Research Unit Report
Exotic meson spectroscopy (Project XYZ)

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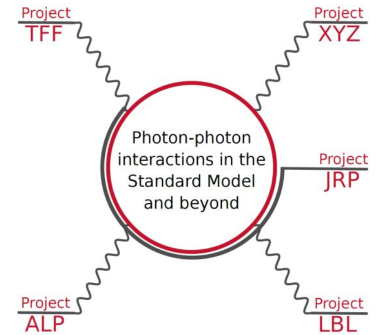
11 – 13 June St. Goar, Workshop of Research Unit FOR5327

In collaboration with I. Danilkin, M. Vanderhaeghen, Y. Guo, T. Liu

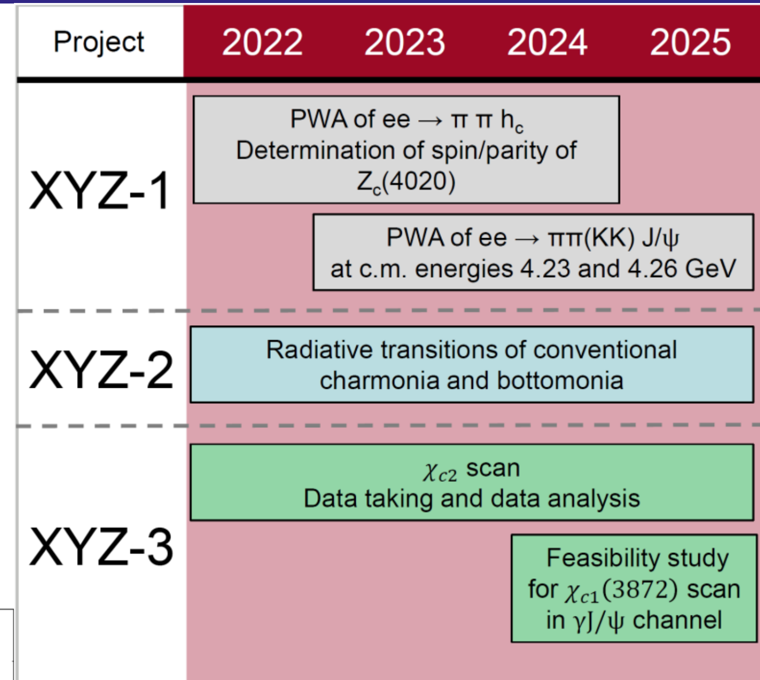


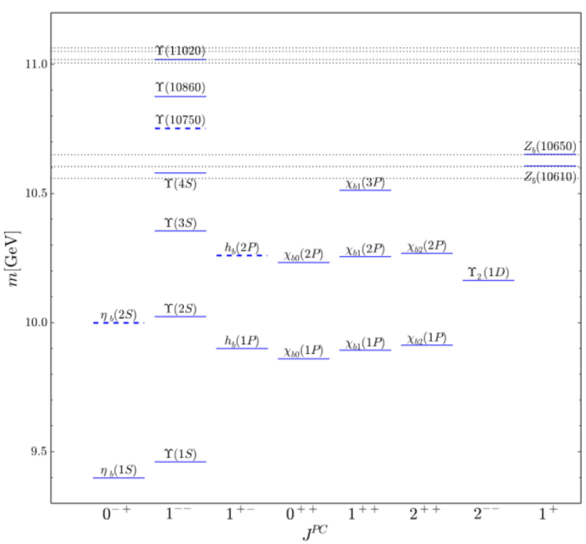
Dispersive analysis

Timeline

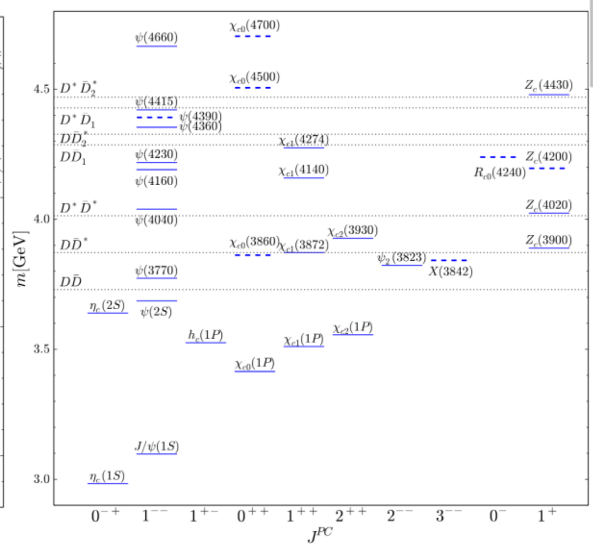
Directions:

- Partial-wave analysis (PWA) of the full BESIII $e^+e^- \to \pi\pi h_c$ data using dispersive techniques and determination of the spin and parity of the $Z_c(4020)$ *(XYZ–1)*
- PWA of the full BESIII data samples of the $e^+e^- \to \pi\pi(K\bar{K})J/\psi$ at cms energies of 4.23 and 4.26 GeV *(XYZ–1)*
- Radiative transitions of vector charmonia and bottomonia using light-by-light (LBL) sum rules *(XYZ–2)*

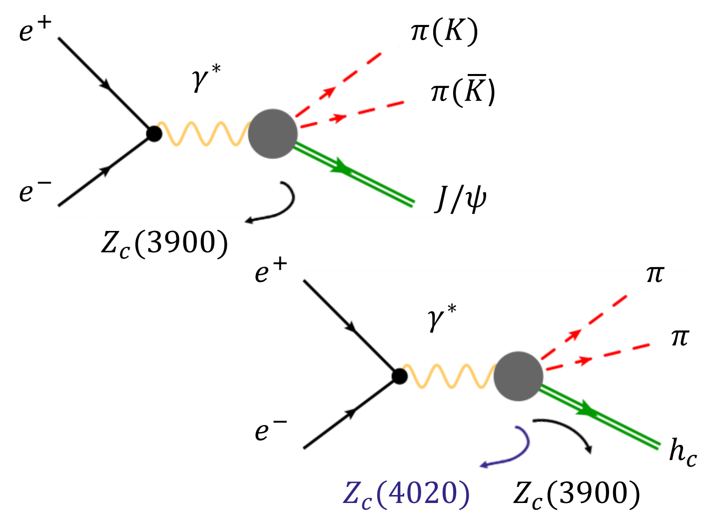




The spectrum of states in the $b\bar{b}$ sector

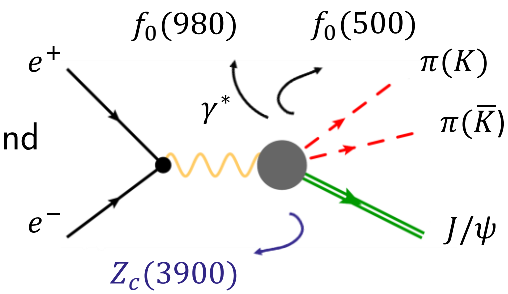


Similar in the $c\bar{c}$ sector Brambilla:2019esw



XYZ – 1: PWA of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

Motivation



- Perform a simultaneous PWA of the existing BESIII data on $e^+e^- \to \pi\pi J/\psi$ and $e^+e^- \to K\bar{K}J/\psi$, which is not included in BESIII:2017bua
- $f_0(500)$ and $f_0(980)$ imply dispersive treatment
- Analyze non-integrated acceptancy corrected data
- The ultimate goal – to constrain more precisely the mass and the width of $Z_c(3900)$

Formalism

We build upon the Dalitz-plot decomposition (DPD) of JPAC:2019ufm

❶ Straightforward to consider any quantum number (QN)

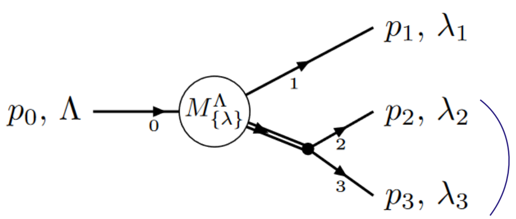
❷ Incorporatable dispersive treatment

❸ Built-in access to angular dependencies

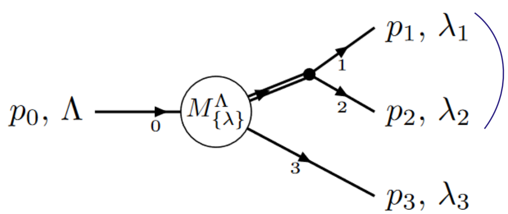
The amplitude for a 3-body decay $(J,\Lambda) \to \{\lambda\}$:

$$M^{\Lambda}_{\{\lambda\}} = \sum_{\nu} \underbrace{D^{J*}_{\Lambda,\nu}(\varphi_1,\theta_1,\varphi_{23})}_{\text{Decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(\{\sigma\})}_{\text{Dalitz-plot function}}$$

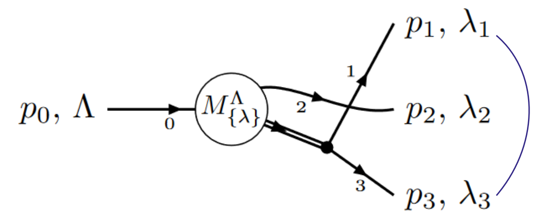
- Model-independent factorization: Wigner D-function of Euler angles $(\varphi_1,\theta_1,\varphi_{23})$ × Mandelstam variables $\{\sigma\}$ function
- Rotation connects the actual frame with the frame of calculation



Decay chain (23)1



Decay – product of subsequent 2-body decays



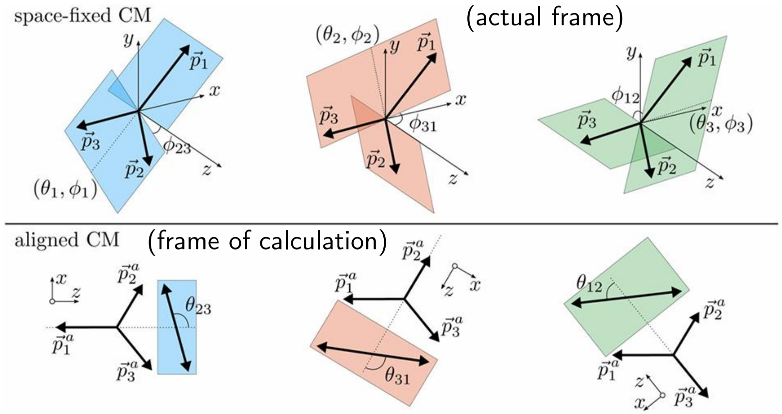
Decay chain (31)2

XYZ – 1: PWA of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

Formalism

$$O^{\nu}_{\{\lambda\}}(\{\sigma\}) = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s\, d^J_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)})$$
$$\times H^{0\to(ij),k}_{\tau,\lambda'_k} X_s(\sigma_k) d^s_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij}) H^{(ij)\to i,j}_{\lambda'_i,\lambda'_j}$$
$$\times d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^1_{k(0)}) d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^2_{k(0)}) d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^3_{k(0)})$$

normalization; connects 3 chains; 2-body helicity coupling; 2-body helicity coupling; boosts connect helicities of final particles



Decay chain (23)1 Decay chain (31)2 Decay chain (12)3

$O^{\nu}_{\{\lambda\}}(\{\sigma\})$ – product of individual 2-body decays $0 \to (ij)k$ and $(ij) \to i,j$ in a rest frame of a decaying particle

$X_s(\sigma_k)$ – energy dependence of resonance (ij) with spin s and helicity τ

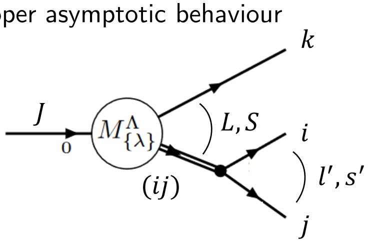
Helicity couplings

$$H^{0\to(ij),k}_{\tau,\lambda'_k} = \sum_{LS} \alpha^{0\to(ij),k}_{LS} \sqrt{\frac{2L+1}{2J+1}} \langle s,\tau; j_k, -\lambda'_k | S, \tau-\lambda'_k\rangle \langle L,0;S,\tau-\lambda'_k|J,\tau-\lambda'_k\rangle p^L B_L$$

$$H^{(ij)\to i,j}_{\lambda'_i,\lambda'_j} = \sum_{l's'} \alpha^{(ij)\to i,j}_{l's'} \sqrt{\frac{2l'+1}{2s+1}} \langle j_i,\lambda'_i; j_j, -\lambda'_j | s', \lambda'_i-\lambda'_j\rangle \langle l',0;s',\lambda'_i-\lambda'_j|s,\lambda'_i-\lambda'_j\rangle p'^{l'} B_{l'}$$

LS-coupling

Blatt-Weisskopf factors to ensure proper asymptotic behaviour



cms momentum of a final particle

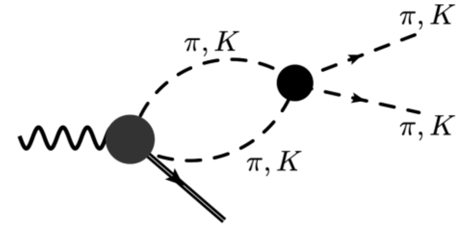
$\alpha^{0\to(ij),k}_{LS}$ and $\alpha^{(ij)\to i,j}_{l's'}$ – fit parameters

Functions $X_s(\sigma_k)$ are the only model-dependent elements

- Breit-Wigner parametrization for narrow resonances $(Z_c(3900), Z_c(4020), \ldots)$
- K-matrix parametrization (for overlapping resonances)
- Generalization: Flatte formula $(Z_c(3900), Z_c(4020), f_0(980))$
- Dispersive treatment $(f_0(500), f_0(980), f_2(1270), \ldots)$

XYZ – 1: PWA of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

Formalism: Dispersive approach



❶ Incorporates final-state interaction $\pi\pi/K\bar{K}$ ❷ Accurately reflects phase shifts

❸ Doesn't violate unitarity (unlike a combination of Breit-Wigner for $f_0(500)$ and Flatte parametrization for $f_0(980)$)

❹ Use standard Muskhelishvili-Omnes formalism: contribution from crossed-channel rescattering (corresponds to the left-hand cuts) can be absorbed just in the subtraction polynomial – minimum fit parameters Danilkin:2020kce

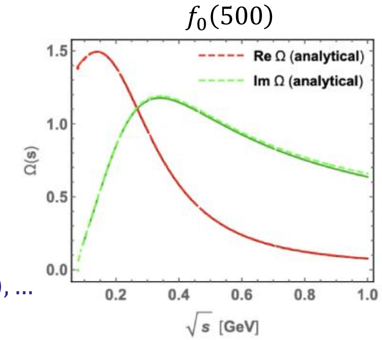
Single-channel case

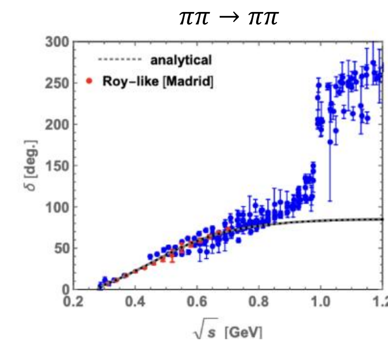
$X(\sigma) \to (a + b\sigma)\Omega(\sigma)$ with Omnes function

$$\Omega(\sigma) = \exp\left(\frac{\sigma}{\pi}\int_{\sigma_{th}}^{\infty}\frac{d\sigma'}{\sigma'}\frac{\delta(\sigma')}{\sigma'-\sigma}\right)$$

phase shift

e.g. $f_0(500), f_2(1270), \ldots$





Phase shift can be taken from different dispersive analyses

Danilkin:2022cnj

Coupled-channel case

$X(\sigma) \to (a + b\sigma)\Omega_{\pi\pi\to\pi\pi}(\sigma) + (c + d\sigma)\Omega_{\pi\pi\to K\bar{K}}(\sigma)$

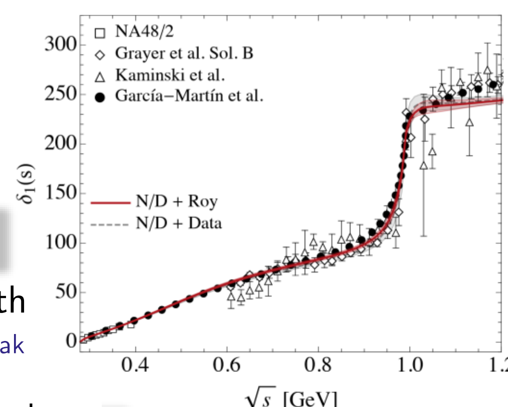
$X(\sigma) \to (a + b\sigma)\Omega_{K\bar{K}\to\pi\pi}(\sigma) + (c + d\sigma)\Omega_{K\bar{K}\to K\bar{K}}(\sigma)$

with Omnes matrix Ω_{nm}

e.g. $f_0(500)\&f_0(980)$

Simultaneous description of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

the same set of subtraction constants



Based on a data-driven N/D ansatz – set of integral equations for N solved with the input from the left-hand cuts as conformal mapping Danilkin:2020pak

- Currently we focus on the S-wave contribution

a, b and c, d – fit parameters

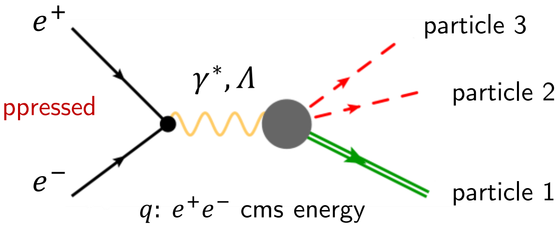
XYZ – 1: PWA of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

Formalism: Cross-section

- The formalism needs to be extended to 2 → 3 process

$$\frac{d\sigma}{d\cos\theta_1\, d\varphi_{23}\, d\sigma_1\, d\sigma_2} = \frac{e^2}{64(2\pi)^4 q^2} \sum_{\{\lambda\}} \left(\left|M^{\Lambda=+1}_{\{\lambda\}}\right|^2 + \frac{2m_e^2}{q^2} \left|M^{\Lambda=0}_{\{\lambda\}}\right|^2 \right)$$

extremely suppressed



Identical particles

- Final-state permutation symmetry is not naturally implemented in JPAC:2019ufm

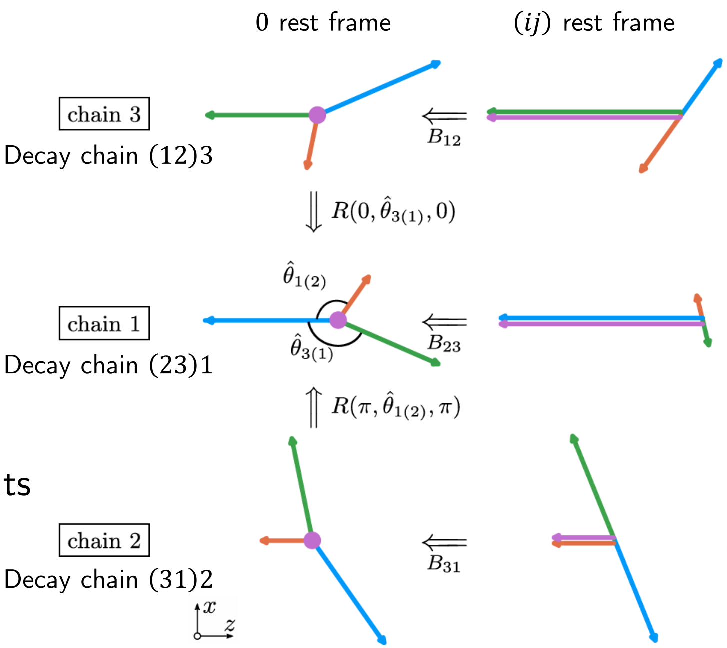
$$H^{(31)\to 3,1}_{\lambda_3',\lambda_1'}(\sigma_2) = (-1)^{s+\lambda_2'-\lambda_1'} H^{(12)\to 1,2}_{\lambda_1',\lambda_2'}(\sigma_3 \to \sigma_2) \;\forall\; \text{particle } 2 = \text{particle } 3$$

e.g. $2 = \pi^+,\ 3 = \pi^-$

Similarly with any other combination

❶ Allows to reduce the number of fit parameters – LS-couplings

Frame of calculation: permutation is important



The entire approach was verified with Lagrangian calculation for various quantum numbers and reactions: Ermolina:2024uln

Application

- Consider $Z_c(3900) = 1^+$ – Breit-Wigner parametrization
- Coupled-channel dispersive treatment of $f_0(500)\&f_0(980)$

LS ($l's'$) combinations	
$\gamma^* \to Z_c^\pm \pi^\mp$	(0,1), (2,1)
$Z_c^\pm \to J/\psi\pi^\mp$	(0,1), (2,1)
$\gamma^* \to f_0 J/\psi$	(0,1), (2,1)
$f_0 \to \pi^+\pi^-$	(0,0)

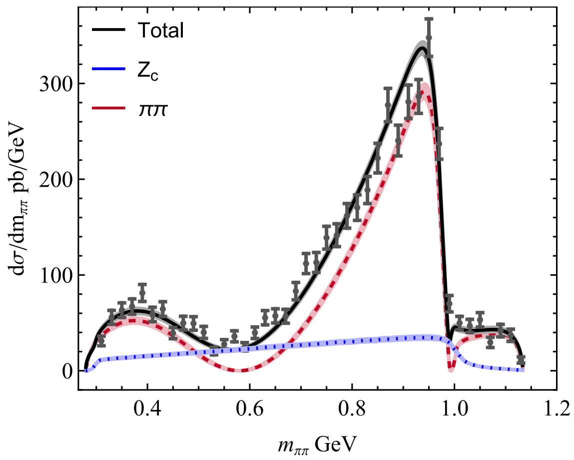
- S-wave in each vertex
- 5 fit parameters: subtraction constants a, b, c, d + 1 LS-coupling (others absorbed in normalization)

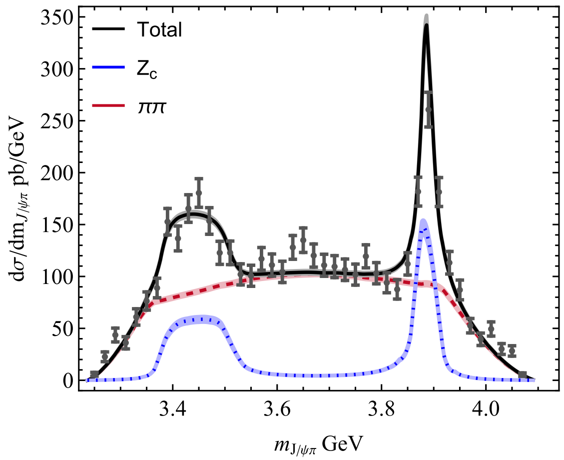
XYZ – 1: PWA of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

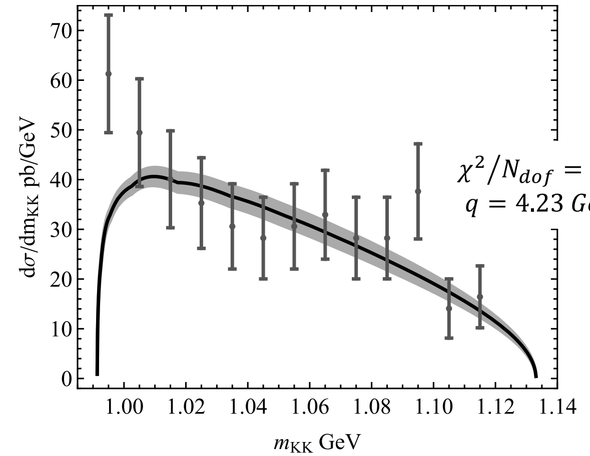
Results: Invariant mass distributions

Ermolina:2024uln

$\pi\pi$ data: BESIII:2017bua $K\bar{K}$ data: BESIII:2022joj

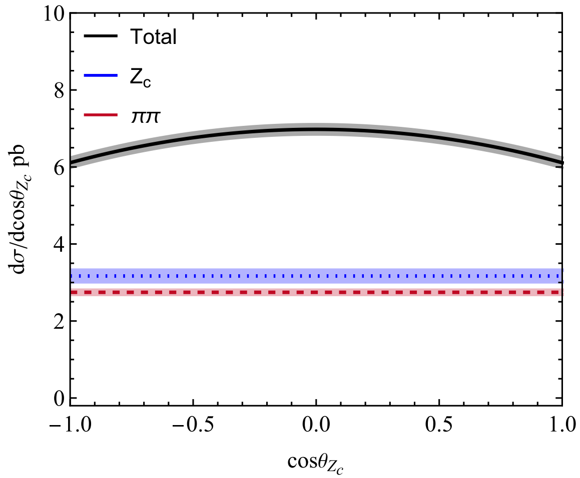


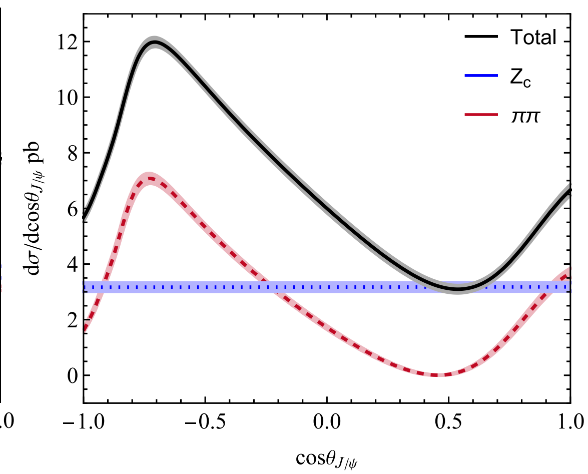




Angular distributions

- Range of $m_{J/\psi\pi} = (3.86; 3.92)\ GeV$ to separate $Z_c(3900)$ resonance





- Fixed mass and width of $Z_c(3900)$

Left: polar angle distribution of $Z_c(3900)$ in $\gamma^* \to Z_c^{\pm}\pi^{\mp}$

Right: helicity angle distribution of J/ψ in $Z_c^{\pm} \to J/\psi\pi^{\mp}$

1. Rescattering strongly affects the shape
2. $\cos\theta_{Z_c}$ – symmetric
   $\cos\theta_{J/\psi}$ – asymmetric

XYZ – 1: PWA of $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

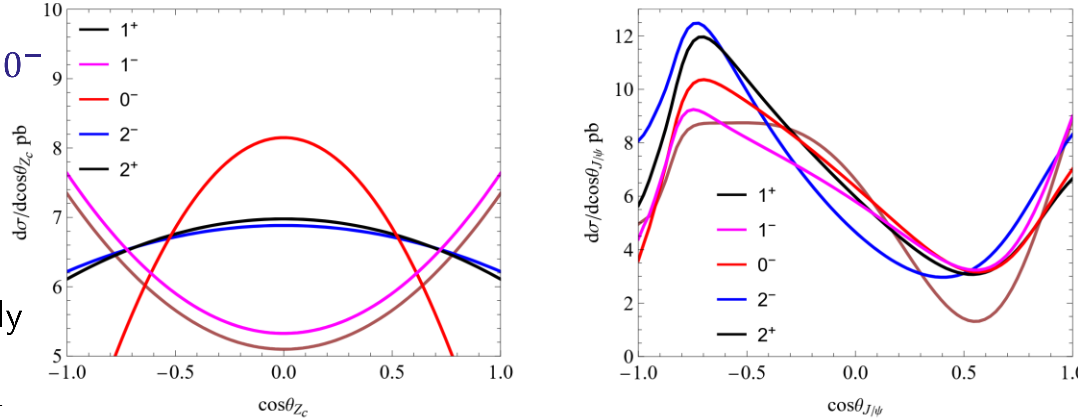
Results: Different QN

- Consider any QN of $Z_c(3900)$: $1^+, 1^-, 2^+, 2^-, 0^-$
- Coupled-channel for $f_0(500)\&f_0(980)$
- Minimal partial wave in each vertex
- Still 5 fit parameters for any QN
- Invariant mass distribution differ insignificantly

LS $(l's')$ combinations

1^-		0^-	
$\gamma^* \to Z_c^\pm\pi^\mp$	(1,1)	$\gamma^* \to Z_c^\pm\pi^\mp$	(1,0)
$Z_c^\pm \to J/\psi\pi^\mp$	(1,1)	$Z_c^\pm \to J/\psi\pi^\mp$	(1,1)
2^-		**2^+**	
$\gamma^* \to Z_c^\pm\pi^\mp$	(1,2), (3,2)	$\gamma^* \to Z_c^\pm\pi^\mp$	(2,2)
$Z_c^\pm \to J/\psi\pi^\mp$	(1,1), (3,1)	$Z_c^\pm \to J/\psi\pi^\mp$	(2,1)

Range of $m_{J/\psi\pi} = (3.86; 3.92)\ GeV$ to distinguish $Z_c(3900)$ resonance $q = 4.23\ GeV$

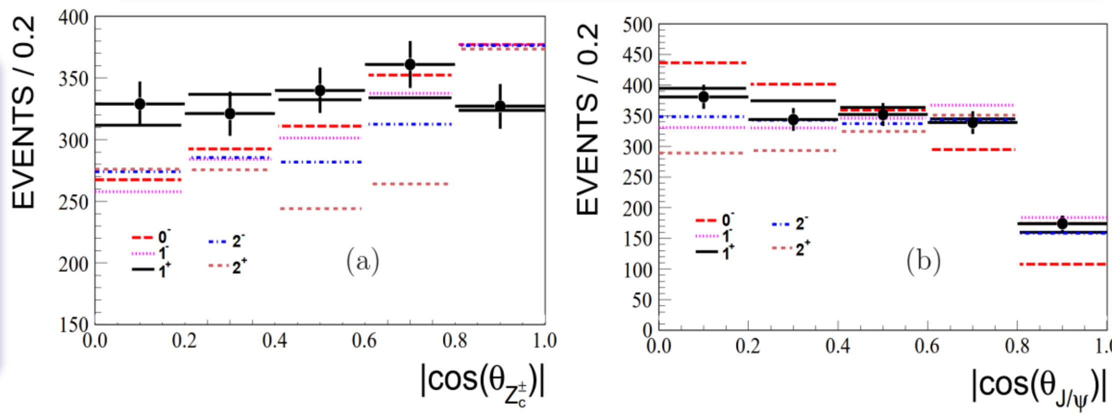


3 fixed parameters: mass and width of $Z_c(3900)$, scale parameter in Blatt-Weisskopf factor

❶ Rescattering strongly affects the shape

❷ $\cos\theta_{Z_c}$ – symmetric
$\cos\theta_{J/\psi}$ – asymmetric

Only modulus is plotted in BESIII:2017bua – no full picture



Prospects

❶ Established and validated the formalism to determine resonant QN

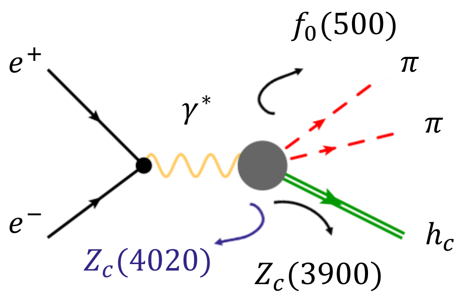
❷ Published & Ready-to-use Ermolina:2024uln

❸ Full acceptancy-corrected data is required for the constraint of $Z_c(3900)$ mass and width

XYZ – 1: PWA of $e^+e^- \to \pi\pi h_c$

Motivation

- The observation of $Z_c(4020)$ resonance at 3 data points of e^+e^- cms energies (along with $Z_c(3900)$) BESIII:2013ouc
- Take a look into the nature of the exotic states, which do not fit in the conventional charmonium predictions
- Collaborative effort with Yuping Guo and Tong Liu
- The ultimate goal – to determine spin and parity J^P of $Z_c(4020)$ from angular distributions



Formalism

❶ Tho whole formalism established above can be used to study $Z_c(4020)$ Ermolina:2024uln

LS ($l's'$) combinations

	1^+	1^-	2^-	2^+
$\gamma^* \to Z_c^\pm \pi^\mp$	(0,1), (2,1)	(1,1)	(1,2), (3,2)	(2,2)
$Z_c^\pm \to h_c \pi^\mp$	(1,1)	(0,1), (2,1)	(2,1)	(1,1), (3,1)
$\gamma^* \to f_0 h_c$	(1,1)	(1,1)	(1,1)	(1,1)
$f_0 \to \pi^+\pi^-$	(0,0)	(0,0)	(0,0)	(0,0)

❷ Single channel dispersive approach for $f_0(500)$

❸ 3 (4) fit parameters for any QN of $Z_c(4020)$
+ 1 (2) to include $Z_c(3900)$

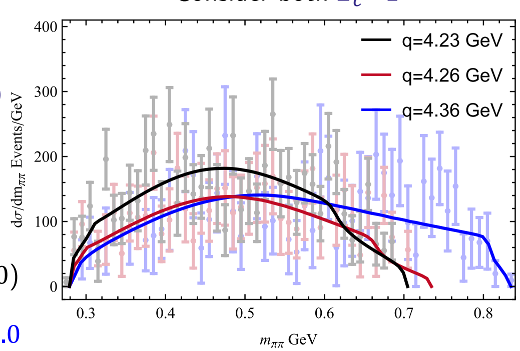
subtraction constants a, b + 1 (2) LS -couplings
(depending on a partial wave; others absorbed in normalization)

Application

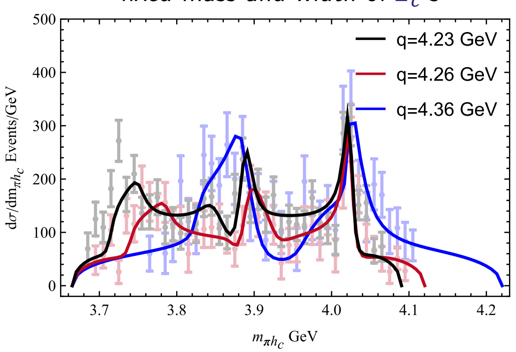
- Breit-Wigner parametrization for $Z_c(3900)$ & $Z_c(4020)$
- Minimal partial wave in each vertex
- 3 fit parameters – subtraction constant a and LS-coupling for each Z_c (for the minimal fitting put $b = 0$)

$\chi^2/N_{dof} = 1.6$ $\chi^2/N_{dof} = 0.9$ $\chi^2/N_{dof} = 1.0$

Consider both Z_c=1^+



fixed mass and width of Z_c's



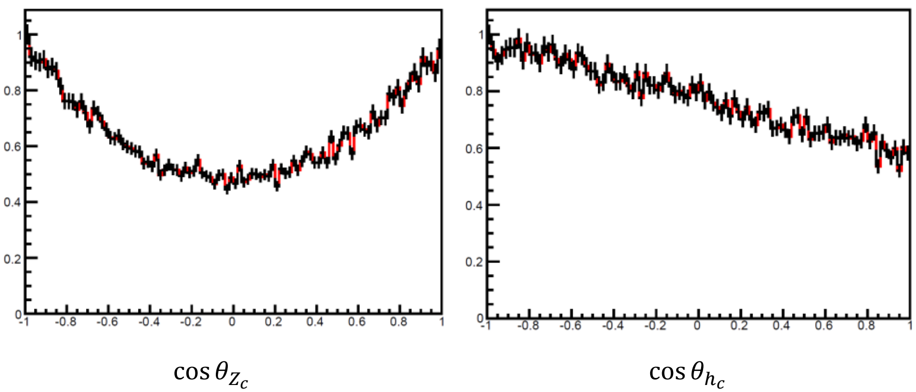
XYZ – 1: PWA of $e^+e^- \to \pi\pi h_c$

Results

$Z_c(4020) = 1^+$ is the dominating hypothesis

1. Fitting the data with different QN hypotheses of $Z_c(4020)$: **see talk of Yuping Guo**
2. 3 formalisms were tested: conventional helicity formalism, DPD and covariant tensor formalism

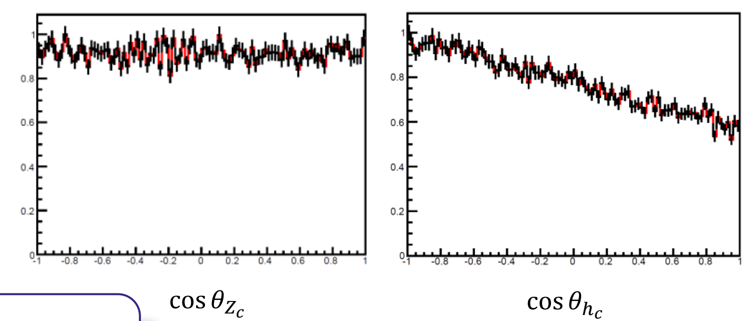
- For the 1^+ case, all 3 provide identical results
- For the 1^- case, DPD and conventional helicity formalism are identical and differ from the covariant tensor formalism
- The tensor $2^\pm$ hypotheses contain inconsistencies



Angular distributions for distinguished $Z_c(4020) = 1^+$ with a minimal partial wave, normalized to 1

further study to claim

by Tong Liu



Angular distributions for $Z_c(4020) = 1^-$ with minimal partial wave, normalized to 1

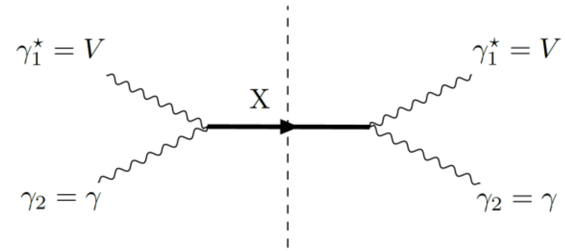
- As in earlier case, the main difference arises in the polar angle of Z_c

Prospects

DPD & dispersive

1. Finalize the fitting implementing the discussed formalism
2. Establish QN J^P, favoured by the data
3. Compare different J^P angular distributions to the measured configurations
4. Ultimately determine spin and parity of $Z_c(4020)$

XYZ – 2: LBL sum rules for quarkonia



Motivation

- Light-by-light sum rule (SR) has been tested for radiative transitions of low-lying bottomonium states
- Can be applied to charmonium states and not low-lying states
- The ultimate goal – to investigate the nature of exotic states in the quarkonia spectra

Formalism

We build upon the formalism, established in Ananyev:2020uve

Pascalutsa:2012pr

total helicity cross-sections

$$\int_{s_0}^{\infty} \frac{ds}{(s - q_1^2)} (\sigma_2 - \sigma_0)\Big|_{q_2^2 = o} = 0$$

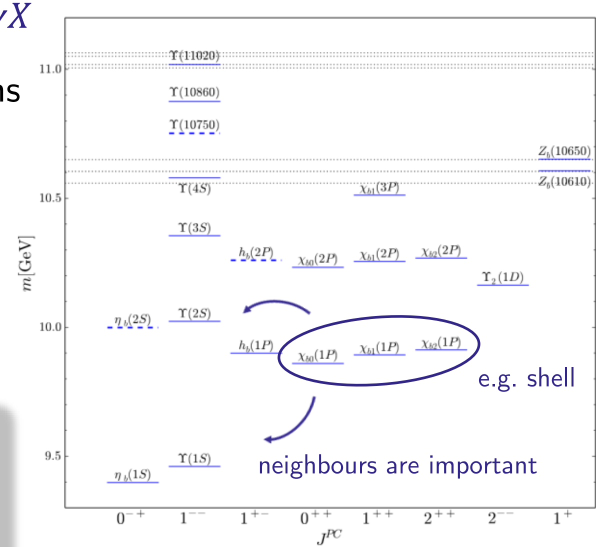
particle-production threshold   virtual photon   real photon

1. LBL sum rule: for the process $\gamma^*\gamma \to X$ (sum over all allowed final states)
2. Unitarity allows to relate $\mathcal{Im}$ part of the helicity amplitude $\gamma V \to \gamma V$ to the $X \to \gamma V$ or $V \to \gamma X$
3. Sum rule can be rewritten in terms of helicity radiative widths $\Gamma_{\Lambda=0,2}$
4. Approximately 0 for each shell in non-relativistic model

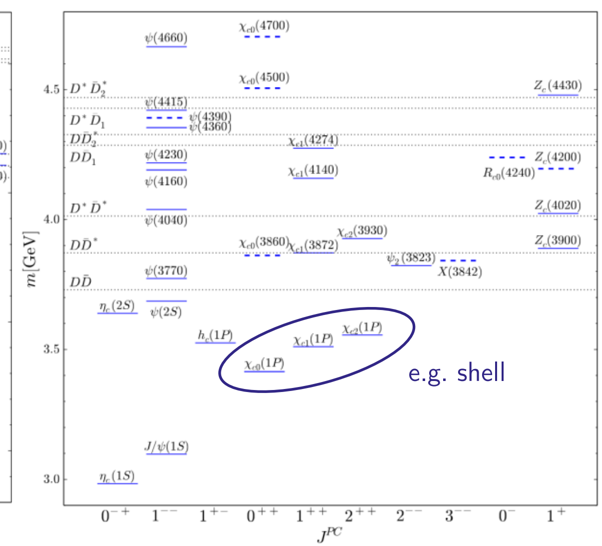
Application

- $\Gamma_\Lambda = r_\Lambda \cdot \Gamma^{EM}$ with unpolarized width $\Gamma^{EM} = \sum_\Lambda \Gamma_\Lambda$ calculated

experimentally   theoretically



The spectrum of states in the $b\bar{b}$ sector



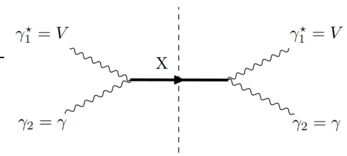
The spectrum of states in the $c\bar{c}$ sector

XYZ – 2: LBL sum rules for quarkonia

Application

- SR for the bottomonium shell Ananyev:2020uve

	m_X $[MeV]$	Γ $[keV]$	Sum rule $[\mu b]$
$\chi_{b0}(1P) \to \gamma\Upsilon(1S)$	9859	$24.2^{+5.7}_{-0.4}$	$-1.6^{+0.0}_{-0.4}$
$\chi_{b1}(1P) \to \gamma\Upsilon(1S)$	9893	$30.2^{+6.4}_{-0.7}$	$-2.3^{+0.1}_{-0.5}$
$\chi_{b2}(1P) \to \gamma\Upsilon(1S)$	9912	$36.1^{+4.1}_{-3.5}$	$4.0^{+0.5}_{-0.4}$
Subtotal:			$0.2^{+0.5}_{-1.2}$
$\Gamma\,[\Upsilon(1S) \to \gamma\eta_b(1S)] \approx 10$ eV (in quark model)			



breaking of pattern

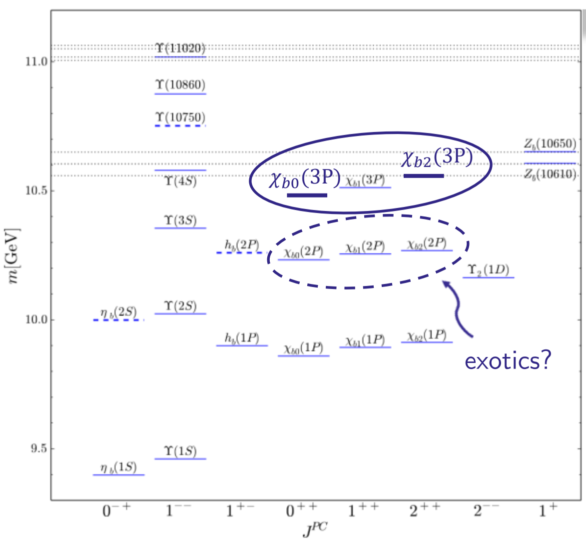
- SR for the charmonium shell (based on experimental data)

	m_X $[MeV]$	Γ $[keV]$	Sum rule $[\mu b]$
$\chi_{c0}(1P) \to \gamma J/\psi$	3414.71 ± 0.30	148.05 ± 14.72	-20.44 ± 2.03
$\chi_{c1}(1P) \to \gamma J/\psi$	3510.67 ± 0.05	288.12 ± 17.54	-28.12 ± 1.71
$\chi_{c2}(1P) \to \gamma J/\psi$	3556.17 ± 0.07	384.15 ± 22.32	46.53 ± 2.70
Subtotal:			$-2.03 \pm \mathbf{3.79}$
$J/\psi \to \gamma\eta_c(1S)$	3096.900 ± 0.006	1.31 ± 0.13	-11.08 ± 1.13
$\eta_c(2S) \to \gamma J/\psi$	3637.8 ± 0.6	208.8 ± 25.2	-6.39 ± 0.77
$\chi_{c0}(2P) \to \gamma J/\psi$	3862 ± 50 3922.1 ± 1.8	?	negative & only $\Lambda = 0$
$\chi_{c1}(2P) \to \gamma J/\psi$	3871.64 ± 0.06	11.9 ± 5.2	-0.20 ± 0.09
$\chi_{c2}(2P) \to \gamma J/\psi$	3922.5 ± 1.0	?	positive

2 experimentally found states

X(3872) measured exotic

In quark model:
- SR $[\chi_{c0}(2P) \to \gamma J/\psi] \approx -0.03$
- SR $[\chi_{c0}(2P) \to \gamma J/\psi] \approx [2; 5]$ range

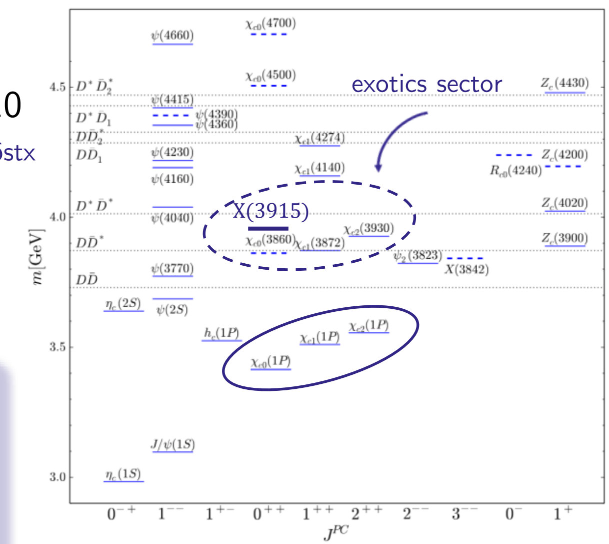


The spectrum of states in the $b\bar{b}$ sector

- X(3872) unpolarized width is up to 10 times greater in quark model Deng:2016stx
- Account for relativistic effects

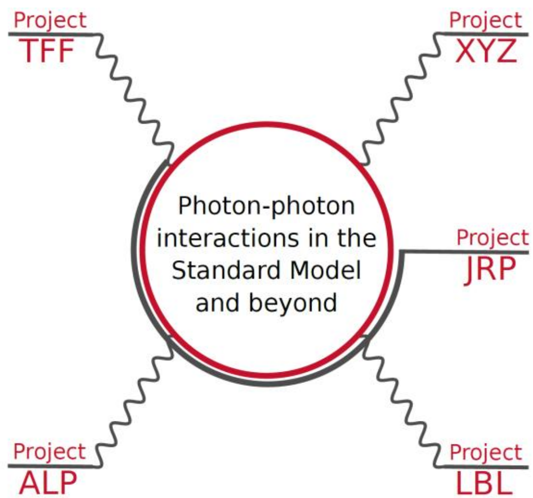
Possible directions

1. A way to constrain exotics in the $c\bar{c}$ sector
2. A way to analogously probe exotics & high-lying $b\bar{b}$ sector



The spectrum of states in the $c\bar{c}$ sector

Thank you for your attention



Dispersive analysis