

New developments in theory (I)

(focus on lattice QCD)

Christoph Lehner
(Uni Regensburg & Brookhaven National Laboratory)

December 11, 2020 - KHuK Jahrestagung

Overview of lattice QCD research output by German groups in 2020

This list is likely incomplete and I apologize for missed contributions

Berlin/Bonn:

- ▶ Lattice continuum-limit study of nucleon quasi-PDFs (arXiv:2011.00964)
- ▶ Parton distribution functions of Δ^+ on the lattice (Phys.Rev.D 102 (2020) 1, 014508)
- ▶ Improvement, generalization, and scheme conversion of Wilson-line operators on the lattice in the auxiliary field approach (Phys.Rev.D 101 (2020) 7, 074509)
- ▶ Moments of nucleon generalized parton distributions from lattice QCD simulations at physical pion mass (Phys.Rev.D 101 (2020) 3, 034519)
- ▶ Ruling Out the Massless Up-Quark Solution to the Strong CP Problem by Computing the Topological Mass Contribution with Lattice QCD (Phys.Rev.Lett. 125 (2020) 23, 232001)
- ▶ Nucleon axial and pseudoscalar form factors from lattice QCD at the physical point (arXiv:2011.13342)

Berlin/DESY:

- ▶ [Asymptotic behavior of cutoff effects in Yang–Mills theory and in Wilson's lattice QCD \(Eur.Phys.J.C 80\(2020\)3, 200\)](#)
- ▶ Flavor decomposition for the proton helicity parton distribution functions (arXiv:2009.13061)
- ▶ Unpolarized and helicity generalized parton distributions of the proton within lattice QCD (arXiv:2008.10573)
- ▶ Parton distribution functions from lattice QCD using Bayes-Gauss-Fourier transforms (Phys.Rev.D 102 (2020) 9, 094508)
- ▶ On Estimation of Thermodynamic Observables in Lattice Field Theories with Deep Generative Models (arXiv:2007.07115)
- ▶ Complete flavor decomposition of the spin and momentum fraction of the proton using lattice QCD simulations at physical pion mass (Phys.Rev.D 101 (2020) 9, 094513)
- ▶ Avoiding the sign-problem in lattice field theory (arXiv:2002.06456)
- ▶ Nucleon strange electromagnetic form factors (Phys.Rev.D 101 (2020) 3, 031501)

[More details on the highlighted topics in this talk](#)

Overview page 2/4

Berlin/Wuppertal:

- ▶ Non-perturbative renormalization by decoupling (Phys.Lett.B 807 (2020) 135571)

Bielefeld:

- ▶ Sensitivity of the Polyakov loop and related observables to chiral symmetry restoration (arXiv:2008.11678)
- ▶ Skewness, kurtosis and the fifth and sixth order cumulants of net baryon-number distributions from lattice QCD confront high-statistics STAR data (Phys.Rev.D 101 (2020) 7, 074502)

Bielefeld/DESY/Jena/Münster/Regensburg:

- ▶ Continuum extrapolation of Ward identities in $\mathcal{N} = 1$ supersymmetric SU(3) Yang–Mills theory (Eur.Phys.J.C 80 (2020) 6, 548)

Bielefeld/Frankfurt:

- ▶ Pion condensation in the early Universe at nonvanishing lepton flavor asymmetry and its gravitational wave signatures (arXiv:2009.02309)

Bielefeld/Giessen:

- ▶ Spectral functions and dynamic critical behavior of relativistic Z_2 theories (PRD102(2020)094510)
- ▶ Spectral functions and critical dynamics of the O(4) model from classical-statistical lattice simulations (Nucl.Phys.B 950 (2020) 114868)

Bielefeld/Regensburg:

- ▶ Magnetic susceptibility of QCD matter and its decomposition from the lattice (JHEP 07 (2020) 183)

Bonn:

- ▶ Relativistic NN-particle energy shift in finite volume (arXiv:2010.11715)
- ▶ Scattering of two and three physical pions at maximal isospin from lattice QCD (arXiv:2008.03035)
- ▶ The ρ -resonance with physical pion mass from $N_f = 2$ lattice QCD (arXiv:2006.13805)
- ▶ Hadron-Hadron Interactions from $N_f = 2 + 1 + 1$ Lattice QCD: The ρ -resonance (Eur.Phys.J.A 56 (2020) 2, 61)
- ▶ New method for calculating electromagnetic effects in semileptonic beta-decays of mesons (JHEP 10 (2020) 179)

[More details on the highlighted topics in this talk](#)

Frankfurt:

- ▶ Computation of the quarkonium and meson-meson composition of the $\Upsilon(nS)$ states and of the new $\Upsilon(10753)$ Belle resonance from lattice QCD static potentials (arXiv:2008.05605)
- ▶ Tetraquark interpolating fields in a lattice QCD investigation of the $D_{s0}^*(2317)$ meson (PRD101(2020)034502)
- ▶ Bottomonium resonances with $I = 0$ from lattice QCD correlation functions with static and light quarks (PRD101(2020)034503)

Frankfurt/Jena:

- ▶ Baryons in the Gross-Neveu model in 1+1 dimensions at finite number of flavors (PRD102(2020)114501)
- ▶ Inhomogeneous phases in the Gross-Neveu model in 1+1 dimensions at finite number of flavors (PRD101(2020)094512)

Giessen:

- ▶ Numerical Study of the Chiral Separation Effect in Two-Color QCD at Finite Density (arXiv:2012.05184)
- ▶ Electric conductivity in finite-density SU(2) lattice gauge theory with dynamical fermions (Phys.Rev.D 102 (2020) 9, 094510)
- ▶ A density of states approach to the hexagonal Hubbard model at finite density (Phys.Rev.D 102 (2020) 5, 054502)

Jena/Regensburg:

- ▶ Measurement of the mass anomalous dimension of near-conformal adjoint QCD with the gradient flow (<https://arxiv.org/abs/2011.02815>)
- ▶ Lattice simulations of a gauge theory with mixed adjoint-fundamental matter (2008.02855)
- ▶ Partial Deconfinement at Strong Coupling on the Lattice (2005.04103)
- ▶ Monte Carlo simulations of overlap Majorana fermions (Phys.Rev.D 102 (2020) 1, 014503)
- ▶ Thermal phase transition in Yang-Mills matrix model (JHEP 01 (2020) 053)

Jülich/Wuppertal:

- ▶ QCD Crossover at Finite Chemical Potential from Lattice Simulations (Phys.Rev.Lett. 125 (2020) 5, 052001)
- ▶ Off-diagonal correlators of conserved charges from lattice QCD and how to relate them to experiment (Phys.Rev.D 101 (2020) 3, 034506)
- ▶ Ab-initio calculation of the proton and the neutron's scalar couplings for new physics searches (arXiv:2007.03319)
- ▶ [Leading-order hadronic vacuum polarization contribution to the muon magnetic moment from lattice QCD](#) (arXiv:2002.12347)

More details on the highlighted topics in this talk

LMU:

- ▶ [openQ*D code: a versatile tool for QCD+QED simulations \(Eur.Phys.J.C 80 \(2020\) 3, 195\)](#)

Mainz:

- ▶ [Rho resonance, timelike pion form factor, and implications for lattice studies of the hadronic vacuum polarization \(PRD101\(2020\)054504\)](#)
- ▶ [Hadronic light-by-light contribution to \$\(g - 2\)_\mu\$ from lattice QCD with SU\(3\) flavor symmetry \(Eur.Phys.J.C 80 \(2020\) 9, 869\)](#)
- ▶ [Rate of photon production in the quark-gluon plasma from lattice QCD \(PRD102\(2020\)091501\)](#)

Regensburg (RBC):

- ▶ [Direct CP violation and the \$\Delta I = 1/2\$ rule in \$K \rightarrow \pi\pi\$ decay from the standard model \(PRD102\(2020\)054509\)](#)
- ▶ [Consistency of hadronic vacuum polarization between lattice QCD and the R-ratio \(PRD101\(2020\)074515\)](#)
- ▶ [Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment from Lattice QCD \(PRL124\(2020\)132002\)](#)

Regensburg (RQCD):

- ▶ [Light-cone distribution amplitudes of pseudoscalar mesons from lattice QCD \(JHEP\(2020\)37\)](#)
- ▶ [Nucleon axial structure from lattice QCD \(JHEP05 \(2020\) 126\)](#)
- ▶ [Double parton distributions in the pion from lattice QCD \(arXiv:2006.14826\)](#)

Reviews with contributions from several German groups:

- ▶ [The anomalous magnetic moment of the muon in the Standard Model \(Phys.Rept. 887 \(2020\) 1-166\)](#)
- ▶ [FLAG 2019 \(Eur.Phys.J.C 80 \(2020\) 2, 113\)](#)

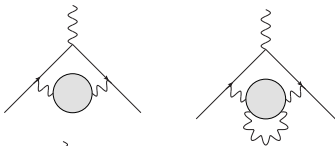
[More details on the highlighted topics in this talk](#)

Highlighted papers contribute to the understanding
of hadronic contributions to the muon $g - 2$

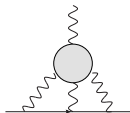
There is a tension of 3.7σ for the muon

$$a_{\mu}^{\text{E821}} - a_{\mu}^{\text{SM}} = 27.4 \underbrace{(2.7)}_{\text{HVP}} \underbrace{(2.6)}_{\text{HLbL}} \underbrace{(0.1)}_{\text{other}} \underbrace{(6.3)}_{\text{E821}} \times 10^{-10}$$

Hadronic Vacuum Polarization (HVP)



Hadronic Light-by-Light (HLbL)



There are also interesting results for the electron $g-2$, topic for another talk

New experiment: Fermilab E989

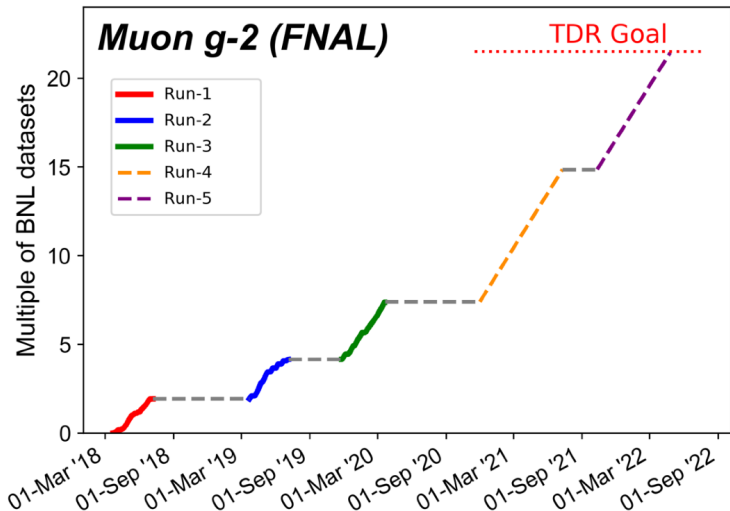


$$a_{\mu}^{\text{E821}} - a_{\mu}^{\text{SM}} = 27.4 \underbrace{(2.7)}_{\text{HVP}} \underbrace{(2.6)}_{\text{HLbL}} \underbrace{(0.1)}_{\text{other}} \underbrace{(6.3)}_{\text{E821}} \times 10^{-10}$$

$$\delta a_{\mu}^{\text{E989}} \rightarrow 1.6 \times 10^{-10}$$

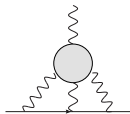
Need to improve uncertainties on HVP and HLbL contributions

Accumulated and projected statistics of E989 experiment (Fig. from Esra Barlas-Yucel):



First results (Run-1) anticipated to be published around February 2021

HLbL contribution

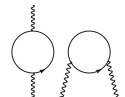


7 quark-level topologies of direct lattice calculation

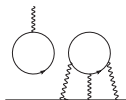
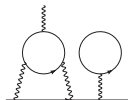
Hierarchy imposed by QED charges of dominant up- and down-quark contribution



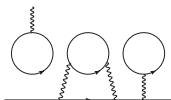
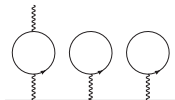
$$Q_u^4 + Q_d^4 = 17/81$$



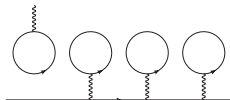
$$(Q_u^2 + Q_d^2)^2 = 25/81$$



$$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$$



$$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$$



$$(Q_u + Q_d)^4 = 1/81$$

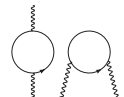
Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

7 quark-level topologies of direct lattice calculation

Hierarchy imposed by QED charges of dominant up- and down-quark contribution

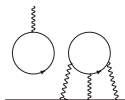
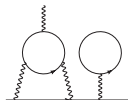


$$Q_u^4 + Q_d^4 = 17/81$$

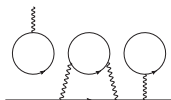
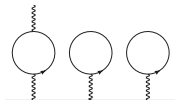


$$(Q_u^2 + Q_d^2)^2 = 25/81$$

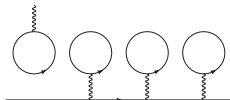
Dominant diagrams in top row: connected and leading disconnected diagram



$$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$$



$$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$$



$$(Q_u + Q_d)^4 = 1/81$$

Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

Development of HLbL lattice methodology (I)

- ▶ QED non-perturbatively and momentum-space
(PRL114(2015)012001)
- ▶ QED perturbatively and position-space
 - ▶ QED_L (PRD93(2016)014503, PRL118(2017)022005): $1/L^2$ finite-volume errors (with linear extent L); noise reduction through importance sampling
 - ▶ QED_∞: exponential finite-volume errors (PRL115(2015)222003, EPJ Web Conf. 175(2018)06023), subtraction prescriptions to reduce systematic errors (PRD96(2017)034515, arXiv:1811.08320)

Color code: Mainz, RBC/Regensburg

Development of HLbL lattice methodology (II)

- ▶ [PRL118\(2016\)022005](#): Physical-pion mass for leading connected+disconnected diagrams at finite volume and lattice spacing $a_{\mu}^{\text{HLbL}} = 5.35(1.35) \times 10^{-10}$
- ▶ [PRD98\(2018\)074501](#): Forward scattering amplitude $(\gamma^* \gamma^* \rightarrow \gamma^* \gamma^*)$
- ▶ [Phys. Rev. D 100, 034520 \(2019\)](#): Pion-pole contribution $a_{\mu}^{\text{HLbL}, \pi^0} = 5.97(36) \times 10^{-10}$

Color code: Mainz, RBC/Regensburg

HLbL from lattice QCD at SU(3)-symmetric limit and $a \rightarrow 0$ and $V \rightarrow \infty$ (Eur. Phys. J. C 80, 869 (2020)):

MITP/20-036
CERN-TH-2020-109

Hadronic light-by-light contribution to $(g - 2)_\mu$ from lattice QCD with SU(3) flavor symmetry

En-Hung Chao,¹ Antoine Gérardin,² Jeremy R. Green,³
Renwick J. Hudspith,¹ and Harvey B. Meyer^{1,4}

¹PRISMA+ Cluster of Excellence & Institut für Kernphysik,
Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany

²Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France.

³Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland

⁴Helmholtz Institut Mainz, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany

(Dated: July 15, 2020)

We perform a lattice QCD calculation of the hadronic light-by-light contribution to $(g - 2)_\mu$ at the SU(3) flavor-symmetric point $m_\pi = m_K \simeq 420 \text{ MeV}$. The representation used is based on coordinate-space perturbation theory, with all QED elements of the relevant Feynman diagrams implemented in continuum, infinite Euclidean space. As a consequence, the effect of using finite lattices to evaluate the QCD four-point function of the electromagnetic current is exponentially suppressed. Thanks to the SU(3)-flavor symmetry, only two topologies of diagrams contribute, the fully connected and the leading disconnected. We show the equivalence in the continuum limit of two methods of computing the connected contribution, and introduce a sparse-grid technique for computing the disconnected contribution. Thanks to our previous calculation of the pion transition form factor, we are able to correct for the residual finite-size effects and extend the tail of the integrand. We test our understanding of finite-size effects by using gauge ensembles differing only by their volume. After a continuum extrapolation based on four lattice spacings, we obtain $a_\mu^{\text{hlbl}} = (65.4 \pm 4.9 \pm 6.6) \times 10^{-11}$, where the first error results from the uncertainties on the individual gauge ensembles and the second is the systematic error of the continuum extrapolation. Finally, we estimate how this value will change as the light-quark masses are lowered to their physical values.

Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment from Lattice QCD

Thomas Blum,^{1,2} Norman Christ,³ Masashi Hayakawa,^{4,5} Taku Izubuchi,^{6,2}

Luchang Jin^{1,2,*}, Chulwoo Jung,⁶ and Christoph Lehner^{7,6}

¹*Physics Department, University of Connecticut, 2152 Hillside Road, Storrs, Connecticut 06269-3046, USA*

²*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*

³*Physics Department, Columbia University, New York, New York 10027, USA*

⁴*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

⁵*Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan*

⁶*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

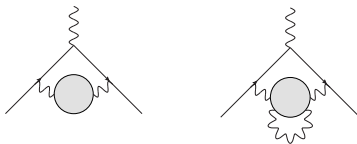
⁷*Universität Regensburg, Fakultät für Physik, 93040 Regensburg, Germany*



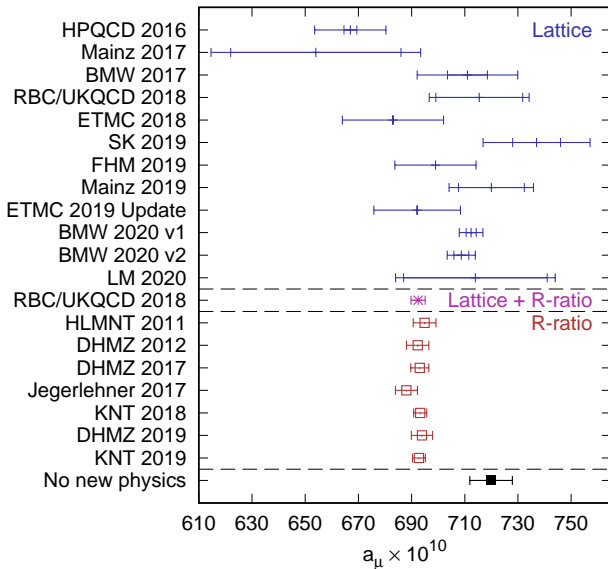
(Received 18 December 2019; accepted 27 February 2020; published 1 April 2020)

We report the first result for the hadronic light-by-light scattering contribution to the muon anomalous magnetic moment with all errors systematically controlled. Several ensembles using 2 + 1 flavors of physical mass Möbius domain-wall fermions, generated by the RBC and UKQCD collaborations, are employed to take the continuum and infinite volume limits of finite volume lattice QED + QCD. We find $a_{\mu}^{\text{HLbL}} = 7.87(3.06)_{\text{stat}}(1.77)_{\text{sys}} \times 10^{-10}$. Our value is consistent with previous model results and leaves little room for this notoriously difficult hadronic contribution to explain the difference between the standard model and the BNL experiment.

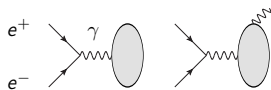
HVP contribution



Status of HVP determinations



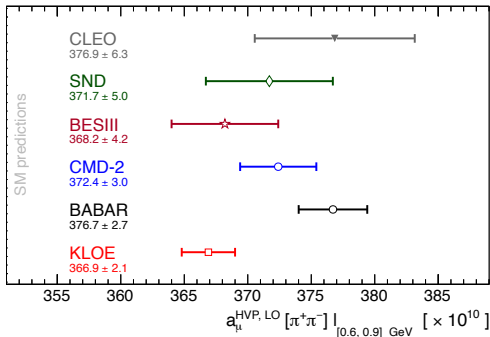
Tensions in R-ratio data ($e^+e^- \rightarrow \pi^+\pi^-$)



$e^+e^- \rightarrow \text{hadrons}(\gamma)$

$$J_\mu = V_\mu^{I=1, I_3=0} + V_\mu^{I=0, I_3=0}$$

Phys.Rept. 887 (2020) 1-166

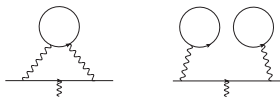


Comparison of results for $a_\mu^{\text{HVP, LO}}[\pi\pi]$, evaluated between 0.6 GeV and 0.9 GeV for the various experiments.

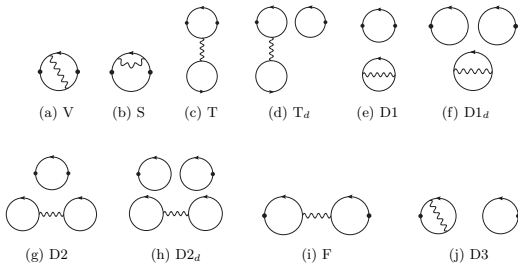
Diagrams

(for a lattice QCD+QED calculation)

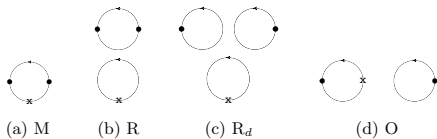
Isospin
limit



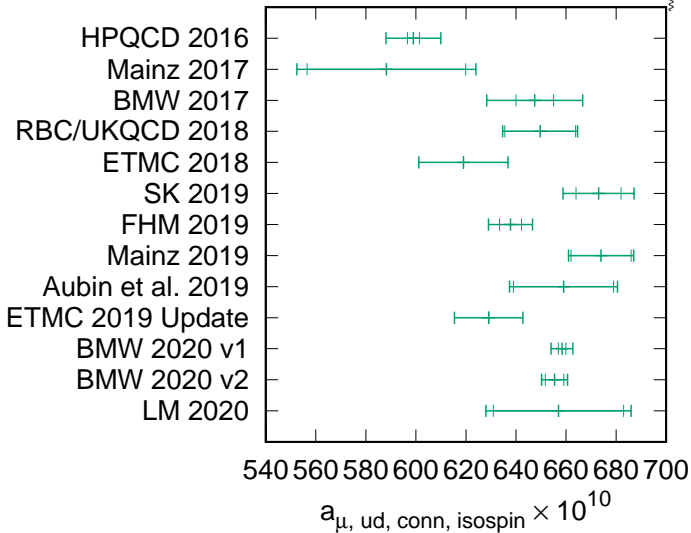
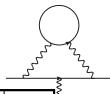
QED
corrections



Strong
isospin
breaking



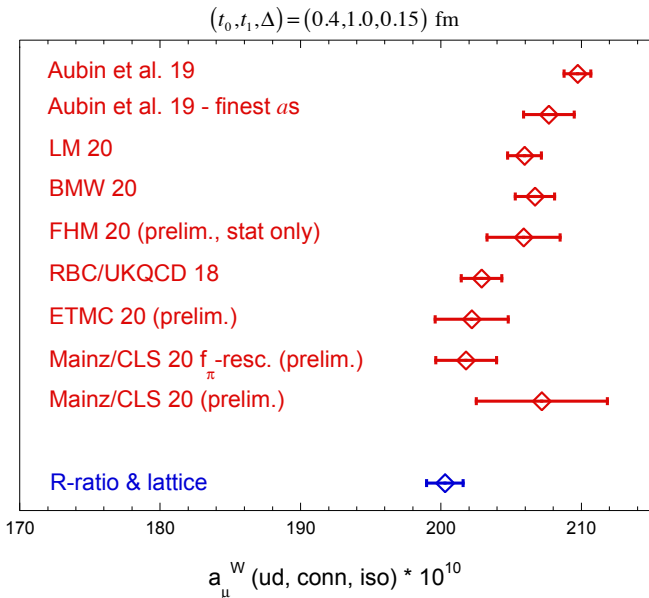
Up, down; isospin symmetric limit; $m_\pi = m_\pi^0$



Some tensions to be understood

Held a workshop three weeks ago to focus, amongst others, on cross-check of simple Euclidean time windows ([PRL121\(2018\)022003](#)):

<https://indico.cern.ch/event/956699/>



Conclusions and Outlook

- ▶ Broad research output from the lattice community in Germany
- ▶ For this talk focussed on $g-2$ HVP + HLbL progress, where many German groups contribute at the forefront
- ▶ Significant progress over the last years in developing methodology and results, commensurate with experimental progress (Fermilab E989, J-PARC E34, e^+e^- experiments, ...)