The normal-beam spin asymmetry in elastic electron scattering and its QED corrections

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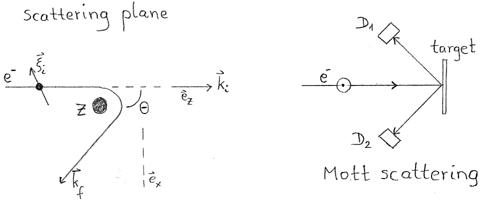
Experimental situation

Scattering of spin-polarized electrons from target nucleus



Asymmetry from relative counting-rate difference:

$A = p \cdot S$ $\qquad$ p = degree of beam polarization

S from theory $\Longrightarrow$ p from experiment

1. Leading-order theory: Potential scattering

Transition amplitude $= \langle \Psi_f | V_T | \psi_i \rangle$

Relativistic prescription of impinging electron:

$$\psi_i(\boldsymbol{r}) \sim u_{k_i} e^{i\boldsymbol{k}_i\boldsymbol{r}}, \qquad u_{k_i} \sim \begin{pmatrix} 1 \\ \frac{\boldsymbol{k}_i c}{E_i + mc^2}\,\boldsymbol{\sigma} \end{pmatrix} \chi_i$$

$\chi_i = a_{1/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_{-1/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ describes initial-state polarization $\boldsymbol{\zeta}_i$

Scattering amplitude for polarized electrons $A_{fi} = \chi_f^+ \, \hat{f} \, \chi_i$

Scattering operator for potential scattering from spin-zero nuclei:

$$\hat{f}(k_i, \theta) = A + B\, \boldsymbol{n} \cdot \boldsymbol{\sigma} \qquad \text{normal } \boldsymbol{n} \upuparrows \boldsymbol{k}_i \times \boldsymbol{k}_f$$

$\boldsymbol{\sigma}$ induces spin-flip

Scattering cross section for normal spin polarization ($\zeta_i \upuparrows \boldsymbol{n}$) and unpolarized scattered electrons:

$$\frac{d\sigma_{\mathrm{coul}}}{d\Omega}(\zeta_i) = \sum_{\sigma_f} |A_{fi}|^2 = \left(\frac{d\sigma_{\mathrm{coul}}}{d\Omega}\right)_{\mathrm{unpol}} (1 + S\,\zeta_i \boldsymbol{n})$$

Sherman function $S = \dfrac{2\,\mathrm{Re}\,\{AB^*\}}{|A|^2 + |B|^2}$ requires $B \neq 0$

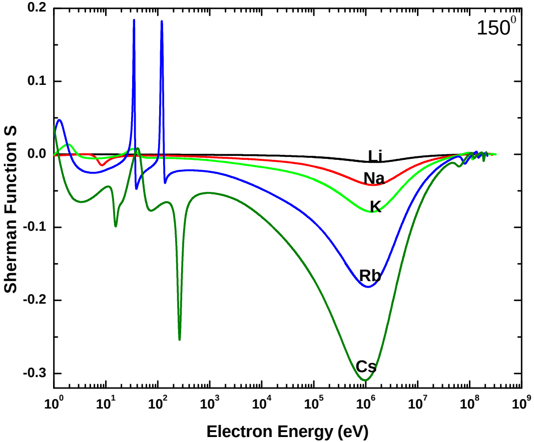
(relativistic effect)

General case:

$$S = \frac{d\sigma/d\Omega(\uparrow) - d\sigma/d\Omega(\downarrow)}{d\sigma/d\Omega(\uparrow) + d\sigma/d\Omega(\downarrow)} = \frac{(1+S) - (1-S)}{(1+S) + (1-S)}$$

Mott polarimetry: replace $\uparrow$ / $\downarrow$ by right/left detector

Sherman function for alkali atoms



A.K.F.Haque et al. (2022)

Electron-nucleus distance:

$$d \sim \frac{1}{q} \sim \frac{1}{2k_i \sin(\theta/2)}$$

$$\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f$$

momentum transfer

1 eV	1 keV	1 MeV	300 MeV
	electronic shells	Rutherford scattering	Composed nucleus

Large S $\Leftrightarrow$ heavy targets, backward scattering angles (1-10 MeV)

2. Recoil corrections to potential scattering

Recoil energy of nucleus: $E_R \approx \frac{q^2}{2M}$, M = target mass number

Final electron energy: $E_f = E_i - E_R$

Reduced collision energy: $\overline{E}_i = \sqrt{E_i E_f}$

Modified cross section from phase-shift theory:

$$\frac{d\sigma_{\text{coul}}}{d\Omega} = \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|} \frac{1}{f_{\text{rec}}} \left(|A|^2 + |B|^2 \right)\Big|_{\overline{E_i}}$$

$$f_{\text{rec}} \approx 1 + \frac{E_R}{E_i}$$

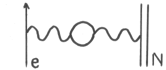
3. Two-photon corrections to potential scattering

(a) First-order Born approximation for QED

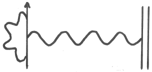
Leading-order process: $|$ − − − − − − $\|$

e N

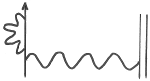
plus two-photon processes:



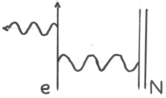
vacuum polarization



vertex correction



self energy



bremsstrahlung
($\omega \leq$ detector resolution)

Scattering amplitude

$$A_{fi}^{(1)} = A_{fi}^{\mathrm{Born}} + A_{fi}^{\mathrm{vac}} + A_{fi}^{\mathrm{vs}}$$

Differential cross section

$$\frac{d\sigma^{(1)}}{d\Omega} = \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|} \frac{1}{f_{\mathrm{rec}}} \sum_{\sigma_f} \left[\underbrace{|A_{fi}^{(1)}|^2_{\mathrm{linearized}}}_{|A_{fi}^{\mathrm{Born}}|^2 + 2\,\mathrm{Re}\,\{A_{fi}^{*\mathrm{Born}}(A_{fi}^{\mathrm{vac}} + A_{fi}^{\mathrm{vs}})\}} + \frac{d\sigma^{\mathrm{soft}}}{d\Omega} \right]$$

$$A_{fi}^{\mathrm{Born}} = \frac{\sqrt{E_i E_f}}{2\pi c^2} \int d\boldsymbol{r}\, u_{k_f}^{+} e^{-i\boldsymbol{k}_f \boldsymbol{r}}\, V_T(\boldsymbol{r})\, u_{k_i}\, e^{i\boldsymbol{k}_i \boldsymbol{r}}$$

$$= -\frac{2\sqrt{E_i E_f}}{c^2} \frac{Z}{\boldsymbol{q}^2} (u_{k_f}^{+}\, u_{k_i})\, F_L(|\boldsymbol{q}|)$$

with charge form factor

$$F_L(|\boldsymbol{q}|) = -\frac{\boldsymbol{q}^2}{4\pi Z} \int d\boldsymbol{r}\, e^{i\boldsymbol{q}\boldsymbol{r}}\, V_T(\boldsymbol{r})$$

$$(F_L = 1 \ \text{ for Coulomb field } V_T = -Z/r)$$

Following Tsai, Maximon/Tjon, Bucoveanu/Spiesberger:

$$A_{fi}^{\mathrm{vac}} \;=\; M_{fi}^{\mathrm{vac}}(q)\; A_{fi}^{\mathrm{Born}}$$

$$q \;=\; \text{4-momentum transfer:}$$
$$q^2 \;=\; (E_f - E_i)^2/c^2 \;-\; \boldsymbol{q}^2$$

$$A_{fi}^{\mathrm{vs}} \;=\; M_{fi}^{\mathrm{vs}}(q)\; A_{fi}^{\mathrm{Born}} \;+\; \underbrace{N_{fi}^{\mathrm{vs}}(q)\;(u_{k_f}^{+}\,\gamma_0\,\boldsymbol{\alpha}\boldsymbol{q}\;u_{k_i})}_{\text{small}}$$

$$\frac{d\sigma^{\mathrm{soft}}}{d\Omega} \;=\; W_{fi}^{\mathrm{soft}}(q)\;|A_{fi}^{\mathrm{Born}}|^2$$

Differential scattering cross section:

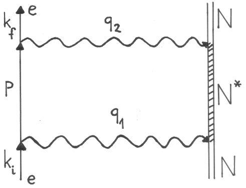
$$\frac{d\sigma^{(1)}}{d\Omega} \;\approx\; \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|}\,\frac{1}{f_{\mathrm{rec}}}\sum_{\sigma_f}|A_{fi}^{\mathrm{Born}}|^2\left(1 + 2M_{fi}^{\mathrm{vac}} + 2M_{fi}^{vs} + \; W_{fi}^{\mathrm{soft}}\right)$$

Spin asymmetry: $\quad S^{\mathrm{Born}} = 0 \;\Longrightarrow\; S^{(1)} = 0$

(b) Second-order Born approximation

| ===== |
e N

(i) Second-order elastic scattering (nucleus in ground state)
(ii) Dispersion (nucleus transiently excited)



Virtual nuclear excitation to state L^π , energy ω_L

Dispersion amplitude (Friar, Rosen 1972)

$$A_{fi}^{\text{box}} = \frac{\sqrt{E_i E_f}}{\pi^2\ c^3} \sum_{L\geq 1,\omega_L} \int d\boldsymbol{p} \frac{1}{(q_2^2+i\epsilon)(q_1^2+i\epsilon)} \sum_{\mu,\nu=0}^{3} t_{\mu\nu}\ T^{\mu\nu}_{L,\omega_L}$$

$$T^{\mu\nu}_{L,\omega_L} = \langle 0|J^\mu(\boldsymbol{q}_2)|L\omega_L\rangle\langle L\omega_L|J^\nu(\boldsymbol{q}_1)|0\rangle, \quad t_{\mu\nu} \sim \frac{1}{(E_i-\omega_L)^2 - E_p^2 + i\epsilon}$$

(nucleus) (electron)

Photon momenta $\boldsymbol{q}_1 = \boldsymbol{k}_i - \boldsymbol{p},\ \boldsymbol{q}_2 = \boldsymbol{p} - \boldsymbol{k}_f$

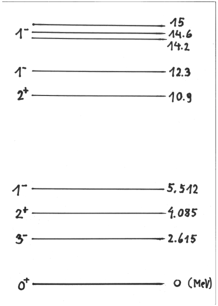
Dispersive cross section

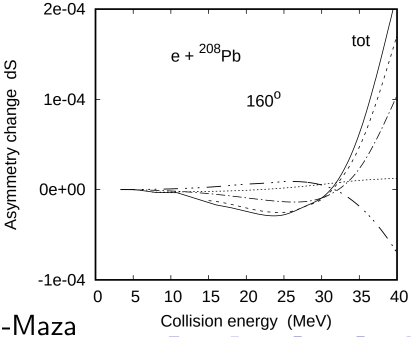
$$\frac{d\sigma^{(2)}}{d\Omega} = \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|}\,\frac{1}{f_{\rm rec}} \sum_{\sigma_f} \left(|A_{\rm coul}|^2 + 2\,\mathrm{Re}\,\{A^*_{\rm coul}\cdot A^{\rm box}_{fi}\}\right)$$

Sherman function $S_{\rm box} = \dfrac{d\sigma^{(2)}(\uparrow)/d\Omega - d\sigma^{(2)}(\downarrow)/d\Omega}{d\sigma^{(2)}(\uparrow)/d\Omega + d\sigma^{(2)}(\downarrow)/d\Omega}$

Spin asymmetry change $\Delta S_{\rm box} = \dfrac{S_{\rm box} - S_{\rm coul}}{S_{\rm coul}}$

Strong nuclear excitations of low multipolarity (^{208}Pb):





L^π ω_L (MeV) Roca-Maza

(c) Nonperturbative treatment of the QED effects

Representation of the QED effects in terms of potentials (instead of transition amplitudes)

Vacuum polarization: Uehling, Klarsfeld, Soff

$$U_e(r) \;=\; -\frac{2}{3\pi c}\int d\boldsymbol{r}'\,\frac{\varrho(r')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\chi(2c|\boldsymbol{r}-\boldsymbol{r}'|)$$

$$\chi(x) \;=\; \int_1^\infty dt\,\frac{e^{-xt}}{t}\,(1+\frac{1}{2t^2})\,(1-\frac{1}{t^2})^{\frac{1}{2}}$$

Born amplitude for scattering from Uehling potential U_e

$$A_{fi}^{\mathrm{U_e-Born}} \;=\; \frac{\sqrt{E_iE_f}}{2\pi c^2}\int d\boldsymbol{r}\;u_{k_f}^+\,U_e(r)\,e^{i\boldsymbol{qr}}\,u_{k_i}$$

$$=\;\frac{2}{3\pi c}\,\boldsymbol{q}^2\int_0^\infty r\,dr\;\chi(2cr)\,j_0(|\boldsymbol{q}|r)\cdot A_{fi}^{\mathrm{Born}} \;=\; A_{fi}^{\mathrm{vac}}$$

Vertex and self-energy correction:

Potential $\Longrightarrow$ Born amplitude
revert: $\Longleftarrow$

Define vs-potential V_{vs} by identifying

$$A_{fi}^{\mathrm{vs}} = \frac{\sqrt{E_i E_f}}{2\pi c^2} \int d\boldsymbol{r}\; u_{k_f}^{+}\, V_{\mathrm{vs}}(r)\, e^{i\boldsymbol{q}\boldsymbol{r}}\, u_{k_i} \qquad = M_{fi}^{\mathrm{vs}} A_{fi}^{\mathrm{Born}}$$

By Fourier back transformation:

$$V_{\mathrm{vs}}(r) = \frac{1}{(2\pi)^3} \int d\boldsymbol{q}\; e^{-i\boldsymbol{q}\boldsymbol{r}}\, A_{fi}^{\mathrm{vs}}\, \frac{1}{A_0}, \quad A_0 = \frac{\sqrt{E_i E_f}}{2\pi c^2}\, u_{k_f}^{+} u_{k_i}$$

$$\approx -\frac{2Z}{\pi} \int_0^{\infty} d|\boldsymbol{q}|\; j_0(|\boldsymbol{q}|r)\; F_L(|\boldsymbol{q}|) \cdot M_{fi}^{\mathrm{vs}}$$

Extended phase-shift analysis:

Solve Dirac equation

$$\left[-ic\boldsymbol{\alpha}\boldsymbol{\nabla} + \gamma_0 mc^2 + V_T(r) + U_e(r) + V_{\rm vs}(r)\right] \, \psi(\boldsymbol{r}) \; = \; E \, \psi(\boldsymbol{r})$$

Partial-wave expansion

$$\psi_i(\boldsymbol{r}) \; = \sum_{m_s=\pm\frac{1}{2}} \sum_{jl} \sqrt{\frac{2l+1}{4\pi}} \, (l0\frac{1}{2}m_s \,|\, jm) \, i^l e^{i\delta_l} \begin{pmatrix} g_l(r) \; Y_{jlm}(\hat{\boldsymbol{r}}) \\ i \, f_l(r) \; Y_{jl'm}(\hat{\boldsymbol{r}}) \end{pmatrix}$$

δ_l = phase shift

Scattering operator $\hat{f} \; = \; A^V \; + \; B^V \; \boldsymbol{n\sigma}$

$$A^V \; = \; -\frac{i}{2|\boldsymbol{k}_i|} \sum_{l=0}^{\infty} \left[(l+1)(e^{2i\delta_{-l-1}} - 1) + l(e^{2i\delta_l} - 1)\right] \; P_l(\cos\theta)$$

$$B^V \; = \; \frac{1}{2|\boldsymbol{k}_i|} \sum_{l=1}^{\infty} \left(e^{2i\delta_{-l-1}} - e^{2i\delta_l}\right) \; P_l^1(\cos\theta)$$

$$\frac{d\sigma^V}{d\Omega} = |A^V|^2 + |B^V|^2, \qquad V = V_T + U_e + V_{\mathrm{vs}}$$

Scattering cross section with QED effects and dispersion

$$\frac{d\sigma^{\mathrm{tot}}}{d\Omega} = \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|} \frac{1}{f_{\mathrm{rec}}} \left[\frac{d\sigma^V}{d\Omega} + 2\ \mathrm{Re}\ \{A^*_{\mathrm{coul}} A^{\mathrm{box}}_{fi}\} + W^{\mathrm{soft}}_{fi} \cdot \frac{d\sigma^V}{d\Omega} \right]$$

For small E_i : $\quad A^{\mathrm{box}}_{fi} \approx 0$

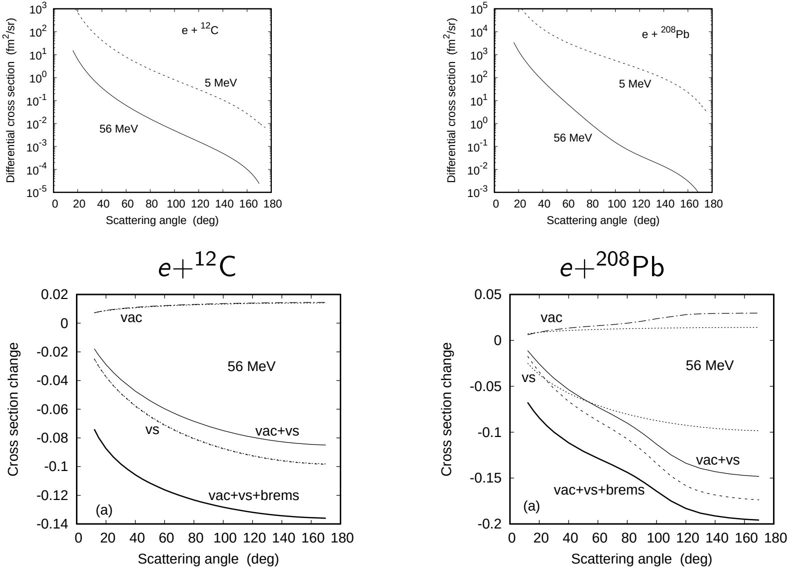
$$\Longrightarrow \quad \frac{d\sigma^{\mathrm{tot}}}{d\Omega} \approx \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|} \frac{1}{f_{\mathrm{rec}}} \left(1 + W^{\mathrm{soft}}_{fi}\right) \frac{d\sigma^V}{d\Omega}$$

Modification of the cross section by radiative effects:

$$\Delta\sigma = \frac{d\sigma^{\mathrm{tot}}/d\Omega - d\sigma_{\mathrm{coul}}/d\Omega}{d\sigma_{\mathrm{coul}}/d\Omega}$$

4. Results for $e+^{12}$C and $e+^{208}$Pb collisions

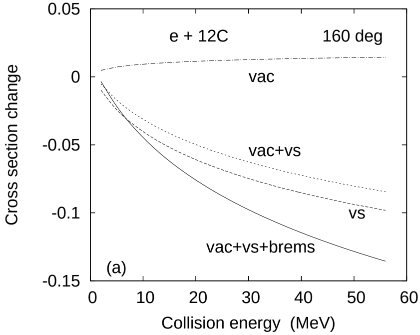
Angular distribution of the differential cross section ($\omega_0 = 1$ MeV)



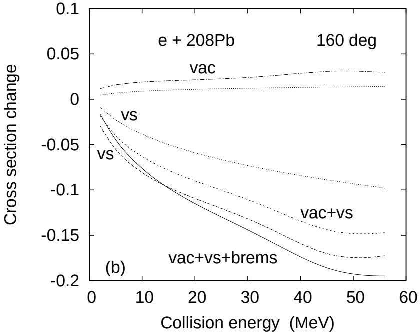
Energy distribution of cross section change

(Bremsstrahlung cut-off frequency $\omega_0 = 1$ MeV)

$e+^{12}$C, 160°



$e+^{208}$Pb, 160°

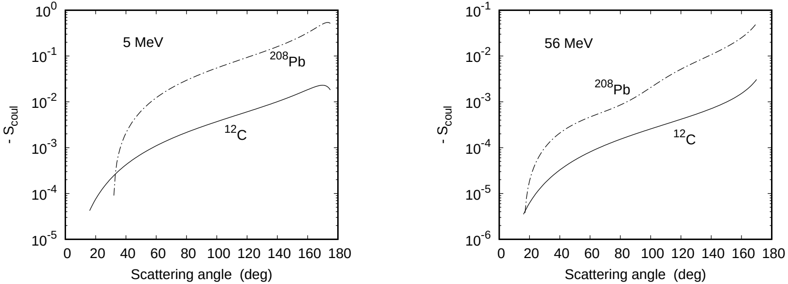


Sherman function

$$S_{\rm tot} = \frac{d\sigma^{\rm tot}/d\Omega(\uparrow) - d\sigma^{\rm tot}/d\Omega(\downarrow)}{d\sigma^{\rm tot}/d\Omega(\uparrow) + d\sigma^{\rm tot}/d\Omega(\downarrow)}$$

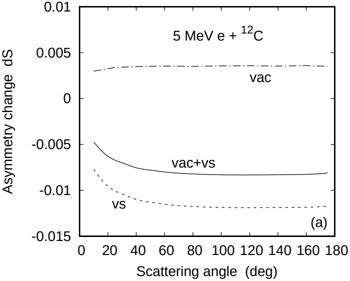
Modification of S by QED effects and dispersion

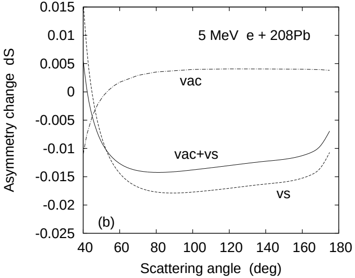
$$\Delta S_{\rm tot} = \frac{S_{\rm tot} - S_{\rm coul}}{S_{\rm coul}}, \qquad S_{\rm coul} = \frac{2\ {\rm Re}\ \{A\,B^*\}}{|A|^2 + |B|^2} \text{ with } V = V_T$$



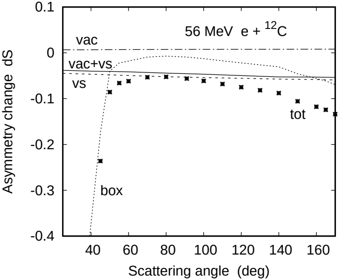
$$E_i \lesssim 10\ {\rm MeV}\ :\quad S_{\rm tot} = S_V = \frac{2\ {\rm Re}\ \{A^V\,B^{V*}\}}{|A^V|^2 + |B^V|^2}, \quad V = V_T + U_e + V_{\rm vs}$$

Angular dependence of the spin-asymmetry change

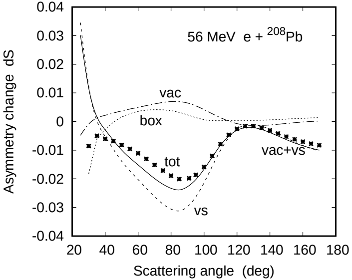




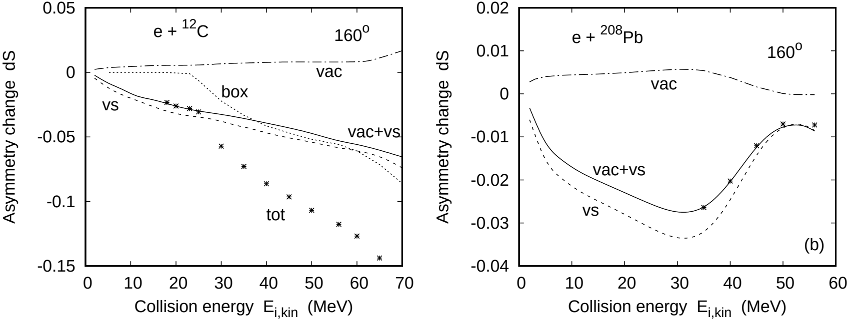
$e + {}^{12}C$



$e + {}^{208}Pb$



Energy dependence of spin-asymmetry change at backward angles



Summary

Nonperturbative treatment of vacuum polarization, vertex + self-energy correction: vital for ΔS

For $\Delta\sigma$: Important for heavy targets in backward hemisphere

Dispersion: unimportant for $\Delta\sigma$ (below 150 MeV)

ΔS: Important for E_i above 30 MeV, particularly for ^{12}C, much smaller for ^{208}Pb

very large at forward angles

Accuracy estimate for 5 MeV $e+{}^{12}$C, ^{197}Au, ^{208}Pb:

	^{12}C, 170°	^{197}Au, 173°	^{208}Pb, 173°
ΔS_{vac}	3.65e-3	3.59e-3	3.87e-3
ΔS_{vs}	-1.18e-2	-1.08e-2	-1.20e-2
ΔS_{QED}	-8.18e-3	-7.28e-3	-8.20e-3

Thank you!