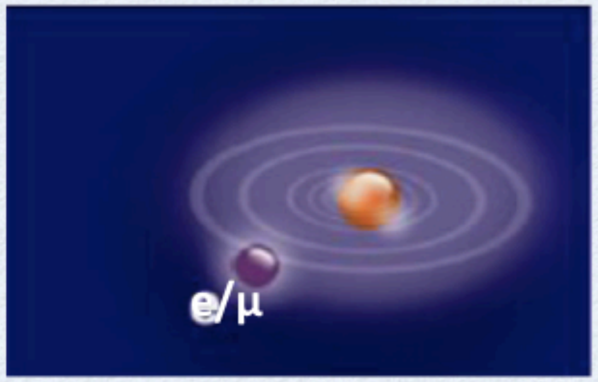
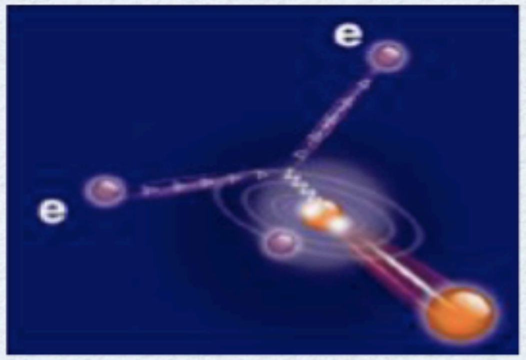




Proton radius: theoretical overview and perspectives

M. Vanderhaeghen

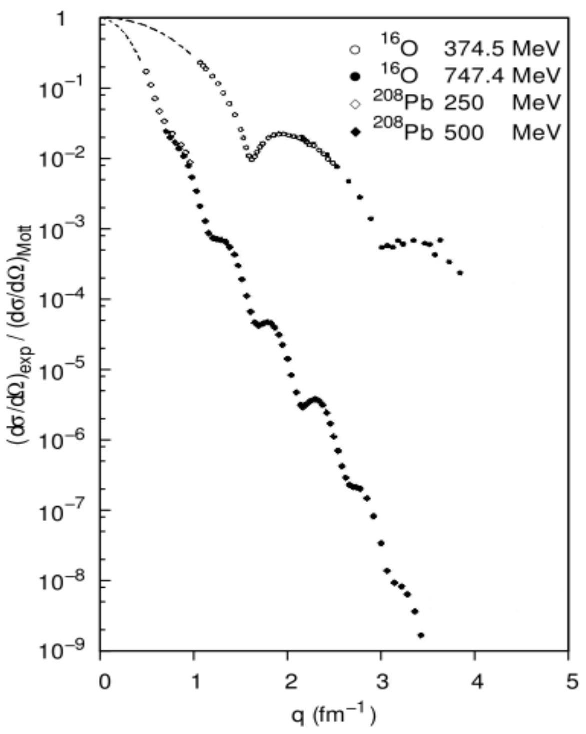
PRES Meeting, March 30-31, 2021, Univ. Mainz

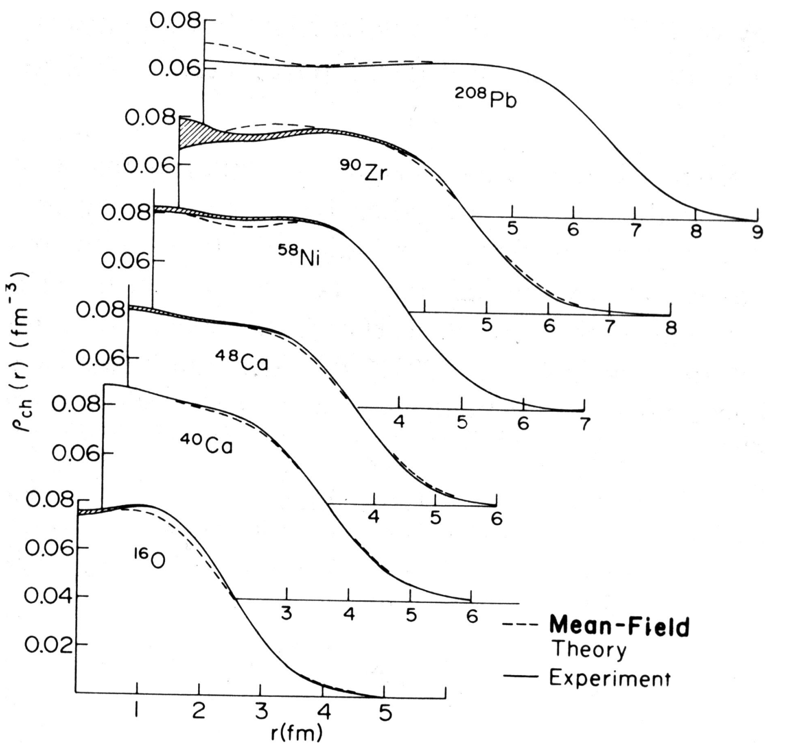
Imaging of atomic nuclei

sizes of nuclei:

as revealed through

elastic electron scattering



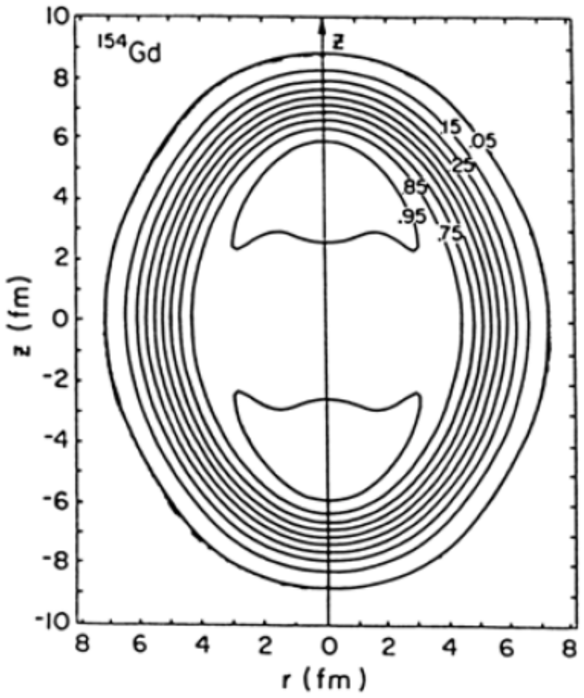


shapes of nuclei:

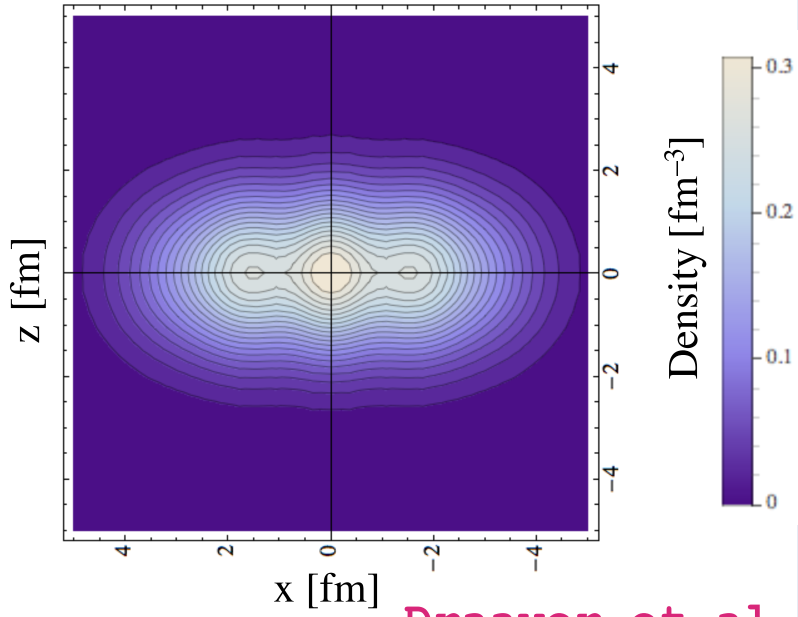
as revealed through

inelastic electron scattering

deformations, coherent states



Hoyle state in ^{12}C



Draayer et al.

Atomic nuclei

- To define and reconstruct a 3dim charge distribution of system of mass M from electron scattering: one needs to localize the system and fix its c.m.

Condition:

Jaffe (2021)

$$1/M \ll R \ll R_E$$

Compton wavelength

localization provided by wave packet

Size of system

- For **atomic nuclei**: $1/M \sim 0.2/A \quad \mathrm{fm}$ for ^{12}C -> 0.02 fm

  $R_E \sim 1.2\ A^{1/3} \quad \mathrm{fm}$ for ^{12}C -> 2.5 fm

  $1/M \lll R_E$ ✓

For static (non-relativistic) systems: the 3D Fourier transform of **form factors** gives the distribution of electric charge and magnetization

$$G_E(Q^2) = 1 - \frac{1}{6} R_E^2 Q^2 + \mathcal{O}(Q^4) \longrightarrow$$

Charge radius: $$R_E^2 \equiv \frac{\int d^3\vec{r}\, r^2\, \rho_{3\mathrm{d}}(r)}{\int d^3\vec{r}\, \rho_{3\mathrm{d}}(r)}$$

From nuclei to nucleons

- For **proton**:

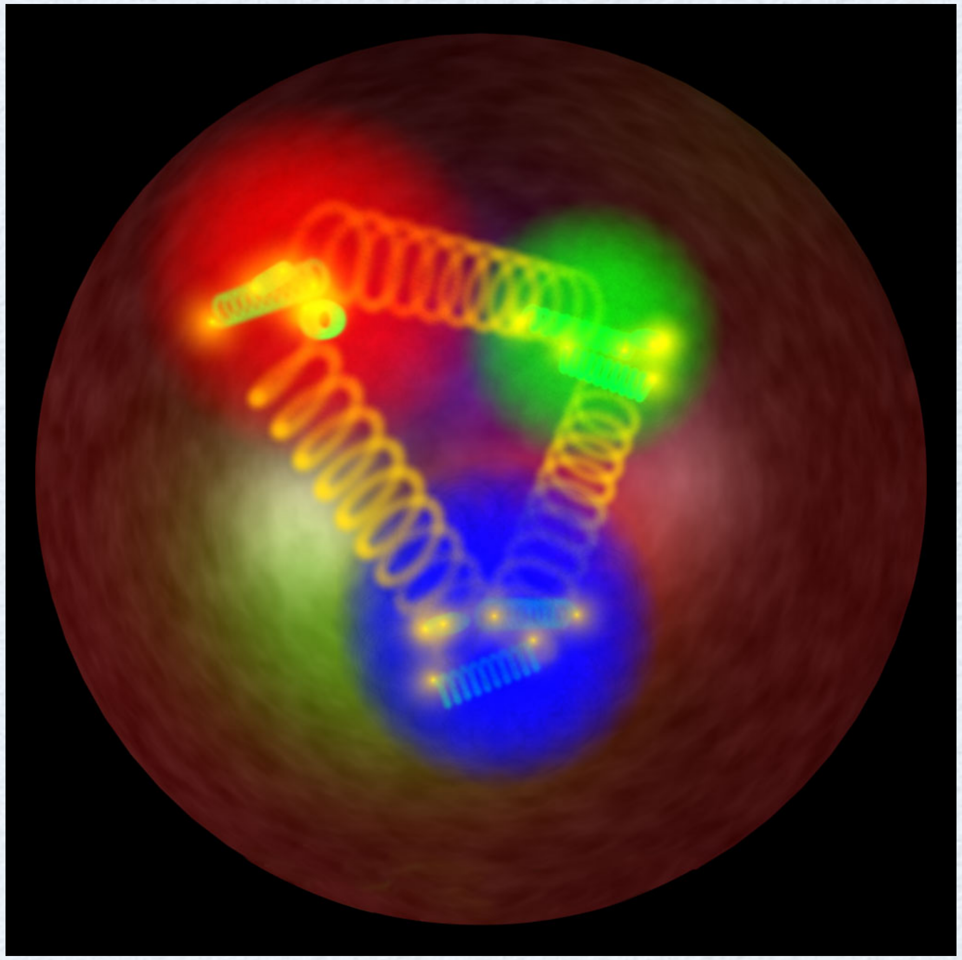
  Compton wavelength: $1/M \sim 0.2 \quad \text{fm}$

  Size of proton: $R_E \sim 0.85 \ \text{fm}$

  Condition $1/M \lll R_E$ ✗

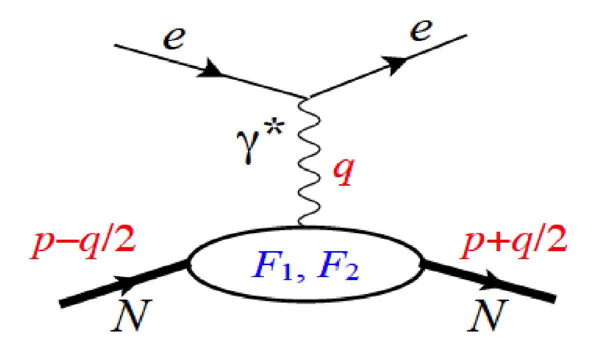
- For relativistic systems as proton: interpretation of Fourier transform of **form factors** as a 3-dim charge or magnetization distribution cannot be defined precisely

  Jaffe (2021)



Proton electromagnetic form factors

- **elastic electron-proton scattering**
  in the *1-photon exchange approximation:*
  parameterized by 2 **form factors (FFs)**



$$\langle p+\frac{q}{2},\lambda'\,|\,J^\mu(0)\,|\,p-\frac{q}{2},\lambda\rangle \;=\; \bar{u}(p+\frac{q}{2},\lambda')\left[\underset{\uparrow}{F_1}(Q^2)\gamma^\mu + \underset{\uparrow}{F_2}(Q^2)\frac{i}{2M}\sigma^{\mu\nu}q_\nu\right]u(p-\frac{q}{2},\lambda)$$

Dirac FF **Pauli FF**

for proton: $F_1(Q^2=0)=1$ $F_2(Q^2=0)=\kappa_p=1.79$

- equivalently: in experiment one uses **Sachs FFs** with $\tau \equiv \frac{Q^2}{4M^2}$

$$\begin{aligned} G_M(Q^2) &= F_1(Q^2)+F_2(Q^2) \longrightarrow \\ G_E(Q^2) &= F_1(Q^2)-\tau F_2(Q^2) \longrightarrow \end{aligned}$$

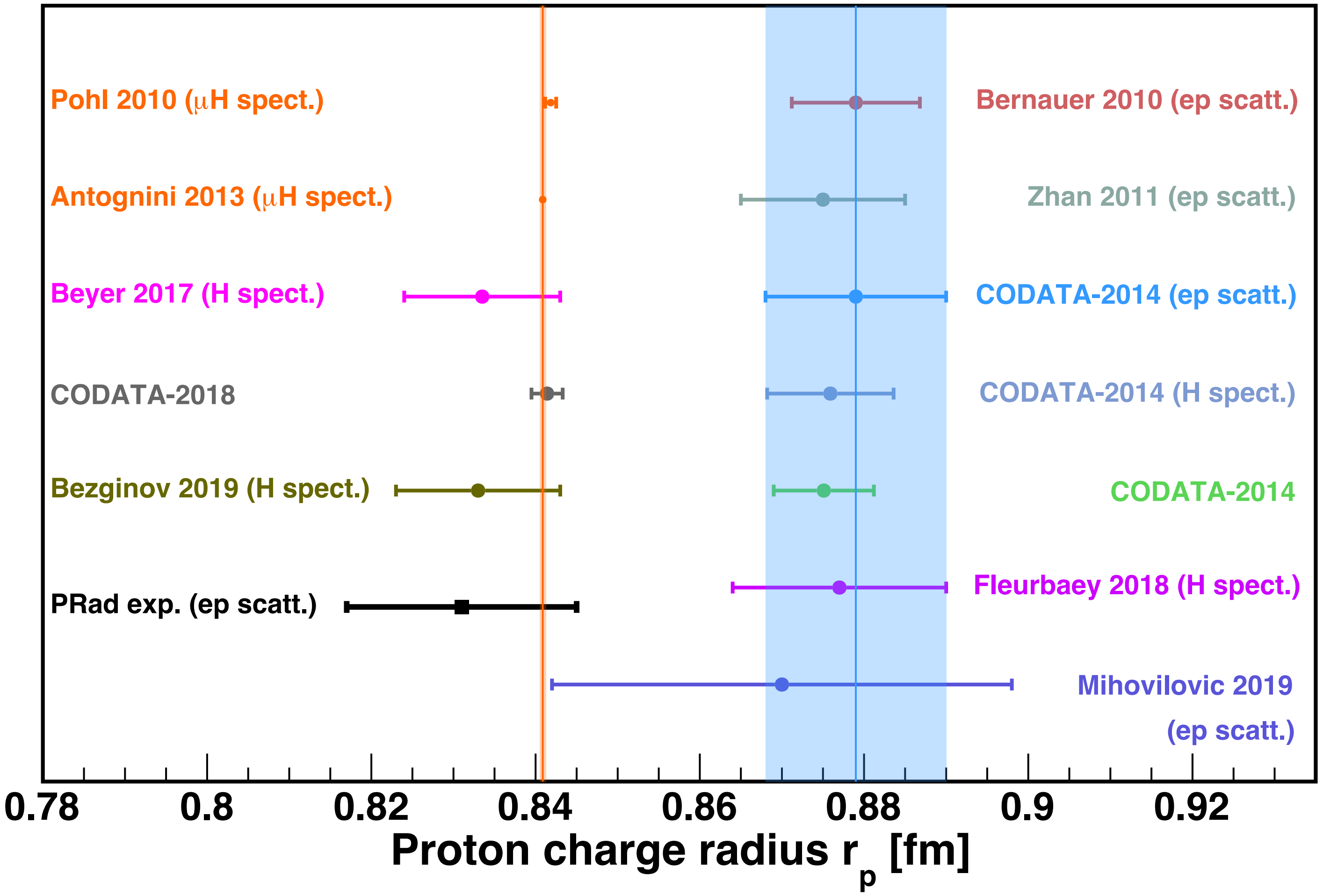
magnetic FF
electric FF

$$G_E(Q^2) = 1 - \frac{1}{6}\langle r_E^2\rangle\,Q^2 + \mathcal{O}(Q^4)$$

charge radius is definition for FF slope!

- In **Hydrogen spectroscopy (Lamb shift)**: **quantity which enters is also slope of G_E !**

Experimental r_{Ep} determinations



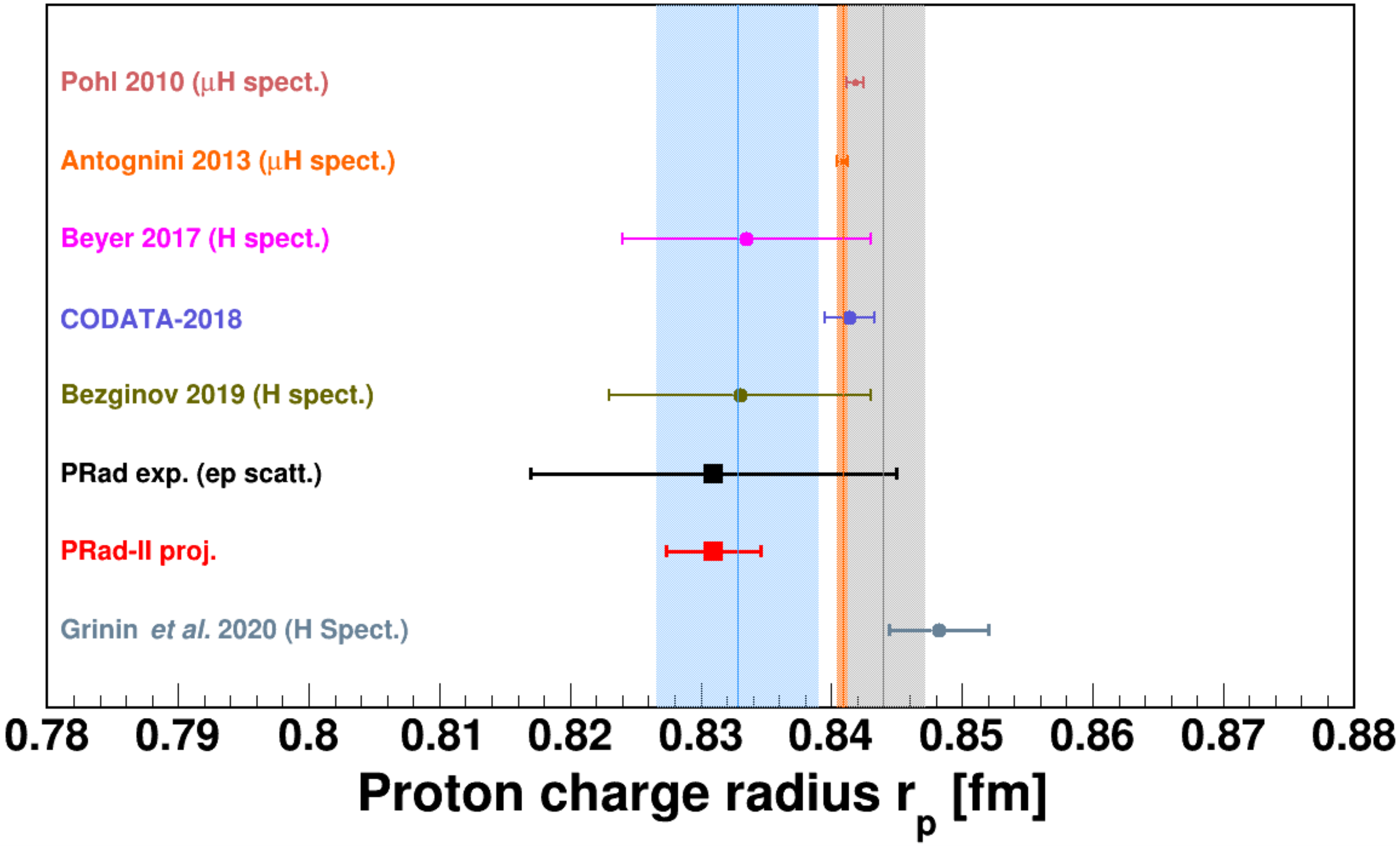
from recent compilation
prepared for Rev. Mod. Phys.

H.Gao, M.Vdh (2021)

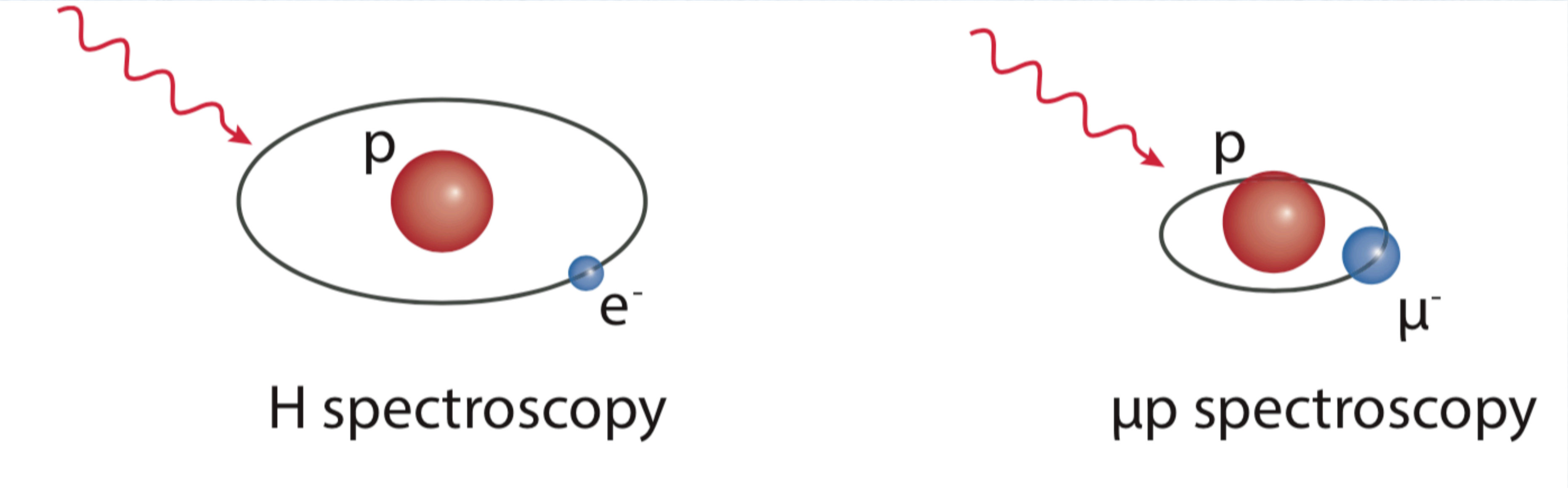
PRad-II plans

PRad-II: new JLab approved experiment `Dutta et al. (2020)`

- improvement of stat. uncertainty by factor 4 in comparison to PRad
- various improvements to reduce syst. uncertainties
- Factor 3.8 reduction in overall exp. uncertainty on proton radius compared to PRad



Hadronic input to muonic atom spectroscopy program



Muonic atom spectroscopy needs nucleon/nuclear input

2S-2P Lamb Shift:

THEORY **EXPERIMENT**

	$\Delta E_{TPE} \pm \delta_{theo}\,(\Delta E_{TPE})$	Ref.	$\delta_{exp}(\Delta_{LS})$	Ref.
μH	33 μeV ± 2 μeV	Antognini et al. (2013)	2.3 μeV	Antognini et al. (2013)
μD	1710 μeV ± 15 μeV	Krauth et al. (2015)	3.4 μeV	Pohl et al. (2016)
μ^3He$^+$	15.30 meV ± 0.52 meV	Franke et al. (2017)	0.05 meV	
μ^4He$^+$	9.34 meV ± 0.25 meV −0.15 meV ± 0.15 meV (3PE)	Diepold et al. (2018) Pachucki et al. (2018)	0.05 meV	Krauth et al. (2020)

μH:

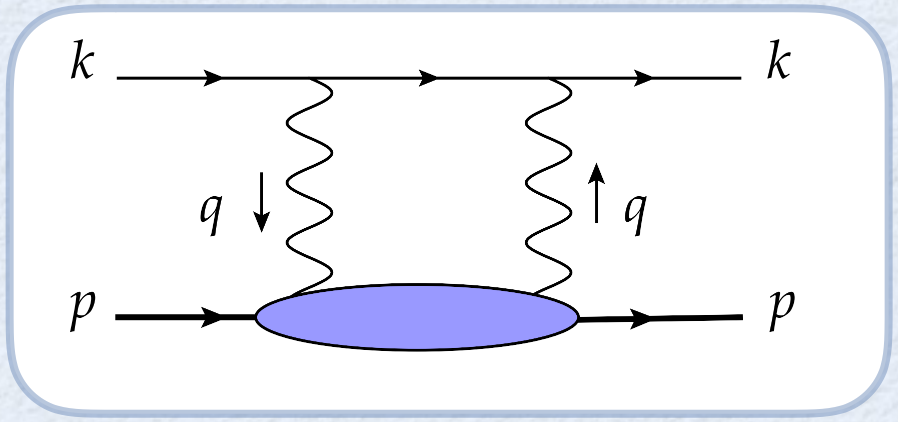
present accuracy comparable with experimental precision

Future: factor 5 improvement planned

μD, μ^3He$^+$, μ^4He$^+$:

present accuracy factor 5-10 worse than experimental precision

Two-photon exchange: hadronic corrections



$$T^{\mu\nu}(p,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

- ➢ Two-photon exchange (TPE): lower blob contains both elastic (nucleon) and inelastic states
- ➢ Lamb shift: described by unpolarized amplitudes T_1, T_2 : functions of energy ν and Q^2
- ➢ Hyperfine splitting: described by polarized amplitudes S_1, S_2
- ➢ Imaginary parts: directly proportional to nucleon structure functions F_1, F_2 resp. g_1, g_2
- ➢ Real parts: obtained as dispersion integral over the imaginary parts modulo a subtraction function in case of T_1

$$\Delta E = \Delta E^{el} + \Delta E^{subtr} + \Delta E^{inel}$$

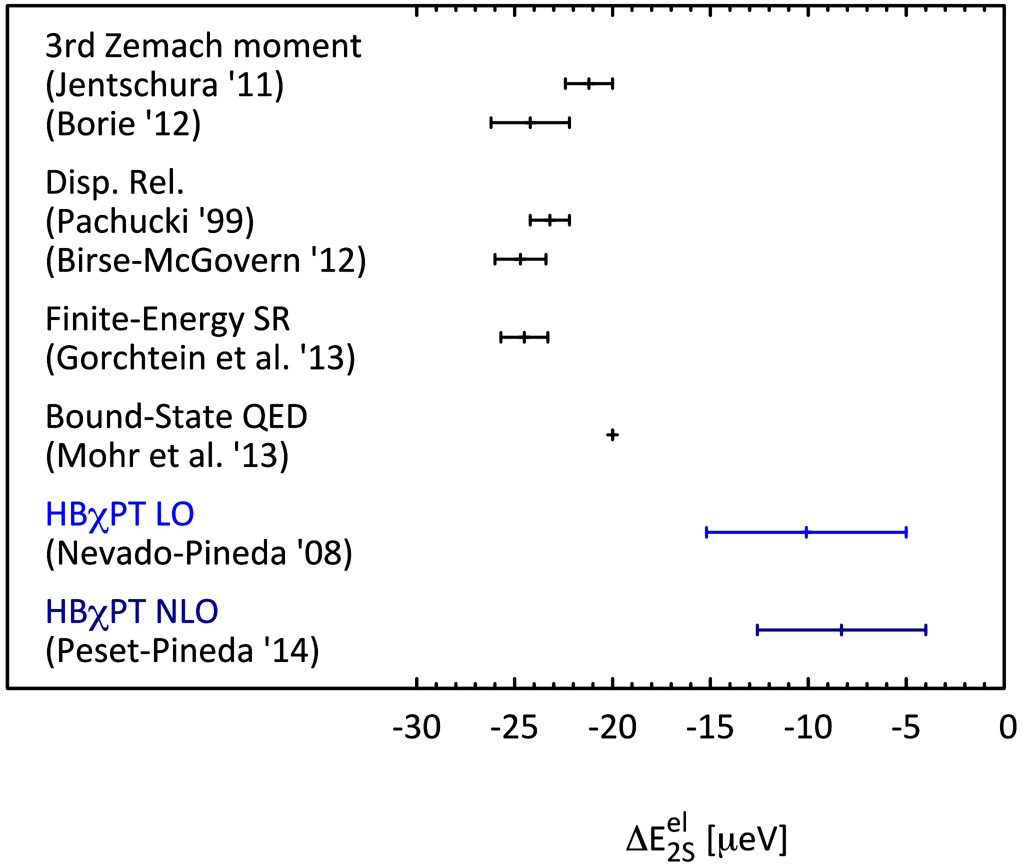
→ Elastic state: involves **nucleon form factors**

→ Subtraction: involves **nucleon polarizabilities**

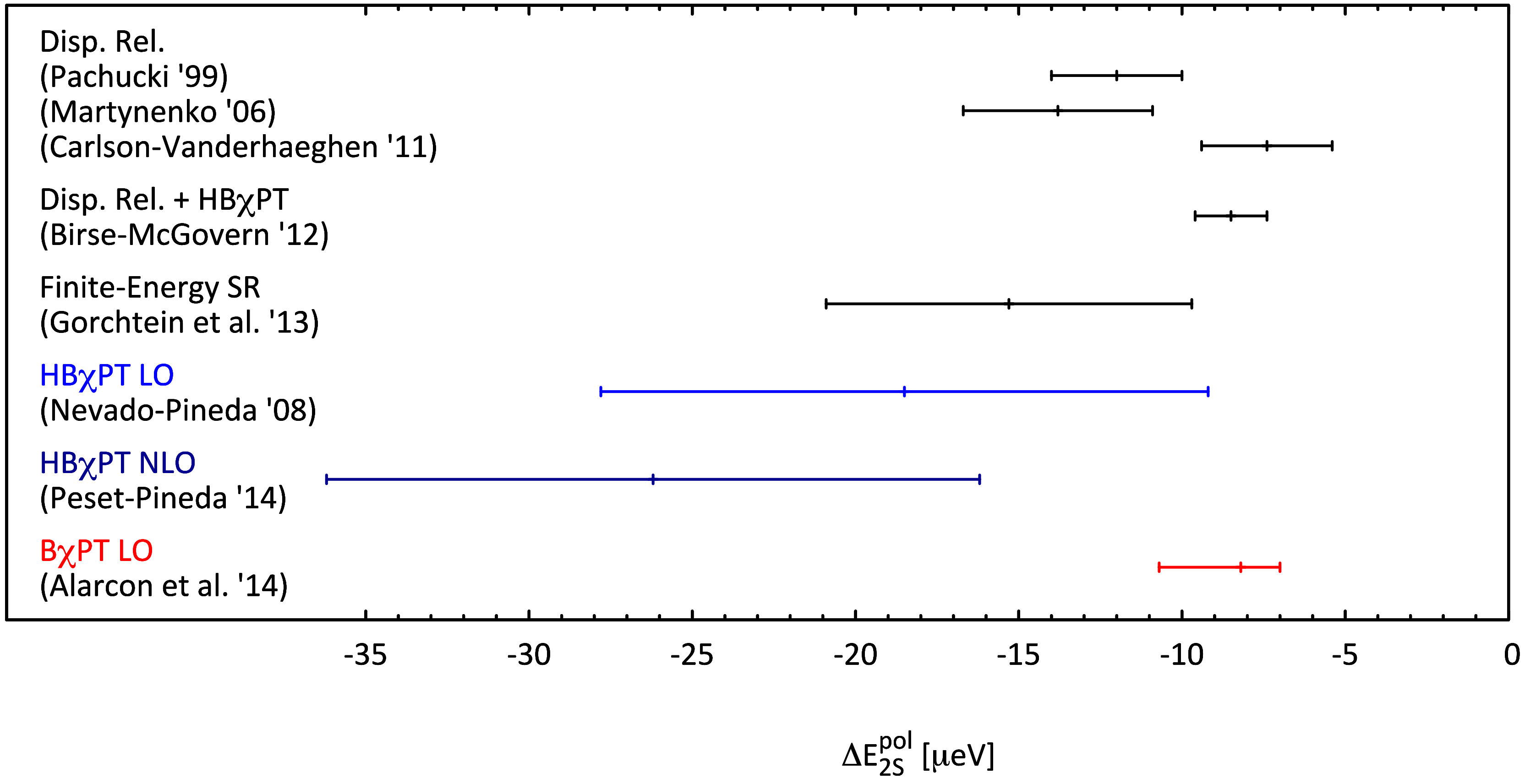
→ Inelastic state: involves **nucleon structure functions**

Hadron/Nuclear physics input needed !

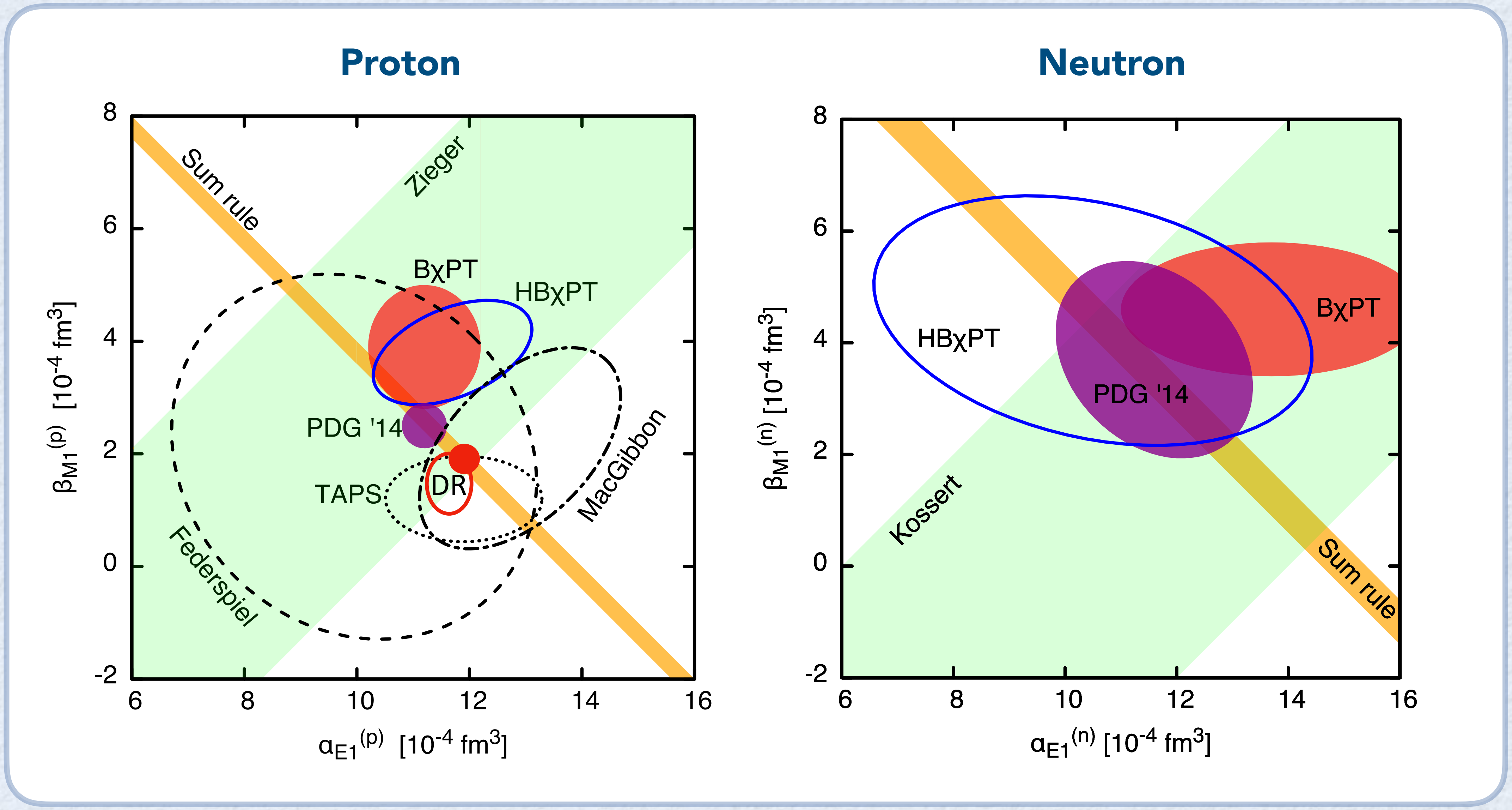
TPE elastic correction:



TPE polarizability correction:



TPC program A2@MAMI: neutron polarizabilities



$\alpha_E = (11.2 \pm 0.4) \times 10^{-4}$ fm^3

PDG '14 values

$\alpha_E = (11.8 \pm 1.1) \times 10^{-4}$ fm^3

$\beta_M = (2.5 \pm 0.4) \times 10^{-4}$ fm^3

$\beta_M = (3.7 \pm 1.2) \times 10^{-4}$ fm^3

New A2@MAMI consisted data set (factor 5 increase of world data) in final stage of analysis

➔ will consolidate PDG errors

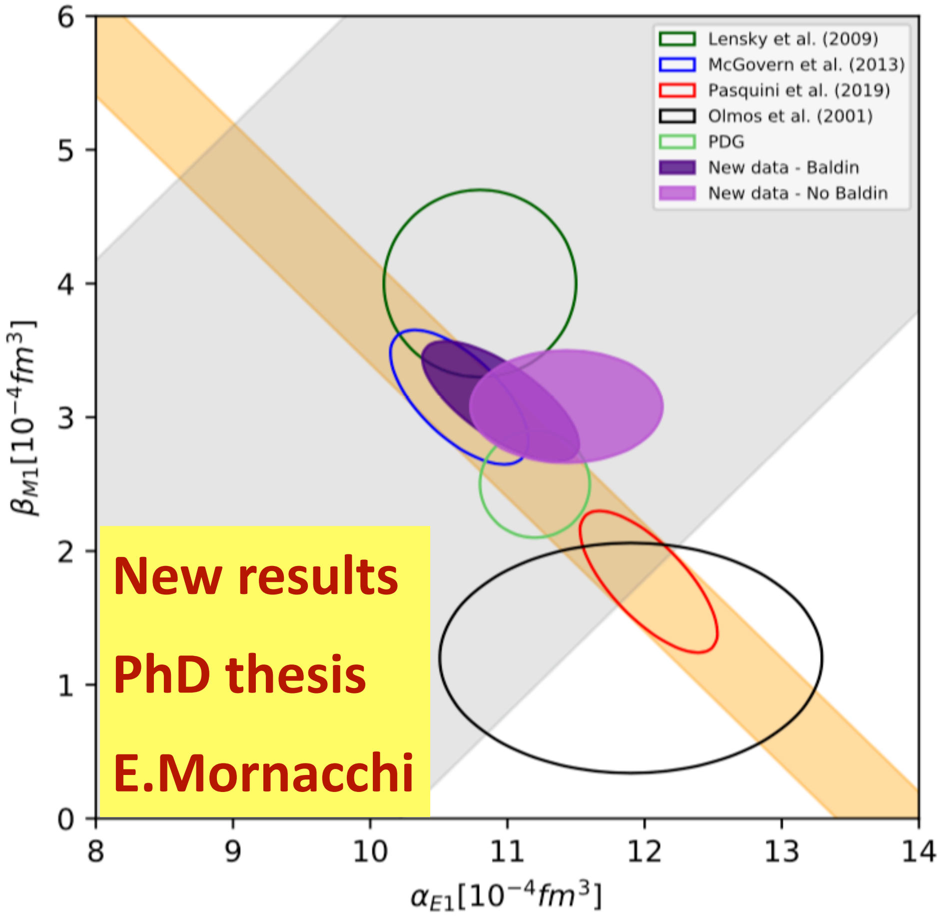
A reduction of error by factor of 2

➔ precision comparable with proton

➔ impact light muonic atom program

TPC program A2@MAMI: neutron polarizabilities

Proton

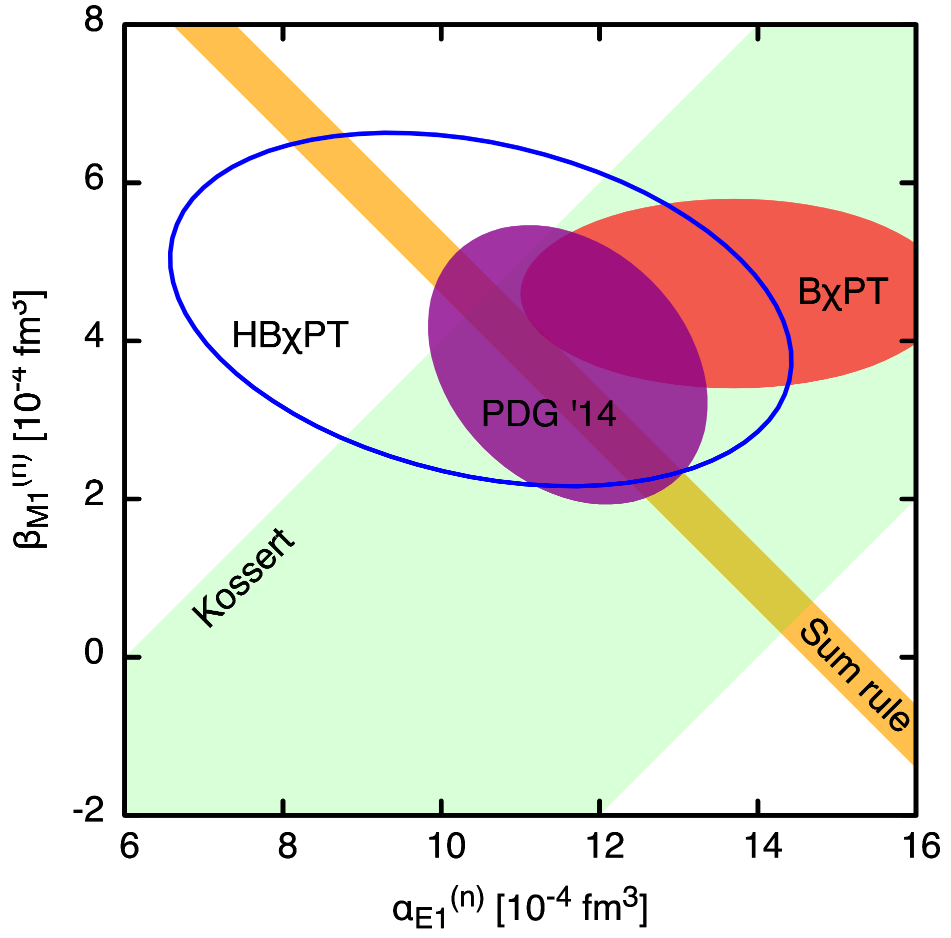


New results

PhD thesis

E.Mornacchi

Neutron



$\boldsymbol{\alpha_E}$ = (11.2 ± 0.4) × 10^{-4} fm^3

$\boldsymbol{\beta_M}$ = (2.5 ± 0.4) × 10^{-4} fm^3

PDG '14 values

$\boldsymbol{\alpha_E}$ = (11.8 ± 1.1) × 10^{-4} fm^3

$\boldsymbol{\beta_M}$ = (3.7 ± 1.2) × 10^{-4} fm^3

New A2@MAMI consisted data set (factor 5 increase of world data) in final stage of analysis

➔ will consolidate PDG errors

A reduction of error by factor of 2

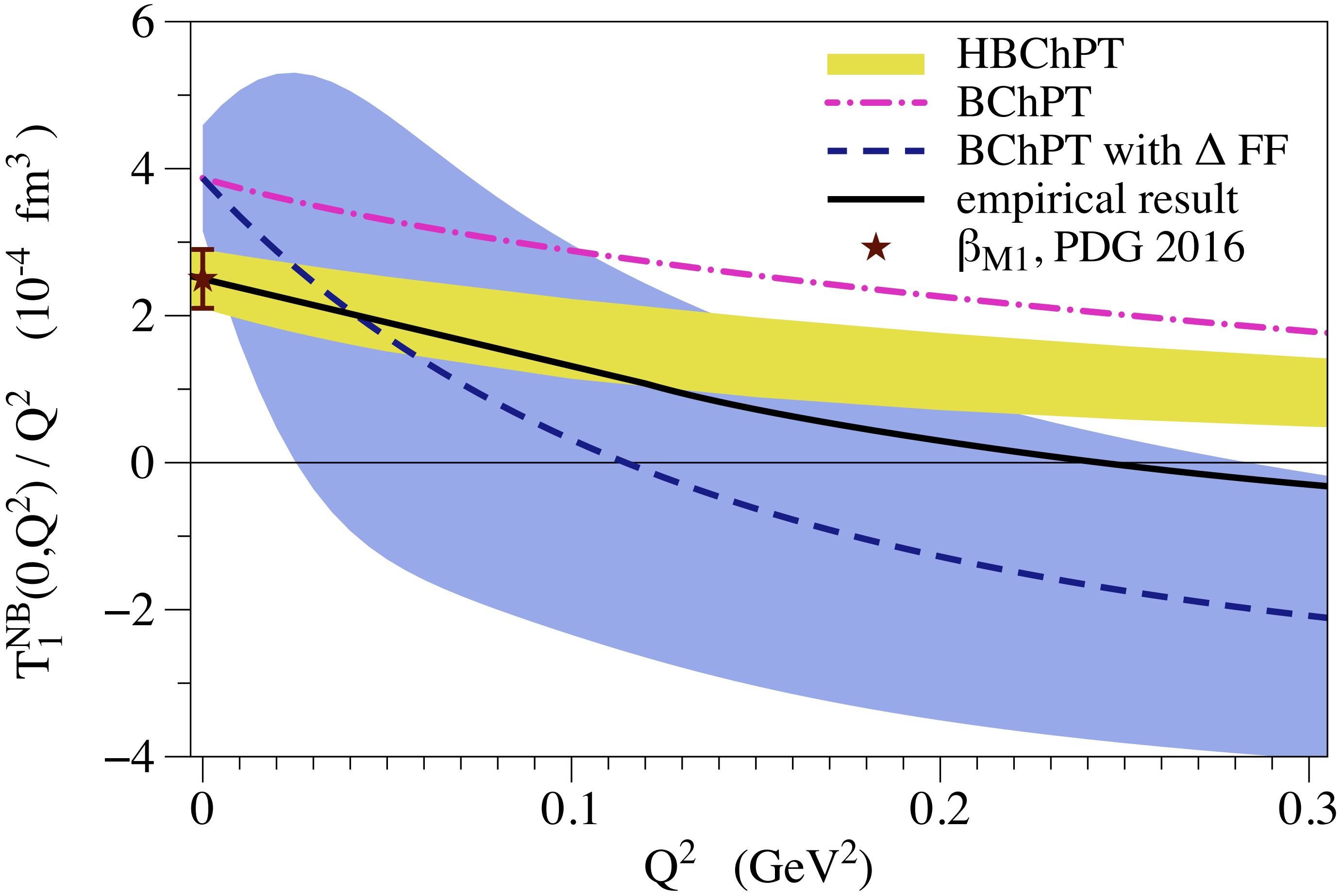
➔ precision comparable with proton

➔ impact light muonic atom program

LS: Improved determinations of subtraction function

Lensky,Hagelstein, Pascalutsa,Vdh (2018)

Subtraction function: Q^2 dependence



To improve on uncertainty due to subtraction function: 3 avenues

- ➢ Full NLO calculation in Baryon ChPT **Pascalutsa et al.**
- ➢ New formalism for lattice determination of subtraction function **Hagelstein, Pascalutsa(2020)**
- ➢ Empirical determination of Q^4 term using dilepton production process

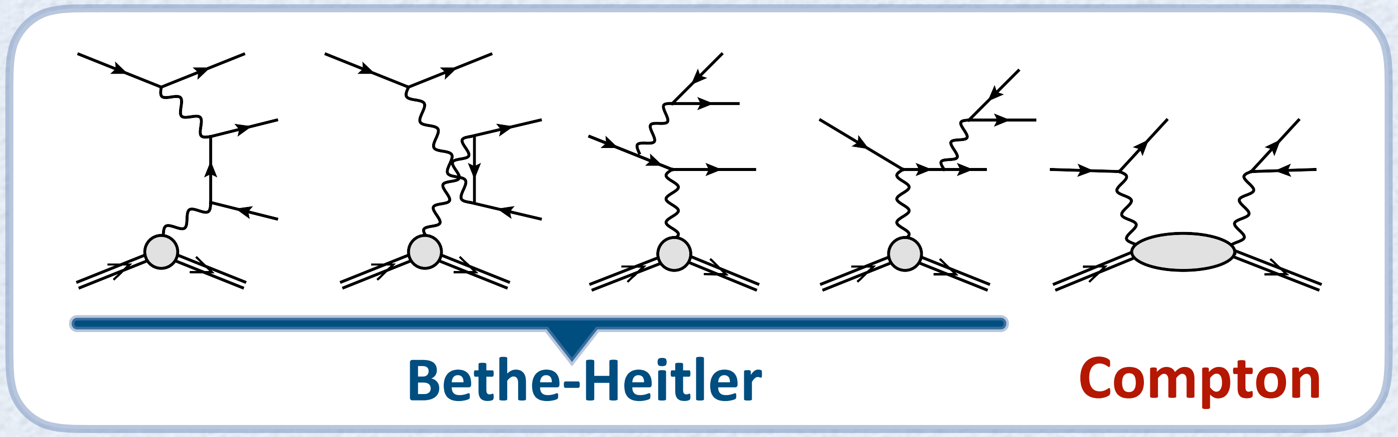
Empirical determination of Q^4 term through $e^- p \to e^- p\, l^- l^+$

$$T_1^{NB}(0,Q^2) = \quad \beta_{M1}\, Q^2 + \left(\frac{1}{6}\beta_{M2} + 2\beta'_{M1} + \alpha_{\rm em}\, b_{3,0} + \frac{1}{(2M)^2}\beta_{M1}\right)\, Q^4 + \mathcal{O}(Q^6)$$

Subtraction function T_1

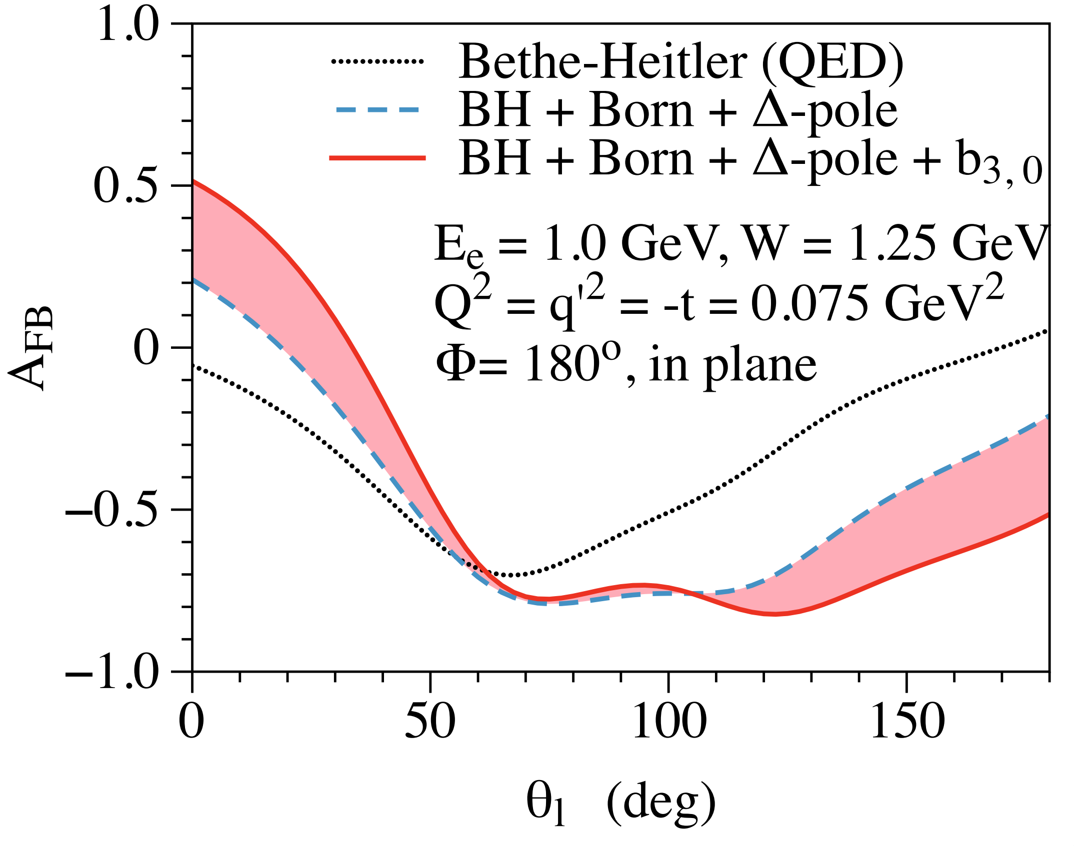
only empirical unknown

- Forward-backward asymmetry of **$e^- p \to e^- p\, l^- l^+$** : allows access to low-energy structure constant **$b_{3,0}$**



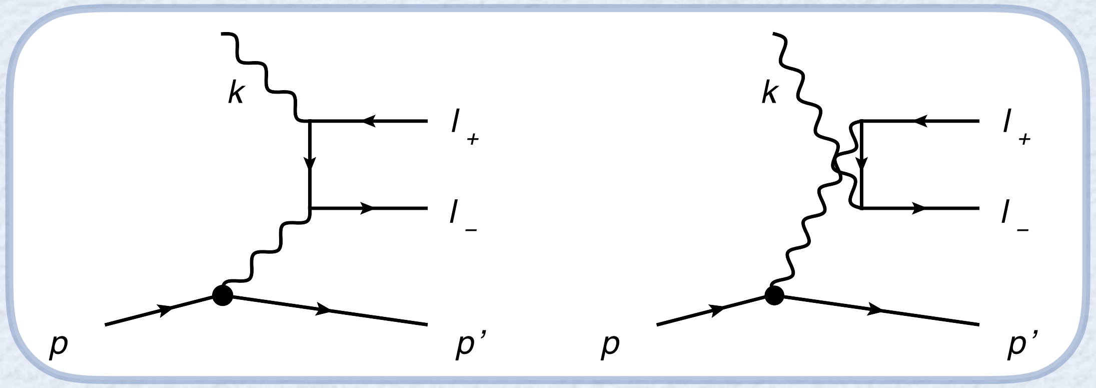
- First CLAS12@JLab data on high-energy timelike Compton (2020): FB asymmetry in 10 - 30% range

$e^- p \to e^- p\, l^- l^+$: forward-backward asymmetry



`Pauk, Carlson, Vdh(2020)`

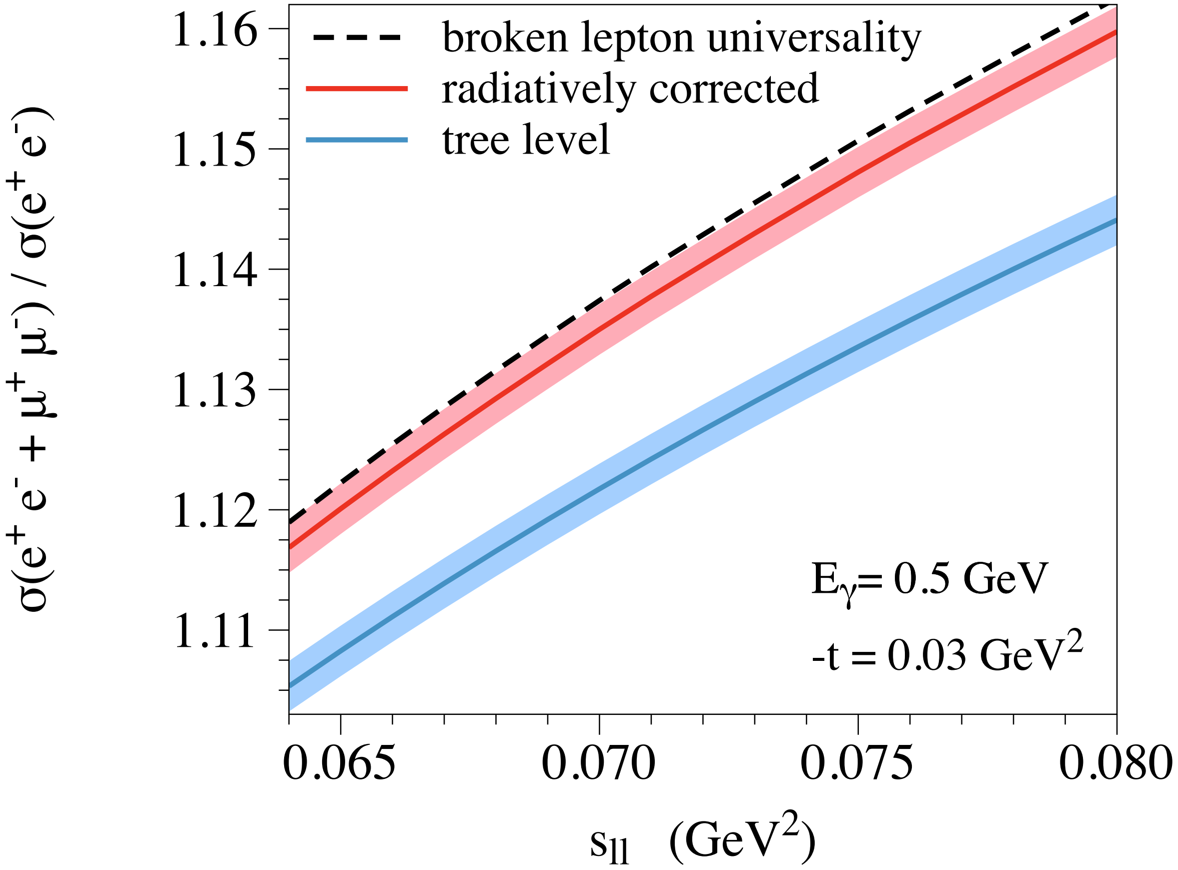
Dilepton photoproduction $\gamma p \to p\, l^- l^+$:
proton form factor, lepton universality



- $e^- e^+$ photoproduction using **active TPC + Crystal Ball @MAMI**
- full one-loop radiatively corrected amplitude needed
- $e^- e^+$ vs $\mu^- \mu^+$ photoproduction: cross section ratio above/below dimuon threshold : different proton radius values correspond to **0.2% effect** in ratio
- Complements ongoing muon scattering program (**MUSE, COMPASS**)

cross section ratio:

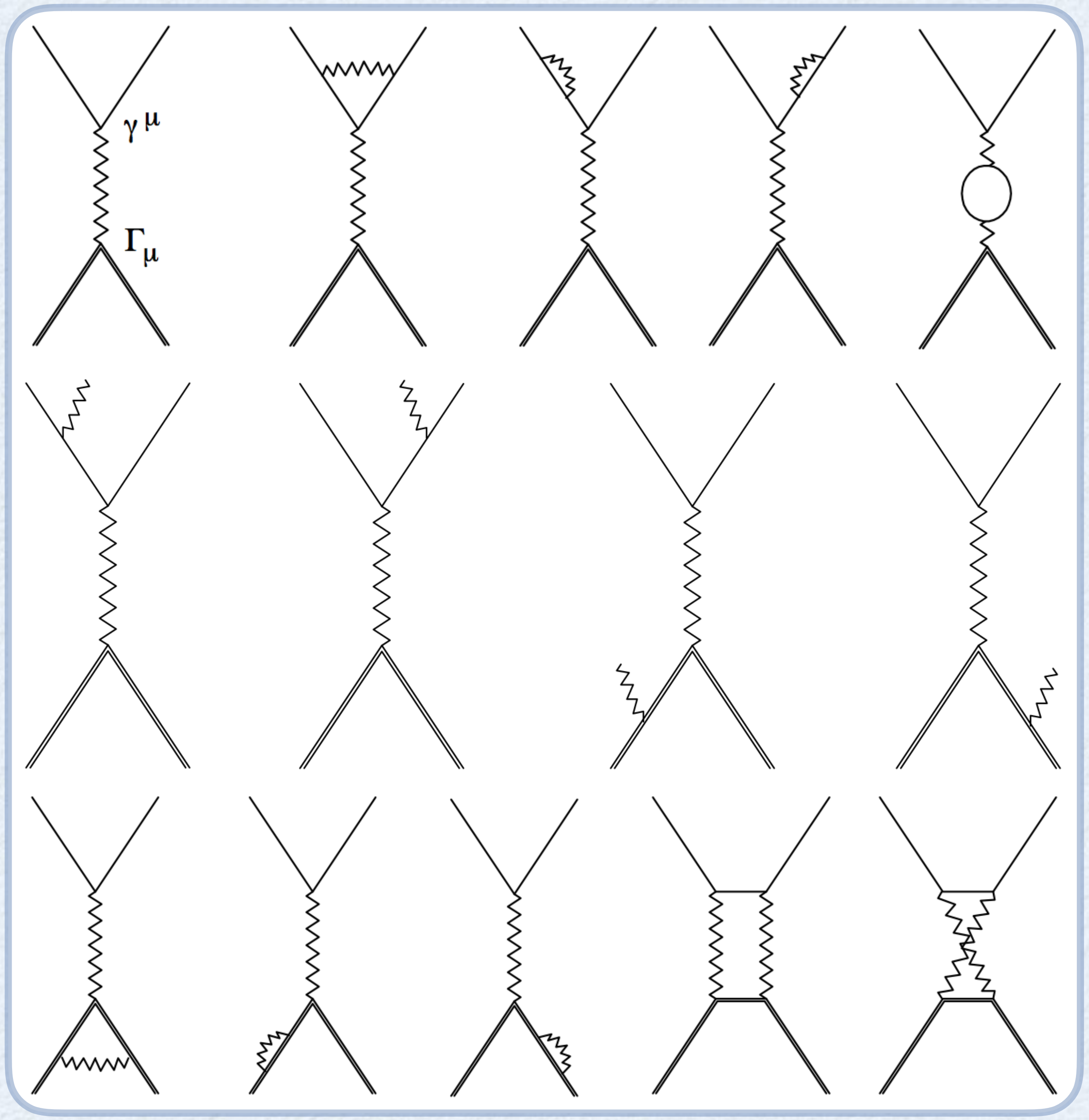
$\gamma\, p \to p\, (e^- e^+ + \mu^- \mu^+) \,/\, \gamma\, p \to p\, e^- e^+$

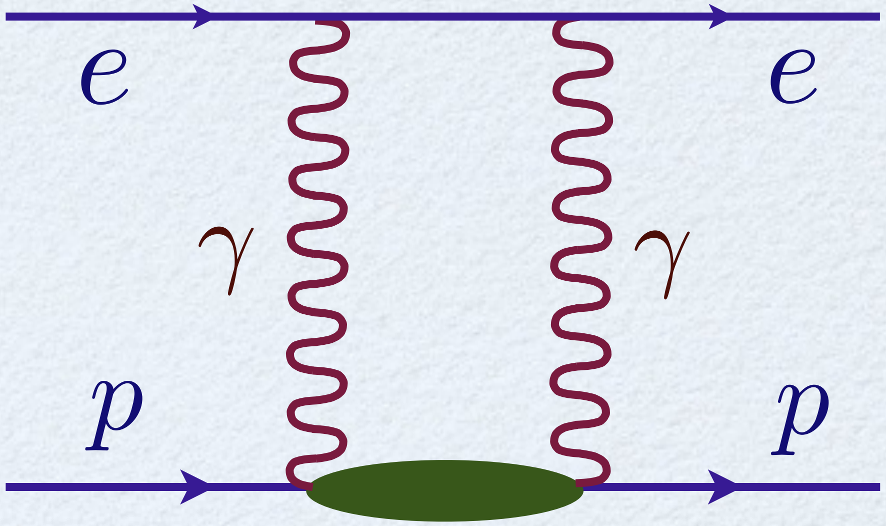


Pauk, Vdh (2015)

Heller, Tomalak, Wu, Vdh (2018, 2019)

Radiative corrections in lepton-nucleon scattering





status of radiative corrections

- Accuracy goal PRES: 0.2% absolute precision on d$\boldsymbol{\sigma}$ /dt

- For experiment with recoil proton detection: **Fadin & Gerasimov (2019)**

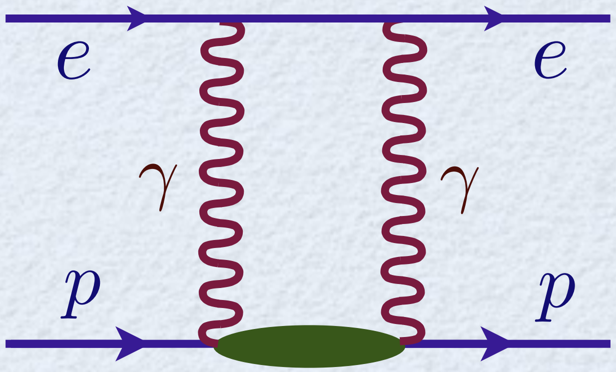
  to 1-loop: real and virtual radiative corrections on electron side cancel
  both for log(Q^2 /m^2) terms and constant terms

  beyond 1-loop: cancellation for log terms

- Vacuum polarization is dominant contribution: **A. Arbuzov**

  Can be accurately calculated: 1 - 1.6 % in Q^2 range: 0.001 - 0.02 GeV^2

- Largest uncertainty from subdominant two-photon exchange between e and p



2γ-exchange at low Q^2

Feshbach inelastic elastic

$$\delta_{2\gamma} \sim a\,\sqrt{Q^2} + b\,Q^2 \ln Q^2 + c\,Q^2 \ln^2 Q^2$$

McKinley, Feshbach (1948)

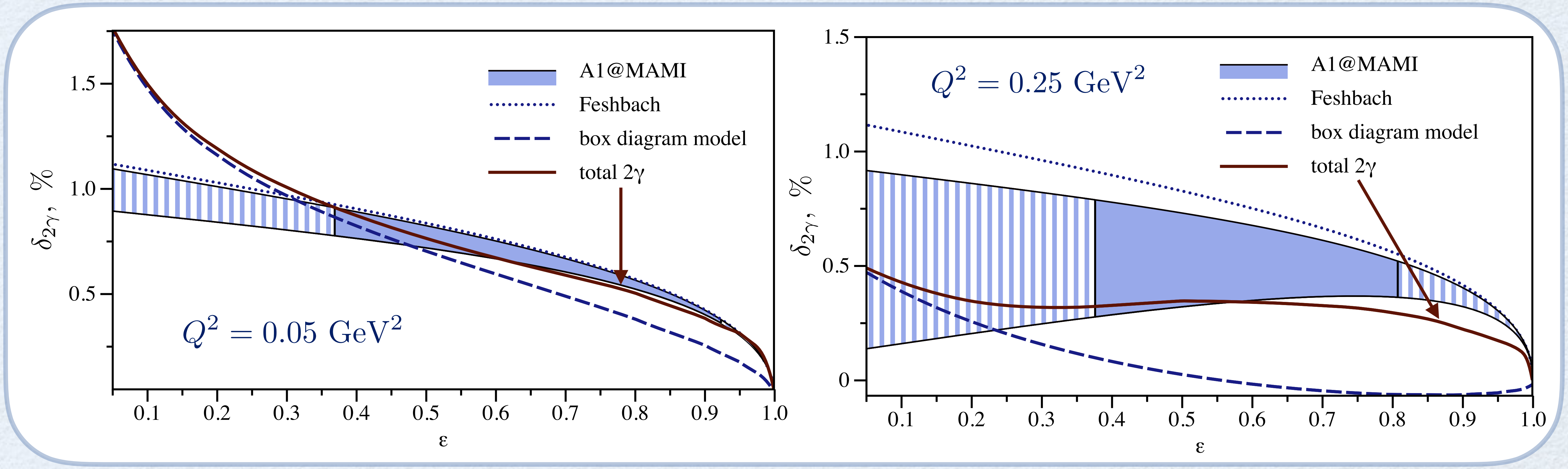
R.W.Brown (1970)

M. Gorchtein (2013)

unpolarized proton structure

Tomalak, Vdh (2016)

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$

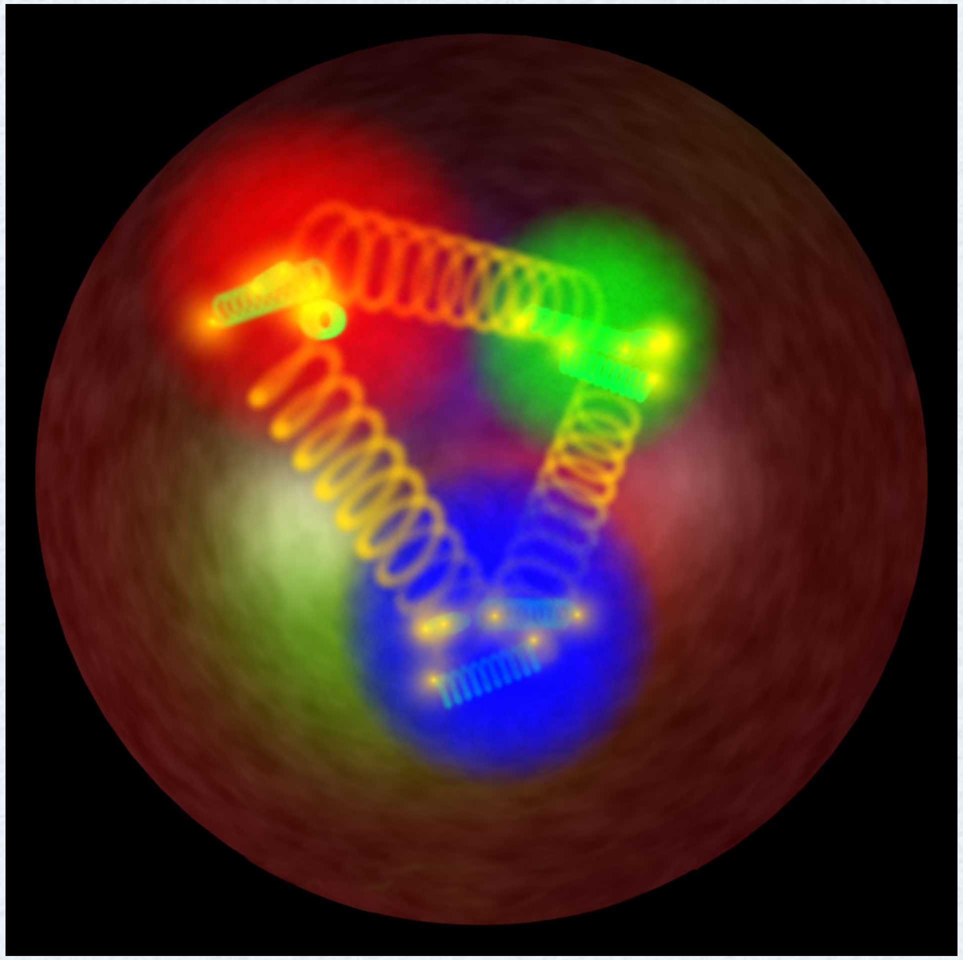


2ɣ at large ε agrees with empirical fit

r_E extraction ✓

TPE varies between 0.05% to 0.2% in Q^2 range: 0.001 - 0.02 GeV^2

How to properly define a charge density for nucleons ?



PHYSICAL
REVIEW
LETTERS

Member Subscription Copy
Library or Other Institutional Use Prohibited Until 2013

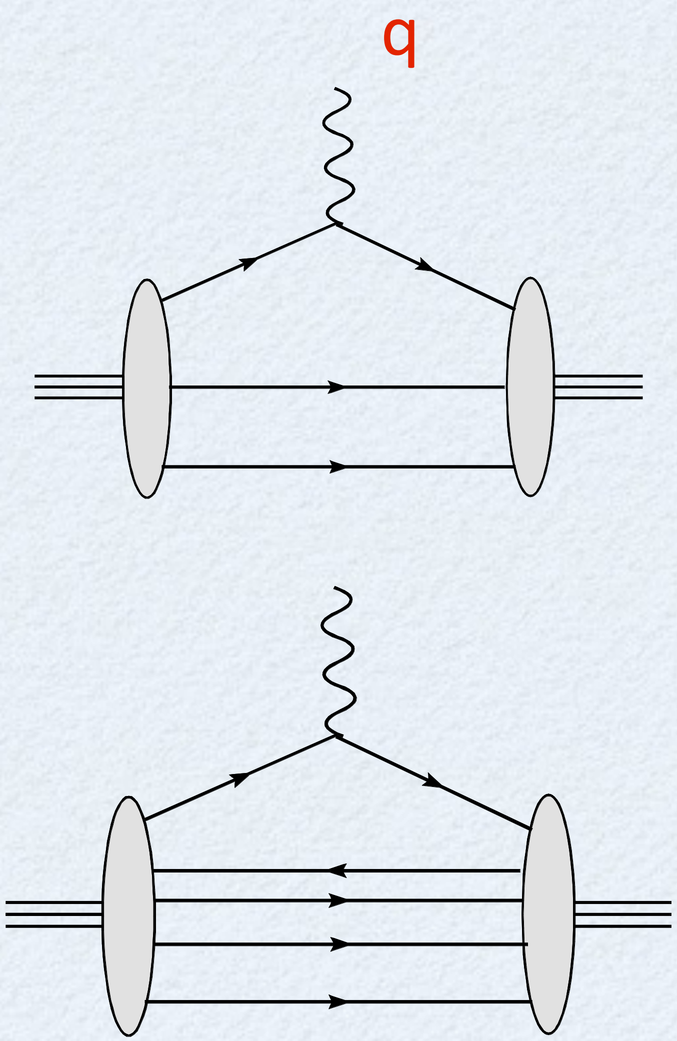
Articles published week ending 25 JANUARY 2008

Published by the
American Physical Society

APS physics

Volume 100, Number 3

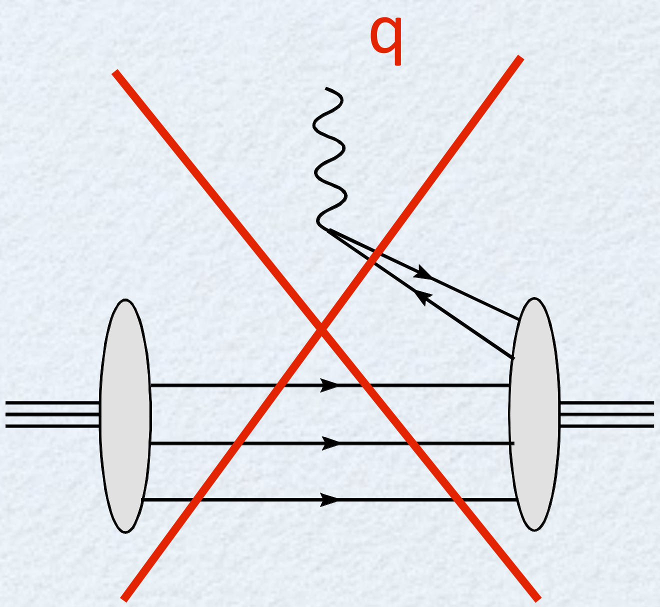
Interpretation of nucleon form factor as density



overlap of wave function

Fock components

with same number of quarks



overlap of wave function
Fock components

with different number of quarks

NO probability / charge density

interpretation

absent in a light-front frame!

$$q^+ = q^0 + q^3 = 0$$

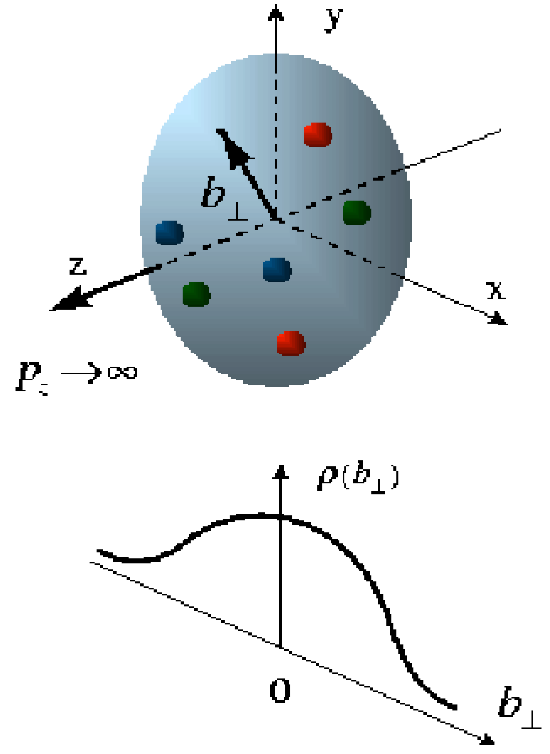
quark transverse charge densities in nucleon

transverse c.m. can be fixed in a light-front frame !

longitudinally polarized nucleon

$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi}\, Q J_0(bQ) F_1(Q^2)$$

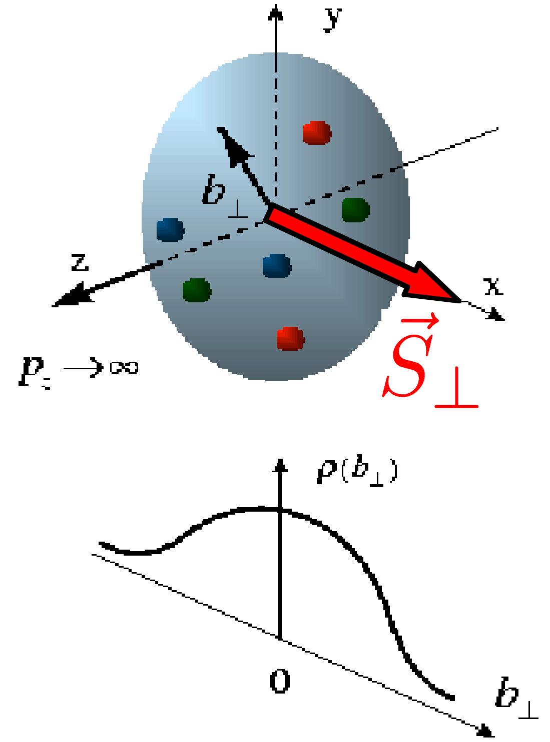


Soper (1997) **Burkardt (2000)** **Miller (2007)**

transversely polarized nucleon

$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2}\rangle$$

$$= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi}\, \frac{Q^2}{2M} J_1(bQ) F_2(Q^2)$$

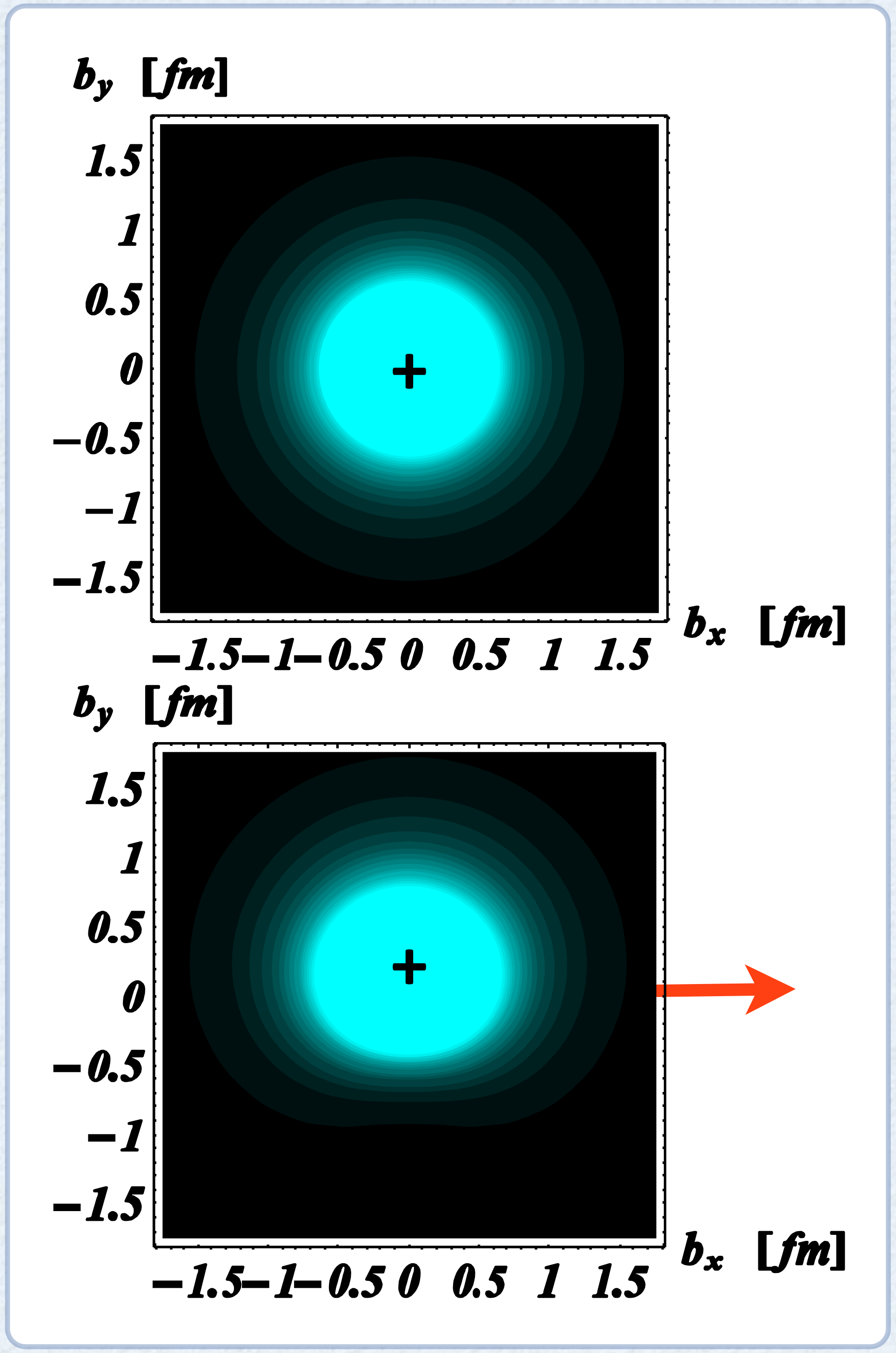


dipole field pattern

Carlson, Vdh (2007)

spatial imaging of nucleons

proton



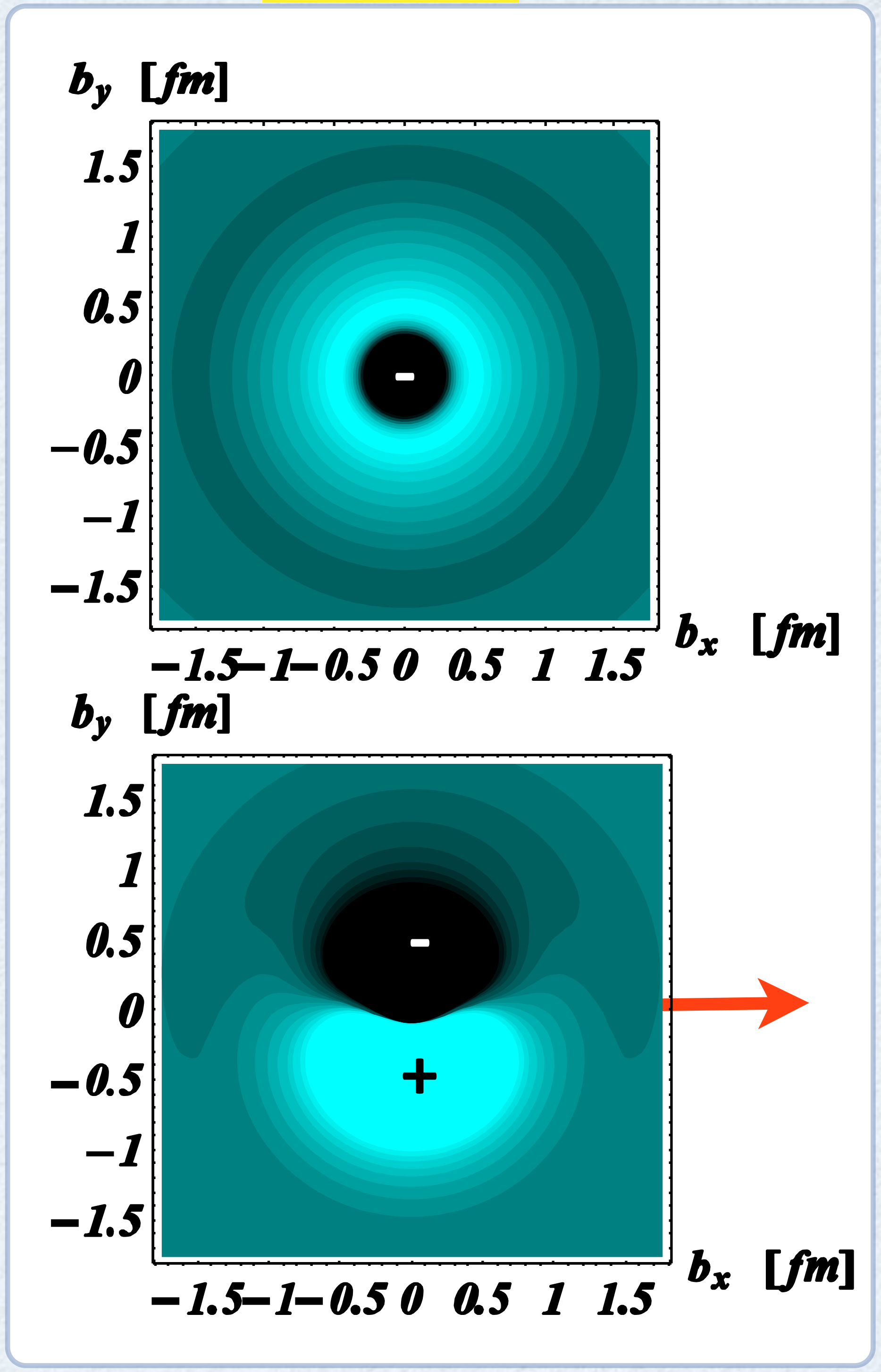
induced

electric dipole

moment:

$$d_y = \kappa \frac{e}{2M}$$

neutron



Miller(2007) Carlson, Vdh(2007)

Radii of charge distribution for nucleons

Radius of 2-dim transverse distribution of quarks of flavor q in **proton**:

$$\langle b^2\rangle^q = \frac{\int d^2\mathbf{b}\,\mathbf{b}^2\,\rho^q(b)}{\int d^2\mathbf{b}\,\rho^q(b)} = -4\frac{F_1'^q(0)}{F_1^q(0)}$$

with: $F_1'^q(0) \equiv \frac{dF_1^q}{dQ^2}\Big|_{Q^2=0}$

Isospin symmetry:

$$F_{1p} = e_u F_1^u + e_d F_1^d$$
$$F_{1n} = e_u F_1^d + e_d F_1^u$$

$$\langle b^2\rangle_p = \frac{4}{3}\langle b^2\rangle^u - \frac{1}{3}\langle b^2\rangle^d = -4F_{1p}'(0)$$
$$\langle b^2\rangle_n = \frac{2}{3}\langle b^2\rangle^d - \frac{2}{3}\langle b^2\rangle^u = -4F_{1n}'(0)$$

Charge radii for proton and neutron:

		r_E^2 (fm^2)	$-\frac{3\kappa_N}{2M^2}$ (fm^2)	$-6F_1'(0)$ (fm^2)	$\langle b^2\rangle$ (fm^2)
Cui et al.(2021)	proton (e-p)	0.717 ± 0.014	-0.1189	0.598 ± 0.014	0.399 ± 0.009
Antognini et al. (2013)	proton (μH)	0.7071 ± 0.0007		0.5882 ± 0.0007	0.3921 ± 0.0005
	neutron (PDG)	-0.1161 ± 0.0022	0.1266	0.0105 ± 0.0022	0.0070 ± 0.0015

H.Gao, M.Vdh (2021)

Summary and outlook

➡ **electron scattering:**

- new PRad proton r_E result is consistent with muonic hydrogen spectroscopy value
- PRad-II aims at factor 3.8 improvement on proton charge radius r_E
- Plans in Mainz: PRES, MAGIX@MESA

➡ **hadronic corrections** to Lamb shift in **muonic atoms:**

- **μH**: present TPE accuracy (2 μeV) is comparable with present Lamb shift accuracy (2.3 μeV)
  Future plan: factor 5 improvement on LS for muonic H
- **μD, μ ^{3}He$^+$, μ ^{4}He$^+$**: present TPE accuracy is 5-10 times worse than Lamb shift accuracy
  Largest uncertainty from neutron polarizability correction

➡ **muon scattering experiments:**

- Ongoing plans: MUSE@PSI, AMBER@COMPASS
- dilepton production complementary: cross section ratio above/below dimuon threshold

➡ **Theoretical understanding:**

For system as proton composed of relativistic quarks and gluons:

- 3-dim charge distribution cannot be defined precisely: r_E is definition of G_E slope at origin !
- instead we can define a 2-dim transverse charge distribution
  as seen by observer which moves with speed of light, and which is obtained as integral
  over quark longitudinal momentum fraction x of partonic distributions
- we can properly define and extract the proton charge radius in this transverse plane