Electromagnetic form factors of the proton and neutron from lattice QCD

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Outline

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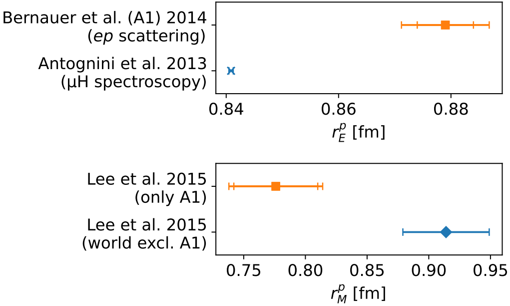
Motivation

- Internal structure of the nucleon still an open research field in subatomic physics
- In particular, there is a discrepancy between different determinations of the electric and magnetic charge radii of the proton
- Electromagnetic form factors of the proton and neutron of high interest

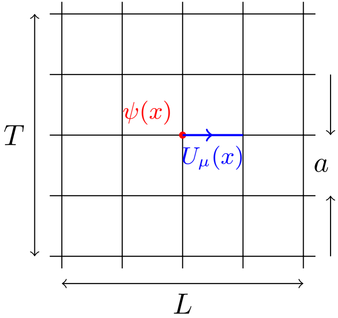


- Full calculation of the proton and neutron form factors from first principles necessitates explicit treatment of the numerically challenging quark-disconnected contributions
- Not included in many previous lattice studies

Outline

QCD on the lattice

- Coupling of QCD is large at large distances / low energies
- Low-energy regime of QCD is hence inaccessible to perturbative methods
- Powerful tool for the non-perturbative study: lattice QCD
- Replace space-time by a four-dimensional Euclidean lattice
- Gauge-invariant UV-regulator for the quantum field theory due to the momentum cut-off
- Path integral becomes finite-dimensional and can be computed numerically
- Allows a systematic extrapolation to the continuum and infinite-volume limit, $a \to 0$ and $V \to \infty$



Ensembles

Coordinated Lattice Simulations (CLS)[1]

- Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions
- $N_f = 2+1$: 2 degenerate light quarks ($m_u = m_d$), 1 heavier strange quark ($m_s > m_{u,d}$)
- $\mathrm{tr}\, M_q = 2m_l + m_s =$ const.
- Tree-level improved Lüscher-Weisz gauge action

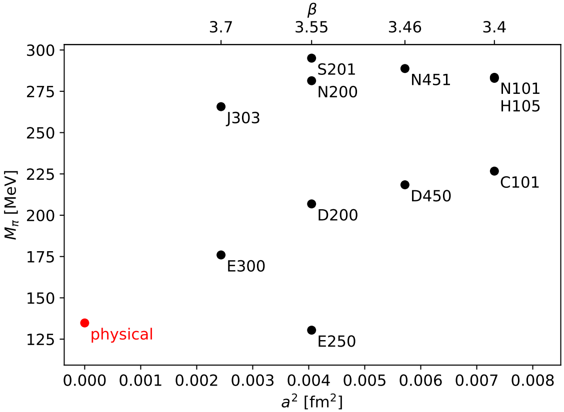
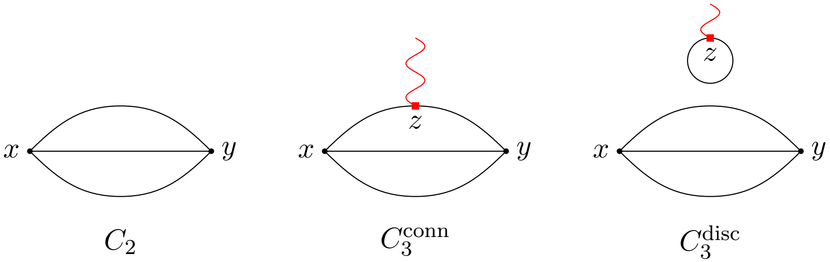


Figure: Overview of the ensembles used in this study

[1] Bruno et al. 2015; Bruno, Korzec, and Schaefer 2017.

Nucleon two- and three-point correlation functions

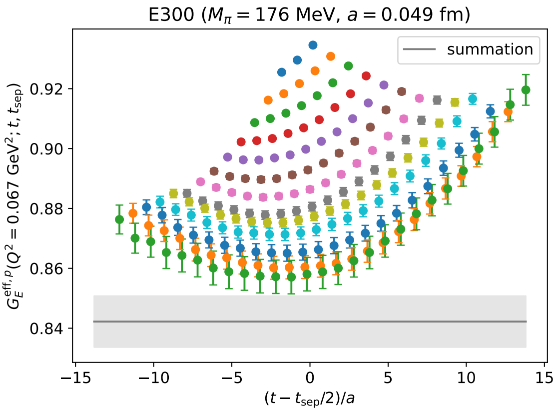


- Measure the two- and three-point correlation functions of the nucleon
- For three-point functions, Wick contractions yield connected and disconnected contribution
- Compute the quark loops via a stochastic estimation using a frequency-splitting technique[2]
- Extract the effective form factors $G_{E,M}^{\mathrm{eff}}$ using the ratio method[3]

[2]Giusti et al. 2019; Cè et al. 2022; [3]Korzec et al. 2009.

Excited-state analysis

- Cannot construct exact interpolating operator for the proton (any hadron) on the lattice
- All possible states with the same quantum numbers contribute
- Effect of heavier excited states suppressed exponentially with the distance between operators in Euclidean time
- For baryons, the relative statistical noise grows also exponentially with the source-sink separation $t_{\text{sep}} = y_0 - x_0$



Excited-state analysis: summation method

- Explicit treatment of the excited-state systematics required
- Summation of the effective form factors over the operator insertion time,

$$S_{E,M}(Q^2; t_{\mathrm{sep}}) = \sum_{t=t_{\mathrm{skip}}}^{t_{\mathrm{sep}}-t_{\mathrm{skip}}} G^{\mathrm{eff}}_{E,M}(Q^2; t, t_{\mathrm{sep}}), \quad t_{\mathrm{skip}} = 2a \tag{1}$$

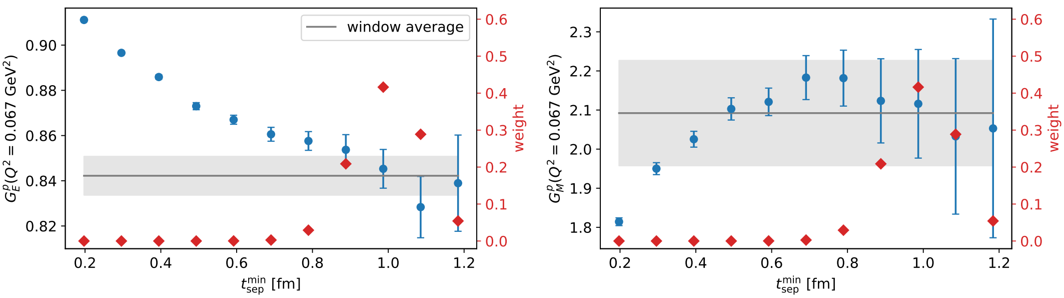
- Parametrically suppresses the effects of excited states ($\propto e^{-\Delta t_{\mathrm{sep}}}$ instead of $\propto e^{-\Delta t}$, $e^{-\Delta(t_{\mathrm{sep}}-t)}$ [Δ: energy gap to lowest-lying excited state]) $\rightarrow$ "summation method"
- For $t_{\mathrm{sep}} \rightarrow \infty$, the slope as a function of t_{sep} is given by the ground-state form factor,

$$S_{E,M}(Q^2; t_{\mathrm{sep}}) \xrightarrow{t_{\mathrm{sep}}\to\infty} C_{E,M}(Q^2) + \frac{1}{a}(t_{\mathrm{sep}} + a - 2t_{\mathrm{skip}}) G_{E,M}(Q^2) \tag{2}$$

Excited-state analysis: window average

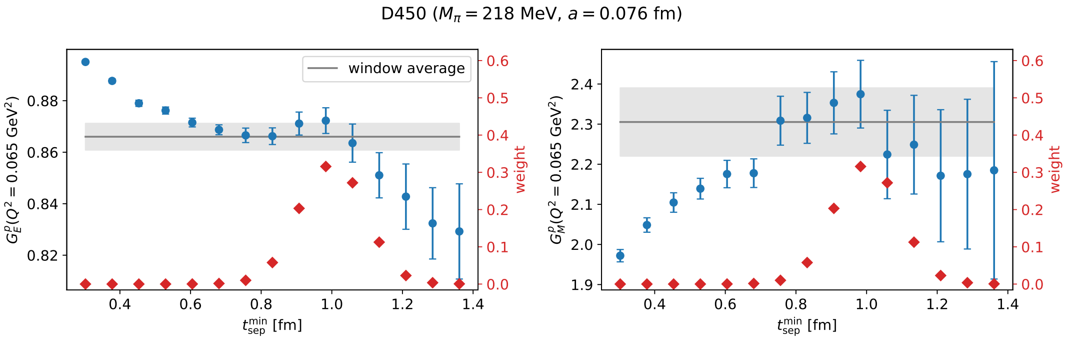
- Apply summation method with varying starting values $t_{\rm sep}^{\rm min}$ for the linear fit
- Perform a weighted average over $t_{\rm sep}^{\rm min}$, where the weights are given by a smooth window function[4]

E300 ($M_\pi = 176$ MeV, $a = 0.049$ fm)



[4] Djukanovic et al. 2022; Agadjanov et al. 2023.

Excited-state analysis: window average



- Reliable detection of the plateau with reduced human bias (same window on all ensembles)
- Conservative error estimate

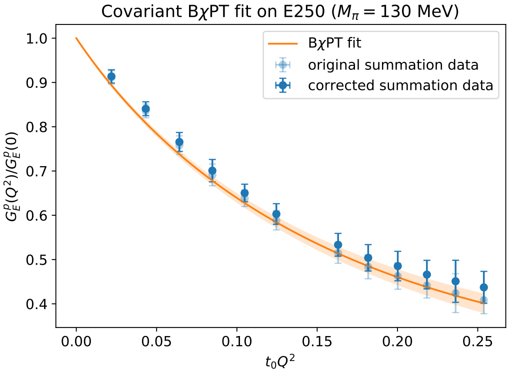
Outline

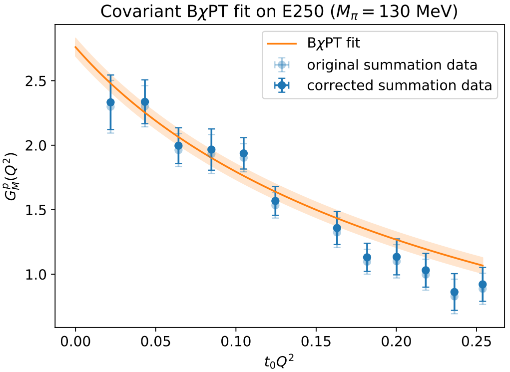
Direct Baryon χPT fits

- Combine parametrization of the Q^2-dependence with the chiral, continuum, and infinite-volume extrapolation
- Simultaneous fit of the pion-mass, Q^2-, lattice-spacing, and finite-volume dependence of the form factors to the expressions resulting from covariant chiral perturbation theory[5]
- Include contributions arising from the ρ meson for both proton and neutron
- For the neutron, also include contributions arising from the ω resonance to introduce additional curvature to the form factors
- Perform fits with various cuts in M_π and Q^2, as well as with different models for the lattice-spacing and finite-volume dependence, in order to estimate systematic uncertainties
- Large number of degrees of freedom $\Rightarrow$ improved stability against lowering the Q^2-cut

[5] Bauer, Bernauer, and Scherer 2012.

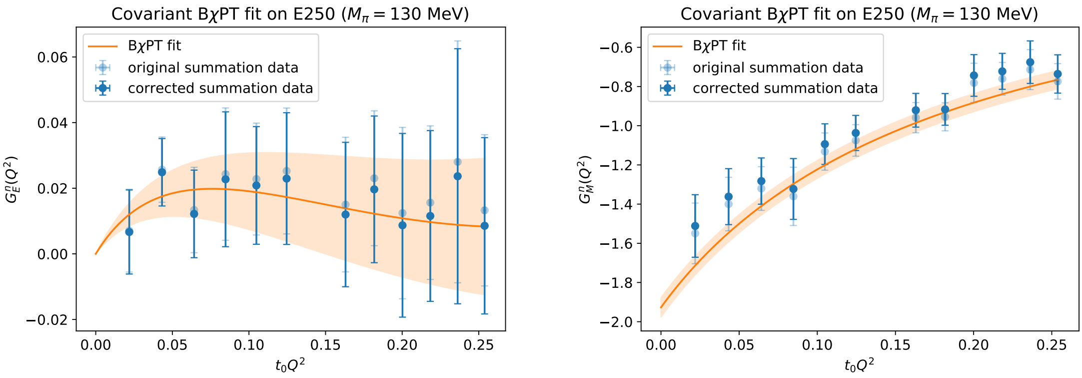
Q^2-dependence of the proton form factors on E250





- Direct BχPT fit describes data very well
- Significantly reduced error due to the inclusion of several ensembles in one fit

Q^2-dependence of the neutron form factors on E250



Outline

Model average

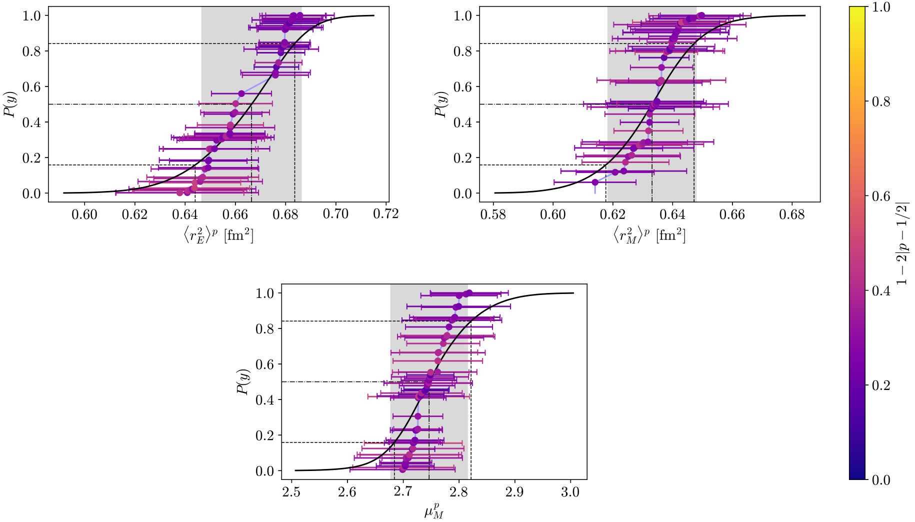
- Perform a weighted average over the results of all fit variations, using weights derived from the Akaike Information Criterion[6],

$$w_i = \exp\left(-\frac{1}{2}\mathrm{BAIC}_i\right) \Bigg/ \sum_j \exp\left(-\frac{1}{2}\mathrm{BAIC}_j\right), \quad \mathrm{BAIC}_i = \chi^2_{\mathrm{noaug,min},i} + 2n_{f,i} + 2n_{c,i}, \tag{3}$$

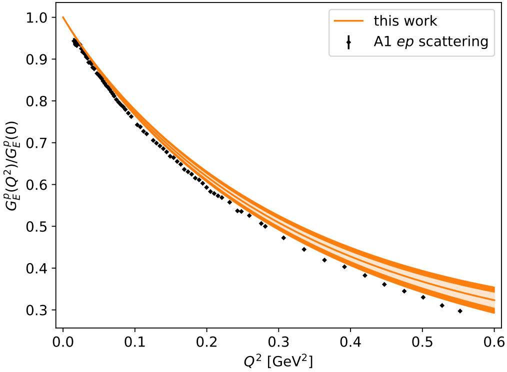
  where n_f is the number of fit parameters and n_c the number of cut data points
- Strongly prefers fits with low n_c, *i.e.*, the least stringent cut in Q^2 $\Rightarrow$ apply a flat weight over the different Q^2-cuts to ensure strong influence of our low-momentum data
- Determine the final cumulative distribution function (CDF) from the weighted sum of the bootstrap distributions[7]
- Quote median of this CDF together with the central $68\,\%$ percentiles

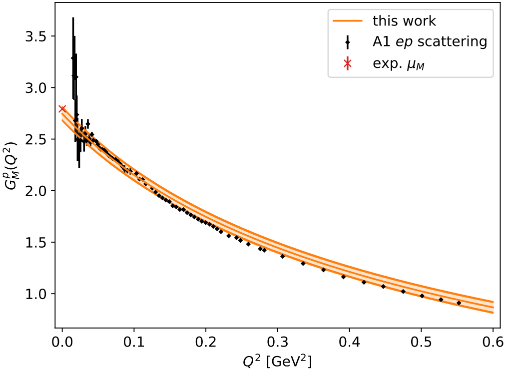
[6] Akaike 1973, 1974; Neil and Sitison 2022; [7] Borsányi et al. 2021.

CDFs of the EM charge radii and magnetic moment of the proton



Model-averaged proton form factors at the physical point

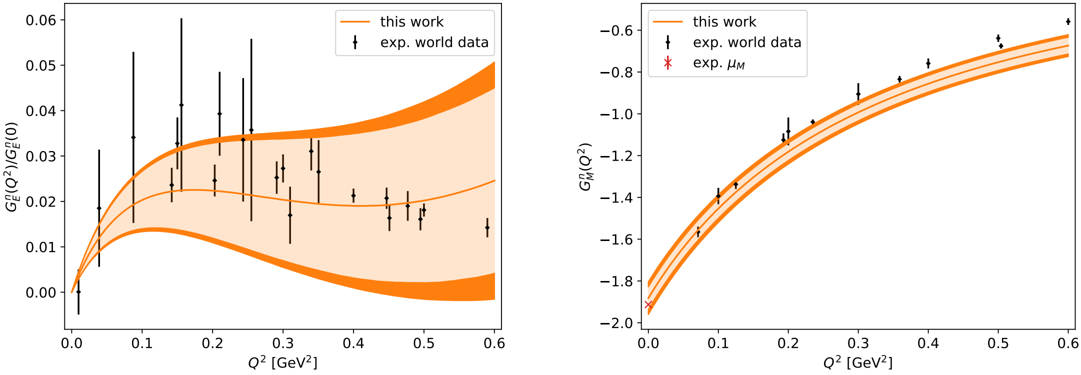




- Mild tension between our result and that of A1[8] for the electric form factor
- Good agreement for the magnetic form factor

[8] Bernauer et al. 2014.

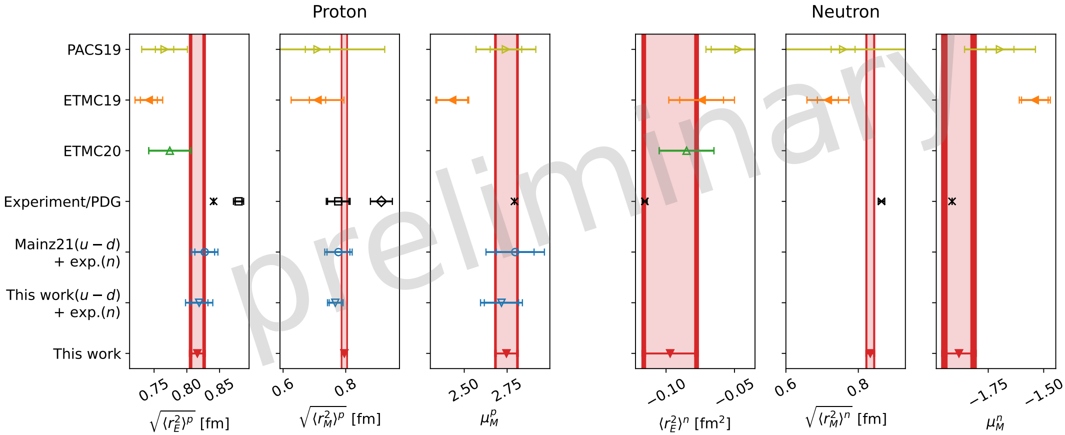
Model-averaged neutron form factors at the physical point



(Mostly) compatible with the collected experimental world data[9] within our errors

[9] Ye et al. 2018.

Electromagnetic charge radii and magnetic moments



Magnetic moments reproduced, low value for $\sqrt{\langle r_E^2\rangle^p}$ clearly favored, $\sqrt{\langle r_M^2\rangle^p}$ agrees with A1

Outline

Hyperfine splitting and the Zemach radius

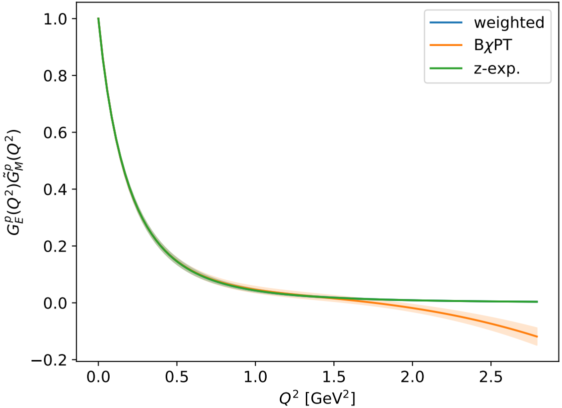
- Determination of nuclear properties from atomic physics
- Magnetic spin-spin interaction between the nucleus and the orbiting lepton gives rise to the hyperfine splitting (HFS)
- Electromagnetic structure of the proton influences the HFS of the s-state of hydrogen
- Relevant parameter deduced from the HFS: Zemach radius[10],

$$r_Z^p = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(\frac{G_E^p(Q^2)G_M^p(Q^2)}{\mu_M^p} - 1\right) = -\frac{2}{\pi}\int_0^\infty \frac{dQ^2}{(Q^2)^{3/2}}\left(\frac{G_E^p(Q^2)G_M^p(Q^2)}{\mu_M^p} - 1\right) \tag{4}$$

[10] Zemach 1956.

Zemach radius from the lattice

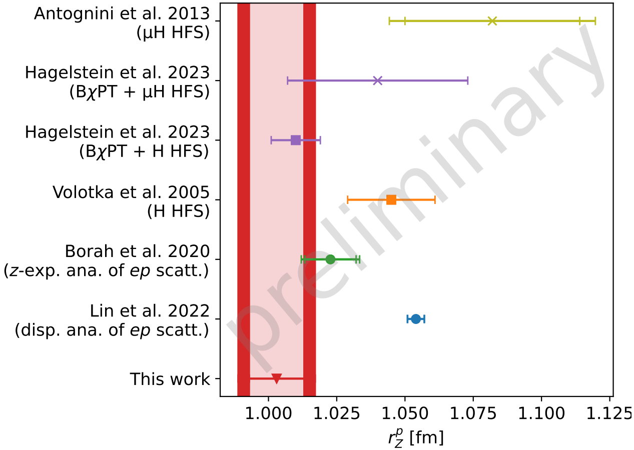
- BχPT only trustworthy up to $Q^2 \approx 0.6\,\text{GeV}^2$
- Tail of the integrand suppressed: contribution of the form factors above $0.6\,\text{GeV}^2$ to r_Z only about $1\,\%$
- Fit a z-expansion[11] to the BχPT fit results up to Q^2_{cut}
- Incorporate the large-Q^2 constraints on the form factors[12]
- For the integration, smoothly replace the BχPT parametrization of the form factors by the z-expansion



[11] Hill and Paz 2010; [12] Lepage and Brodsky 1980; Lee, Arrington, and Hill 2015

Comparison to other studies

- Low value for r_Z^p favored
- Our estimate is not independent from the electromagnetic charge radii (based on the same form factor data)
- Large positive correlation between $\sqrt{\langle r_E^2 \rangle^p}$ and r_Z^p [13]
- Low result for r_Z^p expected, no independent puzzle



[13] Friar and Sick 2005

Outline

Conclusions

- Direct determination of the electromagnetic form factors of the proton and neutron from lattice QCD including all relevant contributions
- Chiral, continuum, and infinite volume extrapolation via matching with the predictions from covariant baryon chiral perturbation theory
- Small electric *and* magnetic charge radii of the proton favored
- Competitive errors, in particular for the magnetic charge radii
- Initial study of the Zemach radius works well and yields a plausible result
- Outlook
  - Increased statistics for the disconnected contribution on our most chiral ensemble E250
  - Investigate some details of the analysis procedure
  - Djukanovic et al. 2023 (in preparation)

Backup slides

Disambiguating the statistical and systematic uncertainties

- Scale the statistical variances of the individual fit results by a factor of $\lambda = 2$
- Repeat the model averaging procedure
- Assumptions:
  - Above rescaling only affects the statistical error of the averaged result
  - Statistical and systematic errors add in quadrature
- Contributions of the statistical and systematic errors to the total error,

$$\sigma^2_{\mathrm{stat}} = \frac{\sigma^2_{\mathrm{scaled}} - \sigma^2_{\mathrm{orig}}}{\lambda - 1}, \quad \sigma^2_{\mathrm{syst}} = \frac{\lambda\sigma^2_{\mathrm{orig}} - \sigma^2_{\mathrm{scaled}}}{\lambda - 1} \tag{5}$$

- Consistency check: results are almost independent of λ (if it is chosen not too small)

z-expansion

- Model-independent description of the Q^2-dependence of the form factors
- Map domain of analyticity of the form factors onto the unit circle,

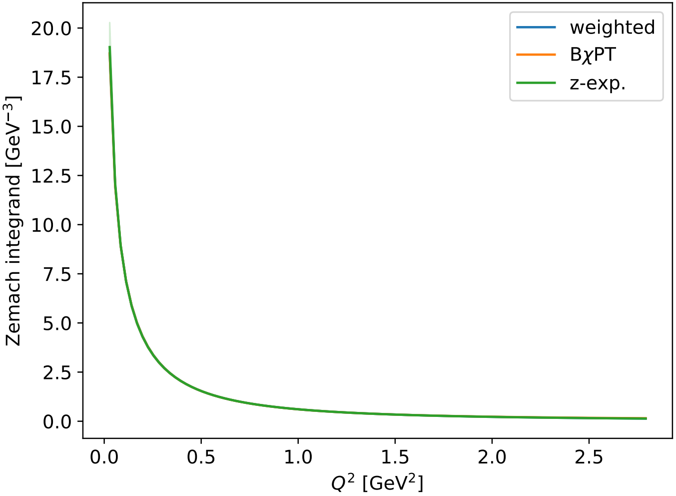
$$z(Q^2) = \frac{\sqrt{\tau_{\text{cut}} + Q^2} - \sqrt{\tau_{\text{cut}} - \tau_0}}{\sqrt{\tau_{\text{cut}} + Q^2} + \sqrt{\tau_{\text{cut}} - \tau_0}}, \tag{6}$$

  where $\tau_{\text{cut}} = 4M_\pi^2$, and we employ $\tau_0 = 0$
- Expand the form factors as

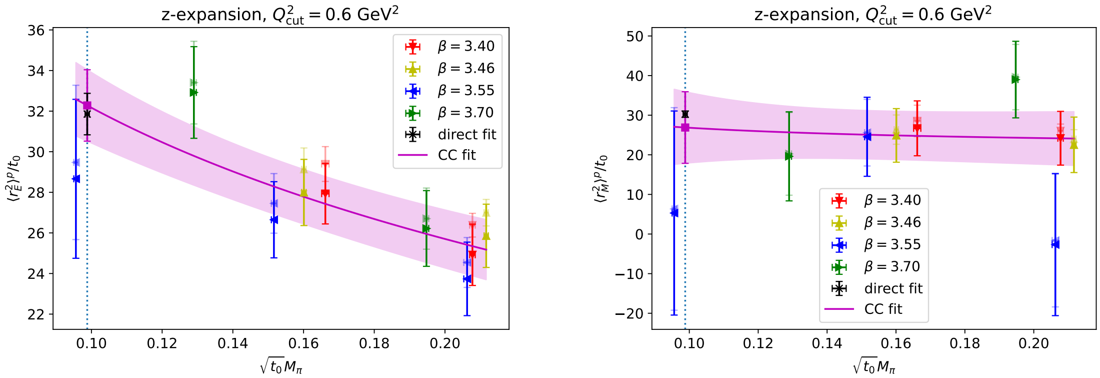
$$\frac{G_E(Q^2)}{G_E(0)} = \sum_{k=0}^{n} a_k z(Q^2)^k, \quad G_M(Q^2) = \sum_{k=0}^{n} b_k z(Q^2)^k \tag{7}$$

- Fix $G_E(0) = a_0 = 1$

Zemach integrand



Crosscheck of direct fits with z-expansion: proton EM charge radii



- Good agreement with direct fits, albeit with significantly larger errors
- Not sufficiently stable against fluctuations on single momenta or ensembles

Crosscheck of direct fits with z-expansion: proton magnetic moment

Significantly smaller than direct fits, which are compatible with experiment