CHALLENGES AT THE PRECISION FRONTIER

A CAUTIONARY TALE ON FINITE-SIZE CONTRIBUTIONS

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MOTIVATION

- Spectroscopy experiments at the precision frontier allow us to study low-energy nuclear structure, test bound-state QED, refine fundamental constants, and potentially find New Physics.
- Theory predictions need to improve in order to match the experimental uncertainties.

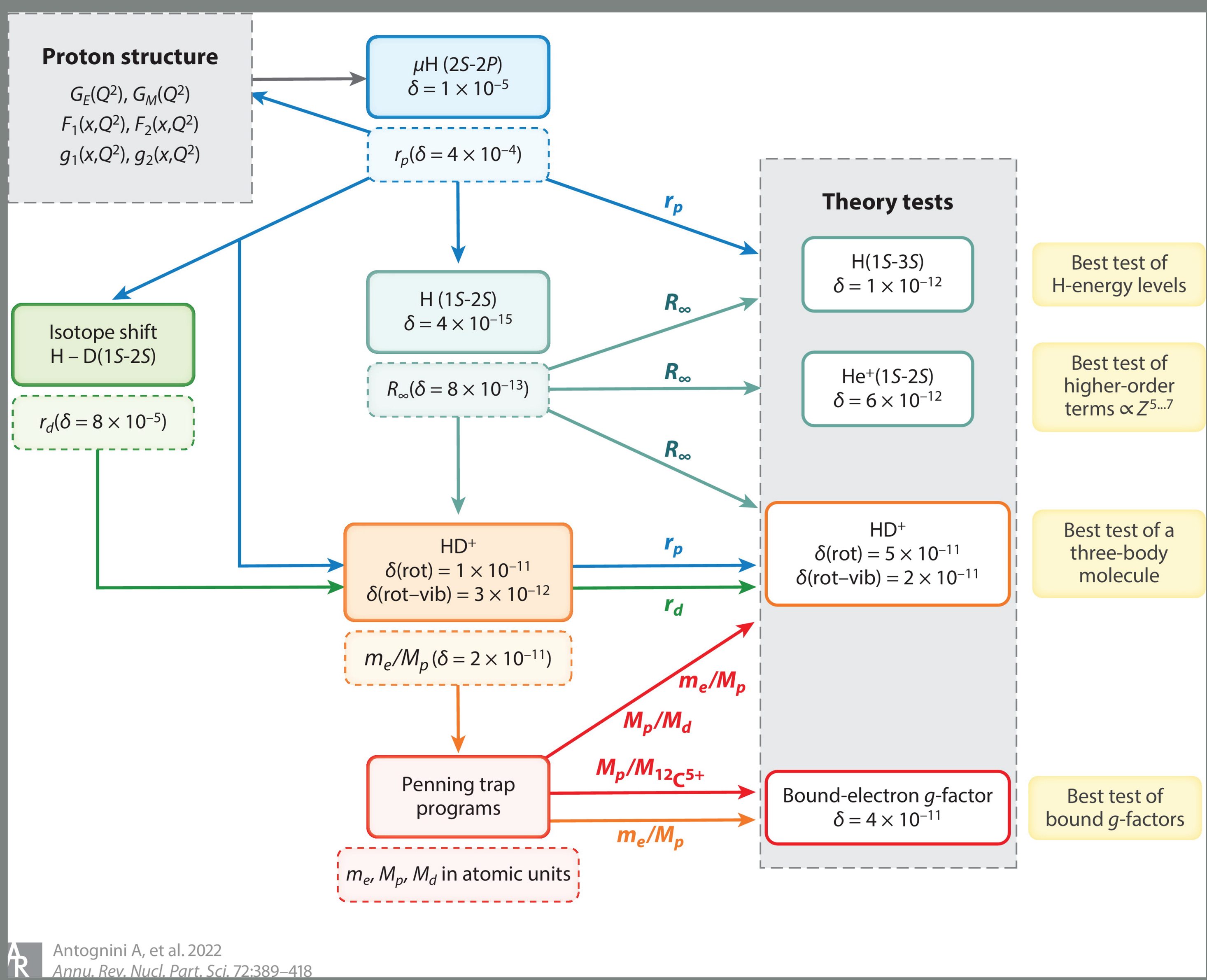
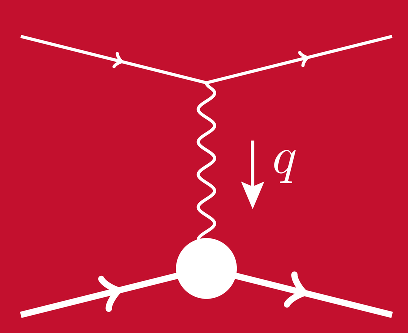


FIG 1: Interplay of experiments and theory at the intersection of atomic, particle and nuclear physics

ONE-PHOTON-EXCHANGE POTENTIAL

- Finite-size contribution to the 2P-2S Lamb shift in (muonic-) hydrogen due to the proton electric Sachs form factor $G_E(Q^2)$:



$$\Delta E_{LS} = \int_0^\infty \mathrm{d}Q\ w(Q)\ G_E(Q^2)$$

$$w(Q) = -\frac{4}{\pi}(Z\alpha)a^2Q^2\frac{1-a^2Q^2}{[1+a^2Q^2]^4}$$

$a = (Z\alpha m_r)^{-1}$ is the lepton Bohr radius, m_r is the reduced mass of the proton-lepton system and $Q^2 = -q^2$ is the squared momentum transfer.

- Finite-size expansion:

$$\Delta E_{LS} = -\frac{(Z\alpha)^4 m_r^3}{12}\left[\langle r^2\rangle_E - Z\alpha m_r\langle r^3\rangle_E\right] + O(\alpha^6)$$

where $\langle r^2\rangle_E$ and $\langle r^3\rangle_E$ are the second and third moments of the proton charge distribution:

$$\langle r^2\rangle_E = -6G_E'(0)$$

$$\langle r^3\rangle_E = \frac{48}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^4}\left[G_E(Q^2) - G_E(0) - Q^2G_E'(0)\right]$$

- The finite-size expansion breaks down in the presence of any non-smooth behavior of the form factor at low Q, due to an enhancement through the weighting functions.

F. Hagelstein & V. Pascalutsa, PRA 91, 040502 (2015); ibid. 93, 026502 (2016)

TOY MODEL

- The weighting function $w(Q)$ peaks at a scale comparable to the inverse Bohr radius of the hydrogen-like system.
- Consider three tiny hand-placed non-smooth contributions to the proton form factor, $\tilde{G}_E^{(i)}(Q^2)$ with $i = 1, 2, 3$, referred to in the following as *form factor models*.

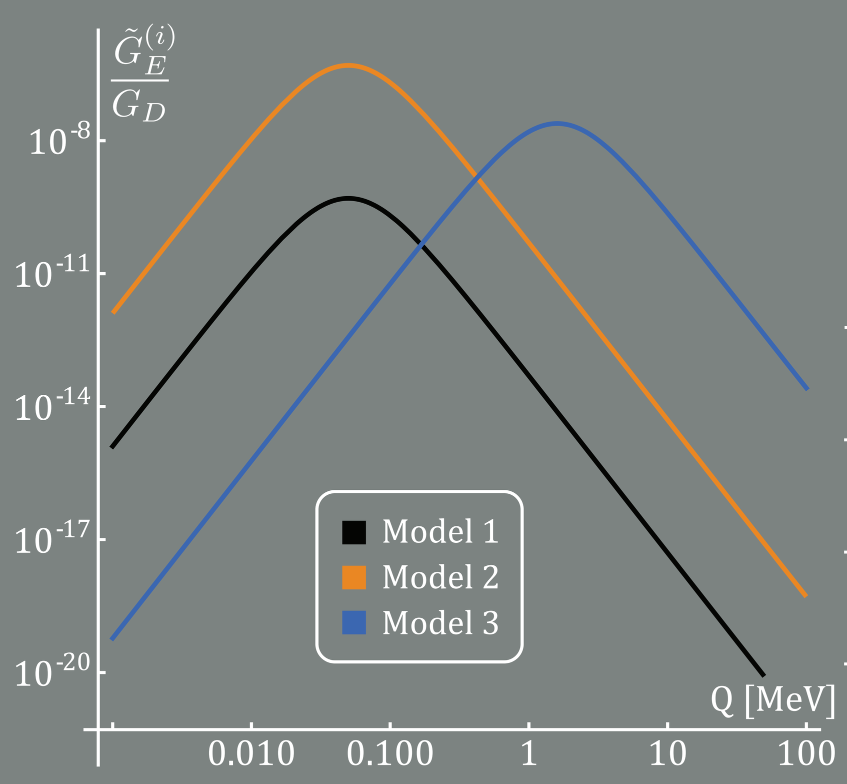


FIG 2: Form factor models normalized to standard dipole $G_D(Q^2) = [\Lambda^2/(\Lambda^2+Q^2)]^2$ with $\Lambda^2 = 0.71\mathrm{GeV}^2$

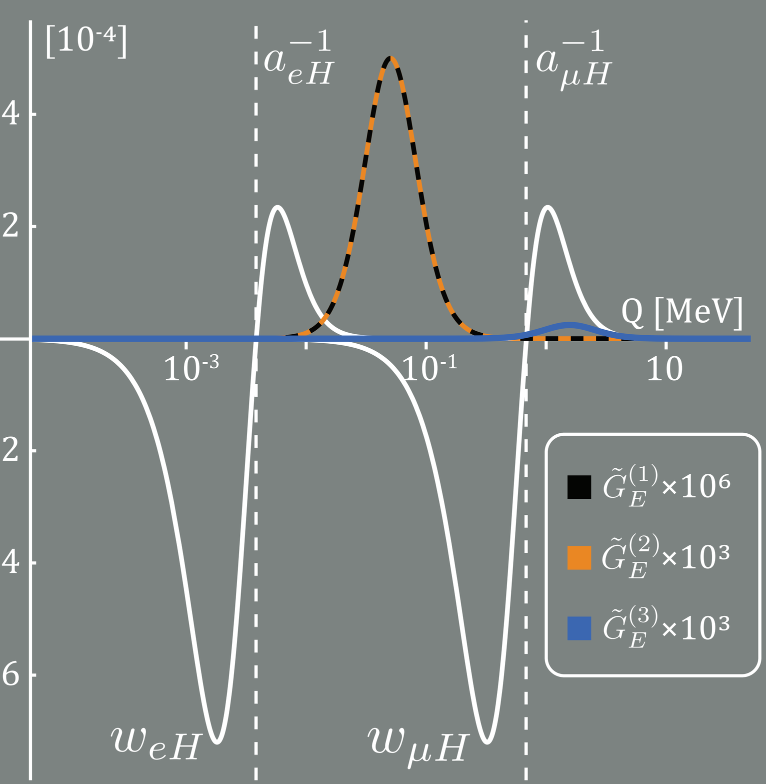


FIG 3: Weighting functions for hydrogen (eH) and muonic hydrogen (μH) compared to form factor models

TABLE 1: Contributions of the form factor models to the Lamb shift. Contributions that would affect experimental measurements based on current experimental accuracy are marked in red

ΔE_{LS}	MODEL 1	MODEL 2	MODEL 3	CURRENT EXPERIMENTAL ACCURACY
eH [10^{-2} neV]	3.2	3150.8	8.2×10^{-3}	1.3
μH [μeV]	-5.4×10^{-3}	-5.4	4.6	2.3

Depending on the placement and magnitude of the contribution, we see a **measurable effect** in the Lamb shift of either eH or μH.

Minute contributions to the form factor in the low-Q region far beyond the present accuracy and reach of scattering experiments can lead to **non-negligible finite-size effects** that should be estimated in an effective-field theory framework.

STANDARD MODEL SCENARIO

- The Standard Model could potentially generate contributions that have a disproportionate effect on the Lamb shift for hydrogen-like atoms.
- It is convenient to express G_E through a dispersion relation:

$$G_E(Q^2) = \frac{1}{\pi}\int_{t_0}^\infty \mathrm{d}t\ \frac{\mathrm{Im}G_E(t)}{t+Q^2}$$

where t_0 is the threshold above which G_E has an imaginary part.

- Consider contributions to G_E whose imaginary part starts at low t_0, for example through a light t-channel cut:

$$t_0 = 4m_e^2 \simeq 1.04\mathrm{MeV}^2$$

FIG 4: A potentially dangerous contribution to the μH Lamb shift

- Even in the SM we need to be careful not to underestimate uncertainties in the theory prediction, but also...

Atomic systems are sensitive to light New Physics at the scale of their inverse Bohr radii!

LIGHT NEW PHYSICS?

DISCUSS!

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