On radiative corrections to elastic electron proton scattering

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Outline

- Types of corrections to *ep* scattering
- Vacuum polarization
- Exponentiation of photonic corrections
- Light pair correction in LLA
- Complete second order NLO corrections
- Monte Carlo codes
- Open questions and Conclusions

Example: MAMI A1 experiment

Mainz Microtron experimental set-up:

— the electron beam energy $E_e \equiv E \lesssim 855$ MeV (1.6 GeV)

— momentum transfer range: $0.003 < Q^2 < 1$ GeV^2

— the outgoing electron energy $E_e{}' \equiv E' > E_e - \Delta E$

— no any other condition: neither on enrgies nor on angles

— experimental precision (point-to-point) $\simeq 0.37\% \quad \to \quad 0.1\%$ (?)

$\Rightarrow$ all effects at least of the 10^{-4} order should be taken into account. That is not a simple task in any case

N.B. $E_e^2 \gg m_e^2, \quad Q^2 \gg m_e^2, \quad (\Delta E)^2 \gg m_e^2, \quad \Delta E \ll E_e$

Ref.: J.C. Bernauer et al. [A1 Coll.] PRC 90 (2014) 015206

Types of RC to elastic *ep* scattering

— Virtual (loop) and/or real emission

— QED, QCD, and (electro)weak effects

— Perturbative and/or non-perturbative contributions

— Perturbative QED effects in $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2)$, ...

— Leading and next-to-leading logarithmic approximations

— Corrections to the electron line, to the proton line, and their interference

— Vacuum polarization, vertex corrections, double photon exchange etc.

— Electron mass effects $\sim m_e^2/Q^2$

Size of RC

Expansion in small and large parameters:

- $\alpha/(2\pi) \approx 0.001$
- $(\alpha/(2\pi))^2 \approx 10^{-6}$
- $m_e^2/Q^2 \approx 2.5 \cdot 10^{-4}$ for $Q^2 = 0.001$ GeV^2
- $L \equiv \ln(Q^2/m_e^2) \approx 16$ the large log for $Q^2 = 1$ GeV^2

  $L \approx 8$ for $Q^2 = 0.001$ GeV^2
- $\ln(\Delta) \sim 5$, where $\Delta = \Delta E_e/E_e \ll 1$

N.B.1. Some $\mathcal{O}(\alpha^2)$ corrections are enhanced with 2nd, 3rd or even 4th power of large logs. So, they should be treated with care.

N.B.2. **The more you cut the more you get!**

First order QED RC (I)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 (1+\delta)$$

The $\mathcal{O}(\alpha)$ QED RC with point-like proton are well known:
Refs.: see e.g. L.C. Maximon & J.A. Tjon, PRC 2000

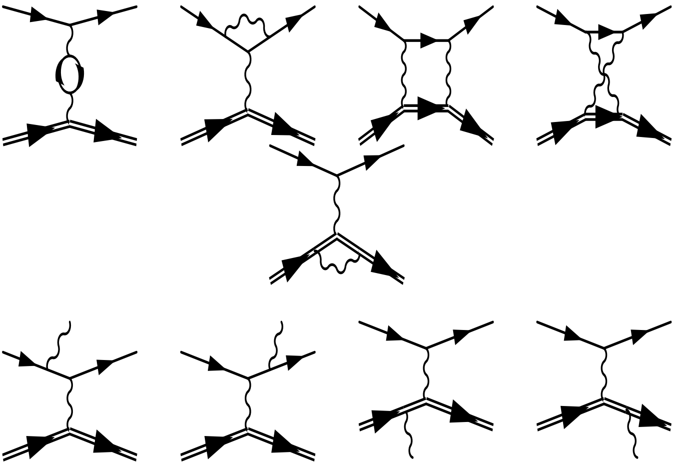
Virtual RC: Vacuum polarization, vertex, and box Feynman diagrams

Real RC: emission off the initial and final electrons and protons

N.B.1. UV divergences are regularized and renormalized

N.B.2. IR divergences cancel out in sum of virtual and real RC

First order QED RC (II)



Vacuum polarization in one-loop

$$\delta^{(1)}_{\rm vac} = \frac{\alpha}{\pi}\frac{2}{3}\left\{\left(v^2 - \frac{8}{3}\right) + v\frac{3-v^2}{2}\ln\left(\frac{v+1}{v-1}\right)\right\}$$

$$\stackrel{Q^2\gg m_l^2}{\longrightarrow} \frac{\alpha}{\pi}\frac{2}{3}\left\{-\frac{5}{3} + \ln\left(\frac{Q^2}{m_l^2}\right)\right\}, \quad v = \sqrt{1+\frac{4m_l^2}{Q^2}}, \quad l = e,\mu,\tau$$

Two ways of re-summation:

1) geometric progression

$$\Rightarrow \alpha(Q^2) = \frac{\alpha(0)}{1-\Pi(Q^2)}, \quad \Pi(Q^2) = \frac{1}{2}\delta^{(1)}_{\rm vac} + \dots$$

2) exponentiation

$$\alpha(Q^2) = \alpha(0)e^{\delta^{(1)}_{\rm vac}/2}$$

the latter option was used by A1 Coll.

Other $\mathcal{O}(\alpha)$ effects

$$\delta^{(1)}_{\mathrm{vertex}} = \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{Q^2}{m^2}\right) - 2 - \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right) + \frac{\pi^2}{6}\right\}$$

$$\begin{aligned}\delta^{(1)}_{\mathrm{real}} &= \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E_s)^2}{E\cdot E'}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] - \frac{1}{2}\ln^2\eta + \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right)\right.\\ &- \left.\frac{\pi^2}{3} + \mathrm{Li}_2\left(\cos^2\frac{\theta_e}{2}\right)\right\}, \quad \eta = \frac{E}{E'}, \quad \Delta E_s = \eta\cdot\Delta E'\end{aligned}$$

Interference δ_1 and radiation off proton δ_2 do not contain the large log.
A1 Coll. applied RC in the exponentiated form:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{exp}}(\Delta E') = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 e^{\delta_{\mathrm{vac}}+\delta_{\mathrm{vertex}}+[\delta_R+\delta_1+\delta_2](\Delta E')}$$

Higher order effects are partially taken into account by exponentiation.
Remind the Yennie-Frautschi-Suura theorem

Multiple soft photon radiation (I)

Exponentiation corresponds to independent emission of soft photons, while the cut on the total lost energy leads to sizable shifts.

For two photons:

$$\mathrm{e}^{\delta_{\mathrm{soft}}} \to \mathrm{e}^{\delta_{\mathrm{soft}}} - \left(\frac{\alpha}{\pi}\right)^2 \frac{\pi^2}{3} (L-1)^2$$

at $Q^2 = 1\ \mathrm{GeV}^2$ this gives $-3.5 \cdot 10^{-3}$

In the leading log approximation

$$\delta^{(3)}_{\mathrm{LLA}} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \frac{1}{6} \left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_{\Delta},$$

$$\left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_{\Delta} = 8\left(P^{(0)}_{\Delta}\right)^3 - 24\zeta(2)P^{(0)}_{\Delta} + 16\zeta(3)$$

$$\Rightarrow \quad \delta^{(3)}_{\mathrm{cut}} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \left[-4\zeta(2)P^{(0)}_{\Delta} + \frac{8}{3}\zeta(3)\right]$$

which is not small and reaches $2 \cdot 10^{-3}$

Multiple soft photon radiation (II)

The exact LLA solution of the evolution equation for the photonic part of the non-singlet structure function in the soft limit is known

$$\mathcal{D}^{\mathrm{NS}}_{\gamma}(z,Q^2)\Big|_{z\to 1} = \frac{\beta}{2}\,\frac{(1-z)^{\beta/2-1}}{\Gamma(1+\beta/2)}\,\exp\left\{\frac{\beta}{2}\left(\frac{3}{4}-C\right)\right\}$$

where C is the Euler constant, $\beta = \frac{2\alpha}{\pi}(\ln\frac{Q^2}{m^2}-1)$

$$\int\limits_{1-\Delta}^{1} \mathrm{d}z\,\mathcal{D}^{\mathrm{NS}}_{\gamma}(z,Q^2) = \exp\left\{\frac{\beta}{2}\ln\Delta + \frac{3\beta}{8}\right\}\frac{\exp(-C\beta/2)}{\Gamma(1+\beta/2)},$$

$$\frac{\exp(-C\beta/2)}{\Gamma(1+\beta/2)} = 1 - \frac{1}{2}\left(\frac{\beta}{2}\right)^2\zeta(2) + \frac{1}{3}\left(\frac{\beta}{2}\right)^3\zeta(3) + \frac{1}{16}\left(\frac{\beta}{2}\right)^4\zeta(4) + \frac{1}{5}\left(\frac{\beta}{2}\right)^5\zeta(5) - \frac{1}{6}\left(\frac{\beta}{2}\right)^5\zeta(2)\zeta(3) + \mathcal{O}(\beta^6)$$

[V. Gribov, L. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 451; 675]

Light pair corrections

A quick estimate can be done within LLA:

$$\delta^{LLA}_{\rm pair} = \frac{2}{3}\left(\frac{\alpha}{2\pi}L\right)^2 P^{(0)}_{\Delta} + \frac{4}{3}\left(\frac{\alpha}{2\pi}L\right)^3 \left\{\left(P^{(0)}\otimes P^{(0)}\right)_{\Delta} + \frac{2}{9}P^{(0)}_{\Delta}\right\} + \mathcal{O}\left(\alpha^2 L, \alpha^4 L^4\right)$$

$$P^{(0)}_{\Delta} = 2\ln\Delta + \frac{3}{2}, \qquad \left(P^{(0)}\otimes P^{(0)}\right)_{\Delta} = \left(P^{(0)}_{\Delta}\right)^2 - \frac{\pi^2}{3}$$

The energy of the emitted pair is limited by the same parameter: $E_{\rm pair} \leq \Delta E$. Both virtual and real e^+e^- pair corrections are taken into account.

Typically, $\mathcal{O}\left(\alpha^2\right)$ pair RC are a few times less than $\mathcal{O}\left(\alpha^2\right)$ photonic ones, see e.g. A.A. JHEP'2001

Complete NLLA corrections (I)

The NLO structure function approach for QED was first introduced in F.A. Berends et al. NPB'1987, and then developed in A.A. & K.Melnikov PRD'2002; A.A. JHEP'2003

The master formula for *ep* scattering reads

$$\mathrm{d}\sigma = \int_{\bar{z}}^{1} \mathrm{d}z \mathcal{D}_{ee}^{\mathrm{str}}(z) \left(\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) + \mathcal{O}\left(\alpha^2 L^0\right) \right) \int_{\bar{y}}^{1} \frac{\mathrm{d}y}{Y} \mathcal{D}_{ee}^{\mathrm{frg}}\left(\frac{y}{Y}\right)$$

where $\mathrm{d}\bar{\sigma}^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the *ep* scattering with a "massless electron" in the $\overline{\mathrm{MS}}$ scheme

N.B. R.D. Bucoveanu and H. Spiesberger, "Second-Order Leptonic Radiative Corrections for Lepton-Proton Scattering," Eur. Phys. J. A 55 (2019) no.4, 57.

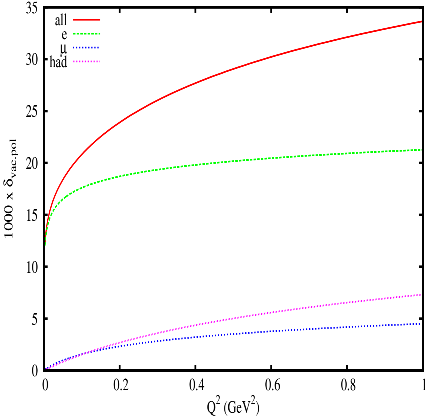
Complete NLLA corrections (II)

$$
\begin{aligned}
\mathrm{d}\sigma^{\mathrm{NLO}} &= \int_{1-\Delta}^{1} \mathcal{D}_{ee}^{\mathrm{str}} \otimes \mathcal{D}_{ee}^{\mathrm{frg}}(z) \Big[\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) \Big] \mathrm{d}z \\
&= \mathrm{d}\sigma^{(0)}(1) \Bigg\{ 1 + 2\frac{\alpha}{2\pi} \Big[L P_{\Delta}^{(0)} + (d_1)_{\Delta} \Big] + 2\left(\frac{\alpha}{2\pi}\right)^2 \Big[L^2 \left(P^{(0)} \otimes P^{(0)} \right)_{\Delta} \\
&+ \frac{1}{3} L^2 P_{\Delta}^{(0)} + 2L(P^{(0)} \otimes d_1)_{\Delta} + L(P_{ee}^{(1,\gamma)})_{\Delta} + L(P_{ee}^{(1,\mathrm{pair})})_{\Delta} \Big] \Bigg\} \\
&+ \mathrm{d}\bar{\sigma}^{(1)}(1)\, 2\frac{\alpha}{2\pi} L P_{\Delta}^{(0)} + \mathcal{O}\left(\alpha^3 L^3\right)
\end{aligned}
$$

$$
(d_1)_{\Delta} = -2\ln^2\Delta - 2\ln\Delta + 2, \quad \ldots
$$

N.B. Method gives complete $\mathcal{O}\left(\alpha^2 L\right)$ results for sufficiently inclusive observables.

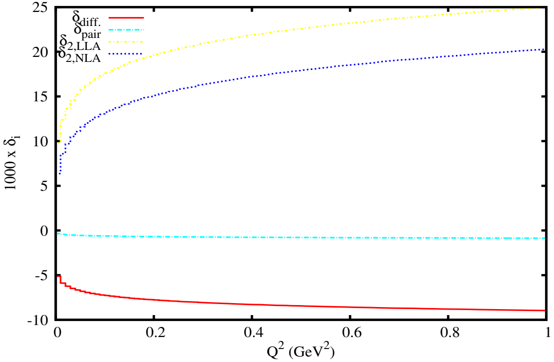
Numerical results: vacuum polarization



Vacuum polarization corrections due to electrons (e), muons (μ), hadrons (had), and the combined effect (all).
Program `AlphaQED` by F. Jegerlehner was used.

Higher-order corrections (1)

$E_{\rm beam} = 800$ MeV, $\quad E_{\rm lost} \leq$ 10 MeV

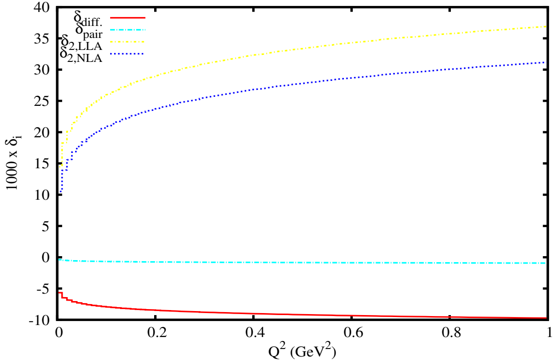


$$\delta_i = \mathrm{d}\sigma^{(i)}/\mathrm{d}\sigma^{(0)}$$

$$\delta_{\rm diff.} = \frac{\mathrm{d}\sigma^{\rm NLO}}{\mathrm{d}\sigma^{(0)}(1)} + \delta^{(3)}_{\rm LLA} + \delta^{(3)}_{\rm LLA,pair} + \delta^{(4)}_{\rm LLA} - \exp\{\delta^{(1)}\}$$

Higher-order corrections (2)

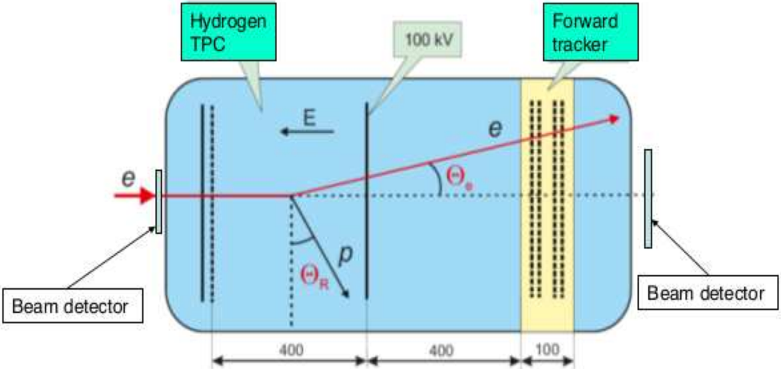
$E_{\rm beam} = 1600$ MeV, $\quad E_{\rm lost} \leq$ 10 MeV



$$\delta_i = \mathrm{d}\sigma^{(i)}/\mathrm{d}\sigma^{(0)}$$

$$\delta_{\rm diff.} = \frac{\mathrm{d}\sigma^{\rm NLO}}{\mathrm{d}\sigma^{(0)}(1)} + \delta^{(3)}_{\rm LLA} + \delta^{(3)}_{\rm LLA,pair} + \delta^{(4)}_{\rm LLA} - \exp\{\delta^{(1)}\}$$

Alternative experimental set-up



Measured quantities:

Recoil energy T_R

Recoil angle Θ_R

Vertex Z coordinate

E scattering angle Θ_e

$$-t = \frac{4\varepsilon_e^2 \sin^2\frac{\vartheta}{2}}{1+\frac{2\varepsilon_e}{M}\sin^2\frac{\vartheta}{2}}$$

$$-t = 2MT_R$$

Leading logs in the new set-up

FSR large log corrections are cancelled out (KLN theorem)

ISR provides an effective reduction of the beam energy.
It affects the the proton Q^2 distribution rather weakly

Some PRELIMINARY results in the collinear leading log approximation were obtained for

$$E_{\mathrm{beam}} = 500\ \mathrm{MeV}, \qquad 0.001 < Q^2 < 0.02\ \mathrm{GeV}^2$$

Q^2 [GeV]	0.001	0.01	0.02
δ_1^{LLA}	$-6.0 \cdot 10^{-4}$	$-2.9 \cdot 10^{-3}$	$-4.6 \cdot 10^{-3}$
δ_2^{LLA}	$-1.1 \cdot 10^{-5}$	$-4.0 \cdot 10^{-5}$	$-5.4 \cdot 10^{-5}$
$\delta_{2+3+\mathrm{pairs}}^{\mathrm{LLA}}$	$-1.3 \cdot 10^{-5}$	$-5.1 \cdot 10^{-5}$	$-7.4 \cdot 10^{-5}$

$$\delta_n(Q^2) = \sigma^{\mathrm{LLA}}(Q^2)/\sigma^{\mathrm{Born}}(Q^2) - 1$$

Recent work

V.S. Fadin and R.E. Gerasimov, "On the cancellation of radiative corrections to the cross section of electron-proton scattering," Phys. Lett. B 795 (2019) 172 [arXiv:1812.10710 [nucl-th]]

1. Proved that the large logs cancell out in the set-up with Q^2 reconstruction from the recoil proton momentum. And that happens as not only in the first order in α but also in higher orders (for the non-singlet channel). But that is obvious (see above) and has been already known.

Cancellation of the large logs is complete only in an idealized situation with certain approximations.

3. *"... the cancellation of the virtual and real corrections turns out to be even stronger. It occurs that in the one-loop approximation the cancellation takes place not only with logarithmic accuracy..."*

The cancellation of RC can't be complete in a realistic experimental set-up. So, we do need a Monte Carlo.

Monte Carlo codes (I)

"A new event generator for the elastic scattering of charged leptons on protons" [A.V. Gramolin, V.S. Fadin, A.L. Feldman et al., J.Phys.G 41 (2014) 115001]

The code contains:

— a library of proton form factors

— vacuum polarization

— complete one-loop QED

— the dependence on m_e^2/Q^2 (not complete)

— double photon exchange treatment

— etc.

Need:

— update higher-order effects:

— higher order leading and next-to-leading RC (for electron variables)*

— complete $\mathcal{O}(\alpha^2)$ to electron line*

— improve technical precision

— tests and tuned comparisons

Monte Carlo codes (II)

"Radiative corrections beyond the ultra relativistic limit in unpolarized ep elastic and Møller scatterings for the PRad Experiment at Jefferson Laboratory" [I. Akushevich, H. Gao, A. Ilyichev, M. Meziane, EPJA 51 (2015) 1]

Advantages:
— both *ep* and *ee* in the same approach
— keep dependence on electron mass
— accurate in $\mathcal{O}(\alpha)$
— what else?

Unclear:
— improper treatment of higher-order effects
— no double photon exchange
— no radiation off proton and up-down interference
— no hadronic vacuum polarization
— tuned comparisons?

Conclusions

1. Application of RC to *ep* scattering experiments is discussed

2. Treatment of higher order QED RC to the electron line is important

3. In particular, effects due to multiple radiation and pair emission in the LLA and NLLA are calculated

4. Vacuum polarization by hadrons should be taken into account

5. The size of the higher order effects make them relevant for the high-precision experiments with electron detection

6. Higher order RC to the electron line should be combined with an advanced treatment of two-photon exchange and other relevant effects

7. Radiative corrections for the experimental set-up with recoil proton detection are essentially $\mathcal{O}(\alpha)$

8. Tuned comparisons of Monte Carlo and semi-analytic codes are required