Muonic Atom Spectroscopy Theory Initiative

from **PUZZLE**

ASTI

to **PRECISION**

Steering Committee

Aldo Antognini

Carl Carlson

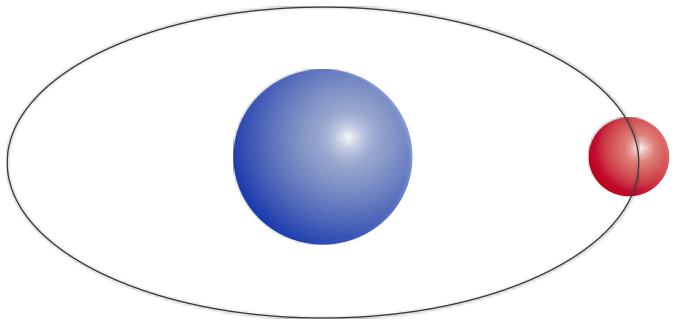
Franziska Hagelstein

Paul Indelicato

Krzysztof Pachucki

Vladimir Pascalutsa

Challenges of the bound-state systems



Many contributions and several expansions

- Radiative corrections
- Relativistic corrections
- Binding effects
- Recoil corrections
- Nuclear structure corrections

all mixed up

We (CREMA collaboration) performed theory compilations, but

Challenges

- Theorist have different approaches
- Many theorist do not compare their results to each other
- How to add up the various contributions from the various authors?
- Double counting, completeness!

We did our own compilations with best effort

- We had great fun to discuss with theorist
- But we are not specialist

Better solution: theory initiative

- Community consensus
- Less prone to mistakes, biases
- Better evaluation of uncertainties
- Easier to have a continuous update
- Expertise from various fields: QED, hadron, nuclear, scattering…..

Example of our (CREMA collaboration) theory compilations……

Table 1
All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The "finite-size to relativistic recoil correction" (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, "1:3" LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, "2:2" LBL		0.00135	0.00115		0.00115	[74,89]
New	"3:1" LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II

(*continued on next page*)

A. Antognini et al. / Annals of Physics 331 (2013) 127–145

...and the most recent update

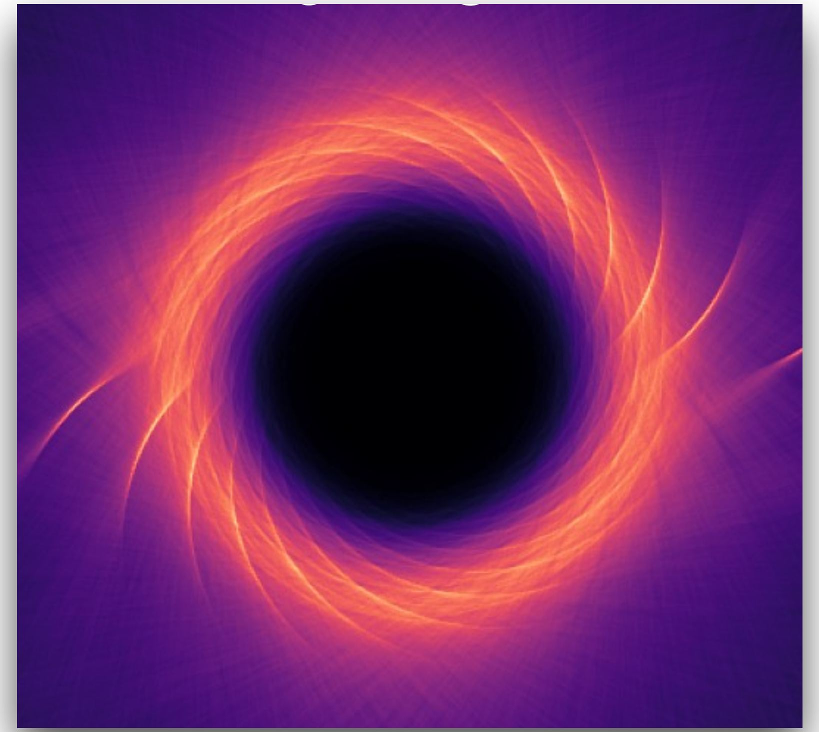
Comprehensive theory of the Lamb shift in light muonic atoms

K. Pachucki, V. Lensky, F. Hagelstein, S. S. Li Muli, S. Bacca, and R. Pohl
Rev. Mod. Phys. **96**, 015001 – Published 24 January 2024

Section	Order	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
III.A	$\alpha(Z\alpha)^2$	eVP$^{(1)}$	205.007 38	227.634 70	1641.886 2	1665.773 1
III.A	$\alpha^2(Z\alpha)^2$	eVP$^{(2)}$	1.658 85	1.838 04	13.084 3	13.276 9
III.A	$\alpha^3(Z\alpha)^2$	eVP$^{(3)}$	0.007 52	0.008 42(7)	0.073 0(30)	0.074 0(30)
III.B	$(Z, Z^2, Z^3)\alpha^5$	Light-by-light eVP	−0.000 89(2)	−0.000 96(2)	−0.013 4(6)	−0.013 6(6)
III.C	$(Z\alpha)^4$	Recoil	0.057 47	0.067 22	0.126 5	0.295 2
III.D	$\alpha(Z\alpha)^4$	Relativistic with eVP$^{(1)}$	0.018 76	0.021 78	0.509 3	0.521 1
III.E	$\alpha^2(Z\alpha)^4$	Relativistic with eVP$^{(2)}$	0.000 17	0.000 20	0.005 6	0.005 7
III.F	$\alpha(Z\alpha)^4$	μSE$^{(1)}$ + μVP$^{(1)}$, LO	−0.663 45	−0.769 43	−10.652 5	−10.926 0
III.G	$\alpha(Z\alpha)^5$	μSE$^{(1)}$ + μVP$^{(1)}$, NLO	−0.004 43	−0.005 18	−0.174 9	−0.179 7
III.H	$\alpha^2(Z\alpha)^4$	μVP$^{(1)}$ with eVP$^{(1)}$	0.000 13	0.000 15	0.003 8	0.003 9
III.I	$\alpha^2(Z\alpha)^4$	μSE$^{(1)}$ with eVP$^{(1)}$	−0.002 54	−0.003 06	−0.062 7	−0.064 6
III.J	$(Z\alpha)^5$	Recoil	−0.044 97	−0.026 60	−0.558 1	−0.433 0
III.K	$\alpha(Z\alpha)^5$	Recoil with eVP$^{(1)}$	0.000 14(14)	0.000 09(9)	0.004 9(49)	0.003 9(39)
III.L	$Z^2\alpha(Z\alpha)^4$	nSE$^{(1)}$	−0.009 92	−0.003 10	−0.084 0	−0.050 5
III.M	$\alpha^2(Z\alpha)^4$	$\mu F_1^{(2)}$, $\mu F_2^{(2)}$, μVP$^{(2)}$	−0.001 58	−0.001 84	−0.031 1	−0.031 9
III.N	$(Z\alpha)^6$	Pure recoil	0.000 09	0.000 04	0.001 9	0.001 4
III.O	$\alpha(Z\alpha)^5$	Radiative recoil	0.000 22	0.000 13	0.002 9	0.002 3
III.P	$\alpha(Z\alpha)^4$	hVP	0.011 36(27)	0.013 28(32)	0.224 1(53)	0.230 3(54)
III.Q	$\alpha^2(Z\alpha)^4$	hVP with eVP$^{(1)}$	0.000 09	0.000 10	0.002 6(1)	0.002 7(1)
IV.A	$(Z\alpha)^4$	r_C^2	$-5.197\,5r_p^2$	$-6.073\,2r_d^2$	$-102.523r_h^2$	$-105.322r_\alpha^2$
IV.B	$\alpha(Z\alpha)^4$	eVP$^{(1)}$ with r_C^2	$-0.028\,2r_p^2$	$-0.034\,0r_d^2$	$-0.851r_h^2$	$-0.878r_\alpha^2$
IV.C	$\alpha^2(Z\alpha)^4$	eVP$^{(2)}$ with r_C^2	$-0.000\,2r_p^2$	$-0.000\,2r_d^2$	$-0.009(1)r_h^2$	$-0.009(1)r_\alpha^2$
V.A	$(Z\alpha)^5$	TPE	0.029 2(25)	1.979(20)	16.38(31)	9.76(40)
V.B	$\alpha^2(Z\alpha)^4$	Coulomb distortion	0.0	−0.261	−1.010	−0.536
V.C	$(Z\alpha)^6$	3PE	−0.001 3(3)	0.002 2(9)	−0.214(214)	−0.165(165)
V.D	$\alpha(Z\alpha)^5$	eVP$^{(1)}$ with TPE	0.000 6(1)	0.027 5(4)	0.266(24)	0.158(12)
V.E	$\alpha(Z\alpha)^5$	μSE$^{(1)}$ + μVP$^{(1)}$ with TPE	0.000 4	0.002 6(3)	0.077(8)	0.059(6)
III	E_{QED}	Point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
IV	$\mathcal{C}r_C^2$	Finite size	$-5.225\,9r_p^2$	$-6.107\,4r_d^2$	$-103.383r_h^2$	$-106.209r_\alpha^2$
V	E_{NS}	Nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
	E_L (exp)	Experiment[a]	202.370 6(23)	202.878 5(34)	1258.598(48)	1378.521(48)
	r_C	This review	0.840 60(39)	2.127 58(78)	1.970 07(94)	1.678 6(12)
	r_C	Previous work[a]	0.840 87(39)	2.125 62(78)	1.970 07(94)	1.678 24(83)

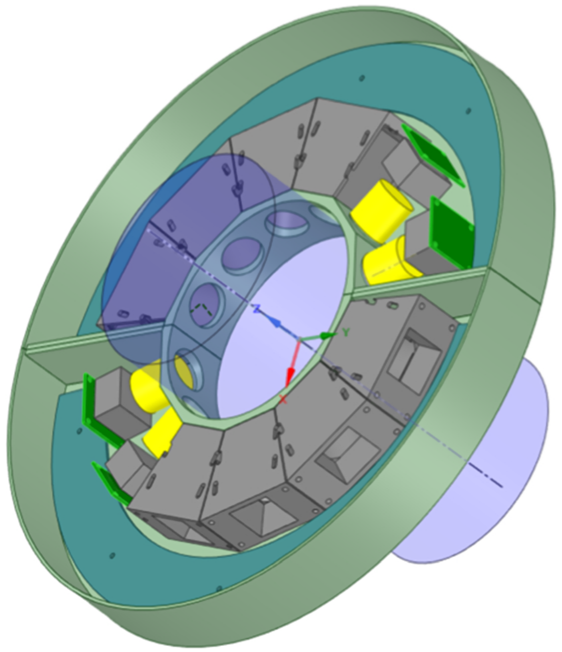
Ongoing activities with muons

Laser spectroscopy of the 1S-HFS in μH



CREMA (PSI)

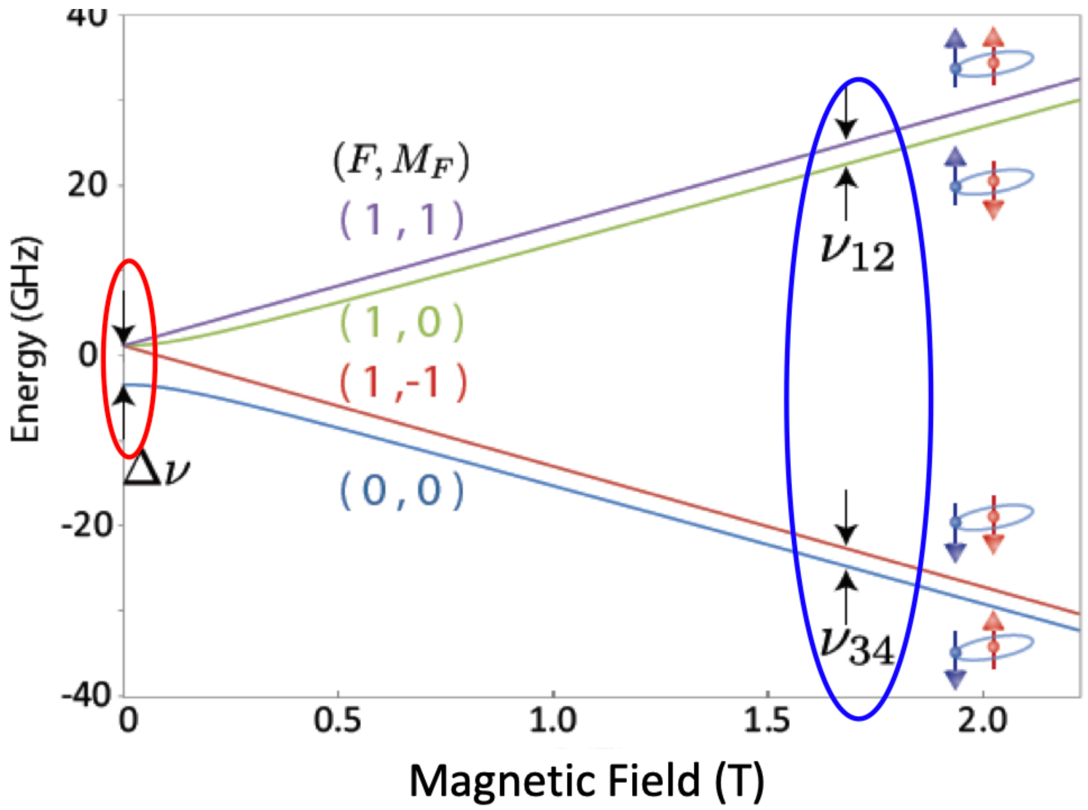
$\delta = 1 \times 10^{-6}$



FAMU (RIKEN-RAL)

$\delta = 1 \times 10^{-5}$

Hyperfine splitting in μ^+e and $(\mu^4\mathrm{He}^{++})e$



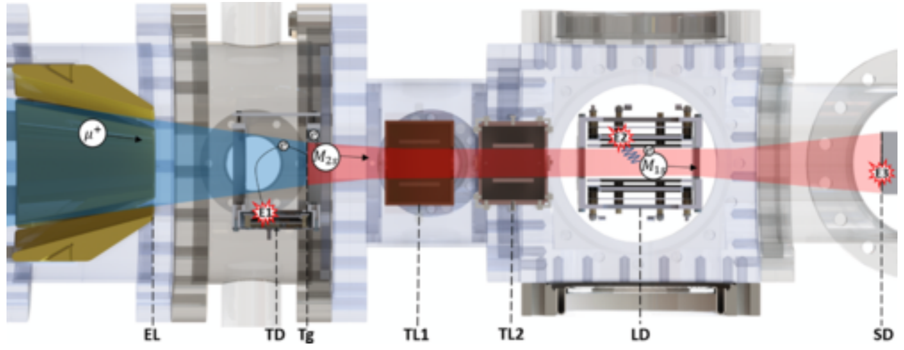
MUSEUM (J-PARC)

$\delta = 2 \times 10^{-9}$

$\nu_{12}+\nu_{34} = \Delta\nu_{HFS}$

$\nu_{12}-\nu_{34} \propto \mu_\mu/\mu_p \propto m_\mu/m_p$

1S-2S and 2S-2P transitions in μ^+e



Mu-Mass (ETH-PSI)

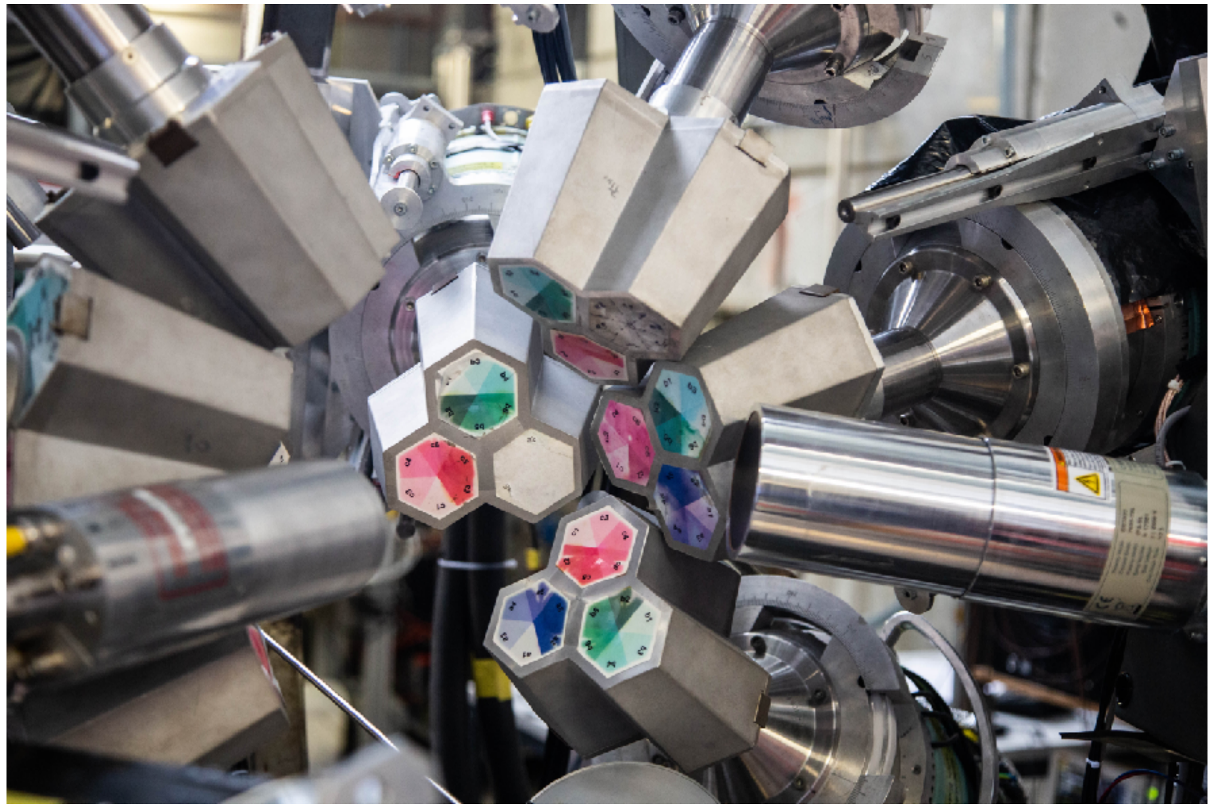
$\delta = 4 \times 10^{-12}$

(J-PARC)

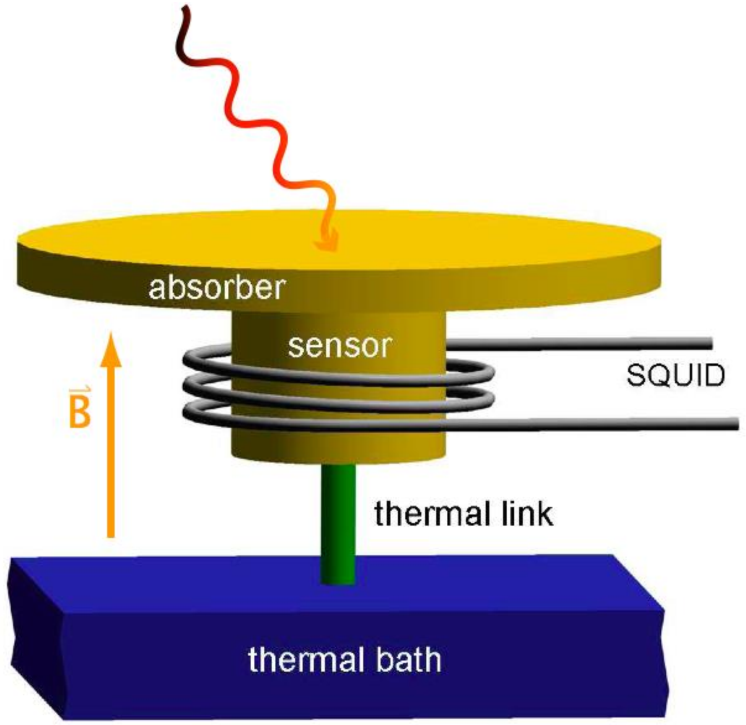
Ongoing activities with muons

muX and Reference Charge Radii

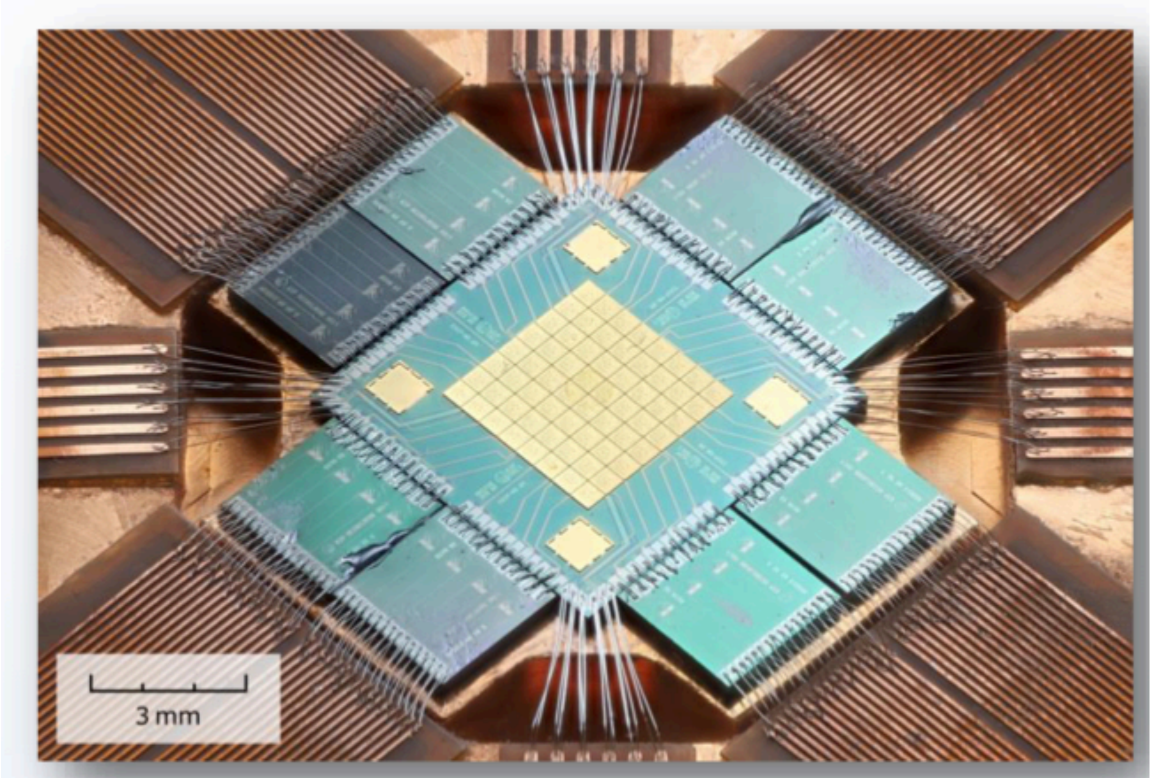
Measurement of muonic x rays from microgram targets to extract absolute charge radii as inputs for APV experiments, laser spectroscopy, nuclear structure investigations



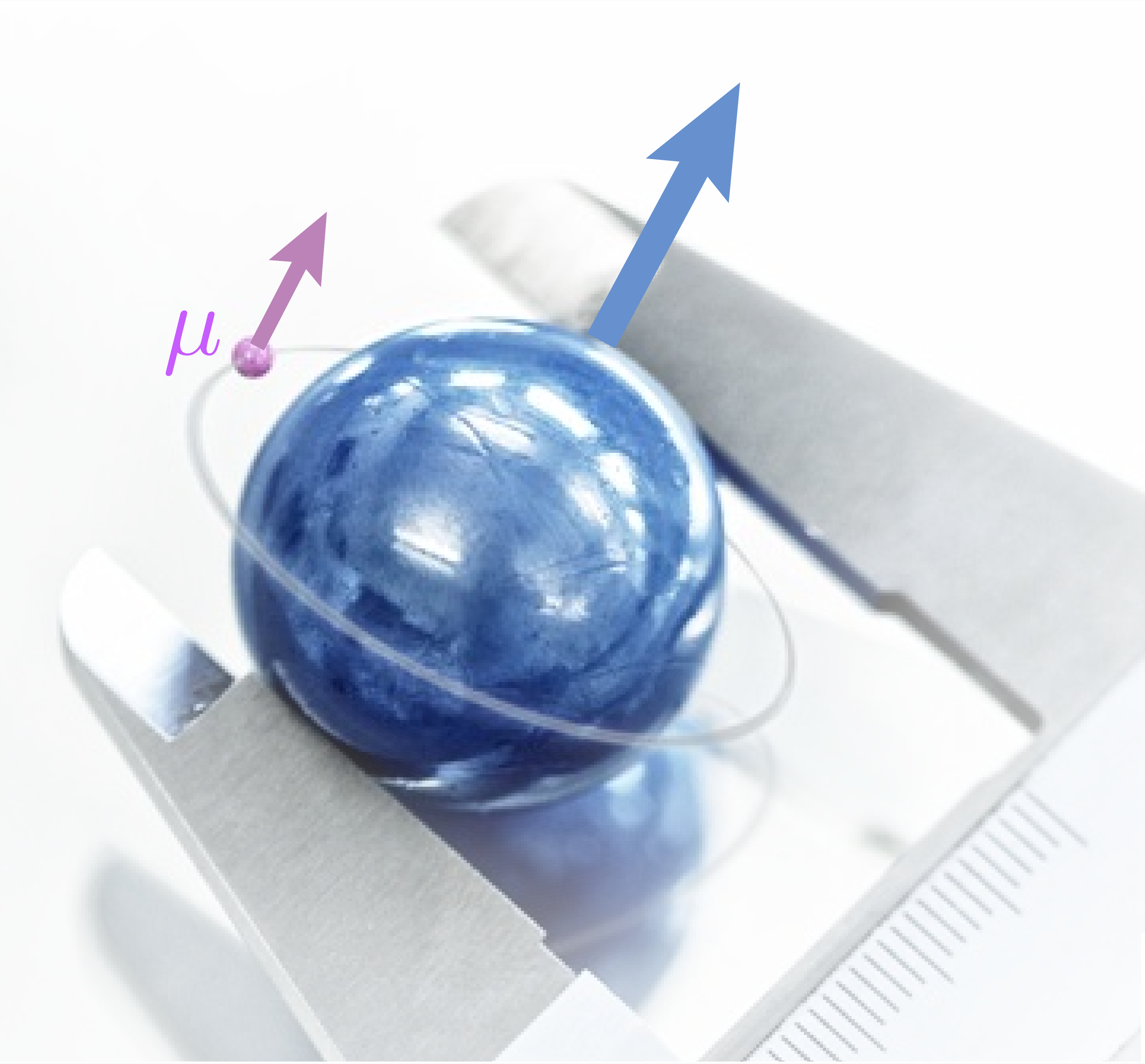
Quartett: charge radii from Lithium to Neon

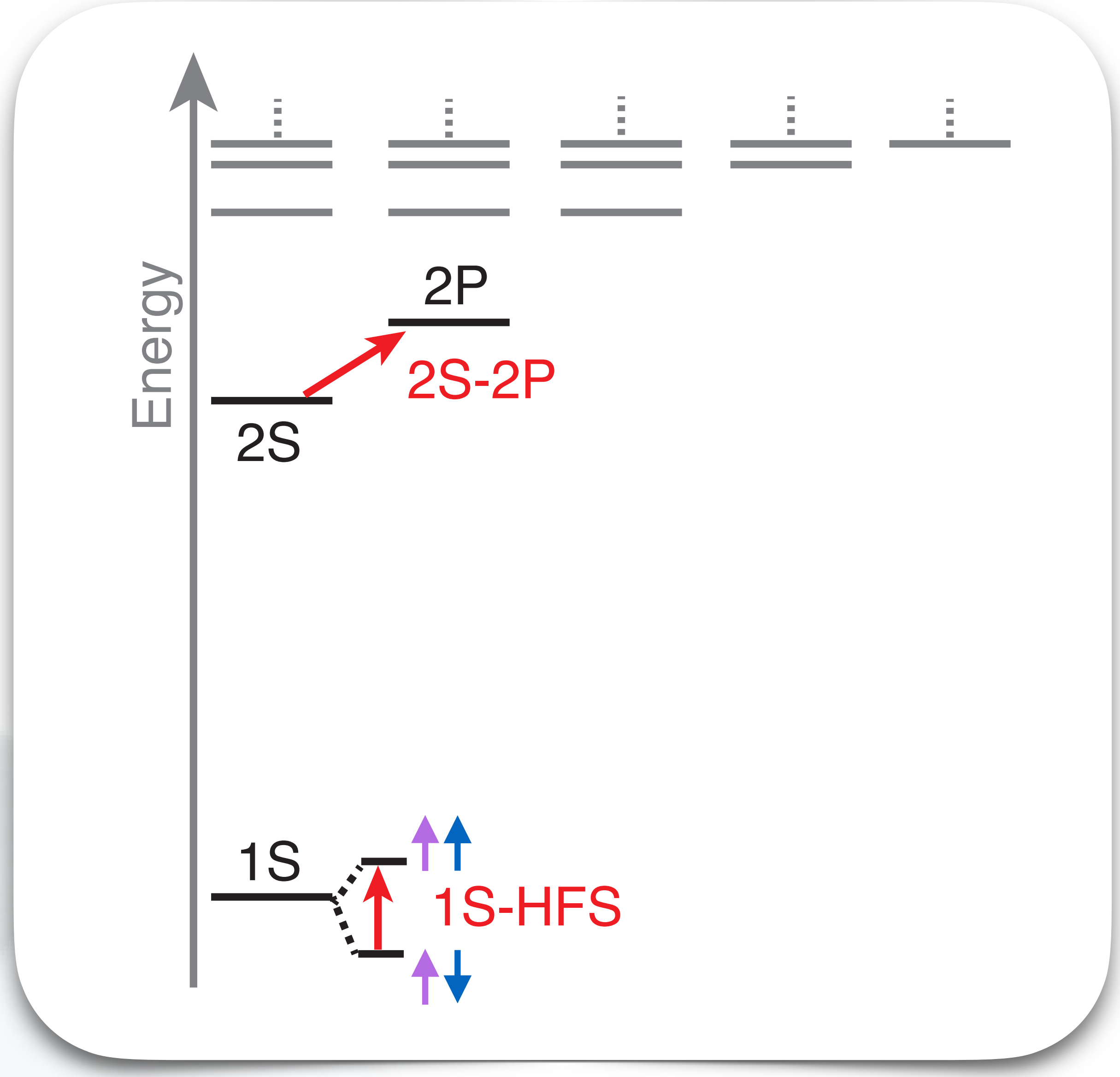


High-resolution measurements using a metallic magnetic calorimeter



One urgent example: 1S hyperfine splitting in muonic hydrogen

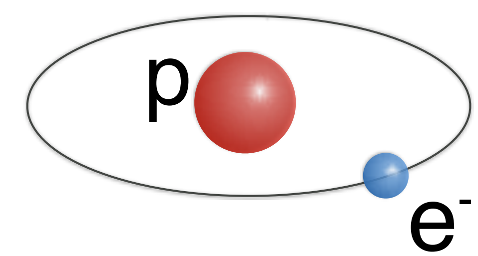


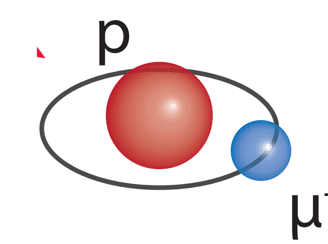


Measure HFS transition with
$\delta \simeq 1 \times 10^{-6}$

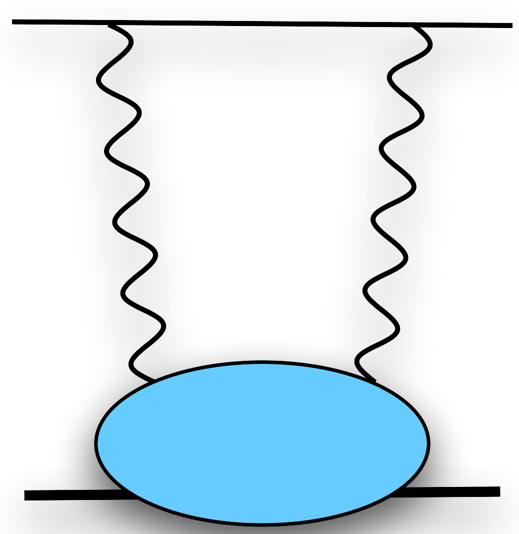
Determine nuclear structure contribution
$\delta \simeq 1 \times 10^{-4}$

Impact of the HFS measurement





Provides information on magnetic structure of the proton



- Spin structure program
- Form factor program
- Chiral perturbation theory
- Dispersion-based th.
- Lattice QCD

Hagelstein, Pascalutsa, Carlson, Martynenko, Tomalak, Faustov, Vanderhaegen, Lensky

Combined with H
→Test of HFS theory with rel. acc. $< 10^{-8}$

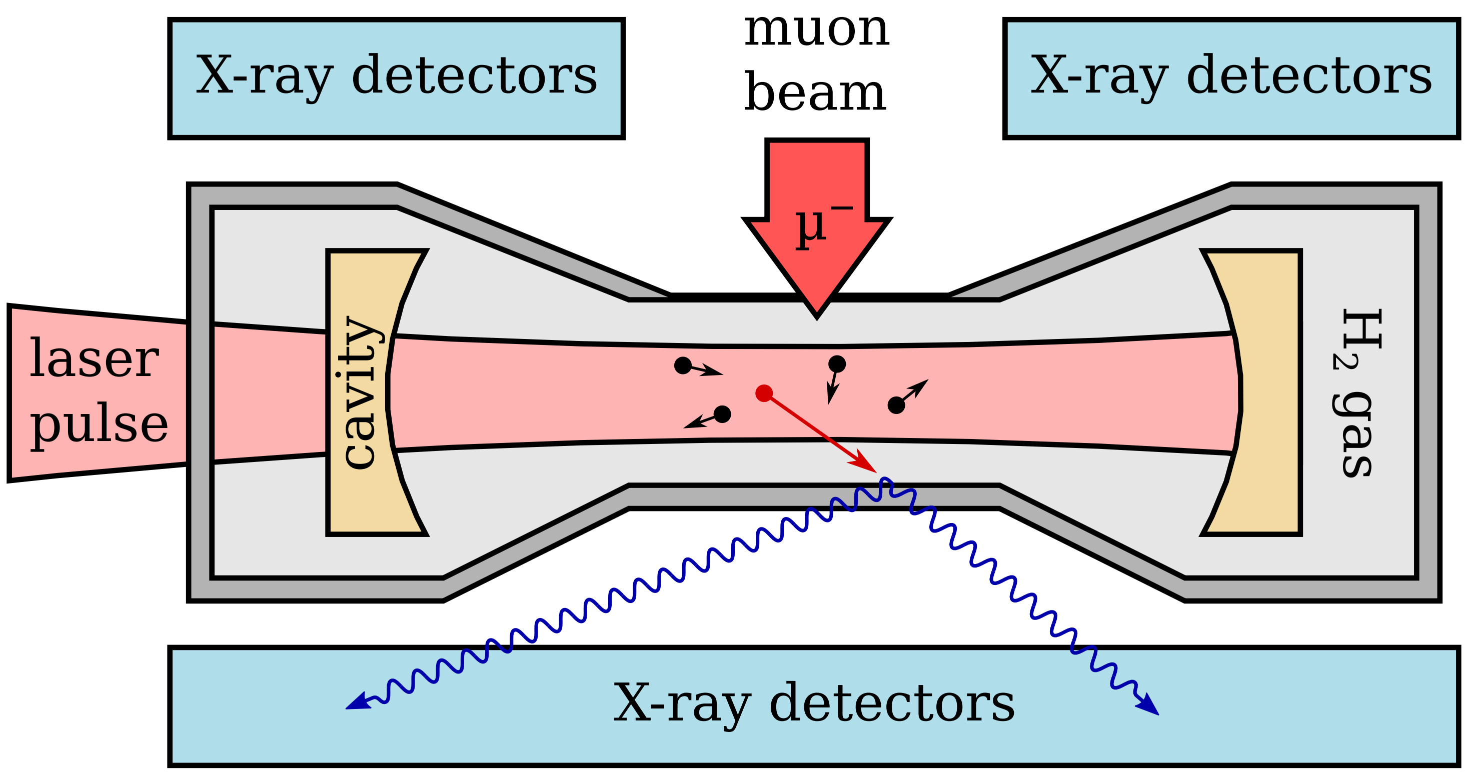
Pachucki, Karshenboim, Indelicato, Eides, Martynenko, Patkos, Yerokhin, …

Sensitive especially to axial-vector BSM contributions

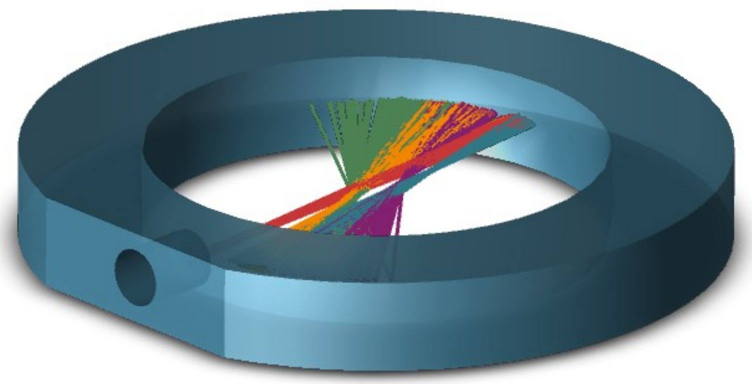
$$V_{\mathrm{HF,A}}(r) = \begin{cases} -\dfrac{2g_A^{(1)}g_A^{(2)}}{3\pi}\left(\dfrac{e^{-m_\phi r}}{r} + \dfrac{2\pi\delta^{(3)}(r)}{m_\phi^2}\right)\mathbf{S_1}\cdot\mathbf{S_2} & \text{for } m_\phi \lesssim a_0^{-1}, \\ -\dfrac{4d_v^{(A)}}{m_1 m_2}\delta^{(3)}(r)\mathbf{S_1}\cdot\mathbf{S_2} & \text{for } m_\phi \sim m_r, \end{cases}$$

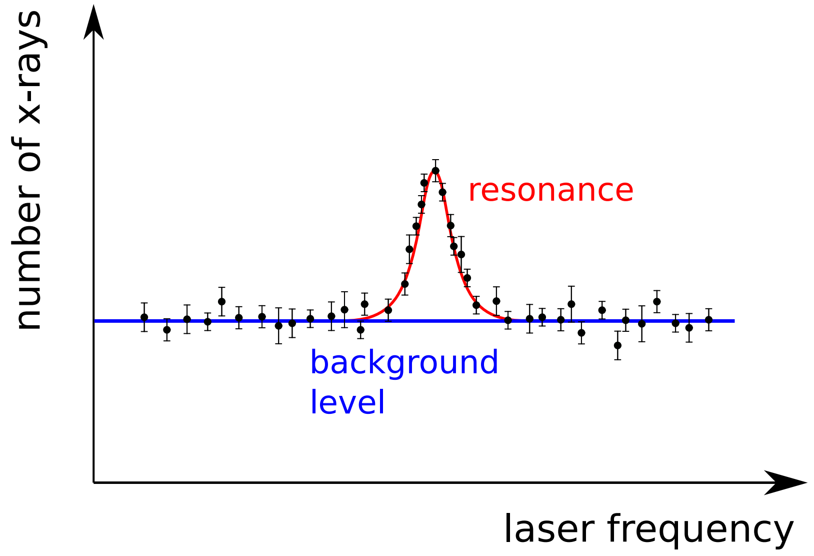
Frugiele & Peset   Stadnik

We need to face a technology leap (compared to 2S-2Pmeasurement)



- 5'000 times larger laser energy density
- Less favourable wavelength:
  6.8 μm instead of 6.0 μm
- 30 times smaller laser bandwidth and 20 times larger number of frequency points
- Indirect signature of a successful laser excitation: larger background (larger statistics need)
- Toroidal multi-pass cavity at cryogenic temperatures





Till now zero photons at 6.8μm but already a dozen of publications on lasers

Optics Express 31, Issue 18, pp. 29558-29572 (2023)

Injection-seeded high-power Yb:YAG thin-disk laser stabilized by the Pound-Drever-Hall method

MANUEL ZEYEN,[1] LUKAS AFFOLTER,[1] MARWAN ABDOU AHMED,[2] THOMAS GRAF,[2] OGUZHAN KARA,[1] KLAUS KIRCH,[1,3] ADRIAN LANGENBACH,[1] MIROSLAW MARSZALEK,[1] FRANÇOIS NEZ,[4] AHMED OUF,[5] RANDOLF POHL,[5,6] SIDDHARTH RAJAMOHANAN,[5] PAULINE YZOMBARD,[4] KARSTEN SCHUHMANN,[1] AND ALDO ANTOGNINI[1,3,*]

Rev. Sci. Instrum. 94, 013001 (2023)

RESEARCH ARTICLE | JANUARY 12 2023

Pound–Drever–Hall locking scheme free from Trojan operating points

Manuel Zeyen ; Lukas Affolter ; Marwan Abdou Ahmed ; Thomas Graf ; Oguzhan Kara ; Klaus Kirch ; Miroslaw Marszalek ; François Nez ; Ahmed Ouf ; Randolf Pohl ; Siddharth Rajamohanan; Pauline Yzombard ; Aldo Antognini ; Karsten Schuhmann

Optics Express 32, Issue 2, pp. 1218-1230 (2024)

A compact 20-pass thin-disk multipass amplifier stable against thermal lensing effects and delivering 330 mJ pulses with $M^2 < 1.17$

MANUEL ZEYEN,[1] LUKAS AFFOLTER,[1] MARWAN ABDOU AHMED,[2] THOMAS GRAF,[2] OGUZHAN KARA,[1] KLAUS KIRCH,[1,3] MIROSLAW MARSZALEK,[1] FRANÇOIS NEZ,[4] AHMED OUF,[5] RANDOLF POHL,[5,6] SIDDHARTH RAJAMOHANAN,[5] PAULINE YZOMBARD,[4] KARSTEN SCHUHMANN[1] AND ALDO ANTOGNINI[1,3,*]

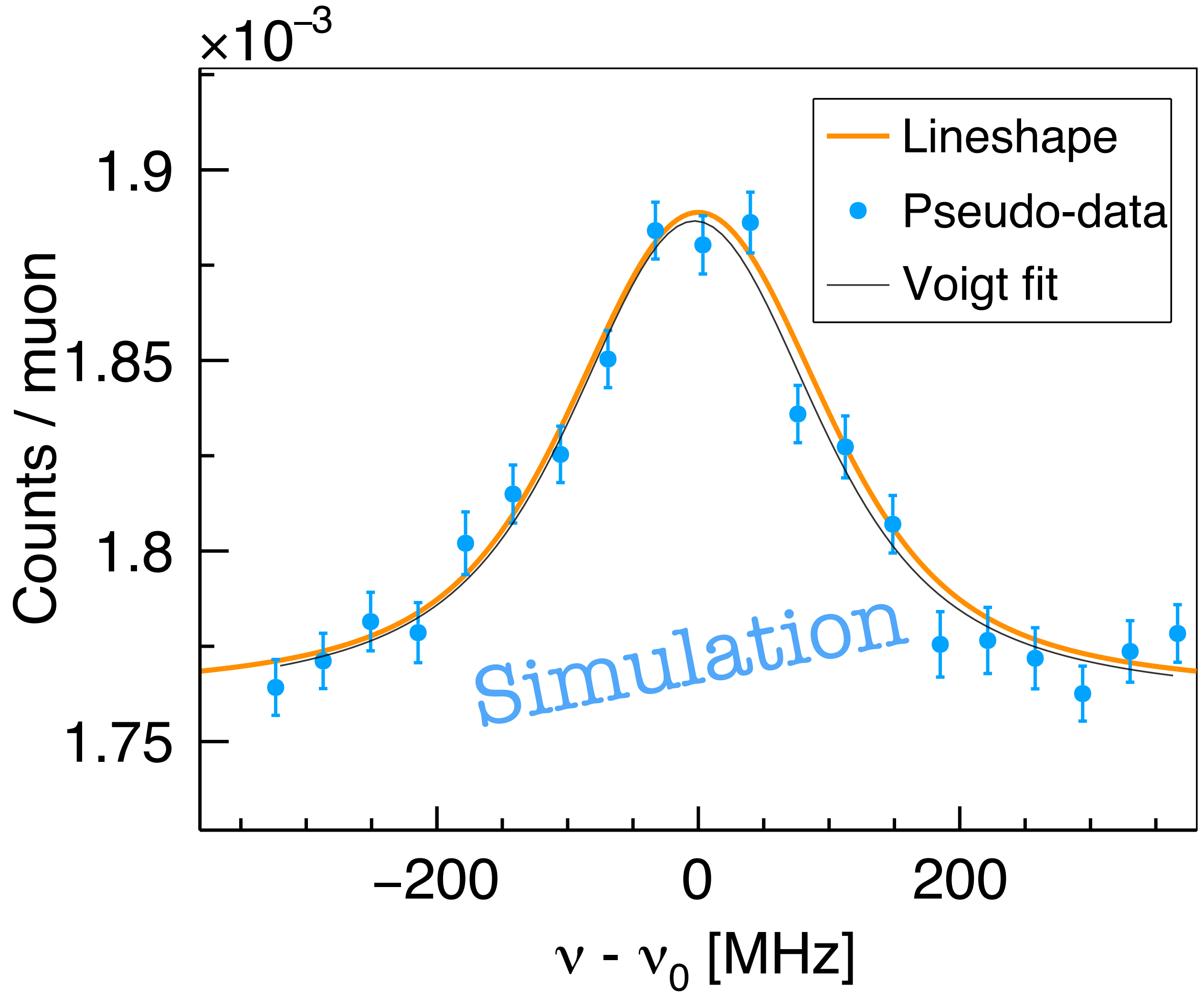
Accepted in applied optics (2024)

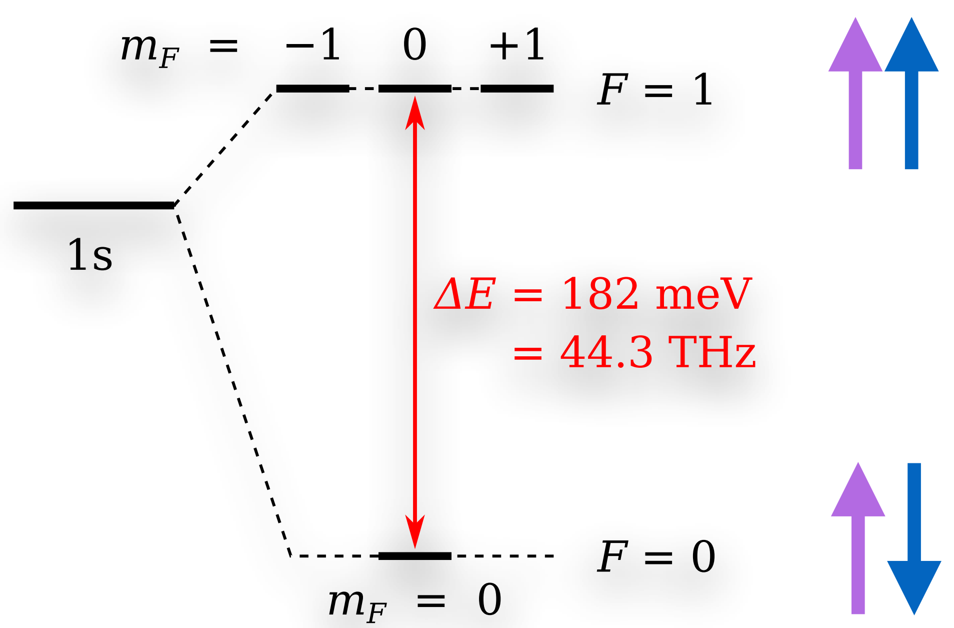
Radiant fluence from ray tracing in optical multipass systems

MIROSŁAW MARSZAŁEK[1,2*], LUKAS AFFOLTER[1], OGUZHAN KARA[1,2], KLAUS KIRCH[1,2], KARSTEN SCHUHMANN[1], MANUEL ZEYEN[1], AND ALDO ANTOGNINI[1,2]

Prospected statistical precision in 3 weeks of data taking

Nuber et al, SciPost Phys. 13, 020 (2022)





Relative accuracies below 10^{-7} are possible

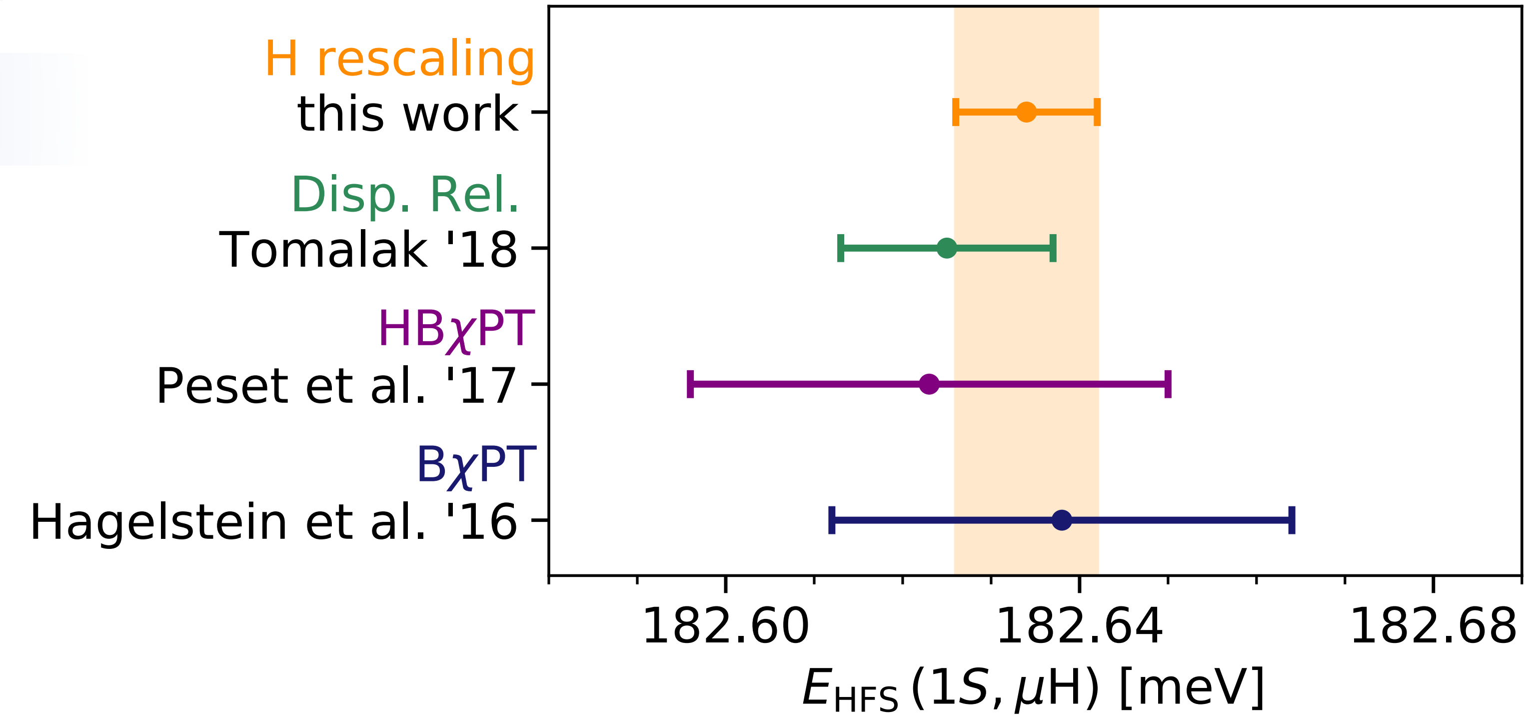
The problem: How long to search for the resonance?

Theory prediction

Antognini, Hagelstein, Pascalutsa, arXiv:2205.10076

$$\Delta E_{1S-HFS} = 182.634(8)\ \text{meV}$$

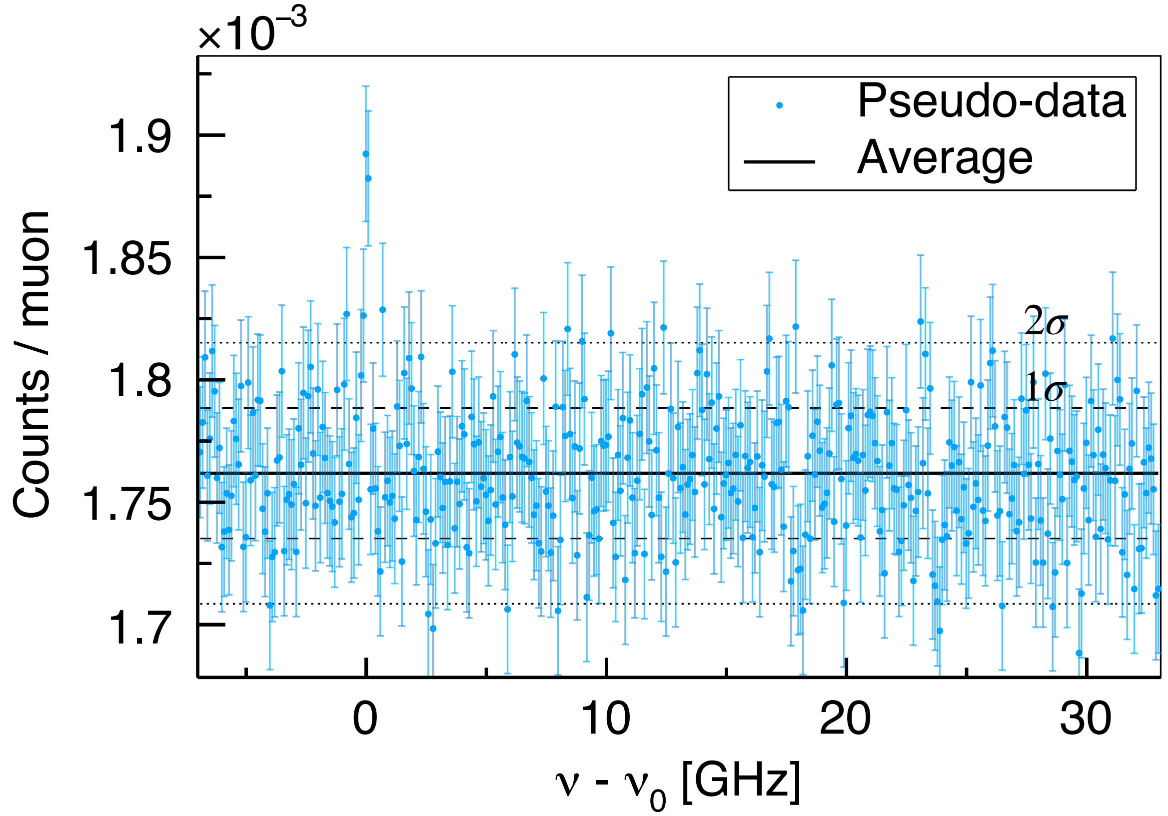
$$\delta = 4 \times 10^{-5}$$



0.16 meV (40 GHz) search range

Simulation of resonance search

- Measure in frequency steps of 100 MHz in a range of 40 GHz
- At each frequency spend 2-3 h
- Include up- and setting up times



→ Easily exceed 10 weeks of beamtime

→ This time can NOT be allocated at PSI

→ Need improved theory prediction

Conclusions

Muonic Atom Spectroscopy Theory Initiative

from PUZZLE
ASTI
to PRECISION

Theory prediction for 1S-HFS in μH needed soon **before** the experiment on the 2×10^{-5} level (1 GHz)

The experiment has the potential to reach the 1×10^{-7} level

Opens the way to test HFS in H $< 1 \times 10^{-8}$ level

Opens the way for investigating the HFS splitting in other systems: $\mu^3\mathrm{He}^+$, $\mu^6\mathrm{Li}^{++}$

First goal of the initiative

Muonic Atom Spectroscopy
Theory Initiative

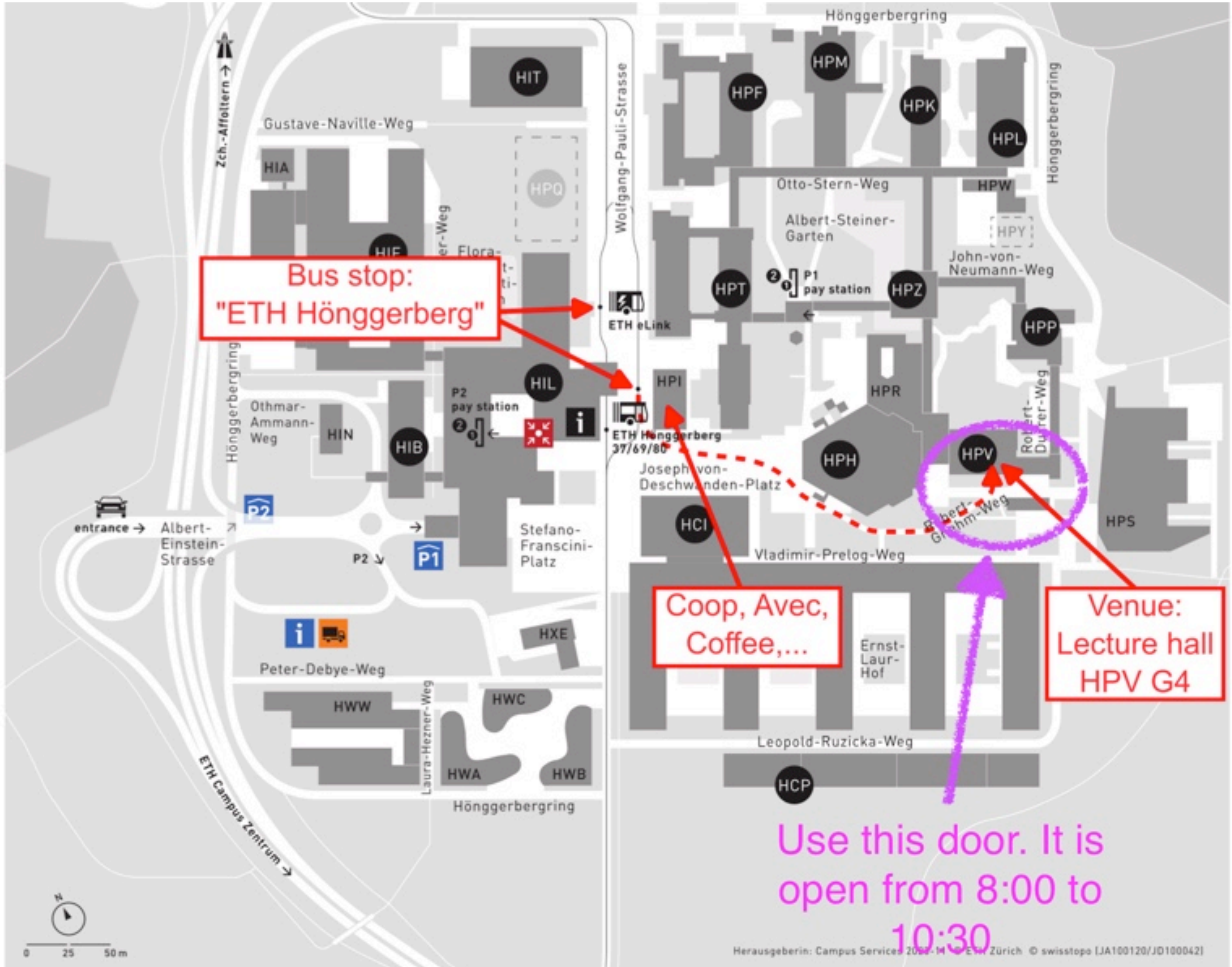
from PUZZLE
ASTI
to PRECISION

Standard Model prediction for the *S*-level hyperfine splitting in light muonic atoms

Muonic Atom Spectroscopy Theory Initiative
(Dated: June 13, 2024)

CONTENTS

For tomorrow Saturday (door is open from 8:00 to 10:30)



Restaurant for this evening (Die Waid at 19:00)