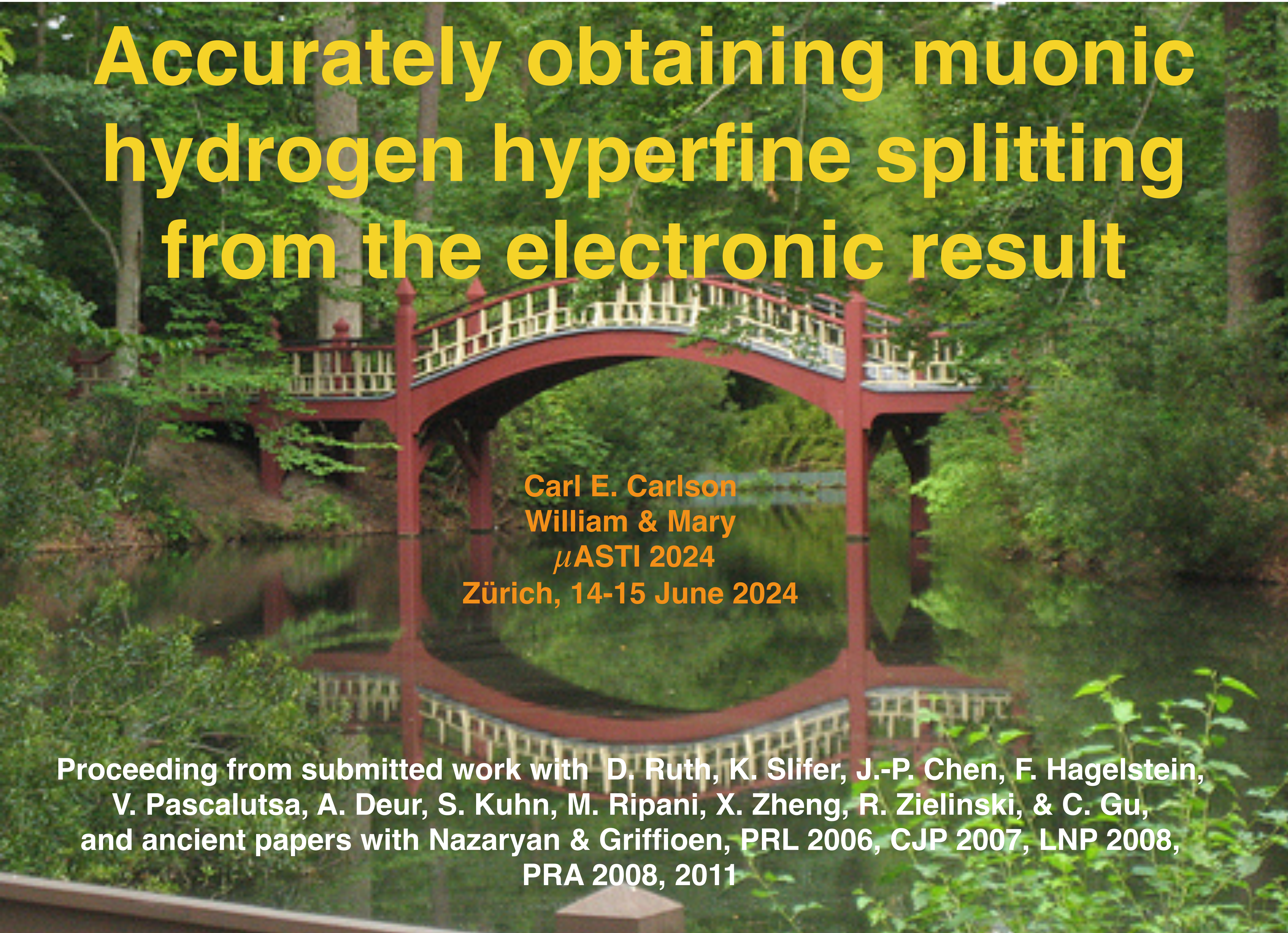

Accurately obtaining muonic hydrogen hyperfine splitting from the electronic result

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William & Mary

μASTI 2024
Zürich, 14-15 June 2024

Proceeding from submitted work with D. Ruth, K. Slifer, J.-P. Chen, F. Hagelstein, V. Pascalutsa, A. Deur, S. Kuhn, M. Ripani, X. Zheng, R. Zielinski, & C. Gu, and ancient papers with Nazaryan & Griffioen, PRL 2006, CJP 2007, LNP 2008, PRA 2008, 2011

Goals of this talk

- Present "stand-alone" calculation of HFS for ℓH, $\ell = \mu$ or e. New data, allows great improvement over earlier results. This work is finished and submitted.

- Explain using measured eH HFS with some scaling and corrections leads for significant reduction is uncertainty limits for μH HFS. Results not checked: will indicate uncertainty limits but not central values.

New input data

- For our calculation, input data includes the spin-dependent proton structure functions g_1 and g_2, measured in polarized inelastic ep scattering

- Functions of W (total CM ep energy) and Q^2 (photon off-shell mass).

- Previously, no g_2 data at all. Now g2p JLab experiment 84 data points, at 4 different Q^2 (Ruth et al., 2022)

- And wonderfully extended set of g_1 data from JLab EG4. 1085 data points, at 25 values of Q^2, range $\approx [0.01, 1.0]$ GeV^2 (Zheng et al., 2021)

For information, the old data

- No old data at all for g_2.
  Wilczek-Wandzura relation could give part of g_2
  and there were data fits (!)

- JLab EG1b g_1 data, available in 2005
  1124 data points at 27 values of Q^2
  range $\approx [0.05, 5.0]$ GeV^2
  (publication Fersch et al., 2017)

- SLAC E155 g_1 data, 24 data points, $Q^2 > 1.2$ GeV^2
  (Anthony et al., 2000)

- Actual data for g_2 and good lower Q^2 g_1 data creates opportunity for much improved calculational result

New planned experiments

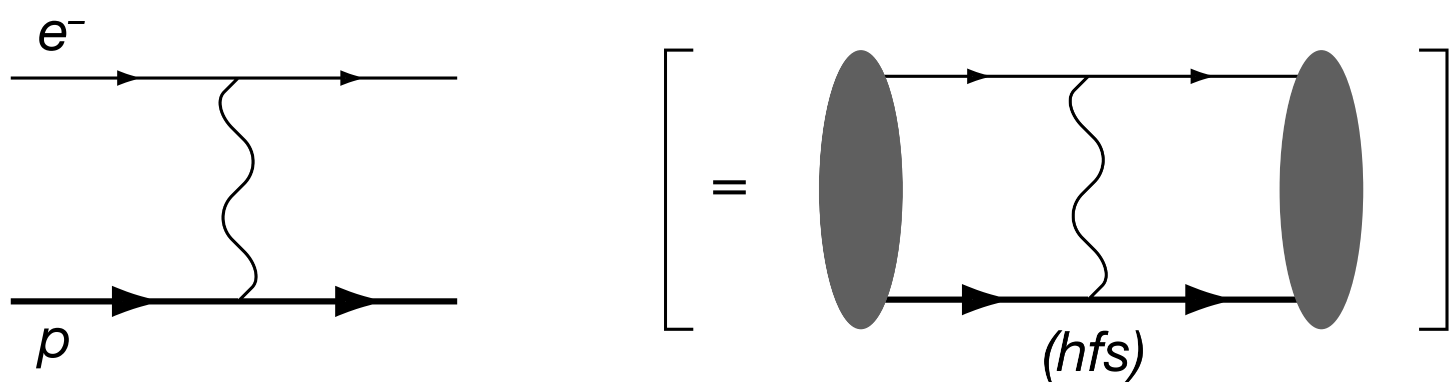
- CREMA, FAMU, & JPARC propose measurement of HFS in ground state μH
- 1S μH splitting is about 182.636 meV
  or wavelength ≈ 6.8 μm (infrared)
  or frequency ≈ 44.2 THz
- Worry about time to run experiment:
  Have laser, frequency width ≈ 100 MHz
- Say spread of prediction is about 0.16 meV (can do better!)
  ➔ spread of frequency prediction is ≈ 40 GHz
  ➔ need ≈ 400 frequency settings of laser to scan HFS region.

Planned experiments run time

- From talks: need 1.4 hour to get 4σ signal above background, and 1 hour to change laser frequency.
- 2.4 hours × 400 = 960 hours ≈ 8 weeks (@ 5 days/week)
Ugh: other groups want the PSI (CREMA's location) also
- ∴ want good theoretical help to reduce the laser scan width
- Anticipate fractional experimental uncertainty upon completion better than 100 MHz/44.2 THz ≈ 2 ppm
- Current best μH HFS splitting measurement is from CREMA (Science, 2013) and is 22.8089 (51) meV for the 2S state,
or ≈ 220 ppm.
- For comparison,
$E_{\text{1S,HFS}}(eH) = h \times 1420.405\,751\,768\,(2)$ MHz or 1.4 ppt

The calculation: lowest order

- H-atom, S-state, spin-dependent splitting
  UG textbook calculation!



- Get $$E_F^p = \frac{8\pi}{3}\frac{\mu_B \mu_p}{a_B^3} = \frac{8\pi}{3}(m_r \alpha)^3 \mu_B \mu_p$$

- $\mu_B = e/(2m_\ell)$ Bohr magneton
  $\mu_p = (1+\kappa_p)\, e/(2m_p)$ exact magnetic moment for proton

- "Fermi energy" ; Can evaluate to about 10-figure accuracy

- Alternate writings, $E_F^p = \dfrac{8\alpha^4}{3}\dfrac{m_\ell^2\,(1+\kappa_p)}{m_p\,(1+m_\ell/m_p)^3} = \dfrac{16\alpha^2}{3}\dfrac{\mu_p}{\mu_B}\dfrac{R_\infty}{(1+m_\ell/m_p)^3}$

Next need corrections

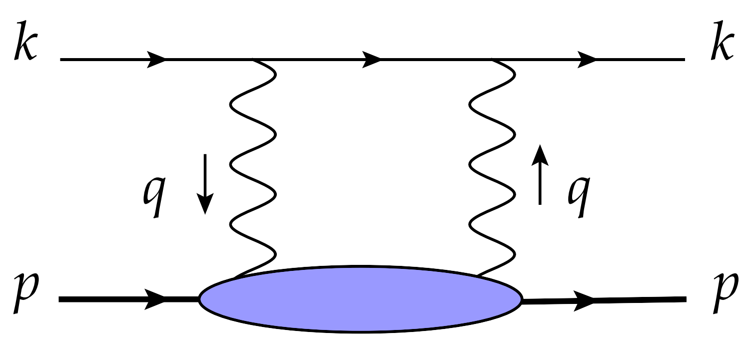
- Write as

$$E^p_{HFS} = E^p_F \left(1 + \Delta_{QED} + \Delta_S + \text{some smaller corrections}\right)$$

- Δ_{QED} well calculated

- "some smaller corrections" won't be discussed

- Δ_S = structure dependent corrections,
  here meaning corrections from 2-γ exchange,

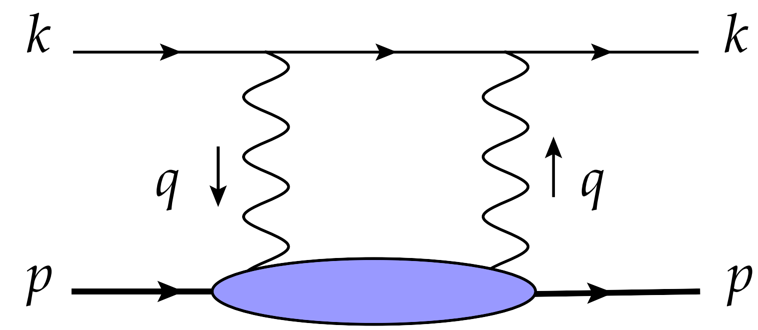


- Conventionally separate as

$$\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$$

NR elastic "Zemach" | Rel. elastic Corrections | Polarizability corrections

2γ corrections



- Not calculable *ab initio*.
  But lower part is forward Compton scattering of off-shell photons, algebraically gotten from

$$T_{\mu\nu}(q,p,S) = \frac{i}{2\pi m_p} \int d^4\xi \; e^{iq\cdot\xi} \langle pS \big| \, T j_\mu(\xi) j_\nu(0) \big| pS \rangle$$

- Spin dependence is in the antisymmetric part

$$T^A_{\mu\nu} = \frac{i}{m_p} \epsilon_{\mu\nu\alpha\beta} q^\alpha \left[H_1(\nu,Q^2)\, S^\beta + H_2(\nu,Q^2) \frac{p\cdot q\, S^\beta - S\cdot q\, p^\beta}{p\cdot q} \right]$$

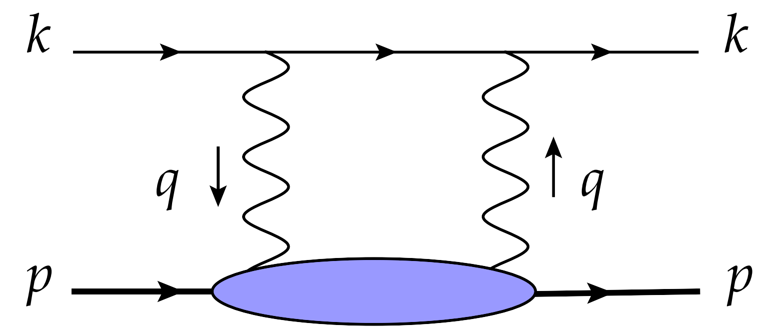
Some use $S_{1,2} = 4\pi^2 \alpha H_{1,2}$

- Imaginary part of above is related to polarized inelastic ep scattering, with

$$\mathrm{Im}\, H_1(\nu,Q^2) = \frac{1}{\nu}\, g_1(\nu,Q^2) \qquad \text{and} \qquad \mathrm{Im}\, H_2(\nu,Q^2) = \frac{m_p}{\nu^2}\, g_2(\nu,Q^2)$$

- Emphasize: g_1 and g_2 are measured at SLAC, HERMES, JLab,…

2γ corrections



- Combine electron part of diagram with Compton bottom, and energy from 2γ exchange

$$\Delta_{\text{pol}} = \left.\frac{E_{2\gamma}}{E_F}\right|_{\text{inel}} = \frac{2\alpha m_e}{(1+\kappa_p)\pi^3 m_p}$$
$$\times \int \frac{d^4Q}{(Q^4 + 4m_e^2 Q_0^2)Q^2} \left\{(2Q^2 + Q_0^2)H_1^{\text{inel}}(iQ_0, Q^2) - 3Q^2 Q_0^2 H_2^{\text{inel}}(iQ_0, Q^2)\right\}$$

- (Wick rotated). Great, but don't know $H_{1,2}$ from data.

- But do know Im parts, and if no subtraction, simple Cauchy (dispersion relation) gives

$$H_1^{\text{inel}}(\nu, Q^2) = \frac{1}{\pi}\int_{\nu_{th}^2}^{\infty} d\nu'^{\,2}\, \frac{\operatorname{Im} H_1(\nu', Q^2)}{\nu'^{\,2} - \nu^2}$$

and similarly for H_2.

Do some integrals analytically, getting

- $$\Delta_{\text{pol}} = \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

- $$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2}\left\{\beta_1\left(\frac{Q^2}{4m_\ell^2}\right)F_2^2(Q^2) + 4m_p\int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2}\tilde{\beta}_1\left(Q^2,\nu,m_\ell\right)g_1(\nu,Q^2)\right\}$$

- $$\Delta_2 = -12m_p\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2}\tilde{\beta}_2\left(Q^2,\nu,m_\ell\right)g_2(\nu,Q^2)$$

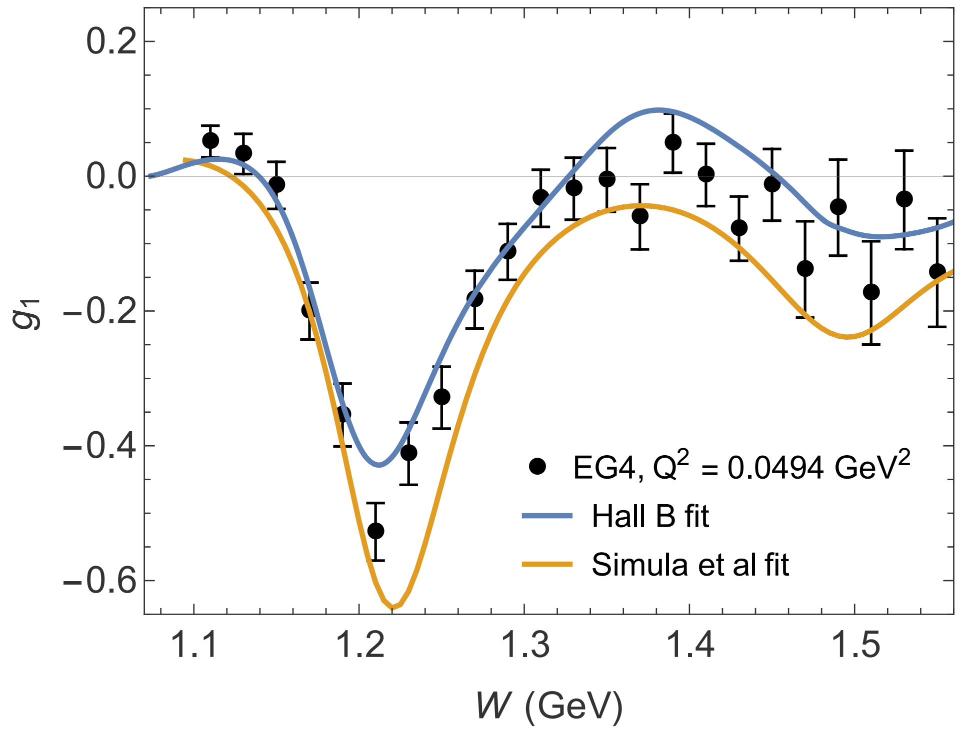
  - $\beta_1(\tau) = -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}$

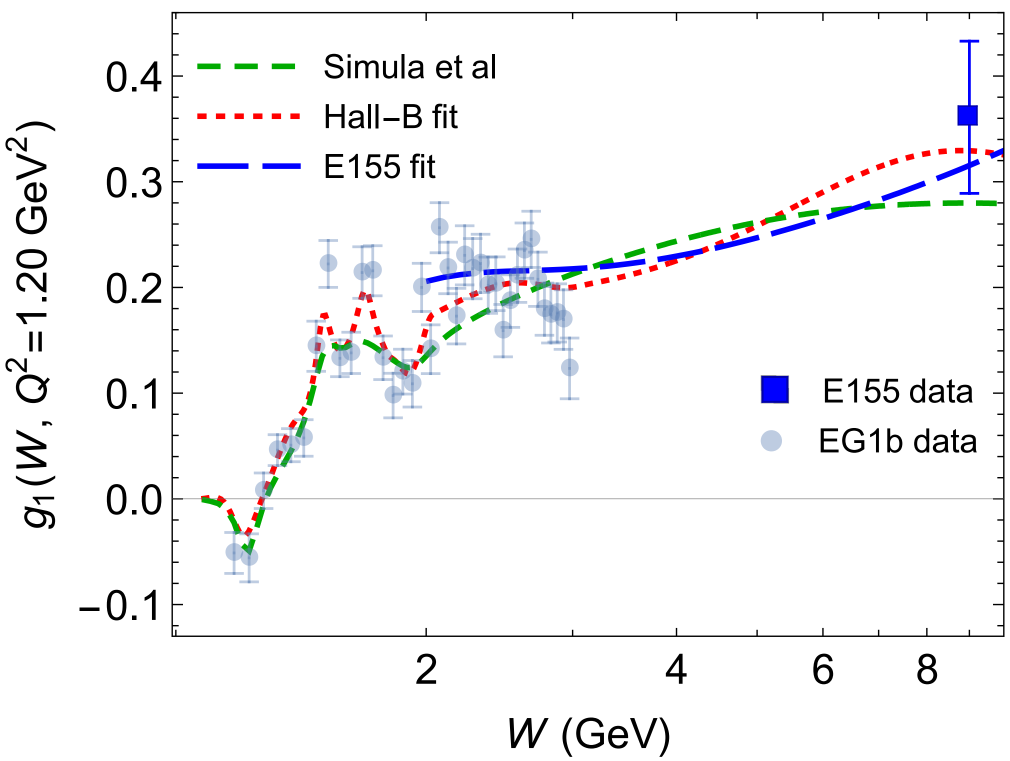
  - $\tilde{\beta}_1$ and $\tilde{\beta}_2$ are known kinematic weighting functions.

Completion of Δ_1 calculation

- More comments on Δ_1 before going to g_2
- ∃ noticeable contributions from outside the data region. Need model or fit to extrapolate. Have fit of Simula et al (PRD, 2002) and
fit of Hall B collaboration (unpub., ca. 2016) and fit of E155 (PLB, 2000, high Q^2, high W only).
- Hall B fits best where we have comparison data

Some fit comparisons





- Generally good agreement among the three fits in scaling region (high Q^2, high W).
- Hall B closer in data region. (They did have EG1b data.)
- We use the Hall B fit for the fill-in contributions (higher W for Q^2in data region, and Q^2 above and below data region).

Δ_1 results today

- $\Delta_1(eH) =$ 4.71 ± 1.02 from data
  + 1.60 ± … high W fill-in, data region
  + 0.12 ± … low Q^2
  + 0.34 ± … high Q^2

  = 6.78 ± 1.02$_{\text{data}}$ ± 0.23$_{\text{fill-in}}$

- Old $\Delta_1(eH) = 8.85 \pm 2.78$

- About -1 unit from newer data and about -1 from updated fill-in choice.

Modern Δ_2, short version

- Thanks to g2p JLab experiment, have data where there was none before

- $\Delta_2(eH) = -1.20_{\text{data}} \pm 0.16_{\text{data}} + \text{fill-in}$
  $= -1.98 \pm 0.16_{\text{data}} \pm 0.38_{\text{fill-in}}$

- Old $\Delta_2(eH) = -0.57 \pm 0.57$

- Big difference from having data.

- Wilczek-Wandzura close to old value, not to data.

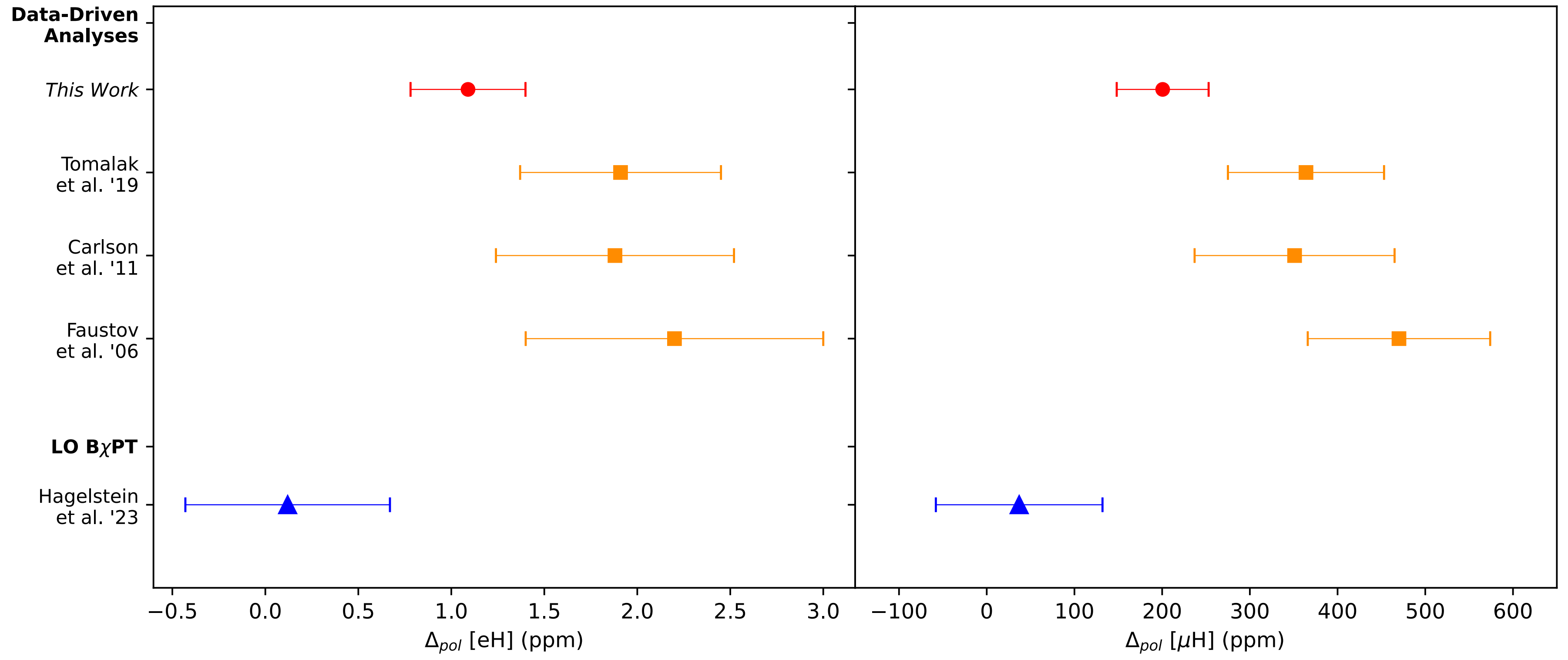
Δ_{pol} results

- Reminders: $$\Delta_{\text{pol}} = \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

$$E^p_{HFS} = E^p_F \left(1 + \Delta_{QED} + \Delta_Z + \Delta_R + \Delta_{\text{pol}} + \text{some smaller corrections}\right)$$

- New results: $\Delta_{\text{pol}}(eH) = 1.09 \pm 0.31$ ppm

  $\Delta_{\text{pol}}(\mu H) = 200.6 \pm 52.4$ ppm



Size of uncertainty

- $\Delta_{\text{pol}}(\mu H) = 200.6 \ \pm \ 52.4$ ppm (new)
- $\Delta_R = 931 \pm 3$ ppm
- $\Delta_Z = -7703 \pm 80$ ppm (AMT, with range to AS and Kelly, from 2008)
- $E_F(\mu H) = 182.443$ meV
- Uncertainties above give (Z-R-pol) 15, 1, 9 μeV, resp.
  (or fractionally 8, small, 5, × 10^{-5})
- (Overall result given on previous slide)

More accurate μH results

- Bootstrap off super accurate *eH* results, $E^{\text{expt}}_{\text{1S-HFS}}(\mathrm{H}) = 1\,420.405\,751\,768(2)$ MHz
- Will refer to as “scaling + corrections”
- Due to Peset and Pineda and to Tomalak (2018) (Presentation here more like Tomalak)
- See also review by Antognini, Hagelstein, & Pascalutsa (2022) and Wednesday evening poster of Vladyslava Sharkovska.

What is it?

- Reminder

$$E_{HFS} = E_F\Big(1 + \underbrace{\Delta_{QED}}_{\text{will quote}} + \underbrace{\Delta_{\mu VP} + \Delta_{hVP} + \Delta_{Weak}}_{\text{known, same for } \mu H \text{ as for } eH} + \underbrace{\Delta_Z + \Delta_R + \Delta_{pol}}_{\Delta_S}\Big)$$

- Can “reverse engineer” $\Delta_S(eH)$, to 7 figure accuracy, using E_{HFS}^{expt} and $E_F(eH)$.

- Need $\Delta_S(\mu H)$.

Tautology, & reworking

- $$\Delta_S(\mu H) = \frac{m_{r\mu}}{m_{re}}\Delta_S(eH) + \left[\Delta_S(\mu H) - \frac{m_{r\mu}}{m_{re}}\Delta_S(eH)\right]$$
- Cannot be wrong! For "improved" Δ_S use
$$\Delta_S^{impr}(\mu H) = \frac{m_{r\mu}}{m_{re}}\Delta_S^{expt}(eH) + \left[\Delta_S(\mu H) - \frac{m_{r\mu}}{m_{re}}\Delta_S(eH)\right]$$
- Scaling for first term, calculation for second, but treat the terms in square bracket as whole.

($m_{r\ell} = m_p m_\ell/(m_p + m_\ell)$ is reduced mass)

Why this scaling?

- The term as a whole is a correction

$$\Delta_S^{corr} = \Delta_S(\mu H) - \frac{m_{r\mu}}{m_{re}}\Delta_S(eH)$$

with a good deal of internal cancellation, both in the central value and in the uncertainty estimates.

- Zemach term is biggest term in Δ_S , and is proportional to the reduced mass

$$\Delta_Z = -2Z\alpha m_{r\ell}R_Z = \frac{8Z\alpha m_{r\ell}}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa_p} - 1\right]$$

and cancels out of Δ_S^{corr} .

Use Δ_2^{corr} as further example

- $$\Delta^{corr}_{\text{pol}} = \frac{\alpha m_\ell}{2\pi(1+\kappa_p)m_p}\left(\Delta_1^{corr} + \Delta_2^{corr}\right)$$

- Note lepton mass factor, so that, e.g.,

$$\Delta_2^{corr} = \Delta_2(\mu H) - \frac{m_p + m_e}{m_p + m_\mu}\Delta_2(eH)$$

- For central values, easy: just subtract already calculated numbers

- Δ_1^{corr} significantly reduced compared to (say) $\Delta_1(\mu H)$

- Uncertainty limits requires some thought, but they are significantly reduced also

Uncertainty limits

- Central value, ab initio

- $$\Delta_2(\ell H) = -24 m_p^2 \int \frac{dQ^2}{Q^4} \int dx\, \tilde{\beta}_2(Q^2, x, m_\ell) g_2(x, Q^2)$$

  (Weighting function $\tilde{\beta}_2$ known, $x = Q^2/(2m_p\nu)$.)

- $$\Delta_2^{corr} = -24 m_p^2 \int \frac{dQ^2}{Q^4} \int dx\, \tilde{\beta}_2^{corr}(Q^2, x, m_\mu, m_e) g_2(x, Q^2)$$

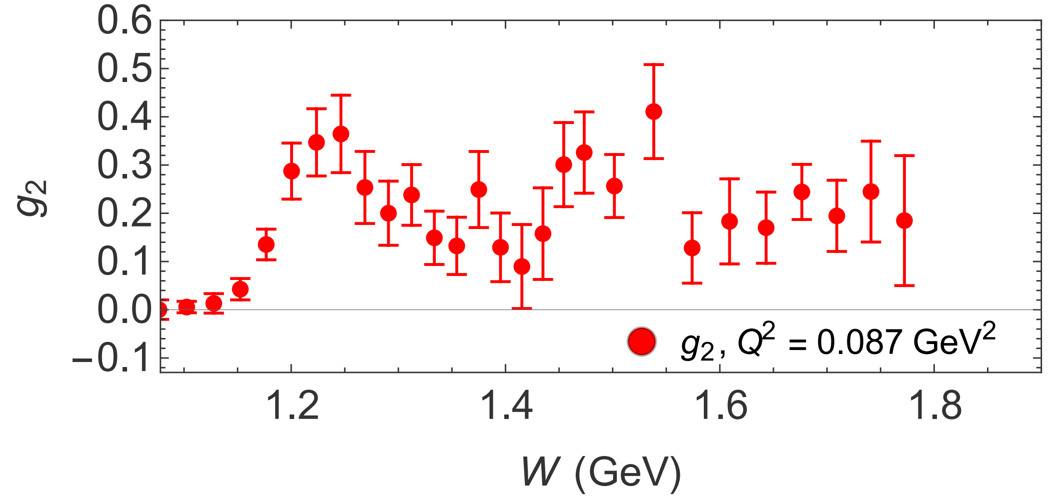
  with

  $$\tilde{\beta}_2^{corr}(Q^2, x, m_\mu, m_e) = \tilde{\beta}_2(Q^2, x, m_\mu) - \frac{m_p + m_e}{m_p + m_\mu} \tilde{\beta}_2(Q^2, x, m_e)$$

- Lots of cancellation in $\tilde{\beta}_2^{corr}$.

Practical

- g_2 data at four values of Q^2 and set of W (initial state energy)



- First integral

$$B_2^{\text{corr}}(Q^2) = \int dx\, \tilde{\beta}_2^{\text{corr}}(Q^2, x, m_\mu, m_e)\, g_2(x, Q^2)$$

$$= \int \frac{dW}{J(W)}\, \tilde{\beta}_2^{\text{corr}}(Q^2, W, m_\mu, m_e)\, g_2(x, Q^2)$$

- the W_i of the data are centers of bins with widths ΔW_i

$$B_2^{\text{corr}}(Q^2) = \sum_i \frac{\Delta W_i}{J(W_i)}\, \tilde{\beta}_2^{\text{corr}}(Q^2, W_i, m_\mu, m_e)\, g_2(W_i, Q^2)$$

for uncertainties

- For central values, same as before

- For uncertainties, should add in quadrature. If δg_2 are uncertainties in data values,

$$\delta B_2^{\mathrm{corr}}(Q^2) = \left\{ \sum_i \left[\frac{\Delta W_i}{J(W_i)} \tilde{\beta}_2^{\mathrm{corr}}(Q^2, W, m_\mu, m_e)\, \delta g_2(W_i, Q^2) \right]^2 \right\}^{1/2}$$

(and likewise for next integral, the Q^2 integral)

- Some results: Δ_2 from g_2 data only

$$\Delta_{2,data}(eH) = -1.205 \pm 0.118$$
$$\Delta_{2,data}(\mu H) = -0.878 \pm 0.078$$

$$\Delta_{2,data}^{corr} = 0.206 \pm 0.033$$

Fill-in

- Need model or fit to get contributions to Δ_2 for Q^2 above and below data region, and for W above measured values in data region.

- Methods: a) Do Padé or other fit to $B_2(Q^2)$ from data region, and use extrapolation to get fill-in.
  b) Use model fits to get $g_2(x, Q^2)$ (and $g_1(x, Q^2)$) everywhere, & use for fill-ins outside data region.

- Already mentioned the two models: Simula et al (2002) [based on good physics but only data was high Q^2 SLAC data for g_1] and "Hall-B fit" [from ca. 2016, had also EG1b JLab g_1 data].

Sample fill-in results for Δ_2

- Use Simula as example, and proceed as for real data: got for contribution above and below data region

$$\begin{aligned}
\Delta_2(\mu H, \text{fill-in}) &= -0.309 \pm 0.129 \\
\Delta_2(eH, \text{fill-in}) &= -0.473 \pm 0.180 \\
\Delta_2^{corr}(\text{fill-in}) &= 0.116 \pm 0.043
\end{aligned}$$

- Uncertainty reduced by factor ca. 4 (rel. to eH).

Modifications from existing work

- $\Delta_2(eH) = -1.98 \pm (0.16)_{data} \pm (0.38)_{model}$
  $\Delta_2(\mu H) = -1.40 \pm (0.11)_{data} \pm (0.31)_{model}$

  $\Delta_2^{corr} = 0.38 \pm (0.06)_{data} \pm (0.10)_{model}$

- $\Delta_1(eH) = 6.78 \pm (1.02)_{data} \pm (0.24)_{model}$
  $\Delta_1(\mu H) = 5.69 \pm (0.84)_{data} \pm (0.20)_{model}$

  $\Delta_1^{corr} = -0.41 \pm (0.34)_{data} \pm (0.06)_{model}$

- $\Delta_{pol}^{corr} = \frac{m_\mu \alpha}{2\pi(1+\kappa_p)m_p}\left(\Delta_1^{corr} + \Delta_2^{corr}\right) = -1.24 \pm 17.0$ ppm

- $\Delta_R^{corr} = -156.7 \pm 3.7$ ppm

Results …

- $\Delta_S^{impr}(\mu H) = \frac{m_{r\mu}}{m_{re}} \Delta_S^{expt}(eH) + \Delta_R^{corr} + \Delta_{pol}^{corr} = -\,157.9 \pm 17.4$ ppm

- Fermi energy $E_F(\mu H) = 182.443$ meV

- Quoted uncertainty in $\Delta_S^{impr}(\mu H)$ leads to 0.003 meV uncertainty in $E_{HFS}^{1S}(\mu H)$ (or $\approx 2 \times 10^{-5}$ fractionally).

Summary

- For separate and uncoupled eH and μH calculations:
  - Dispersive calculation is complete, well defined, and unambiguous.
  - New data reduces uncertainty limits in calculated HFS by more than factor 2.
- Can do better for μH by getting some terms using experimental HFS data for eH
  - Reduces uncertainty limits by about another factor 3 (for μH only).
- Still “tension” with EFT calculation that requires resolution.

Beyond the end

Comments

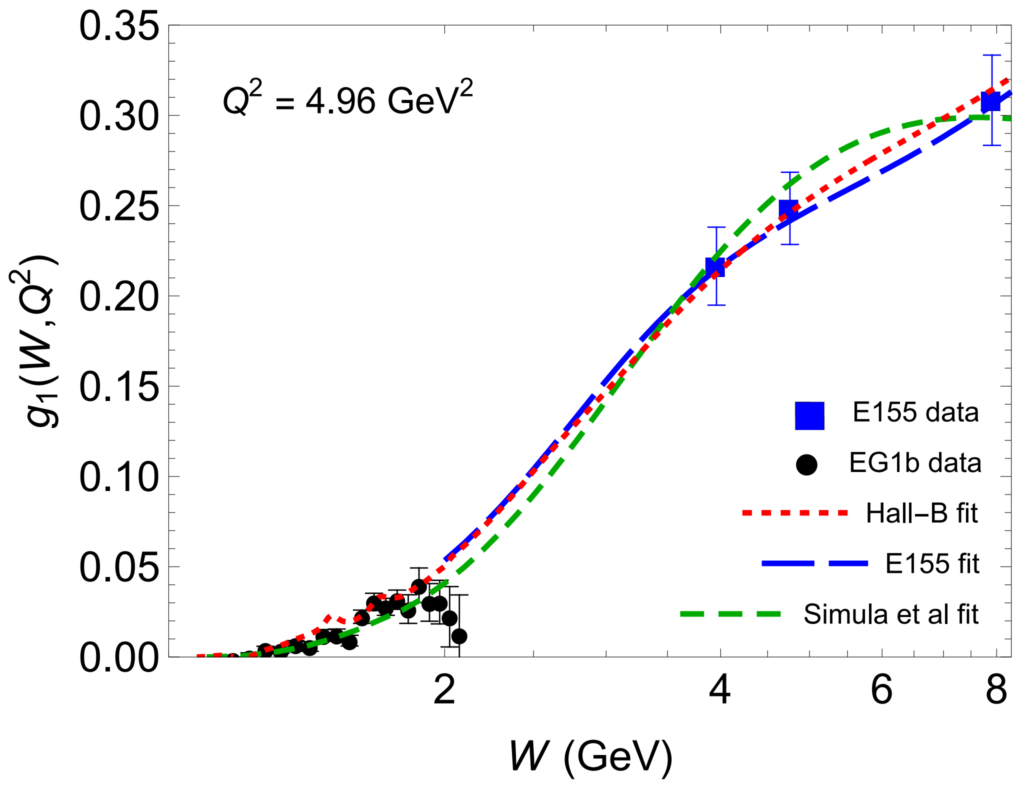
- Early history: begun by Iddings (1965),
  finalized by Drell and Sullivan (1967),
  put in present notation by de Rafael (1971).
  No spin-dependent data existed,
  no nonzero evaluation for > 30 years,
  until Faustov and Martynenko (2002),
  then modern era starts

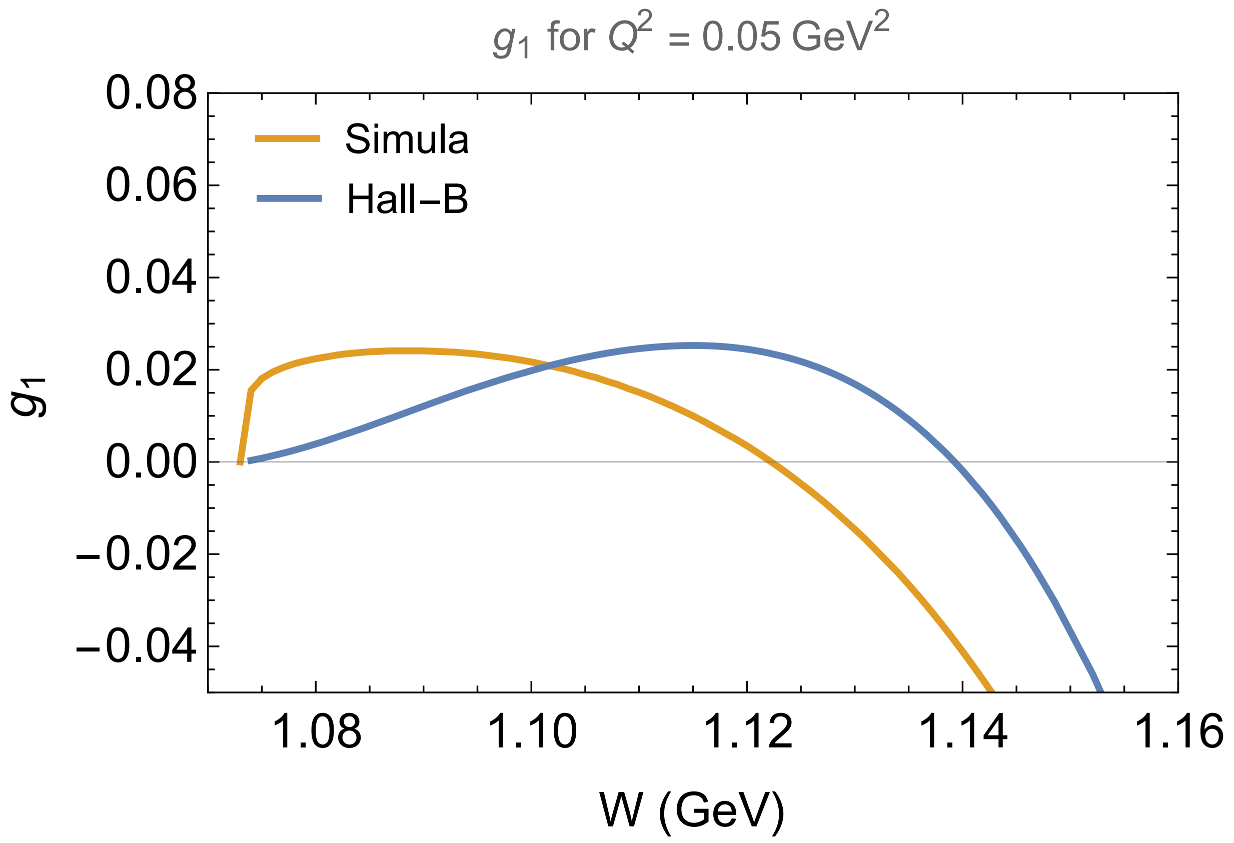
- Someone added something: the F_2^2 term. Not inelastic.
  (Put in here, taken out somewhere else.)
  Thought convenient in 1967, still here in 2024..

- Δ_1 term as written finite in $m_e \to 0$ limit, because of known sum rule, $4m_p \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2} g_1(\nu,0) = -\kappa_p^2$ (DHGHY)

More fit comparisons

- Scaling region; near threshold W.



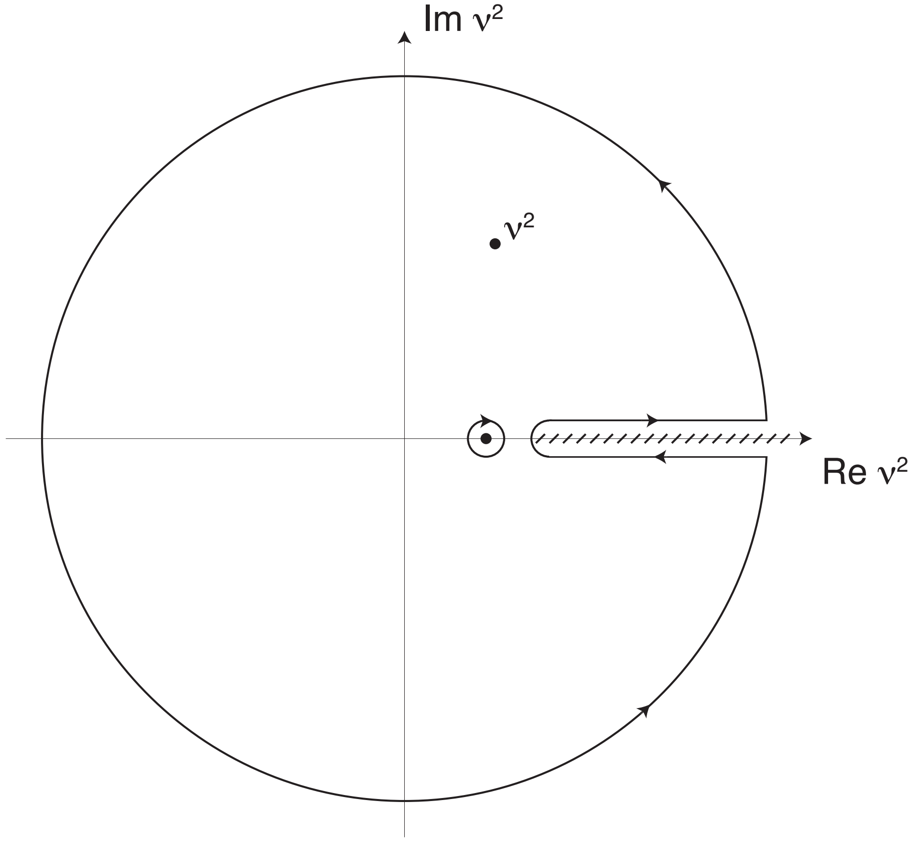


Unsubtracted dispersion relation (DR)?

- Was once openly discussed (< 2006, say), now seems generally thought o.k.

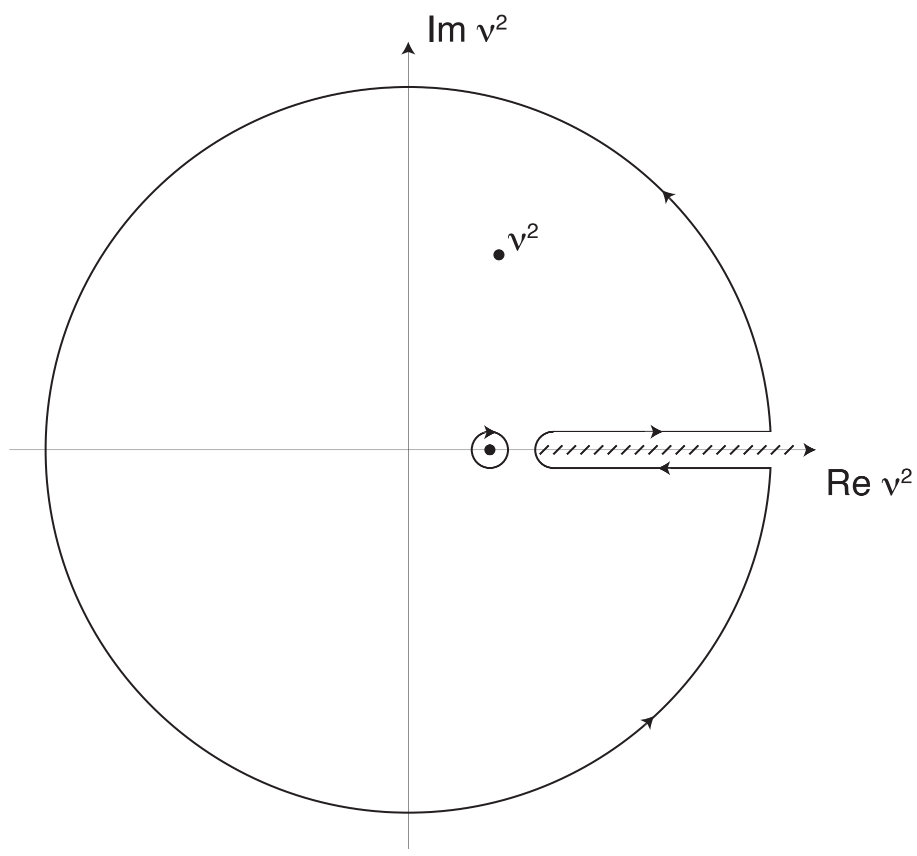
- DR comes from Cauchy integral formula applied with some contour (closed integration path)

$$H_1(\nu, Q^2) = \frac{1}{2\pi i} \oint \frac{H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2$$



- (DR in ν (or ν^2) with Q^2 fixed)

Dispersion relation



- Work into

$$H_1(\nu,Q^2) = \frac{\text{Res } H_1(\nu,Q^2)\Big|_{el}}{\nu_{el}^2-\nu^2} + \frac{1}{\pi}\int_{cut}\frac{\text{Im}\,H_1(\nu',Q^2)}{\nu'^2-\nu^2}d\nu'^2 + \frac{1}{2\pi i}\int_{|\nu'|=\infty}\frac{H_1(\nu',Q^2)}{\nu'^2-\nu^2}d\nu'^2$$

- Drop the $|\nu| = \infty$ term. O.k. if H_1 falls at high ν.

- Can view as standard or as dramatic assumption.

H_1

- The elastic term can be worked out, sticking on-shell form factors at the γp vertices,

$$H_1^{el} = \frac{2m_p}{\pi}\left(\frac{Q^2F_1(Q^2)G_M(Q^2)}{(Q^2-i\epsilon)^2-4m_p^2\nu^2} - \frac{F_2^2(Q^2)}{4m_p^2}\right)$$

- The second term does not fall with ν at fixed Q^2.

- Unsubtracted DR fails for H_1^{el} alone. Overall success requires exact cancelation between elastic and inelastic contributions.

- (In case of interest: $H_2^{el} = -\frac{2m_p}{\pi}\frac{m_p\nu F_2(Q^2)G_M(Q^2)}{(Q^2-i\epsilon)^2-4m_p^2\nu^2}$.)

But then,

- Free quarks if there is at least one large momentum scale. So at high ν, Compton amplitude for proton should be sum of Compton amplitudes for free quarks, which have zero F_2 .

- Regge theory suggests H_1 must fall with ν. See Abarbanel and Nussinov (1967), who show $H_1 \sim \nu^{\alpha-1}$ with $\alpha < 1$.*

- Very similar DR derivation gives GDH sum rule, which is checked experimentally and works, within current experimental uncertainty.

- GDH sum rule also checked in LO and NLO order perturbation theory in QED. Appears to work.

*Footnote 16 of this paper reads: For an indication of the lengthy details involved one may see M. Gell-Mann, M. L. Goldberger, and F. .E Low, Rev. Mod. Phys. 36, 640 (1964).

Resolution?

- In modern times, authors who use experimental scattering data and DR to calculate the 2γ corrections assume an unsubtracted DR works for all of H_1 .
- Reevaluation always possible.
- Proceed to next topic, comparison of data driven evaluations of HFS to evaluations using BχPT to obtain $H_{1,2}$.
- See if subtraction comments come into play.

Side note: how good need we be?

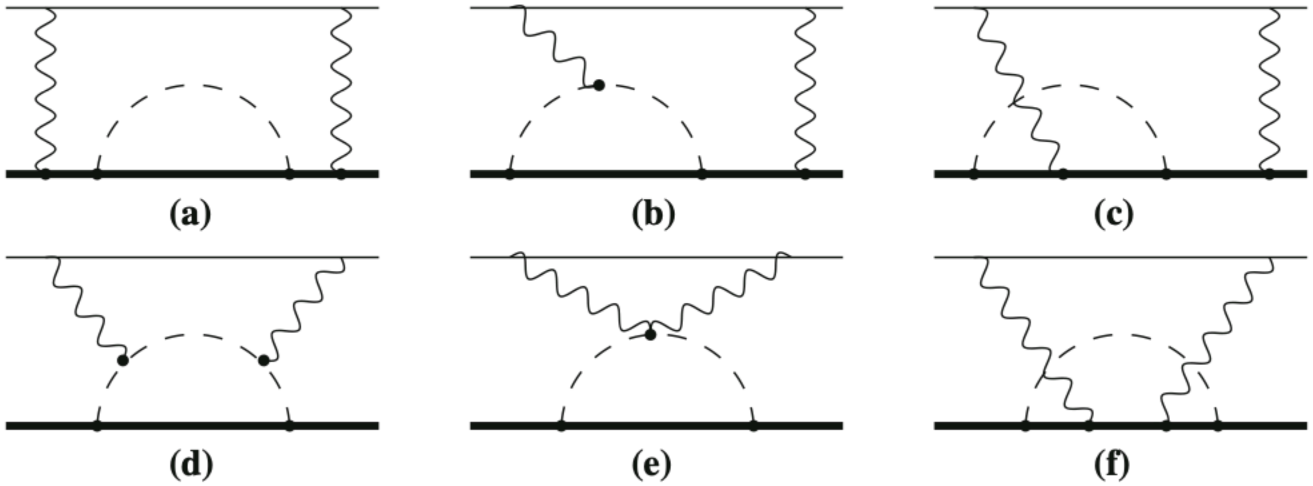
- New measurements of HFS in μH in 1S state are planned.
- May measure to 0.1 ppm (as fraction of Fermi energy). But need theory prediction to help determine starting point of laser frequency scan.
- From 2018 conference at MITP (Mainz), want theory prediction to 25 ppm or better. Better is what we should look for.
- Believe state of art for HFS in 1S μH is from Antognini, Hagelstein, Pascalutsa (2022),

$$E^{\text{1S}}_{\text{HFS}} = 182.634(8)\ \text{meV}$$

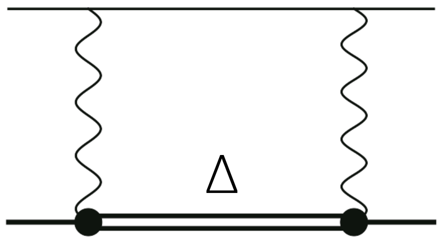
or 44 ppm.

Application of BχPT

- Using chiral perturbation theory, one can calculate beyond the elastic case diagrams like



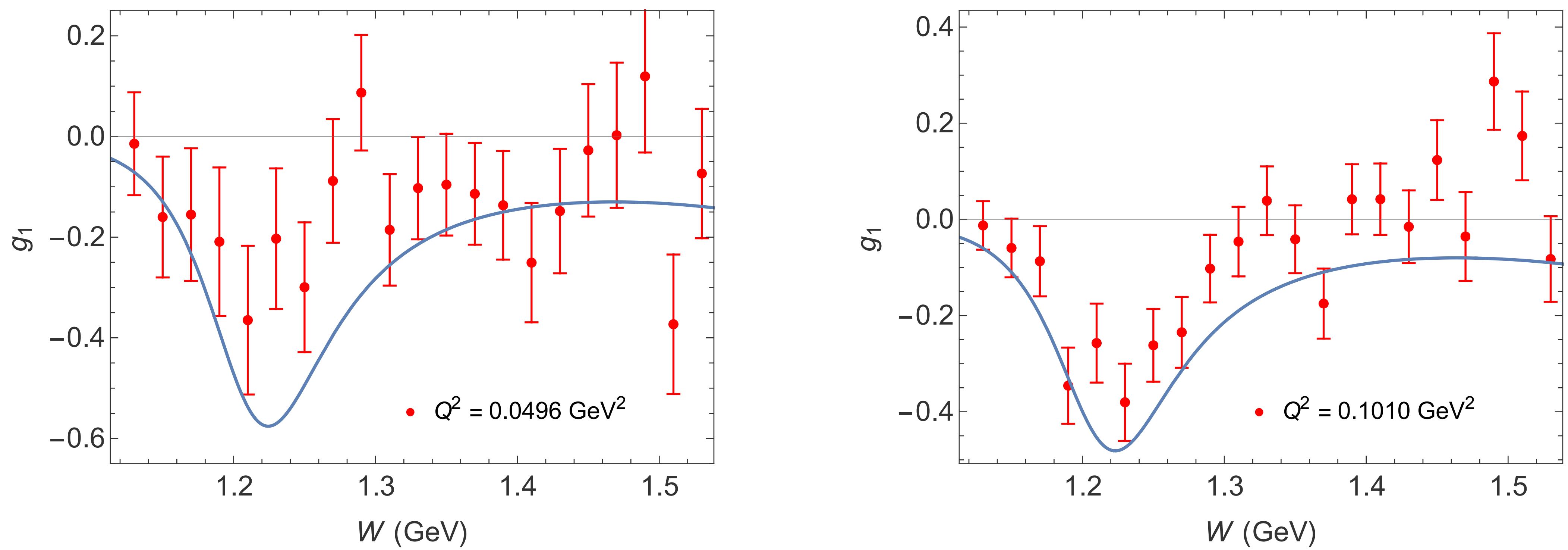
- Or diagrams where there is a Δ-baryon on the hadronic leg,



- These can be used to calculate $H_{1,2}$, at low Q^2 and CM energy W not too far from threshold. Also can get $\gamma^* N \rightarrow \pi N$ or $\gamma^* N \rightarrow \Delta$ and from them obtain $g_{1,2}$ at similarly low kinematics.

g_1 comparison

- Compare g_1 from BχPT (blue lines) to JLab data

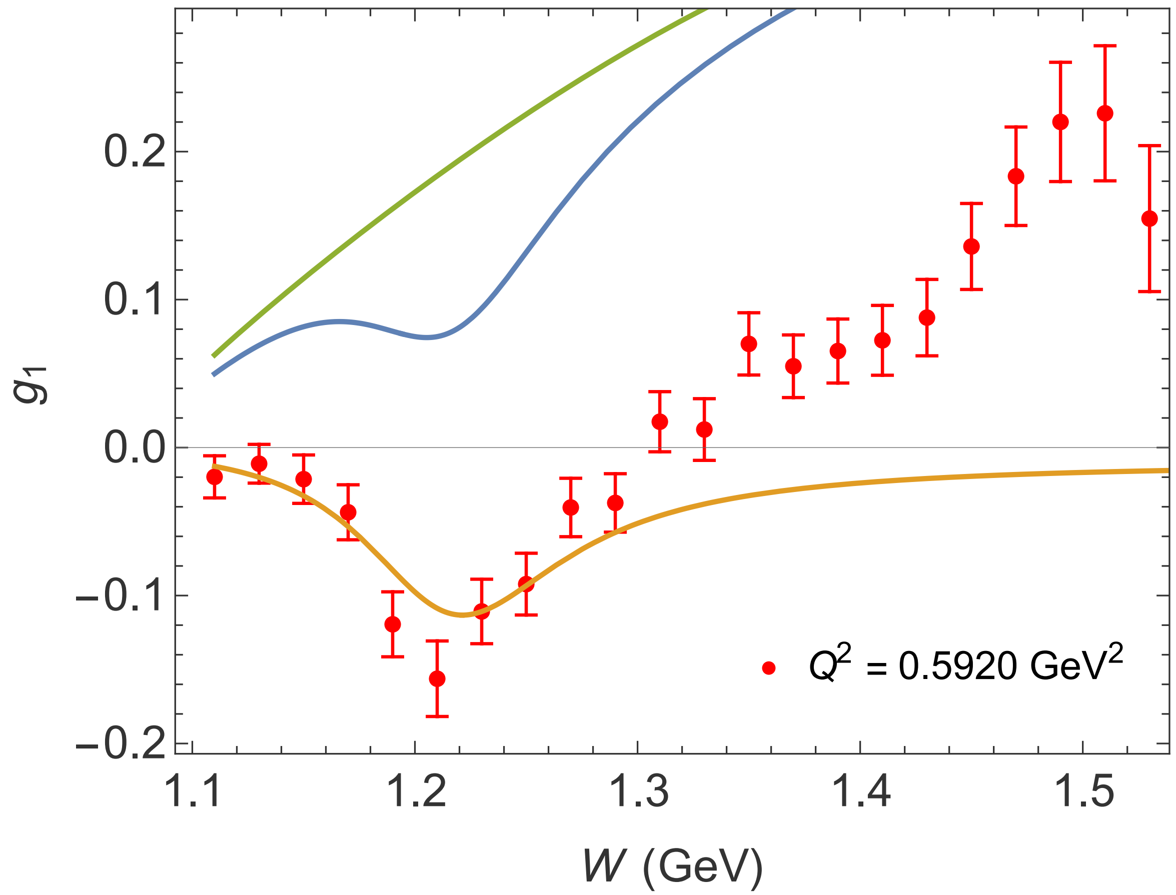


- Plots are “unofficial”: Made by me* and involve spreading Δ pole out using Lorentzian of same total area.

*With greatest thanks to Pascalutsa and Hagelstein for providing code for their gamma N -> pi N

- O.k. This won’t explain difference in Δ_{pol} results.

Another g_1 comparison



- green = proton contribution
- gold = Δ contribution
- blue = sum

Non-pole terms

- Non-pole means ν independent terms in $H_{1,2}$.
- Recall elastic $H_1^{el} = \frac{2m_p}{\pi}\left(\frac{Q^2F_1(Q^2)G_M(Q^2)}{(Q^2-i\epsilon)^2-4m_p^2\nu^2}-\frac{F_2^2(Q^2)}{4m_p^2}\right)$.
- The BχPT results for H_1 with π-N and Δ intermediate states also have non-pole terms.
- To calculate energies for the non-pole terms, cannot use the DR (at least not un-subtracted ones), but can use the expressions on slide 7, which were before any Cauchy trickery was used

Pole and non-pole

- One part: The Δ contribution to μH HFS for 2S state*

$$E_{pol}^{HFS} = -40.69\,\mu\text{eV} \qquad \text{pole}$$
$$= 39.54\,\mu\text{eV} \qquad \text{non-pole}$$
$$= -1.15\,\mu\text{eV} \qquad \text{total}$$

- Lot of cancellation.

- But from asymptotic freedom, or from Regge analysis, or from success of DHG sum rule, expect zero non-pole term. Totality, from elastic and resonances and inelastic terms, needs to add to zero for the ν independent terms.

- Something to talk about.

*from Hagelstein (2016)

One point

- How should one deal with non-zero non-pole terms that result from partial information, when one knows that the non-pole terms are zero when one has complete information?

Δ_{pol} with newest $g_{1,2}$

- Defer to David Ruth (next after next talk).
- Except for comment on handling regions outside the data range.
- Mostly, because of the kinematic factors, the need is for data at low Q^2 and low ν (or W near threshold), and this is where the data is.
- Again, mostly, where there is no data and we use models or interpolations, the contributions to $\Delta_{1,2}$ are not great and the accruing uncertainty is not great.

Δ_{pol} with newest $g_{1,2}$

- An exception may be the very low Q^2 region, where there is no data. For the 2003 data, this was $Q^2 < 0.0452$ GeV^2.

- And there may be a problem when comparing to χPT.

- What we did: reminder

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\}$$

with
$$B_1(Q^2) = \frac{4}{9}\int_0^{x_{\rm th}} dx\,\beta_1(\tau) g_1(x,Q^2)\ .$$

- For very low Q^2 we used

$$B_1(Q^2) = -\frac{\kappa_p^2}{8m_p^2}Q^2 + c_{1B}Q^4 = -\frac{\kappa_p^2}{8m_p^2}Q^2 + 4.94\,Q^4/\text{GeV}^4$$

got by fitting to data $Q^2 < 0.3$ GeV^2

Δ_{pol} with newest $g_{1,2}$

- The region $Q^2 < 0.0492$ GeV^2 contributed about 15% of Δ_1 and (by our estimate) 30% of the uncertainty.

- Use standard expansion for the form factor,

$$F_2(Q^2) = \kappa_p\left(1 - \frac{1}{6}R_{Pauli}^2 Q^2 + \ldots\right)$$

- Get Integrand =

$$\frac{9}{4}\frac{1}{Q^2}\left(F_2^2 + \frac{8m_p^2}{Q^2}B_1\right) = -\frac{3}{4}\kappa_p^2 R_{Pauli}^2 + 8m_p^2 c_{1B}$$

- And $\Delta_1(0 \to Q^2_{low\,data}) \approx$ Integrand $\cdot\, Q^2_{low\,data} \approx 1.35$

Δ_{pol} with newest $g_{1,2}$

- χPT has knowledge of g_1 at low Q^2, and can do the integrals. Do good approximation by expanding the β_1 function for low Q^2.

- Work for a while to get Integrand =

$$-\frac{3}{4}\kappa_p^2 R_{Pauli}^2 + 8m_p^2 c_1 - \frac{5m_p^2}{4\alpha}\gamma_0 + \mathcal{O}(Q^2),$$

- Where $\gamma_0 = \frac{2\alpha}{m_p^2}\int \frac{d\nu}{\nu^4} g_1(\nu,0)$

and c_1 came from

$$I(Q^2) \equiv 4m_p \int \frac{d\nu}{\nu^2} g_1(\nu, Q^2) = -\kappa_p^2 + c_1 Q^2 + \mathcal{O}(Q^4)$$

Δ_{pol} with newest $g_{1,2}$

- Value for known, and doing integrals to get c_1, find

  $\Delta_1(0 \to Q^2_{low\,data}) \approx$ Integrand $\cdot\, Q^2_{low\,data} \approx -0.45$

thanks again to F. Haglestein et al.

- Not even same sign!

- Corresponding numbers for μ are ≈ 0.86 and -0.20

- Remembering $\Delta_{\text{pol}} = \dfrac{\alpha m_\mu}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$, difference gives about 50 ppm or about 15% of discrepancy.

- More to talk about!