The Role of Precise Nuclear Radii in Precision Tests of SM with Nuclei

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Outline

Precision tests of the Standard Model with β-decays

Precise V_{ud} from superallowed decays

Status of isospin-symmetry breaking correction δ_C

Nuclear charge radii constrain δ_C

Summary, Caveats & Outlook

Precision tests of the Standard Model with β-decays

Universality, Completeness & CKM unitarity

Fermi constant from muon lifetime: $G_F = G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

$$\mathscr{L}_{e\mu} = -2\sqrt{2}G_\mu \bar{e}\gamma_\alpha \nu_{eL} \cdot \bar{\nu_{\mu L}}\gamma^\alpha \mu + \mathrm{h.c.}$$

SM: same W-coupling to LH leptons and quarks, but strength shared between 3 generations

$$\mathscr{L}_{eq} = -\sqrt{2}G_\mu \bar{e}\gamma_\mu \nu_{eL} \cdot \bar{U}_i \gamma^\mu (1-\gamma_5) V_{ij} D_j + \mathrm{h.c.}$$

$$U_i = (u, c, t)^T$$
$$D_j = (d, s, b)^T$$

Universality + Completeness of SM (only 3 gen's) —> unitary CKM matrix $V^\dagger V = 1$

Top-row unitarity condition: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

At low energy accessible via β-decas of hadrons, e.g. $n \to pe\bar{\nu}$

$$\mathscr{L}_{e\nu pn} = -\sqrt{2}G_\mu V_{ud} \bar{e}\gamma_\mu \nu_L \cdot \bar{p}\gamma^\mu (g_V^{pn} - g_A^{pn}\gamma_5) n + \mathrm{h.c.}$$

Conserved vector current: $g_V^{pn} = 1 + O((m_d - m_u)^2)$ but $g_A^{ud} = 1 \to g_A^{pn} \approx 1.276$

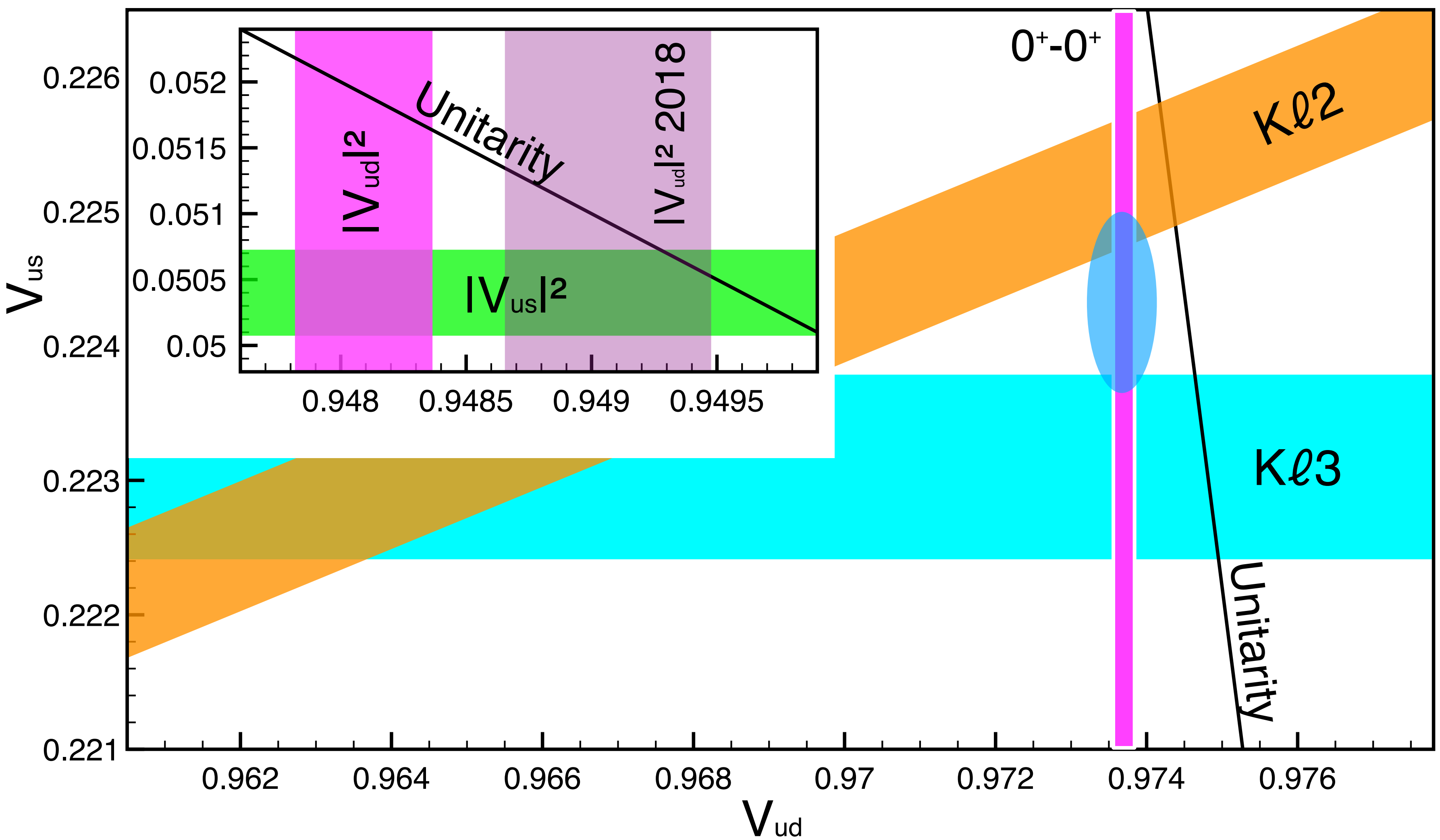
Precise measurements of g_V —> precision tests of EW sector of SM (currently 0.02%)

Get rid of g_A —> superallowed nuclear decays between states $J^P = 0^+$

Top-row CKM unitarity deficit

$$|V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 $\sim 10^{-5}$



Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions

Most precise V_{ud} from superallowed nuclear decays

Status of V_{ud}

0^+-0^+ nuclear decays: long-standing champion

$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.97370\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

Nuclear uncertainty x 3

Neutron decay: discrepancies in lifetime τ_n and axial charge g_A; competitive!

$$|V_{ud}|^2 = \frac{5024.7\text{ s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$$

Single best measurements only

$$|V_{ud}^{\text{free n}}| = 0.9733\,(2)_{\tau_n}(3)_{g_A}(1)_{RC}[4]_{total}$$

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9733\,(3)_{\tau_n}(8)_{g_A}(1)_{RC}[9]_{total}$$

RC not a limiting factor: more precise experiments a-coming

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)}\frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\text{s}^{-1}}$$

$$|V_{ud}^{\pi\ell 3}| = 0.9739\,(27)_{exp}(1)_{RC}$$

Future exp: 1 o.o.m. (PIONEER)

Status of V_{ud}

Major reduction of uncertainties in the past few years

Theory

Universal correction Δ_R^V to free and bound neutron decay

Identified 40 years ago as the bottleneck for precision improvement

Novel approach dispersion relations + experimental data + lattice QCD

$$\Delta_R^V = 0.02467(22)$$

Factor 2 improvement

C-Y Seng et al., Phys.Rev.Lett. 121 (2018) 24, 241804;
C-Y Seng, MG, M.J. Ramsey-Musolf, Phys.Rev. D 100 (2019) 1, 013001;
A. Czarnecki, B. Marciano, A. Sirlin, Phys.Rev. D 100 (2019) 7, 073008
C-Y Seng, X. Feng, MG, L-C Jin, Phys.Rev. D 101 (2020) 11, 111301;
K. Shiells, P. Blunden, W. Melnitchouk, Phys. Rev. D 104 (2021) 033003;
L. Hayen, Phys. Rev. D 103 (2021) 113001

RC to semileptonic pion decay

$$\delta = 0.0332(3)$$

X. Feng, MG, L-C Jin, P-X Ma, C-Y Seng, Phys.Rev.Lett. 124 (2020) 19, 192002

Factor 3 improvement

Experiment

$$g_A = -1.27641(56)$$

Factor 4 improvement

PERKEO-III *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

$$g_A = -1.2677(28)$$

aSPECT *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506*

$$\tau_n = 877.75(28)^{+16}_{-12}$$

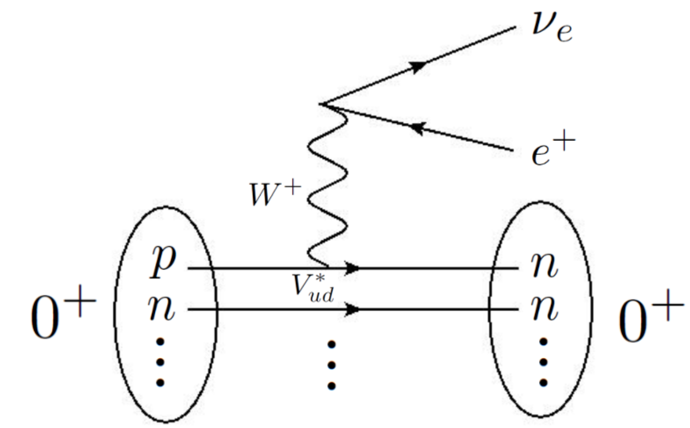
Factor 2-3 improvement

UCNτ *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

Precise V_{ud} from superallowed nuclear decays and BSM searches

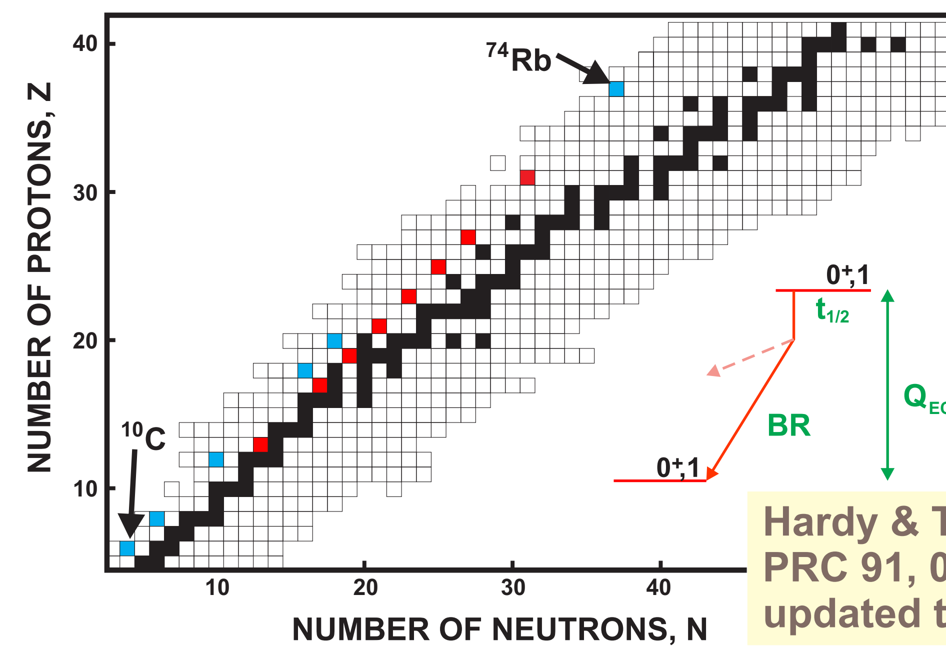
Precise V_{ud} from superallowed decays

Superallowed 0^+-0^+ nuclear decays:
- only conserved vector current
- many decays
- all rates equal modulo phase space

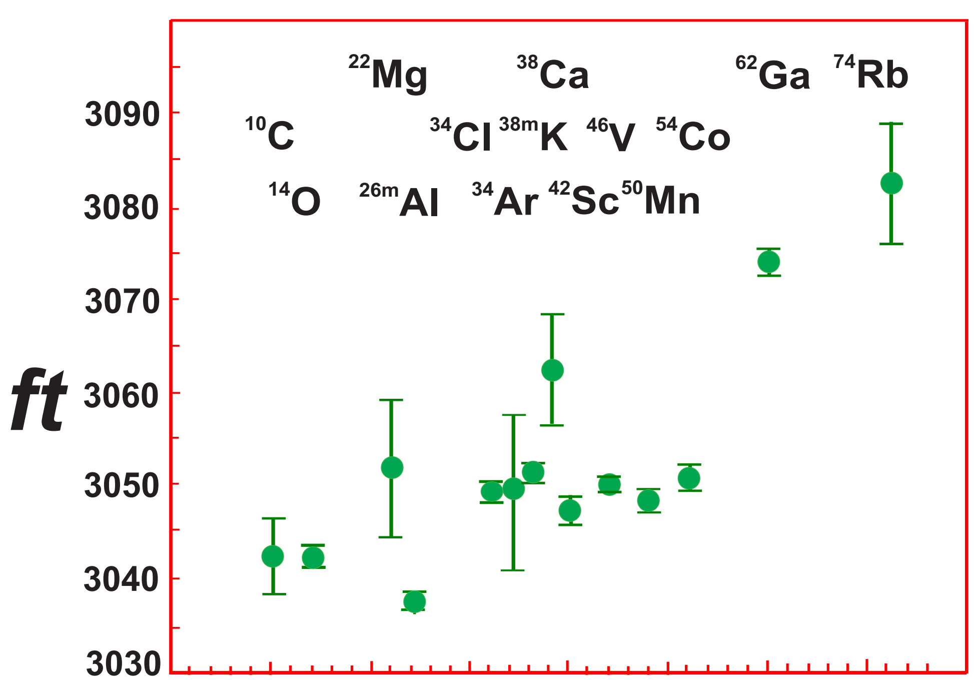


Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- 8 cases with ***ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**



Hardy & Towner PRC 91, 025501 (2015) updated to 2017



ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken

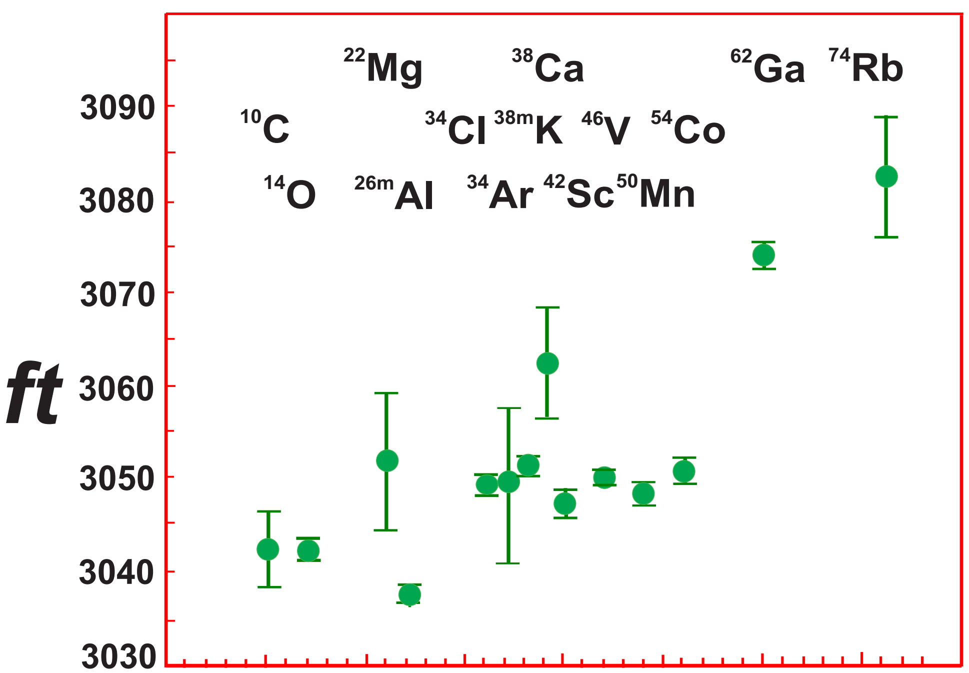
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
   (proton and neutron distribution not the same)

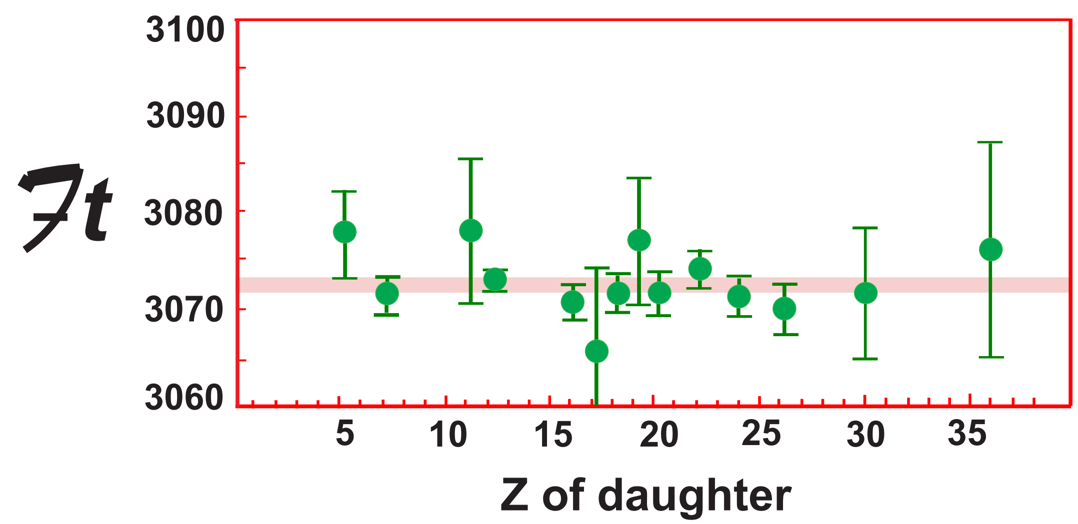
Precise V_{ud} from superallowed decays

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_R^V) = \mathrm{ft}(1 + \delta_R')(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

Outer: QED — Isospin-breaking — Nuclear structure — Universal inner





Average of 14 decays - 0.02%

$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$

Hardy, Towner 1973 - 2020

Status of isospin-breaking correction δ_C

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

τ^+ — Isospin operator

$|i\rangle, |f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states (e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2 (1 - \delta_C)$$

ISB correction is crucial for V_{ud} extraction

HT: shell model with *phenomenological* Woods-Saxon potential locally adjusted to:

- Masses of the isotriplet T=1, 0+ (IMME)
- Neutron and proton separation energies
- Known charge radii of stable isotopes

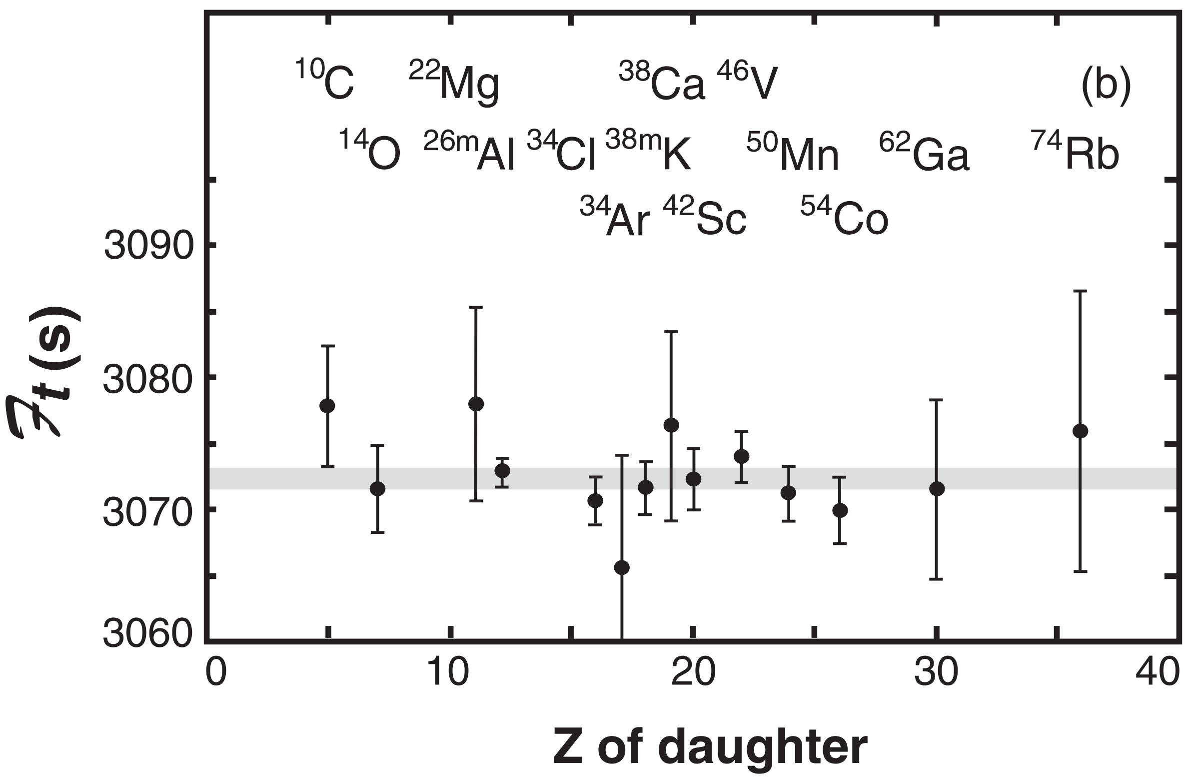
TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

$$\delta_C \sim 0.17\% - 1.6\%\,!$$

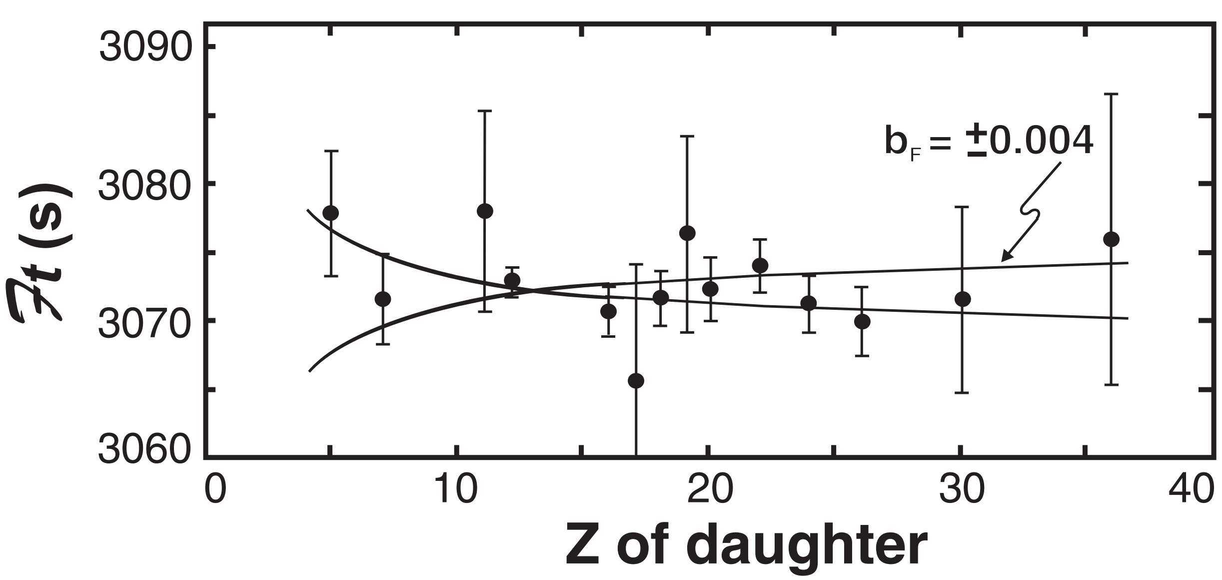
ISB or scalar BSM interactions?



Once all corrections are included:
CVC —> Ft constant

δ_C particularly important for alignment!

Fit to 14 transitions:
Ft constant within 0.02% if using SM-WS



If BSM scalar currents present: "Fierz interference" b_F

$$\mathscr{F}t^{SM} \rightarrow \mathscr{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$Q_{EC} \uparrow$ with Z —> effect of $b_F \downarrow$ with Z
Introduces nonlinearity in the Ft plot

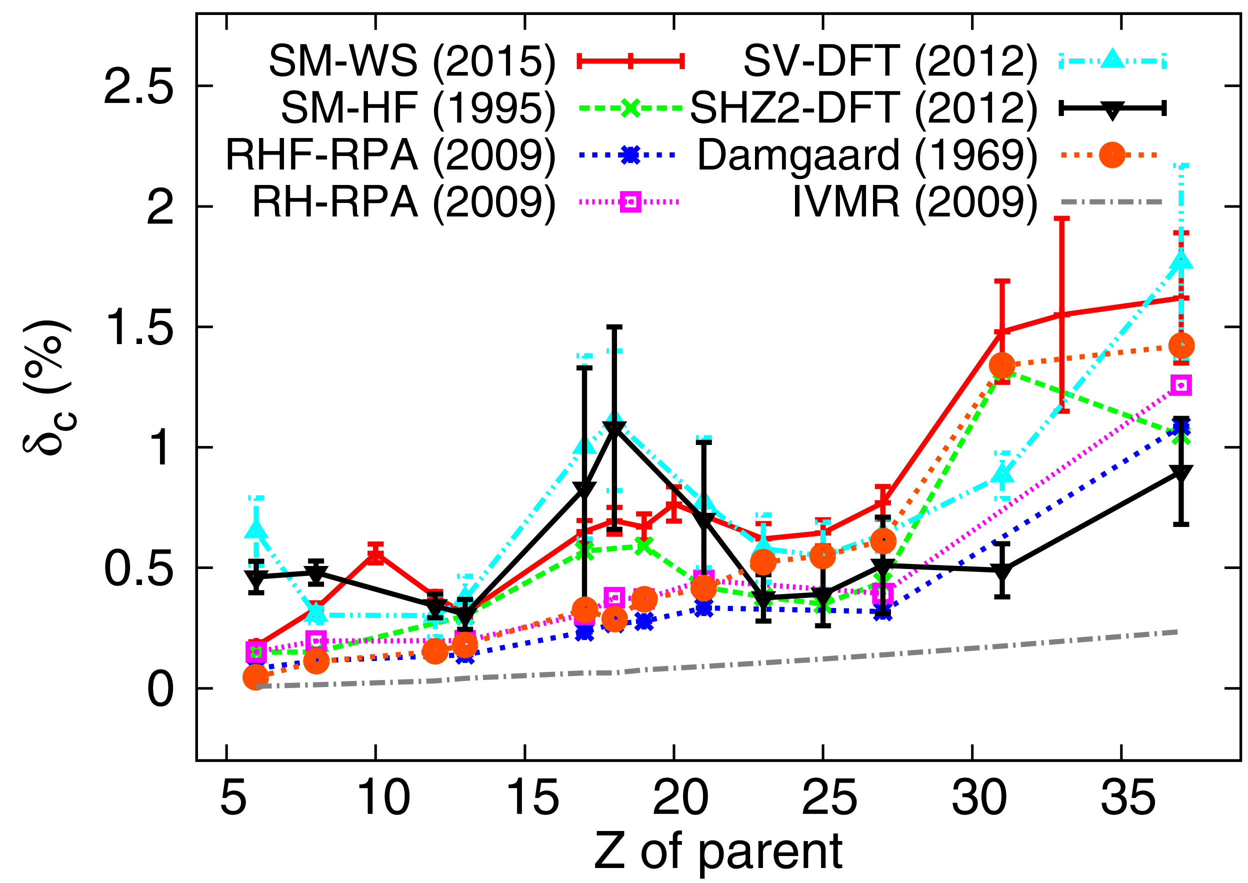
$b_F = -0.0028(26)$ ~ consistent with 0

Nuclear model comparison for δ_C

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

	SM-WS	SM-HF	RPA PKO1	RPA DD-ME2	RPA PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



HT: χ^2 as criterion to prefer SM-WS; V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations (NCSM, GFMC, CC, IMSRG)
Especially interesting for light nuclei accessible to different techniques!

Precise nuclear EW radii constrain δ_C

Phenomenological constraints on δ_C?

Idea: δ_C dominated by Coulomb repulsion between protons (hence C)

Coulomb interaction generates both δ_C and ISB combinations of nuclear radii

Miller, Schwenk 0805.0603; 0910.2790; Auerbach 0811.4742; 2101.06199; Seng, MG 2208.03037; 2304.03800; 2212.02681

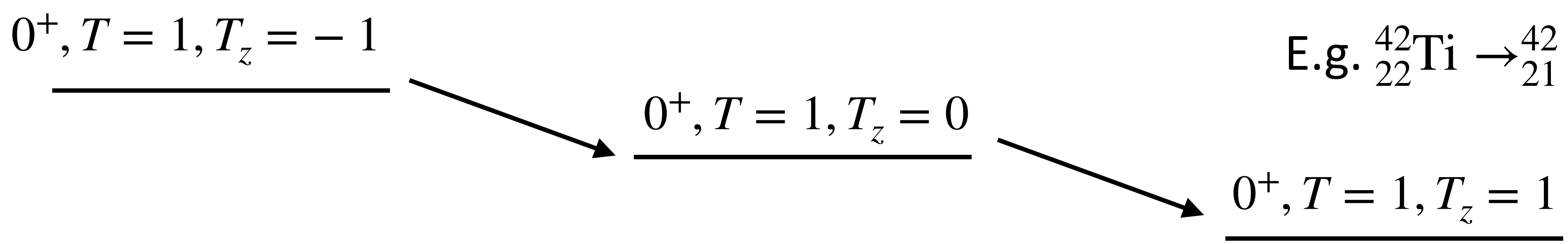
Nuclear Hamiltonian: $H = H_0 + V_{\rm ISB} \approx H_0 + V_C$

Coulomb potential for uniformly charged sphere $$V_C \approx -\frac{Ze^2}{4\pi R_C^3}\sum_{i=1}^{A}\left(\frac{1}{2}r_i^2 - \frac{3}{2}R_C^2\right)\left(\frac{1}{2} - \hat{T}_z(i)\right)$$

ISB due to IV monopole, $V_{\rm ISB} \approx \frac{Ze^2}{8\pi R^3}\sum_i r_i^2 \hat{T}_z(i) = \frac{Ze^2}{8\pi R^3}\hat{M}_0^{(1)}$

Same operator generates nuclear radii $$R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2\left(\frac{1}{2} \mp \hat{T}_z(i)\right)|\phi\rangle}$$

Phenomenological constraints on δ_C?



E.g. ${}^{42}_{22}\mathrm{Ti} \to {}^{42}_{21}\mathrm{Sc} \to {}^{42}_{20}\mathrm{Ca}$

ISB-sensitive combinations of radii: Wigner-Eckart theorem

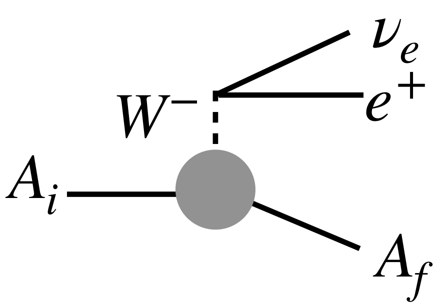
$$\Delta M_A^{(1)} \equiv \langle f | M_{+1}^{(1)} | i \rangle + \langle f | M_0^{(1)} | f \rangle$$

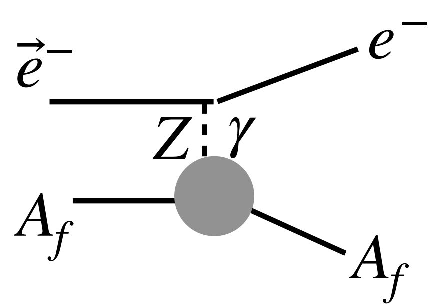
Transition radius From β spectrum

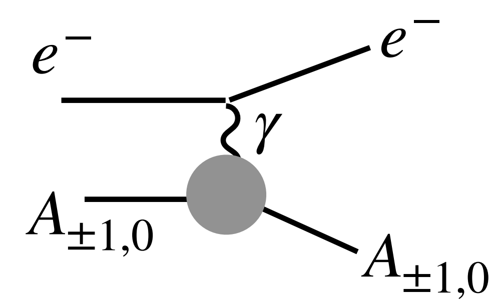
Neutron skin From PVES

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

Charge radii from atomic spectra and electron scattering







$F_{CW}(Q^2) = 1 - R_{CW}^2 Q^2/6 + \ldots$

$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{Q_W}{Z}\frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$F_{Ch}(Q^2) = 1 - R_{Ch}^2 Q^2/6 + \ldots$

Since N ≠ Z for $T_z = \pm 1$ factors $Z_{\pm1,0}$ remove the symmetry energy to isolate ISB

(Usually PVES —> neutron skins —> symmetry energy —> nuclear EOS —> nuclear astrophysics)

Electroweak radii constrain ISB in superallowed β-decay

Employ the correct isospin formalism by **Schwenk, Miller 0805.0603; 0910.2790**

δ_C and radii expressed via the same set of matrix elements

$$\delta_\mathrm{C} = \frac{1}{3}\sum_{a} \frac{|\langle a;0||V||g;1\rangle|^2}{(E_{a,0}-E_{g,1})^2} + \frac{1}{2}\sum_{a\neq g} \frac{|\langle a;1||V||g;1\rangle|^2}{(E_{a,1}-E_{g,1})^2} - \frac{5}{6}\sum_{a} \frac{|\langle a;2||V||g;1\rangle|^2}{(E_{a,2}-E_{g,1})^2} + \mathcal{O}(V^3)$$

$$\Delta M_A^{(1)} = \frac{1}{3}\Gamma_0 + \frac{1}{2}\Gamma_1 + \frac{7}{6}\Gamma_2$$

$$\Delta M_B^{(1)} = \frac{2}{3}\Gamma_0 - \Gamma_1 + \frac{1}{3}\Gamma_2$$

$$\Gamma_T = -\sum_a \frac{|\langle a;T||V||g;1\rangle|^2}{E_{a,T}-E_{g,1}}$$

Different scaling with ISB: $\delta_C \sim \mathrm{ISB}^2,\ \Delta M_A^{(1)} \sim \mathrm{ISB}^1,\ \Delta M_B^{(1)} \sim \mathrm{ISB}^3$

Compare to IMME (masses across an isomultiplet)

$$E(a,T,T_z) = \mathtt{a}(a,T) + \mathtt{b}(a,T)T_z + \mathtt{c}(a,T)T_z^2$$

$$\mathtt{b} \sim \langle a;T,T_z|V^{(1)}|a;T,T_z\rangle\ ,\ \ \mathtt{c} \sim \langle a;T,T_z|V^{(2)}|a;T,T_z\rangle$$

Unlike δ_C, $\Delta M_{A,B}^{(1)}$ — IMME only depends on diagonal m.e. — indirect constraint

Electroweak radii constrain ISB in superallowed β-decay

For numerical analysis: lowest isovector monopole resonance dominates
One ISB matrix element, one energy splitting

Model for δ_C → prediction for $\Delta M^{(1)}_{A,B}$

Transitions	δ_C (%)					$\Delta M_A^{(1)}$ (fm^2)					$\Delta M_B^{(1)}$ (fm^2)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
$^{26m}\mathrm{Al} \to {}^{26}\mathrm{Mg}$	0.310	0.329	0.30	0.139	0.08	-2.2	-2.3	-2.1	-1.0	-0.6	-0.12	-0.12	-0.11	-0.05	-0.03
$^{34}\mathrm{Cl} \to {}^{34}\mathrm{S}$	0.613	0.75	0.57	0.234	0.13	-5.0	-6.1	-4.6	-1.9	-1.0	-0.17	-0.21	-0.16	-0.06	-0.04
$^{38m}\mathrm{K} \to {}^{38}\mathrm{Ar}$	0.628	1.7	0.59	0.278	0.15	-5.4	-14.6	-5.1	-2.4	-1.3	-0.15	-0.42	-0.15	-0.07	-0.04
$^{42}\mathrm{Sc} \to {}^{42}\mathrm{Ca}$	0.690	0.77	0.42	0.333	0.18	-6.2	-6.9	-3.8	-3.0	-1.6	-0.15	-0.17	-0.09	-0.07	-0.04
$^{46}\mathrm{V} \to {}^{46}\mathrm{Ti}$	0.620	0.563	0.38	/	0.21	-5.8	-5.3	-3.6	/	-2.0	-0.12	-0.11	-0.08	/	-0.04
$^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}$	0.660	0.476	0.35	/	0.24	-6.4	-4.6	-3.4	/	-2.4	-0.12	-0.09	-0.06	/	-0.04
$^{54}\mathrm{Co} \to {}^{54}\mathrm{Fe}$	0.770	0.586	0.44	0.319	0.28	-7.8	-5.9	-4.4	-3.2	-2.8	-0.13	-0.10	-0.07	-0.05	-0.05

Can discriminate models if independent information on nuclear radii is available
ΔM_A from measured radii —> test models for δ_C

Charge radii across superallowed isotriplets?
Some are known (but difficult — unstable isotopes, some g.s. are not 0^+)
Typically, precision is not enough to make a quantitative statement — need to improve!

Precise nuclear radii beyond δ_C

Impact of atomic spectra and nuclear radii?

We said that ft-values are experimental — but not quite!
A few theory ingredients are absorbed: Coulomb distortions, nuclear form factors, atomic screening…

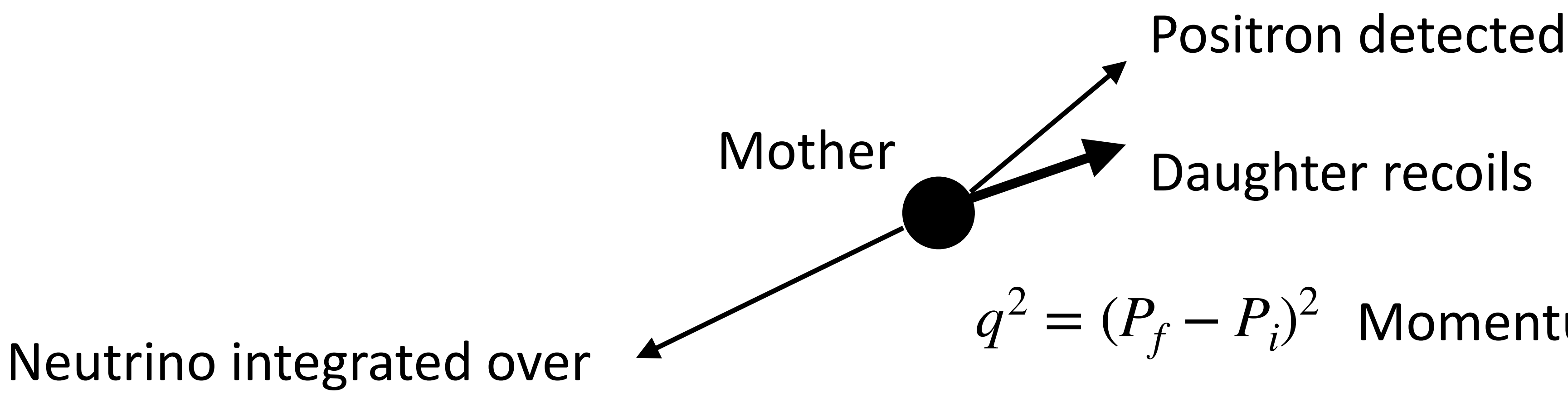
Statistical rate function: $f \approx m_e^{-5} \int_{m_e}^{E_0(Z)} |\vec{p}_e| E_e (E_0 - E_e)^2 F(Z,E_e) S(Z,E_e) C(Z,E_e) \ldots dE_e$

- Fermi Function $F(Z,E_e)$: point Coulomb, finite size, … (pointlike CC transition!)
- Weak CC form factor effect $C(Z,E_e)$: integrating over the neutrino momentum (tree-level)
- Shape factor $S(Z,E_e)$: overlap of CC and charge FF

Fermi function: analytical point-Coulomb $F_0(Z,E_e)$ - regularized at the nuclear radius (def.!)
—> Uniform sphere of radius $R = \sqrt{5/3}R_{Ch}$, can evaluate at origin, finite at origin
—> Correct for the finite surface thickness: employ e.g. 2pF charge density
—> Open question: how important further correcting the charge density (sum of Gaussians?)

Work ongoing with
Chien Yeah Seng (INT/FRIB), Giovanni Carotenuto, Michela Sestu, Matteo Cadeddu, Nicola Cargioli (INFN Cagliari)

Charge radii + isospin symmetry —> CC weak radius



$q^2 = (P_f - P_i)^2$ Momentum transfer

Integrating over neutrino momenta = integrating over q^2

$$ft \equiv ft(q^2 = 0) \int_{\min}^{\max} \frac{F_{CW}(q^2)\, dq^2}{q^2_{\max} - q^2_{\min}}$$

Usual approach (Behrens & Bühring): assume $F_{CW} \approx F_{Ch}^{\text{daughter}}$ —> $R_{CW} = R_{Ch,1}$

But R_{CW} can be expressed via charge radii assuming approximate isospin symmetry

$$R^2_{\mathrm{CW}} = R^2_{\mathrm{Ch},1} + Z_0\left(R^2_{\mathrm{Ch},0} - R^2_{\mathrm{Ch},1}\right) = R^2_{\mathrm{Ch},1} + \frac{Z_{-1}}{2}\left(R^2_{\mathrm{Ch},-1} - R^2_{\mathrm{Ch},1}\right)$$

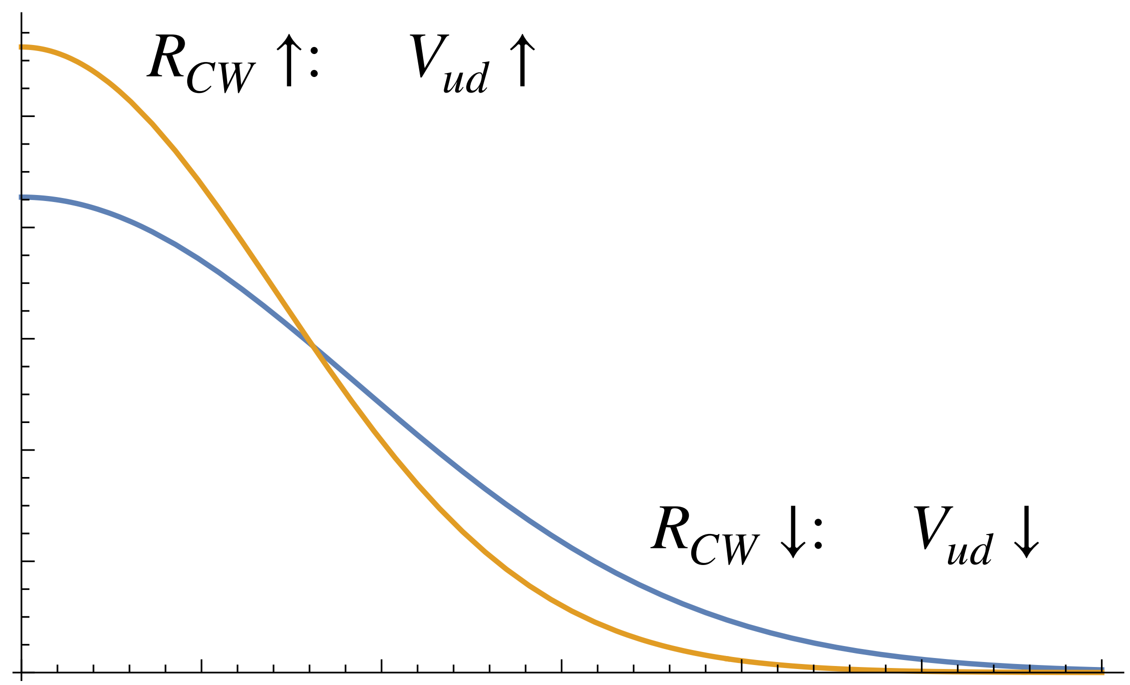
Seng 2212.02681

Charge radii + isospin symmetry —> CC weak radius

A	$R_{\mathrm{Ch},-1}$ (fm)	$R_{\mathrm{Ch},0}$ (fm)	$R_{\mathrm{Ch},1}$ (fm)	$R^2_{\mathrm{Ch},1}$ (fm^2)	R^2_{CW} (fm^2)
10	${}^{10}_{6}$C	${}^{10}_{5}$B(ex)	${}^{10}_{4}$Be: 2.3550(170)[a]	5.546(80)	N/A
14	${}^{14}_{8}$O	${}^{14}_{7}$N(ex)	${}^{14}_{6}$C: 2.50 25(87)[a]	6.263(44)	N/A
18	${}^{18}_{10}$Ne: 2.9714(76)[a]	${}^{18}_{9}$F(ex)	${}^{18}_{8}$O: 2.77 26(56)[a]	7.687(31)	13.40(53)
22	${}^{22}_{12}$Mg: 3.0691(89)[b]	${}^{22}_{11}$Na(ex)	${}^{22}_{10}$Ne: 2.9525(40)[a]	8.717(24)	12.93(71)
26	${}^{26}_{14}$Si	${}^{26m}_{13}$Al	${}^{26}_{12}$Mg: 3.0337(18)[a]	9.203(11)	N/A
30	${}^{30}_{16}$S	${}^{30}_{15}$P(ex)	${}^{30}_{14}$Si: 3.1336(40)[a]	9.819(25)	N/A
34	${}^{34}_{18}$Ar: 3.3654(40)[a]	${}^{34}_{17}$Cl	${}^{34}_{16}$S: 3.2847(21)[a]	10.789(14)	15.62(54)
38	${}^{38}_{20}$Ca: 3.467(1)[c]	${}^{38m}_{19}$K: 3.437(4)[d]	${}^{38}_{18}$Ar: 3.4028(19)[a]	11.579(13)	15.99(28)
42	${}^{42}_{22}$Ti	${}^{42}_{21}$Sc: 3.5702(238)[a]	${}^{42}_{20}$Ca: 3.5081(21)[a]	12.307(15)	21.5(3.6)
46	${}^{46}_{24}$Cr	${}^{46}_{23}$V	${}^{46}_{22}$Ti: 3.6070(22)[a]	13.010(16)	N/A
50	${}^{50}_{26}$Fe	${}^{50}_{25}$Mn: 3.7120(196)[a]	${}^{50}_{24}$Cr: 3.6588(65)[a]	13.387(48)	23.2(3.8)
54	${}^{54}_{28}$Ni: 3.738(4)[e]	${}^{54}_{27}$Co	${}^{54}_{26}$Fe: 3.6933(19)[a]	13.640(14)	18.29(92)
62	${}^{62}_{32}$Ge	${}^{62}_{31}$Ga	${}^{62}_{30}$Zn: 3.9031(69)[b]	15.234(54)	N/A
66	${}^{66}_{34}$Se	${}^{66}_{33}$As	${}^{66}_{32}$Ge	N/A	N/A
70	${}^{70}_{36}$Kr	${}^{70}_{35}$Br	${}^{70}_{34}$Se	N/A	N/A
74	${}^{74}_{38}$Sr	${}^{74}_{37}$Rb: 4.1935(172)[b]	${}^{74}_{36}$Kr: 4.1870(41)[a]	17.531(34)	19.5(5.5)

Effect of large CW radii on ft and V_{ud}

Total decay rate $\sim ft\,|V_{ud}|^2 \sim |V_{ud}|^2 \int_0^{Q^2_{EC}} dQ^2 F_{CW}(Q^2)$



Only total rate measured — if radius underestimated, V_{ud} will come out smaller

Systematic shift by up to 0.1% to some ft values —> may resolve CKM deficit?

Estimated from isospin symmetry — but isospin symmetry broken, how credible?

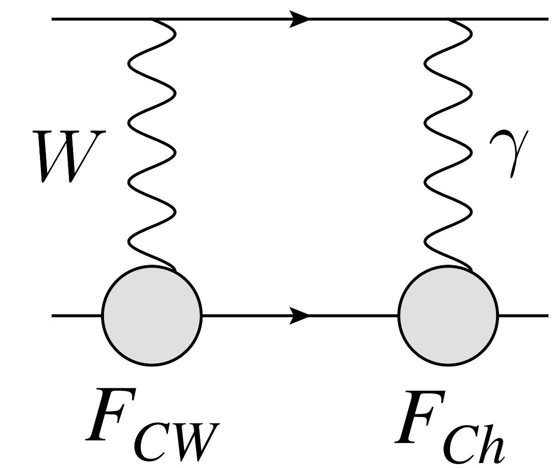
Theory strategy: compute all radii AND δ_C — check pattern, compare to available data, motivate exp.

Shape factor: ~ Friar radius for beta decay

Solution to Dirac equation with nuclear charge/weak densities

Bulk result due to charge and charged-weak radii (and beyond)

TPE approximation won't do — full Dirac eq. solution



$$C(Z,W) = \sum_{k_e,k_\nu,K} \lambda_{k_e} \left\{ M_K^2(k_e,k_\nu) + m_K^2(k_e,k_\nu) - \frac{2\mu_{k_e}\gamma_{k_e}}{k_e W} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right\}$$

Dirac Coulomb radial functions

$$\lambda_{k_e} = \frac{\alpha^2_{-k_e} + \alpha^2_{+k_e}}{\alpha^2_{-1} + \alpha^2_{+1}} \qquad \mu_{k_e} = \frac{\alpha^2_{-k_e} - \alpha^2_{+k_e}}{\alpha^2_{-k_e} + \alpha^2_{+k_e}} \frac{k_e W}{\gamma_{k_e}}$$

M, m —> convolutions of electron radial fn with nuclear FF

Work ongoing with

Chien Yeah Seng (INT/FRIB), Giovanni Carotenuto, Michela Sestu, Matteo Cadeddu, Nicola Cargioli (INFN Cagliari)

Plan: update the ft-values tables — uncertainties!! (nuclear charge radii, FF shape)

Summary, Caveats & Outlook

Summary, Caveats and Outlook

With improved Δ_R^V: for precise V_{ud} <— precise $\mathscr{F}t$ <— precise ft + precise $\delta_C,\ \delta_{NS}$

Precise nuclear radii are crucial ingredients in ft-values and δ_C

For a T=1 triplet with $T_z = (-1,0,1)$: complete set of 8 radii $R_{Ch}^{(-1,0,1)}$, $R_{NW}^{(-1,0,1)}$, $R_{CW}^{(-1,0),(0,1)}$

All 8 radii + δ_C are accessible for theory calculation!

For robust uncertainty: motivate experiment — $R_{Ch}^{(-1,0,1)}$ and $R_{NW}^{(1)}$ for stable daughters

Most precise charge radii from μ-atoms; radii of unstable isotopes from isotope shifts

NC radius — PV electron scattering from stable daughter (e.g. Ca-42 at MESA: Ca-48 planned)

Feasibility study for PVES on C-12: sub-% measurement of weak charge and radius

O. Koschii et al, Phys.Rev.C 102 (2020) 2, 022501

Work ongoing with

Nicola Cargioli, Matteo Cadeddu, Hubert Spiesberger, Jorge Piekarewicz, Xavi Roca-Maza

Summary, Caveats and Outlook

For all this: precise charge radii are a prerequisite!

Where do we take the charge radii from? — Usually from some tables,
e.g. Angeli-Marinova or Fricke-Heilig

A&M do not give much ingredients but have the smallest uncertainties (??)
F&H do give ingredients in detail but credibility of nuclear polarizability??
Example: Ne-20 — NPol = 19(2)eV — from Rinker & Späth (1970's)

Isotope	$E_{exp.}$ [keV]	$E_{theo.}$ [keV]	NPol [keV]	c [fm]	$\langle r^2 \rangle^{1/2}_{model}$ [fm]	α [1/fm]	k	C_z [am/eV]	$R^{\mu}_{k\alpha}$ [fm]	Ref.
^{20}Ne	207.282 5	207.282	0.019	2.9589 24	3.006	0.0329	2.0445	-0.516	3.8656 (26;33)	[Fr92]

Can I reproduce F&H result for NPol? Can I improve it?

1. Estimate with photonuclear sum rules (Berman-Fultz, RMP 47 (1975) 713) + nuclear size:
   NPol(1S) = 20 eV (Z/10)^3 (A/20)^(4/3) — OK(?) accuracy???????? 50-100% — FH claim 10%
2. In light μ-atoms nucleon pol not negligible: rescale the known μH result
   nPol(2S μH) = 13 μeV —> nPol(1S μNe-20) = 13 μeV $\times 2^3 \times 10^3 \times 20 \times (\mu_{Ne}/\mu_H)^4$ ~ 3 eV

Importantly: what NPol is included in e-scattering? How is it calculated? Guess: not at all

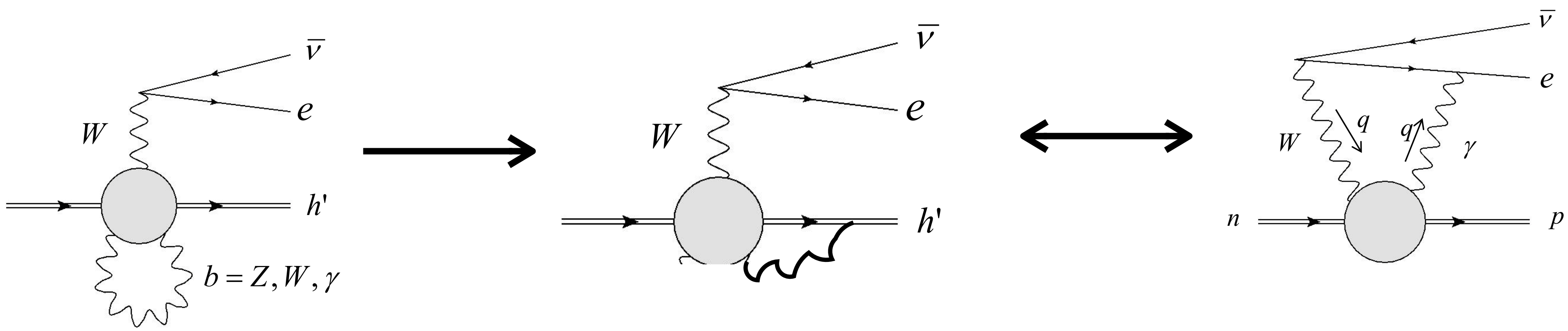
Summary, Caveats and Outlook

NPol (μ-atoms) — δ_{NS} (beta decays) — nuclear γZ-box (neutron skin): same physics

Coulomb corrections extremely important (exact shape of charge distribution)

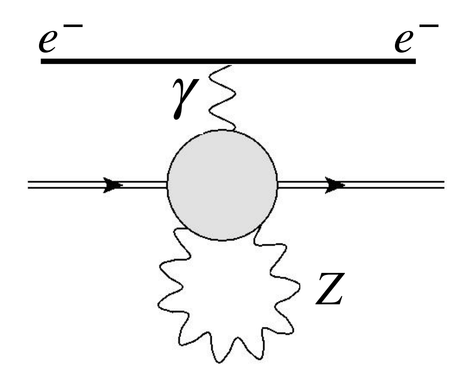
Nuclear radii extracted from μ atoms and from e-scattering — compatible?
Corrections applied to scattering data: Coulomb corrections, NPol, RC, … — compatible?

Vertex corrections: for FF often discussed away in “FF definition” bulked with SE,…
But for beta decays are crucial to cancel UV div of γW-box *Sirlin Rev.Mod.Phys. 50 (1978) 905*



Recently: vertex correction to gA w.r.t. gV may be ~1-2% (usually expected 0.01%)
Cirigliano et al, Phys.Rev.Lett. 129 (2022) 12, 121801

Another example of large vertex correction: anapole moment



Renormalizes the axial FF: major problem for P2@MESA;
NC axial FF $\neq$ CC axial FF
Long-time object of desire in APV (nuclear AM ~ Z^3 - Bouchiats)

Thank you!