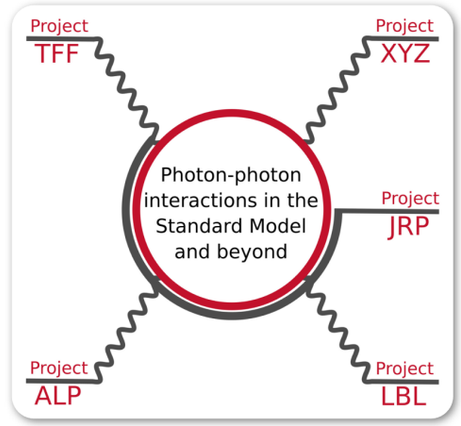
DFG

Workshop of Research Unit FOR5327

11-13 June, 2025 @ St. Goar, Germany



Project TFF – Phenomenology

Photon-photon fusion to three-meson final states

Xiu-Lei Ren

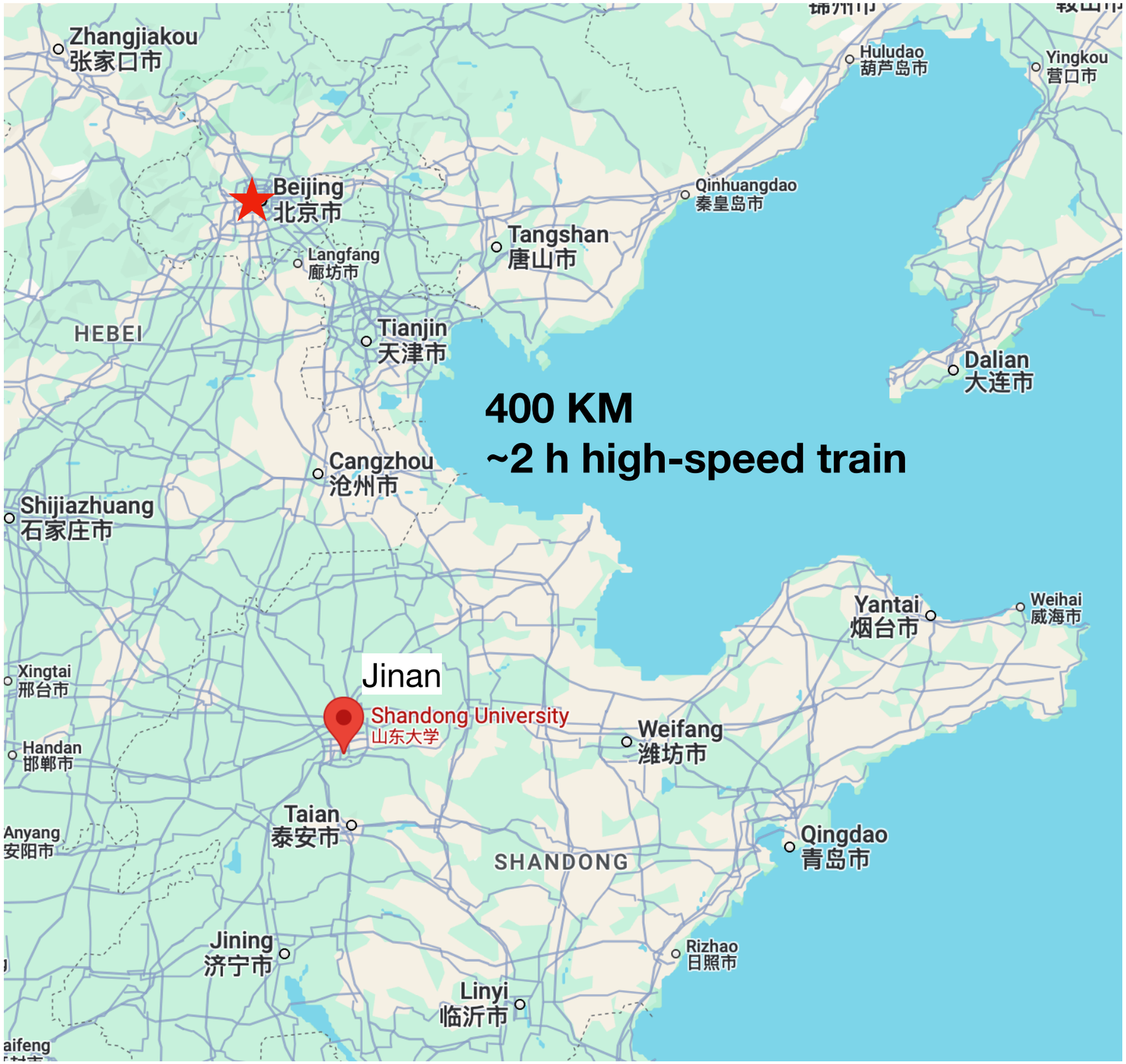
HIM
Helmholtz-Institut Mainz

2025

SHANDONG UNIVERSITY
山东大学

In collaboration with:

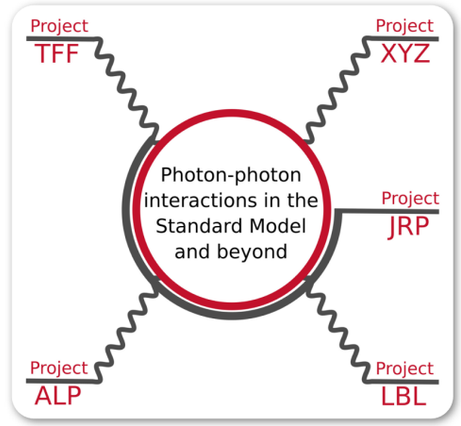
Igor Danilkin & Marc Vanderhaeghen (JGU Mainz)


Zhangjiakou
张家口市
Huludao
葫芦岛市
Yingkou
营口市
Beijing
北京市
Qinhuangdao
秦皇岛市
Tangshan
唐山市
Langfang
廊坊市
HEBEI
Tianjin
天津市
Dalian
大连市
400 KM
~2 h high-speed train
Cangzhou
沧州市
Shijiazhuang
石家庄市
Yantai
烟台市
Weihai
威海市
Xingtai
邢台市
Jinan
Shandong University
山东大学
Weifang
潍坊市
Handan
邯郸市
Taian
泰安市
Anyang
安阳市
Qingdao
青岛市
SHANDONG
Jining
济宁市
Rizhao
日照市
Linyi
临沂市

DFG

Workshop of Research Unit FOR5327

11-13 June, 2025 @ St. Goar, Germany



Project TFF – Phenomenology

Photon-photon fusion to three-meson final states

Xiu-Lei Ren

HIM
Helmholtz-Institut Mainz

2025

SHANDONG UNIVERSITY
山东大学

In collaboration with:

Igor Danilkin & Marc Vanderhaeghen (JGU Mainz)

HLbL contribution from white papers

WP(2020): $92(19) \times 10^{-11}$	
hadronic state	$a_\mu^{\mathrm{HLbL}}\ [10^{-11}]$
scalars+tensors $\gtrsim 1$ GeV	$\sim -1(3)$
axial vectors	$\sim 6(6)$

WP(2025): $103.3(8.8) \times 10^{-11}$	
hadronic state	$a_\mu^{\mathrm{HLbL}}\ [10^{-11}]$
tensors (f_2, a_2)	$\sim -2.5(8)$
axial vectors (f_1, f_1', a_1)	$\sim 12.2(4.3)$

- Total uncertainty of HLbL reduced up to <10%
- The axial vector contributions are still **the major source of uncertainty**

- Most relevant axial vector and tensor mesons

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
decay modes	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi$ (70%)
	$\pi\pi\pi\pi$ (33%)	$\eta\pi\pi$ (4%)	$KK\pi$	$\eta\pi$ (14%)
	$K\bar{K}\pi$ (9%)			$\omega\pi\pi$ (11%)
	$\gamma\rho^0$ (6%)			$K\bar{K}$ (5%)

- Coordinate with BESIII experiments

  TFF-2: Analysis of axial-vector meson TFFs in the channel $\pi^+\pi^-\eta$ at BESIII

- Focus on the photon-photon fusion to three mesons

  $\gamma\gamma \to \pi^+\pi^-\pi^0$ PRD107, 054037 (2023)

  $\gamma\gamma^* \to K\bar{K}\pi$ PRD110, 054004 (2024)

  $\gamma\gamma^* \to \eta\pi^+\pi^-$ PRD110, 094043 (2024)

$\gamma^{(*)}\gamma^{(*)} \to \pi^+\pi^-\pi^0$ process

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
decay modes	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)
	$\pi\pi\pi\pi$ (33%)	$\eta\pi\pi$ (4%)	$KK\pi$	$\eta\pi$ (14%)
	$K\bar{K}\pi$ (9%)			$\omega\pi\pi$ (11%)
	$\gamma\rho^0$ (6%)			$K\bar{K}$ (5%)

XLR, I. Danilkin, M. Vanderhaeghen, PRD107,054037(2023)

Resonances in $\gamma^{(*)}\gamma^{(*)} \to \pi^+\pi^-\pi^0$

- $a_1(1260)$ production is only from $\gamma^{(*)}\gamma^* \to \pi^+\pi^-\pi^0$
  - $a_1(1260)$ is not a well-established resonance

$a_1(1260)$ [j] $I^G(J^{PC}) = 1^-(1^{++})$

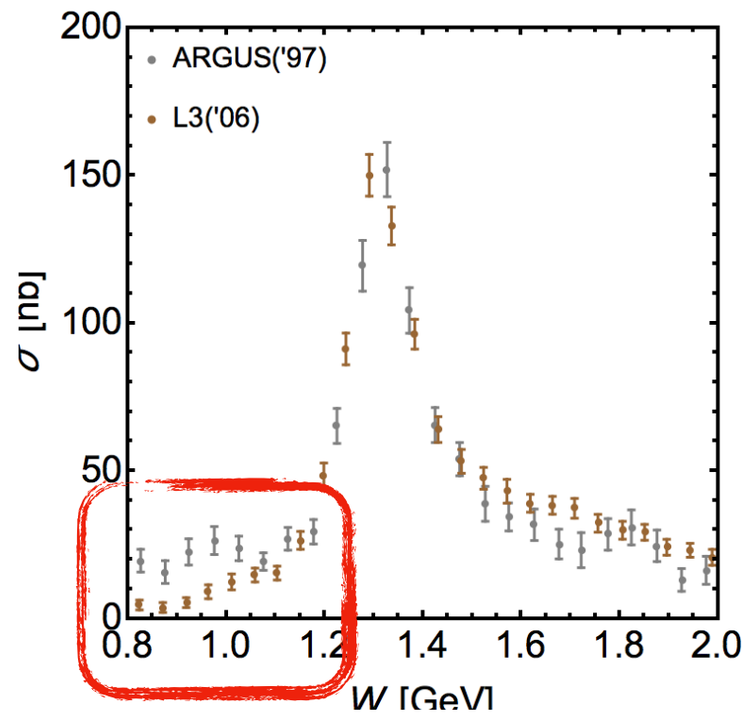
T-Matrix Pole $\sqrt{s} = (1209^{+13}_{-10}) - i(288^{+45}_{-12})$ MeV
Mass (Breit-Wigner) $= 1230 \pm 40$ MeV [i]
Full width (Breit-Wigner) $= 250$ to 600 MeV [i]

- $a_2(1320)$ production is dominant in $\gamma^{(*)}\gamma^{(*)} \to \pi^+\pi^-\pi^0$
  - $a_2(1320)$ is a well-defined resonance

$a_2(1320)$ $I^G(J^{PC}) = 1^-(2^{++})$

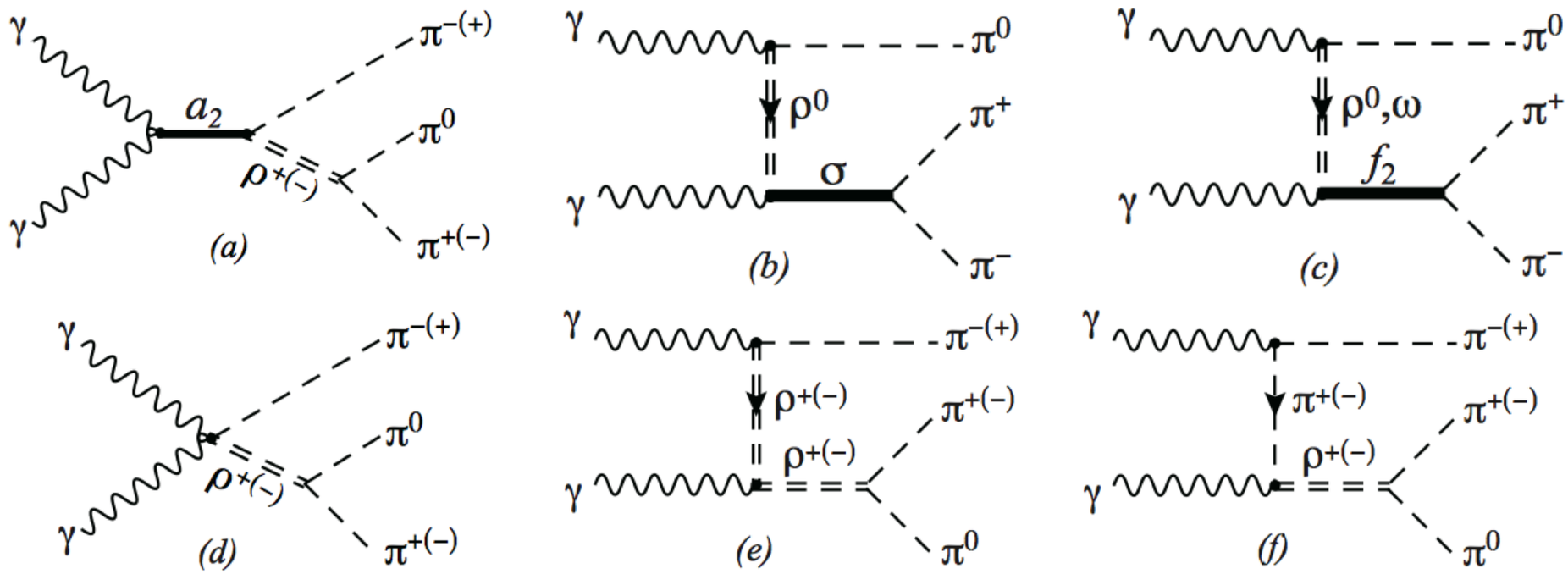
T-Matrix Pole $\sqrt{s} = (1305\text{–}1321) - i(52\text{–}58)$ MeV
Mass (Breit-Wigner) $= 1318.2 \pm 0.6$ MeV (S = 1.2)
Full width $\Gamma = 107 \pm 5$ MeV

- Complicated mechanism involved in the $\pi^+\pi^-\pi^0$ final states
  - $\rho(770), \sigma/f_0(500), f_2(1270)$ resonances
- **As a first step**, we focus on $\gamma\gamma \to \pi^+\pi^-\pi^0$
  - Experimental data: low statistics
    - ✓ ARGUS collab. *Z. Phys. C 74, 469 (1997)*
    - ✓ L3 collab. *PLB 413, 147(1997)*
  - Cross section data shows a significant difference in the low-energy region

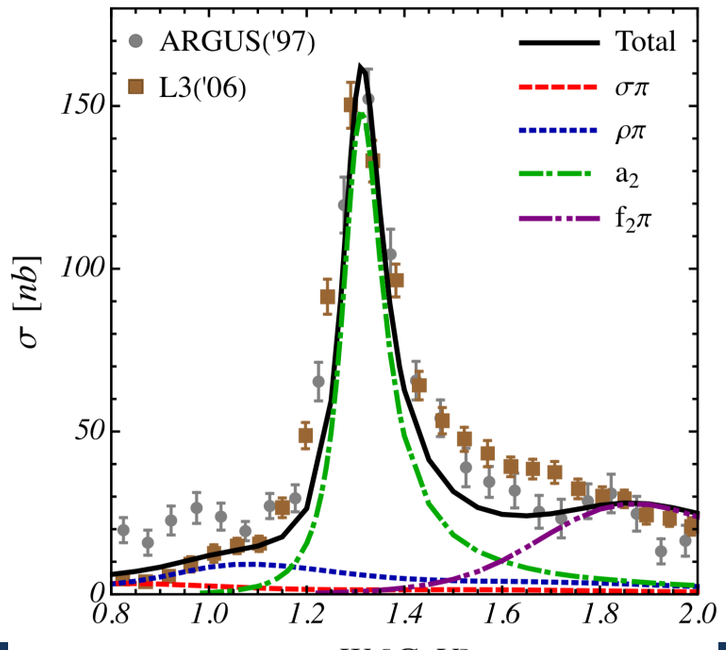


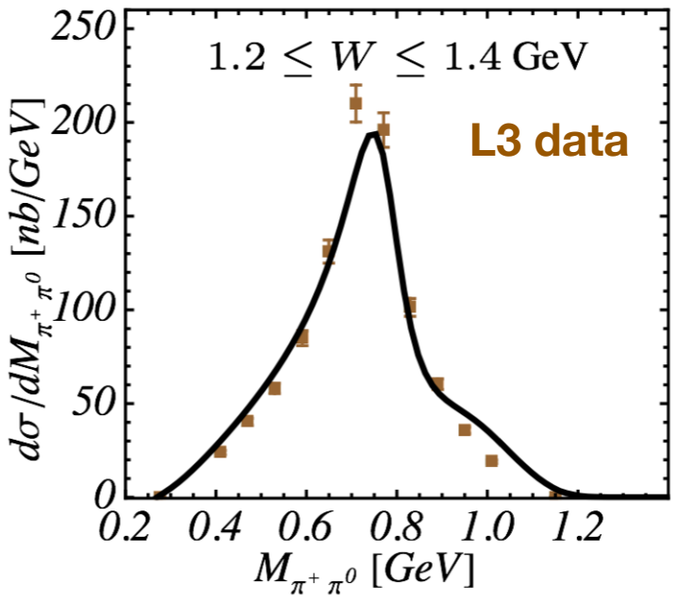
Phenomenological model for $\gamma\gamma \to \pi^+\pi^-\pi^0$

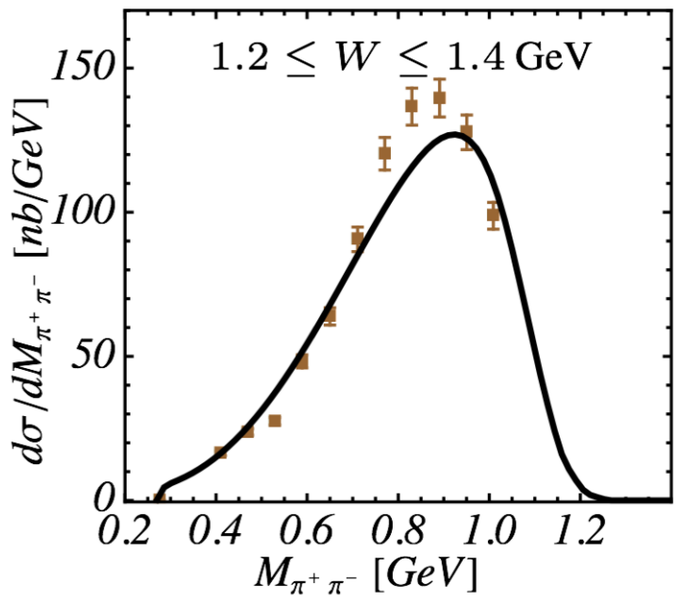
- Cover the low and the intermediate energy region
  - Include all relevant channels within effective Lagrangian method



- Description of ARGUS and L3 data







$\gamma\gamma^* \to K\bar{K}^* \to K\bar{K}\pi$ process

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
decay modes	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)
	$\pi\pi\pi\pi$ (33%)	$\eta\pi\pi$ (4%)	$KK\pi$	$\eta\pi$ (14%)
	$K\bar{K}\pi$ (9%)			$\omega\pi\pi$ (11%)
	$\gamma\rho^0$ (6%)			$K\bar{K}$ (5%)

XLR, I. Danilkin, M. Vanderhaeghen, PRD110 (2024) 094043

$f_1(1420)$ production in $\gamma\gamma^*$ fusion

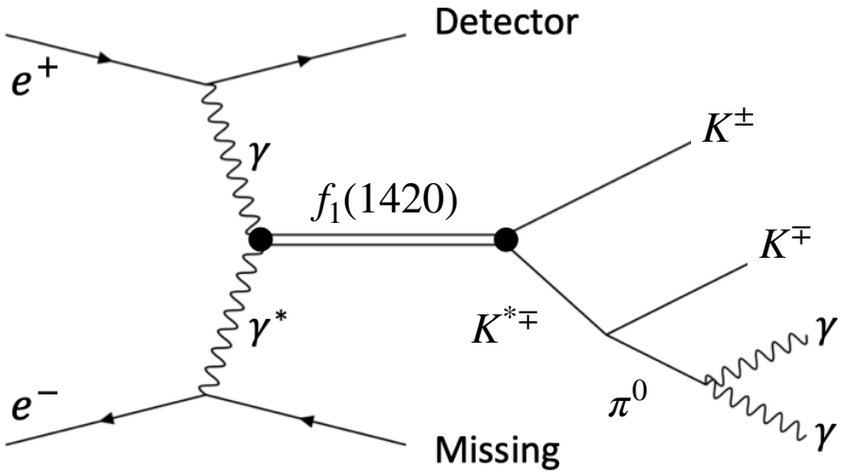
- Ideal channel to study the $f_1(1420)$ TFFs

$$\gamma^{(*)}\gamma^* \to f_1(1420) \xrightarrow{96\,\%} K\bar{K}^*(892) \to K\bar{K}\pi$$

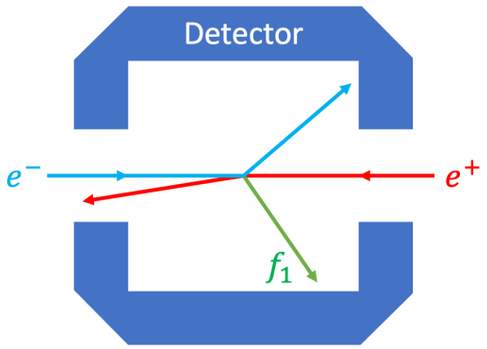
  - L3 experimental data *JHEP 03(2007) 018*

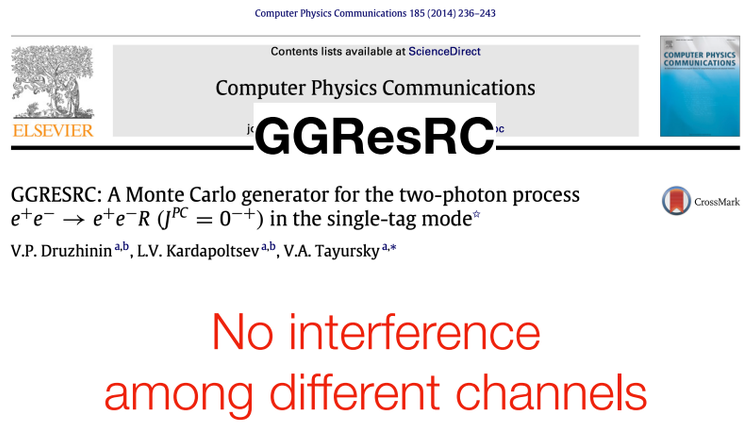
    ✓ $e^+e^- \to e^+e^-\gamma\gamma^* \to e^+e^-K_S^0K^\pm\pi^\mp$ with $Q^2 \in [0.01 - 7.0]\ \mathrm{GeV}^2$

  - Ongoing analysis of BESIII measurement *Master Thesis by N. Effenberger @ Mainz*



✓ Single-tag technique ✓ Monte Carlo generator




Computer Physics Communications 185 (2014) 236–243

Contents lists available at ScienceDirect

Computer Physics Communications

GGResRC

GGRESRC: A Monte Carlo generator for the two-photon process $e^+e^- \to e^+e^-R$ ($J^{PC} = 0^{-+}$) in the single-tag mode☆

V.P. Druzhinin[a,b], L.V. Kardapoltsev[a,b], V.A. Tayursky[a,*]

No interference among different channels

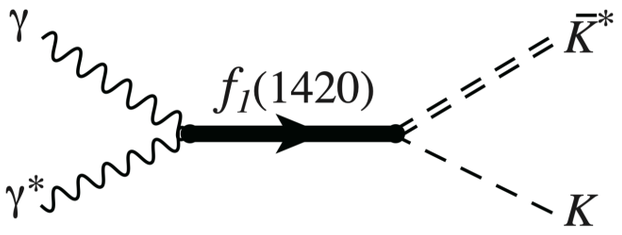
- Provide a more realistic MC generator for data analysis
  - We focus on the $\gamma\gamma^* \to K\bar{K}^*(892)$ process
  - Build up a phenomenological model in the $f_1(1420)$ region

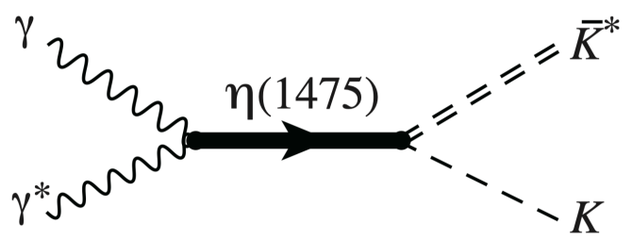
Phenomenological model for $\gamma\gamma^* \to K^{\pm}K^{*\mp}$

- Use effective Lagrangian approach to evaluate the amplitudes

- $f_1(1420)$ TFFs

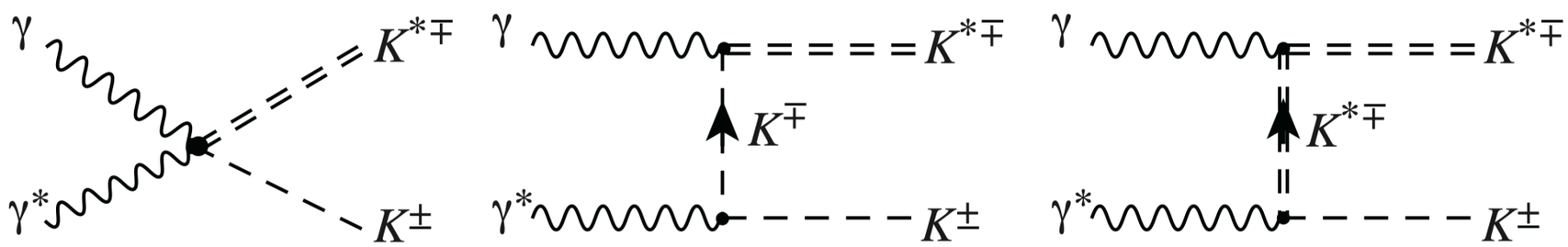
$$F^{TL}_{f_1\gamma^*\gamma^*}(0,Q_2^2) = -F^{TT}_{f_1\gamma^*\gamma^*}(0,Q_2^2) = \frac{F^{TL}_{f_1\gamma^*\gamma^*}(0,0)}{\left(1+Q_2^2/\Lambda_{f_1}^2\right)^2}$$



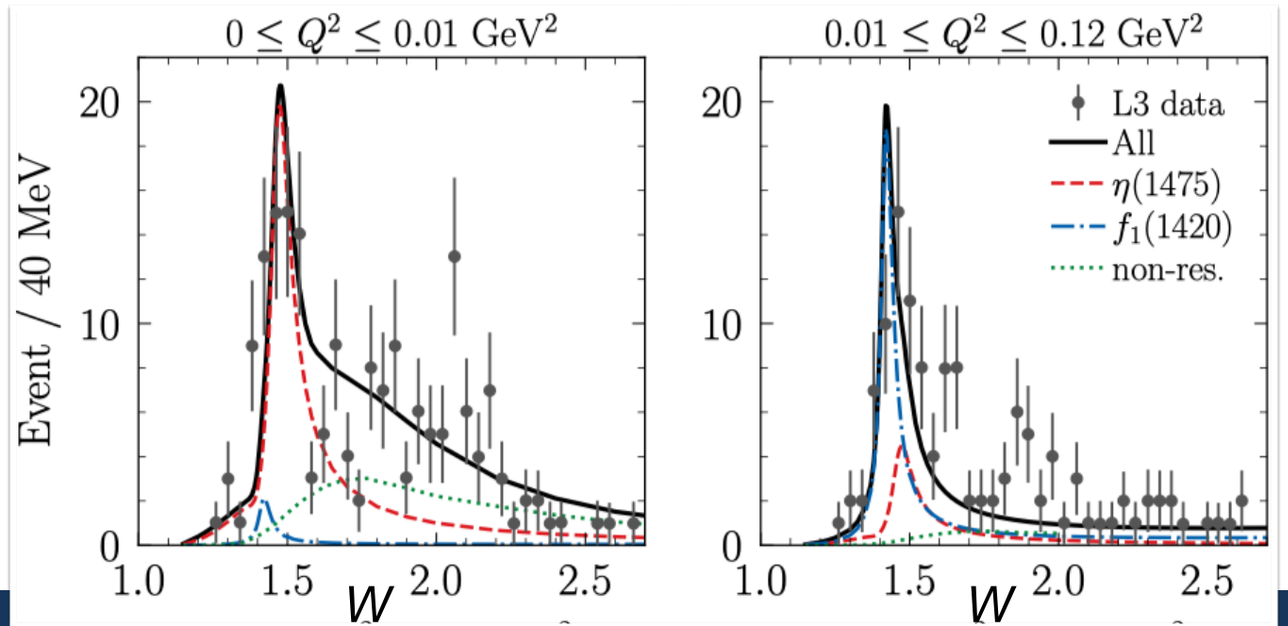


- $\eta(1405)$ TFF:

$$F_{\bar{\eta}\gamma^*\gamma^*}(0,Q_2^2) = \frac{F_{\bar{\eta}\gamma^*\gamma^*}(0,0)}{1+Q_2^2/\Lambda_{\bar{\eta}}^2}$$



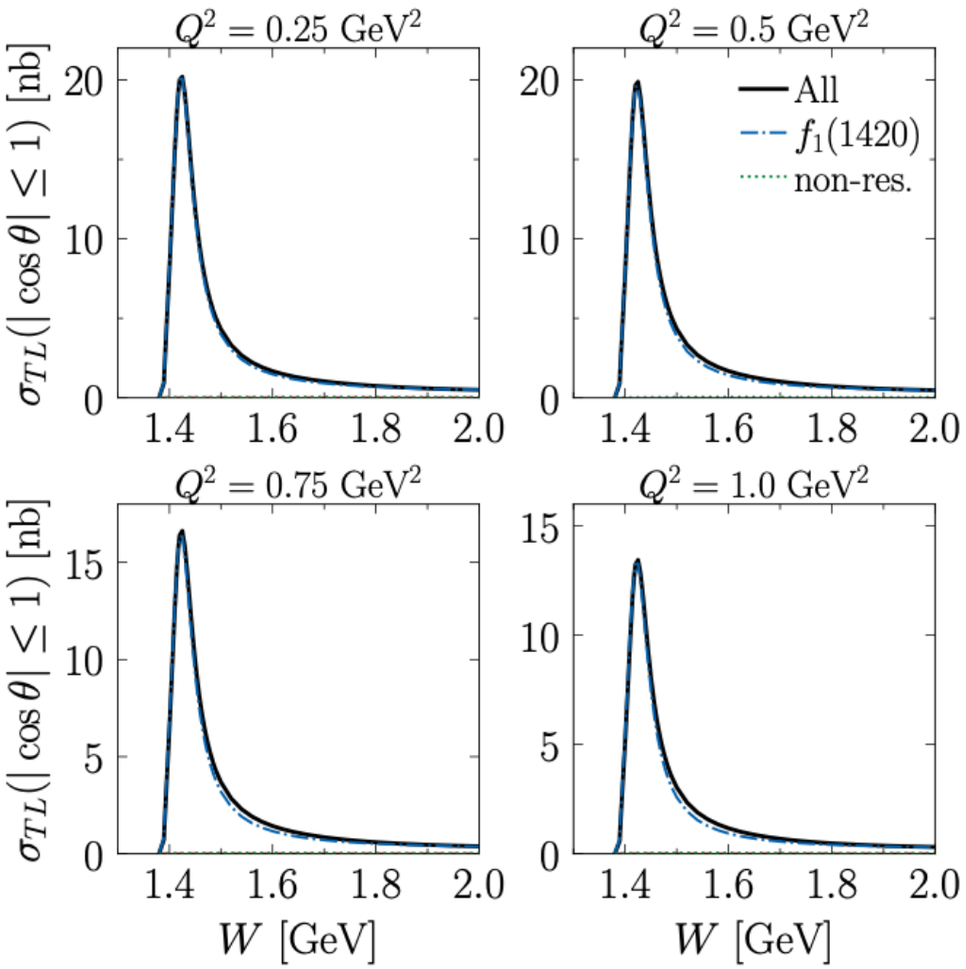
- Reasonable description of L3 events: $e^+e^- \to e^+e^- K_S^0 K^{\pm}\pi^{\mp}$
  - **Constructive interference** among $\eta(1475), f_1(1420),$ and non-res. channels



Theoretical results of the $e^+e^- \to e^+e^- K_S^0 K^{\pm}\pi^{\mp}$ process is sensitive to the f1(1420) TFF

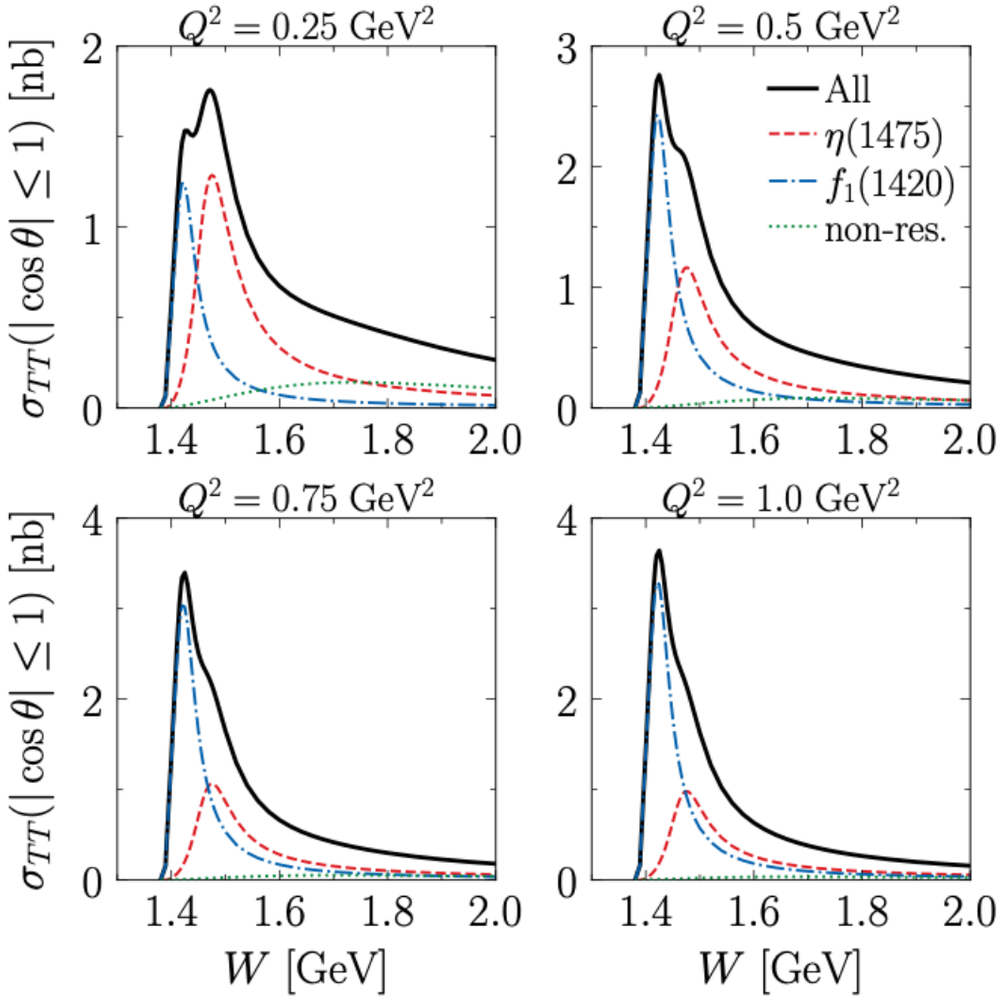
Prediction of cross section $\gamma\gamma^* \to K^{\pm}K^{*\mp}$

Transverse-longitude cross section



- No η(1475) contribution to σ_{TL}
- $f_1(1420)$ chanel dominant
- **Model-independently extract $f_1(1420)$ TFFs**

Transverse cross section



- **Broad peak ~1.45 GeV with Q^2=0.25**
  - Interference of η(1475) and f1(1420)
  - Consistent with a preliminary analysis of BESIII *N. Effenberger Master Thesis*
- Large Q^2 , $f_1(1420)$ channel dominant

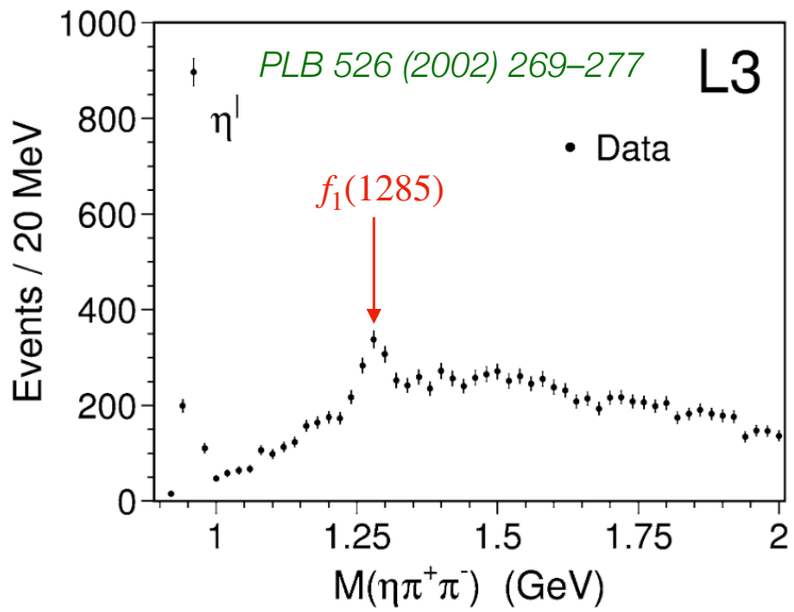
$\gamma\gamma^* \to \eta\pi\pi$ process

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)
decay	$\pi\pi\pi\pi$ (33%)	$\eta\pi\pi$ (4%)	$KK\pi$	$\eta\pi$ (14%)
modes	$K\bar{K}\pi$ (9%)			$\omega\pi\pi$ (11%)
	$\gamma\rho^0$ (6%)			$K\bar{K}$ (5%)

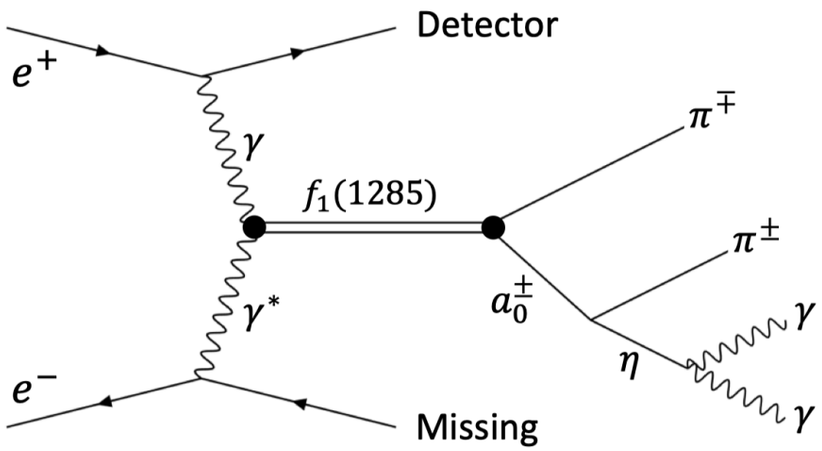
XLR, I. Danilkin, M. Vanderhaeghen, PRD 110 (2024) 094043

$f_1(1285)$ production in $\gamma\gamma^* \to \eta\pi\pi$

- **Clean channel** to study the $f_1(1285)$ TFFs
  - $\eta'(958)$ and $f_1(1285)$ **well separated**
  - $f_1(1285)$ peak vanish (Q^2=0), exclude $\eta(1295)$
- **Existing data:** MarkII, TPC/Two-Gamma, L3
  - Low-statistic / large background
  - No analysis of different helicity states of $f_1(1285)$



- **BESIII on-going analysis** @ Mainz group

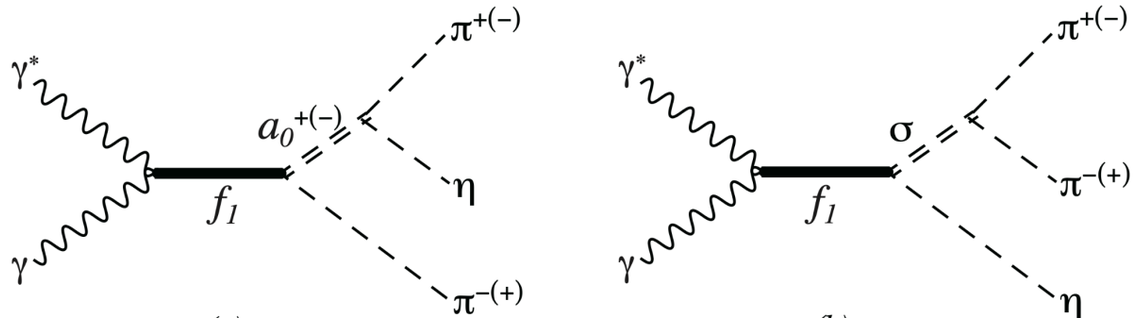


- Feasibility study M.Sc. thesis of D. Becker
- Ph.D. project (2023-) J. Muskalla

- Data analysis needs the Monte Carlo generator, which includes interference among different channels

Phenomenological model for $\gamma\gamma^* \to \eta\pi^+\pi^-$

- Focus on the production mechanism of $f_1(1285)$

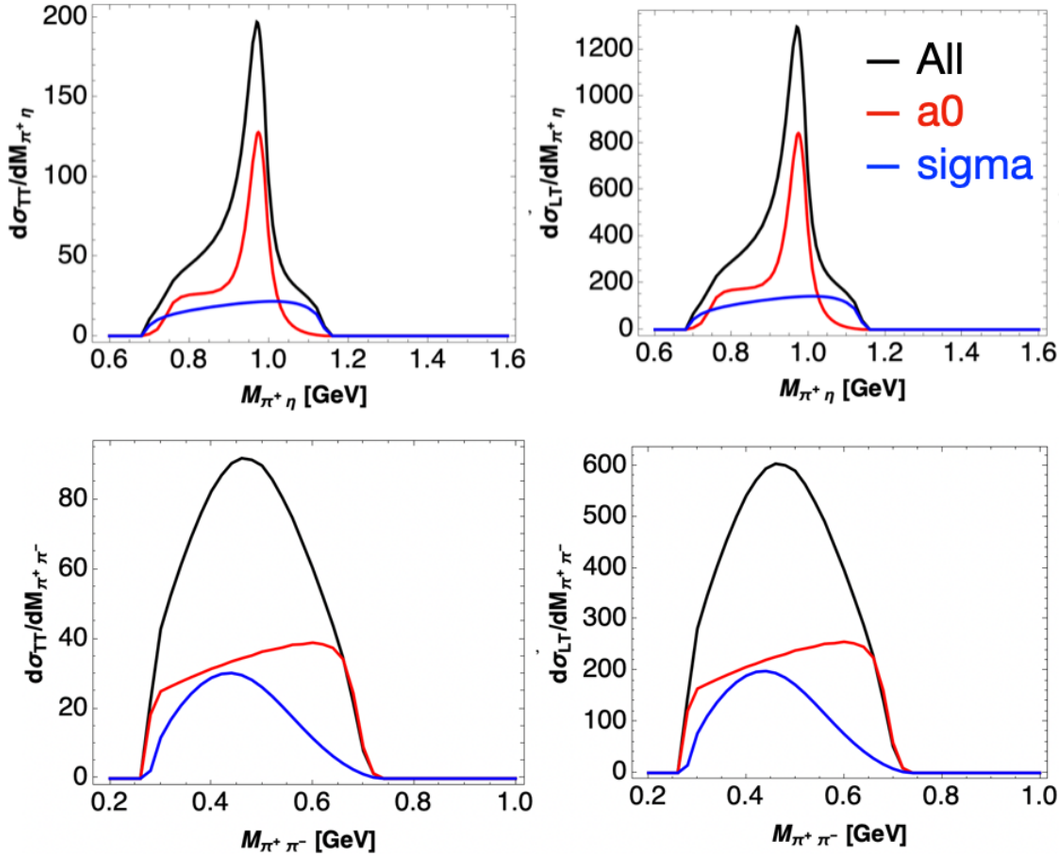


- $f_1(1285)$ TFFs

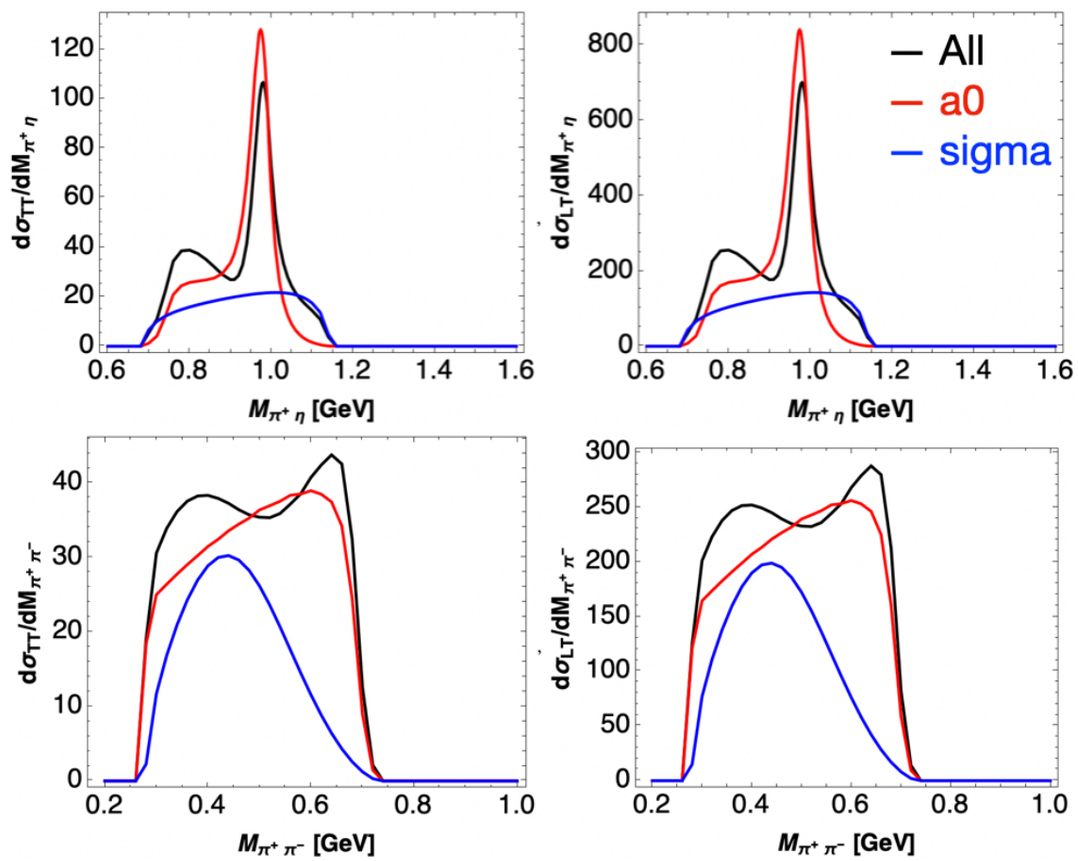
$$F^{TT}_{f_1\gamma^*\gamma^*}(Q_1^2,0) = -F^{LT}_{f_1\gamma^*\gamma^*}(Q_1^2,0) = -\frac{F^{LT}_{f_1\gamma^*\gamma^*}(0,0)}{(1+Q_1^2/\Lambda^2_{f_1})^2}$$

- Prediction for the invariant mass distributions

Constructive interference

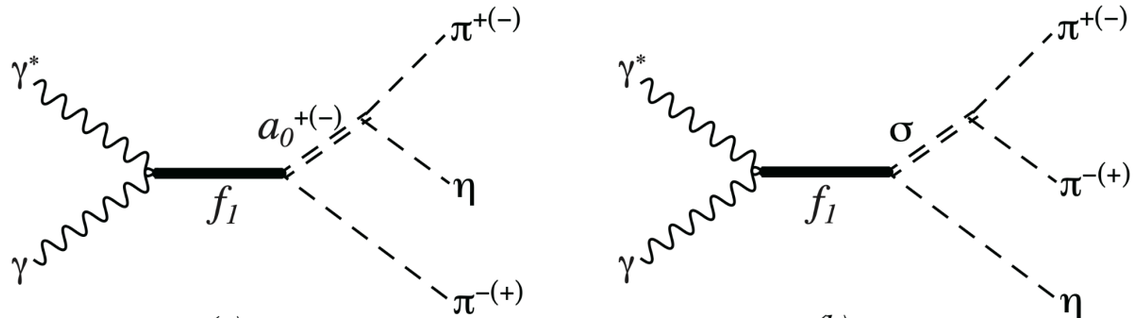


Destructive interference



Phenomenological model for $\gamma\gamma^* \to \eta\pi^+\pi^-$

- Focus on the production mechanism of $f_1(1285)$

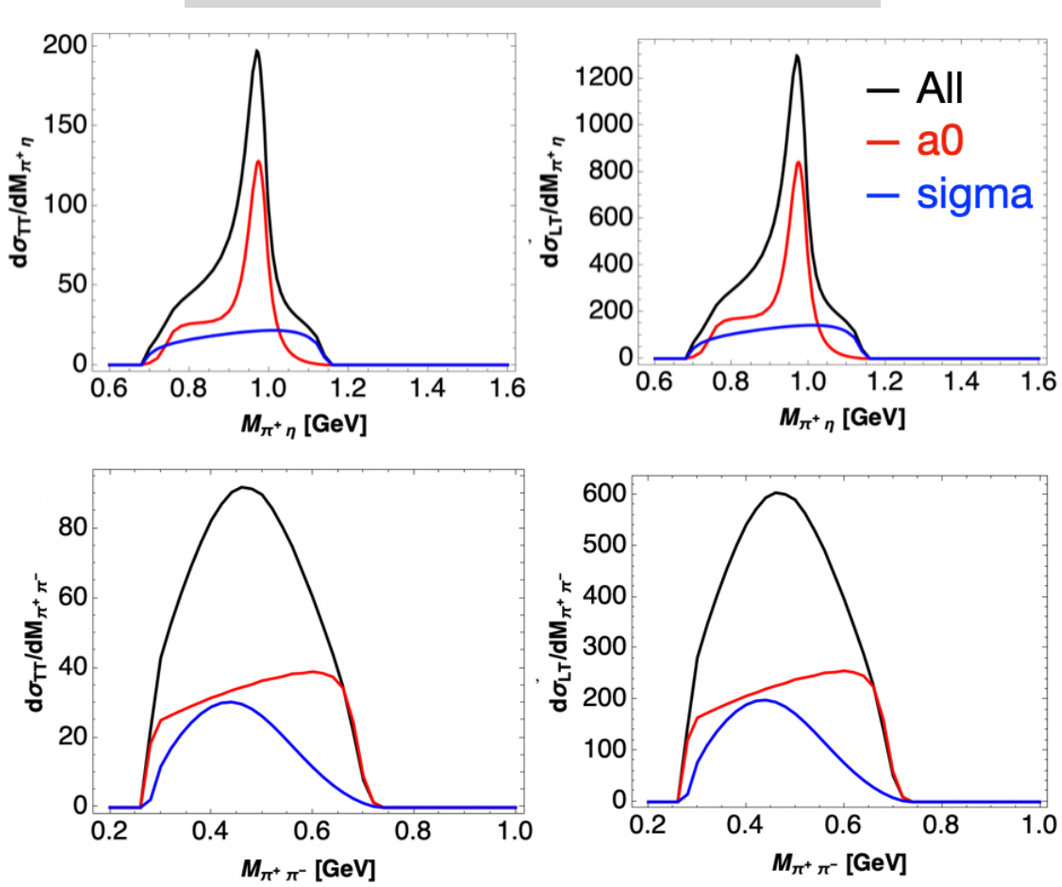


- $f_1(1285)$ TFFs

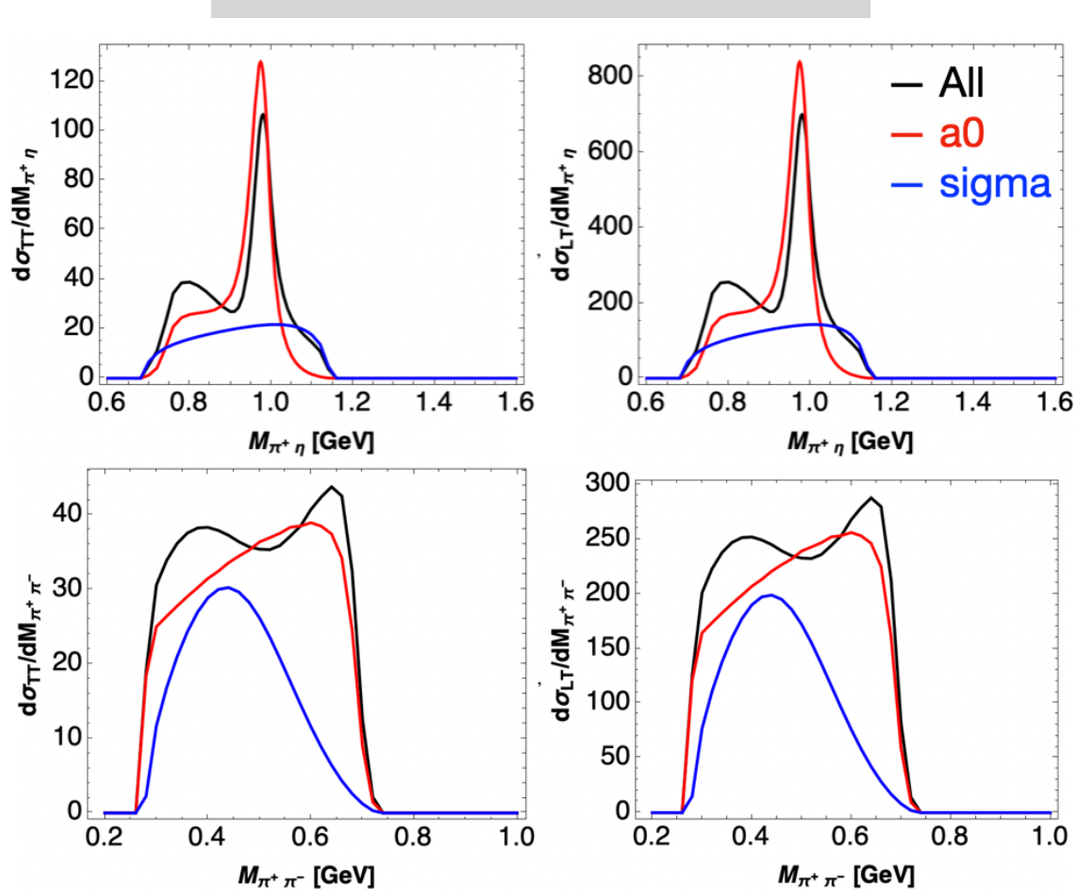
$$F^{TT}_{f_1\gamma^*\gamma^*}(Q_1^2,0) = -F^{LT}_{f_1\gamma^*\gamma^*}(Q_1^2,0) = -\frac{F^{LT}_{f_1\gamma^*\gamma^*}(0,0)}{(1+Q_1^2/\Lambda^2_{f_1})^2}$$

- Prediction for the invariant mass distributions

Constructive interference



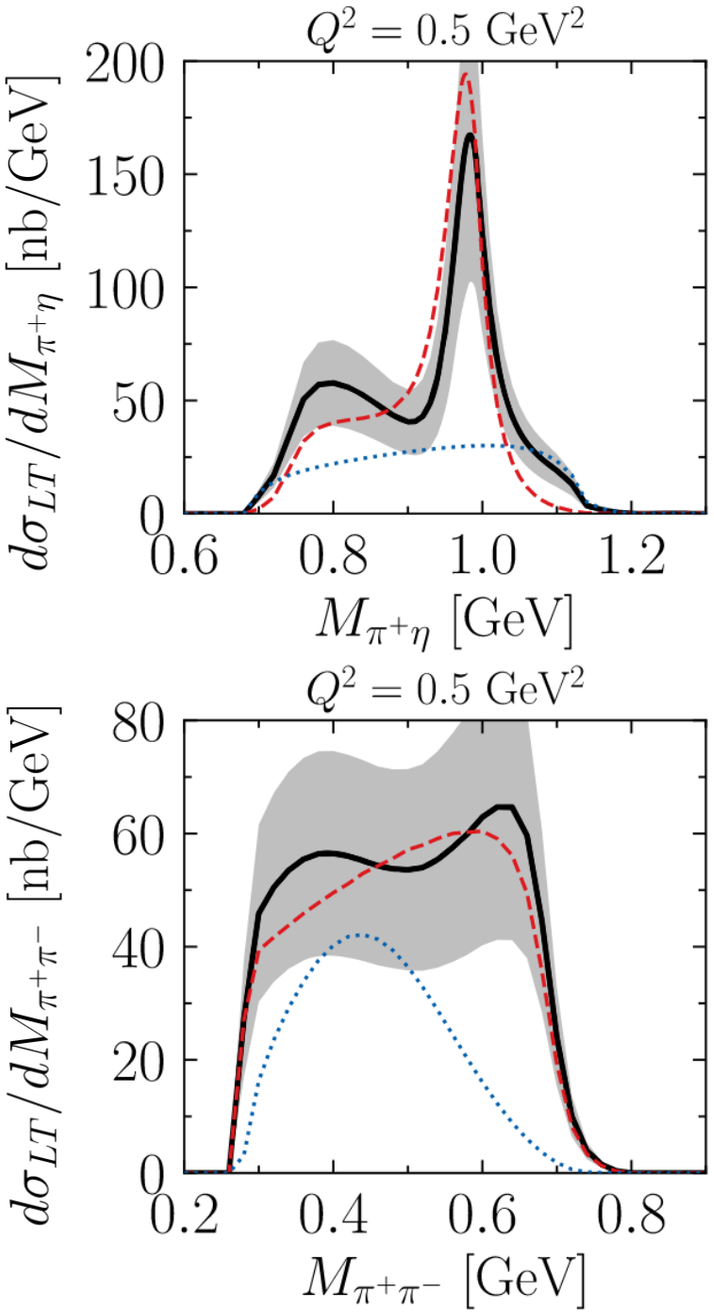
Destructive interference

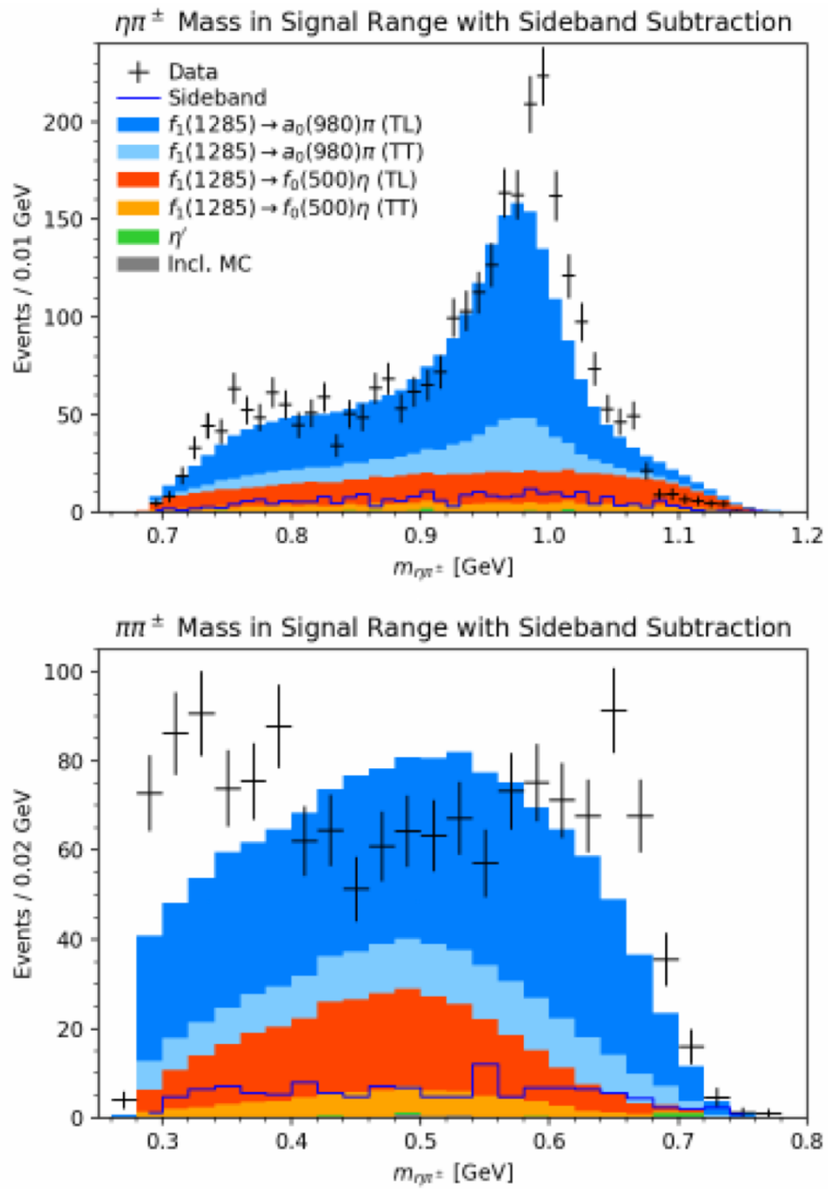


Interference: important to avoid the misinterpretation of the experimental data!

Theoretical Predictions for $\gamma\gamma^* \to \eta\pi^+\pi^-$

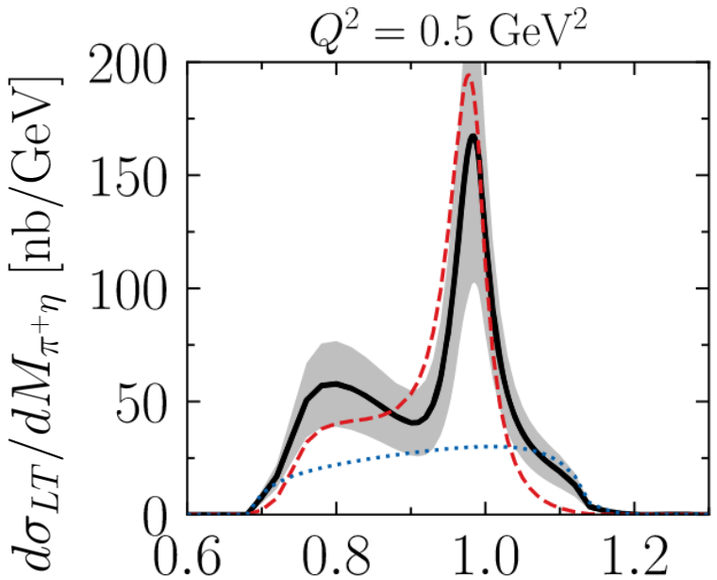
- Destructive interference: hinted by BESIII preliminary data

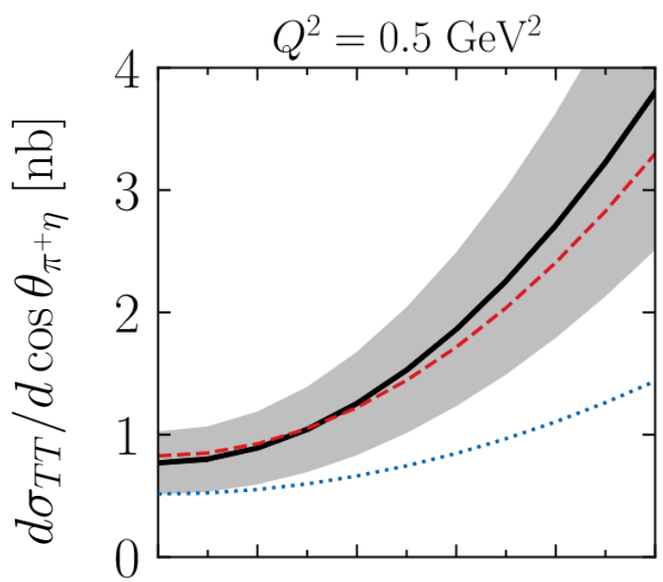


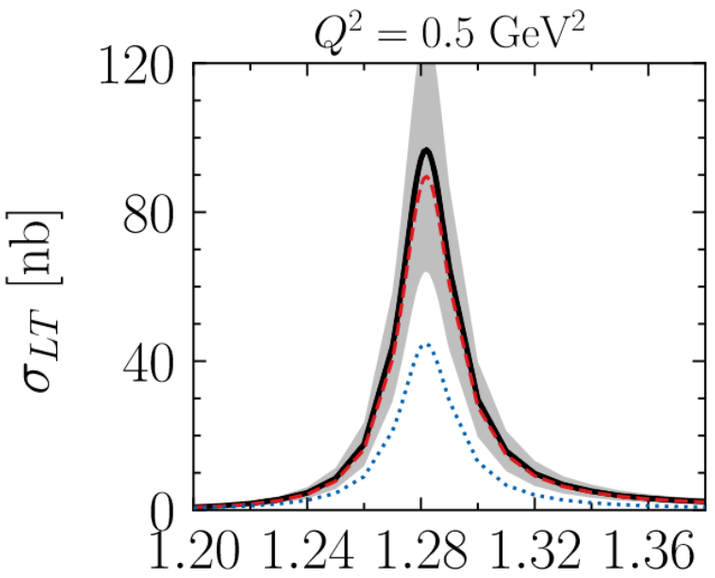


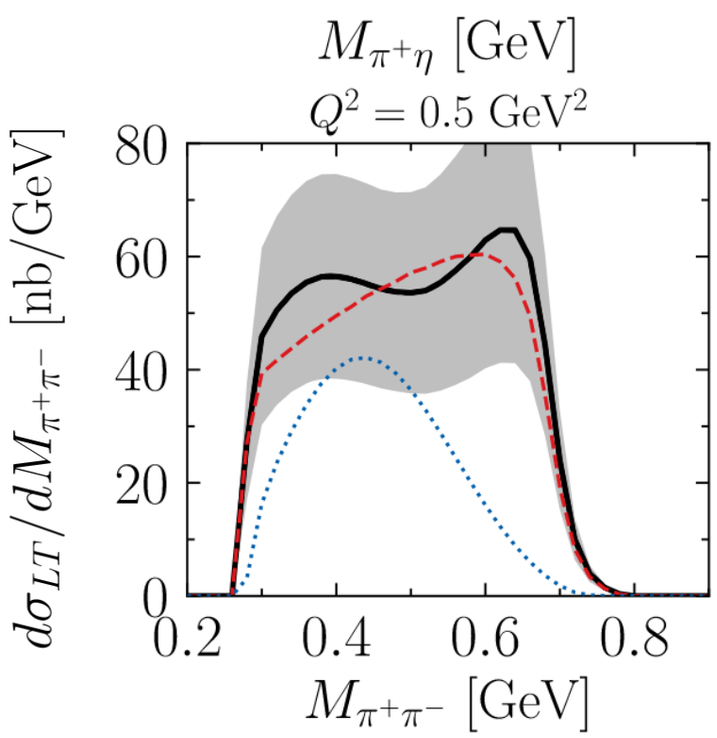
Theoretical Predictions for $\gamma\gamma^* \to \eta\pi^+\pi^-$

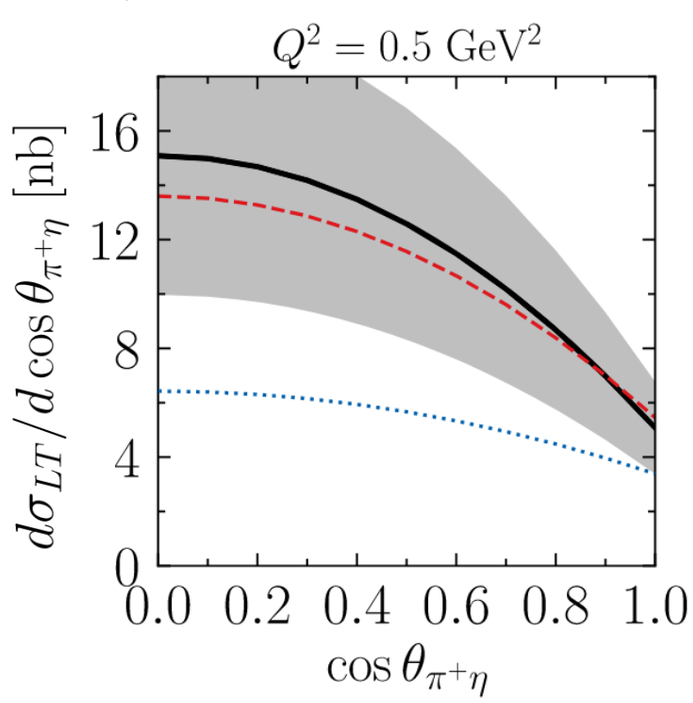
- Destructive interference: hinted by BESIII preliminary data

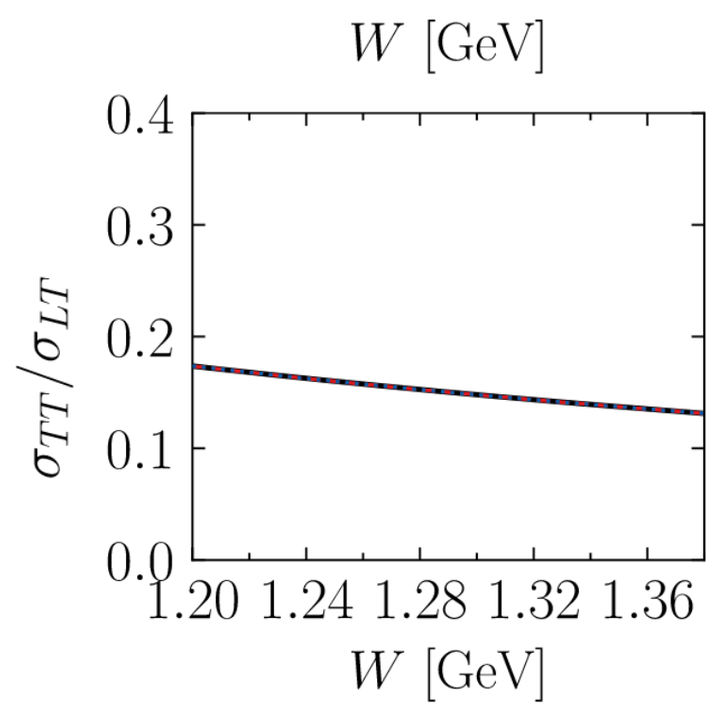








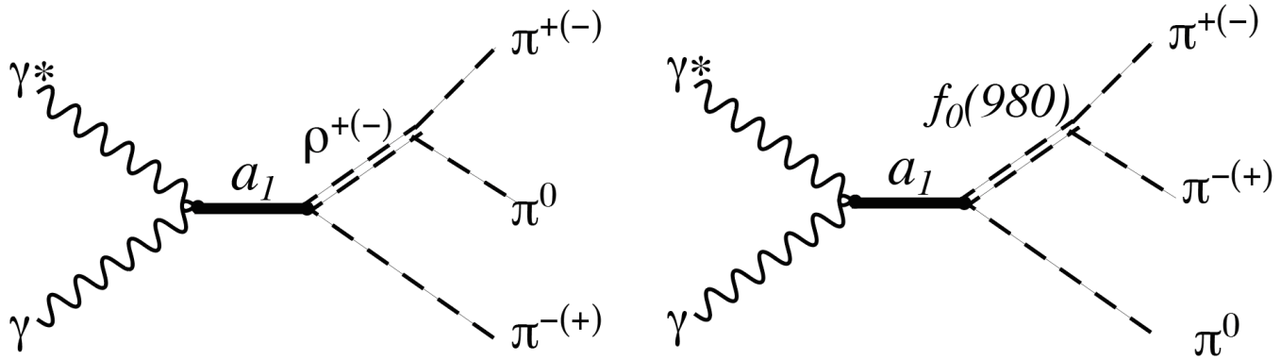




Possible extensions

- Theoretical input for **Monte Carlo Event Generator (HadroTOPS)** of hadronic two-photon scattering in electron-positron colliders
  - Included $\gamma^*\gamma^* \to f_1(1285) \to \eta\pi^+\pi^-$ — joint paper on HadroTOPS
  - Extend $\gamma\gamma^* \to K\bar{K}^*$ study to double virtual case and allow $K^* \to K\pi$
  - Extend $\gamma\gamma \to \pi^+\pi^-\pi^0$ study to single/double virtual case and include $a_1(1260)$ contribution

FP1

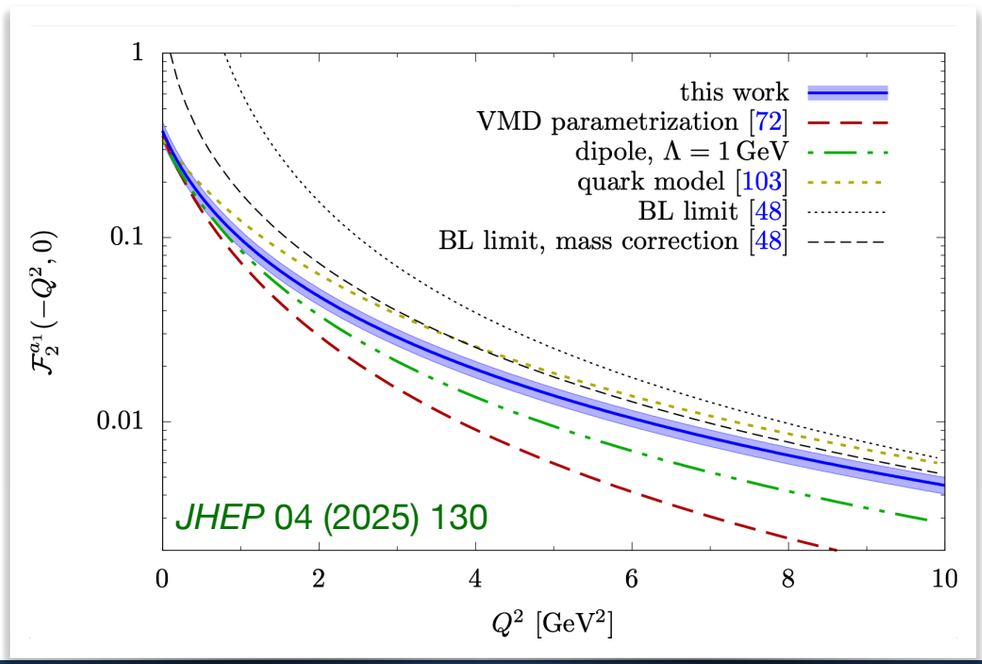


✓ $a_1(1260)$ decay process has been studied in PRD104 (2021) 036008 within the effective Lagrangian approach

✓ Singly-virtual $a_1(1260)$ TFFs: (dipole form)

$$F^{TL}_{a_1\gamma^*\gamma^*}\left(0,Q_2^2\right) = -F^{TT}_{a_1\gamma^*\gamma^*}\left(0,Q_2^2\right) = \frac{F^{TL}_{a_1\gamma^*\gamma^*}(0,0)}{\left(1+Q_2^2/\Lambda^2_{a_1}\right)^2}$$

- asymptotic behavior $\sim 1/Q_2^4$



Possible extensions

- Theoretical input for **Monte Carlo Event Generator (HadroTOPS)** of hadronic two-photon scattering in electron-positron colliders
  - Included $\gamma^*\gamma^* \to f_1(1285) \to \eta\pi^+\pi^-$ — joint paper on HadroTOPS
  - Extend $\gamma\gamma^* \to K\bar{K}^*$ study to double virtual case and allow $K^* \to K\pi$
  - Extend $\gamma\gamma \to \pi^+\pi^-\pi^0$ study to single/double virtual case and include $a_1(1260)$ contribution

) **FP1**

- Theoretical development — **FP2**
  - BESIII data on $\gamma^{(*)}\gamma^* \to \eta\pi^+\pi^-$ will become available in the near future
  - Upgrade the existing phenomenological model of $\gamma^{(*)}\gamma^* \to f_1(1285) \to \eta\pi^+\pi^-$ to a dispersion-relation-based analysis

Summary and outlook

- Phenomenological study of photon-photon fusion to three mesons
  - Focus on the lowest-lying axial vectors and tensors

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)

- Possible extensions
  - Theoretical input for Monte Carlo Event Generator of hadronic two-photon scattering in electron-positron colliders
  - Upgrade the existing phenomenological model to a dispersion-relation-based analysis

Summary and outlook

- Phenomenological study of photon-photon fusion to three mesons
  - Focus on the lowest-lying axial vectors and tensors

	$f_1(1285)$	$f_1(1420)$	$a_1(1260)$	$a_2(1320)$
$I^G(J^{PC})$	$0^+(1^{++})$	$0^+(1^{++})$	$1^-(1^{++})$	$1^-(2^{++})$
	$\eta\pi\pi$ (52%)	$K\bar{K}\pi$ (96%)	$\pi\pi\pi$	$\pi\pi\pi$ (70%)

- Possible extensions
  - Theoretical input for Monte Carlo Event Generator of hadronic two-photon scattering in electron-positron colliders
  - Upgrade the existing phenomenological model to a dispersion-relation-based analysis

Thank you for your attention!