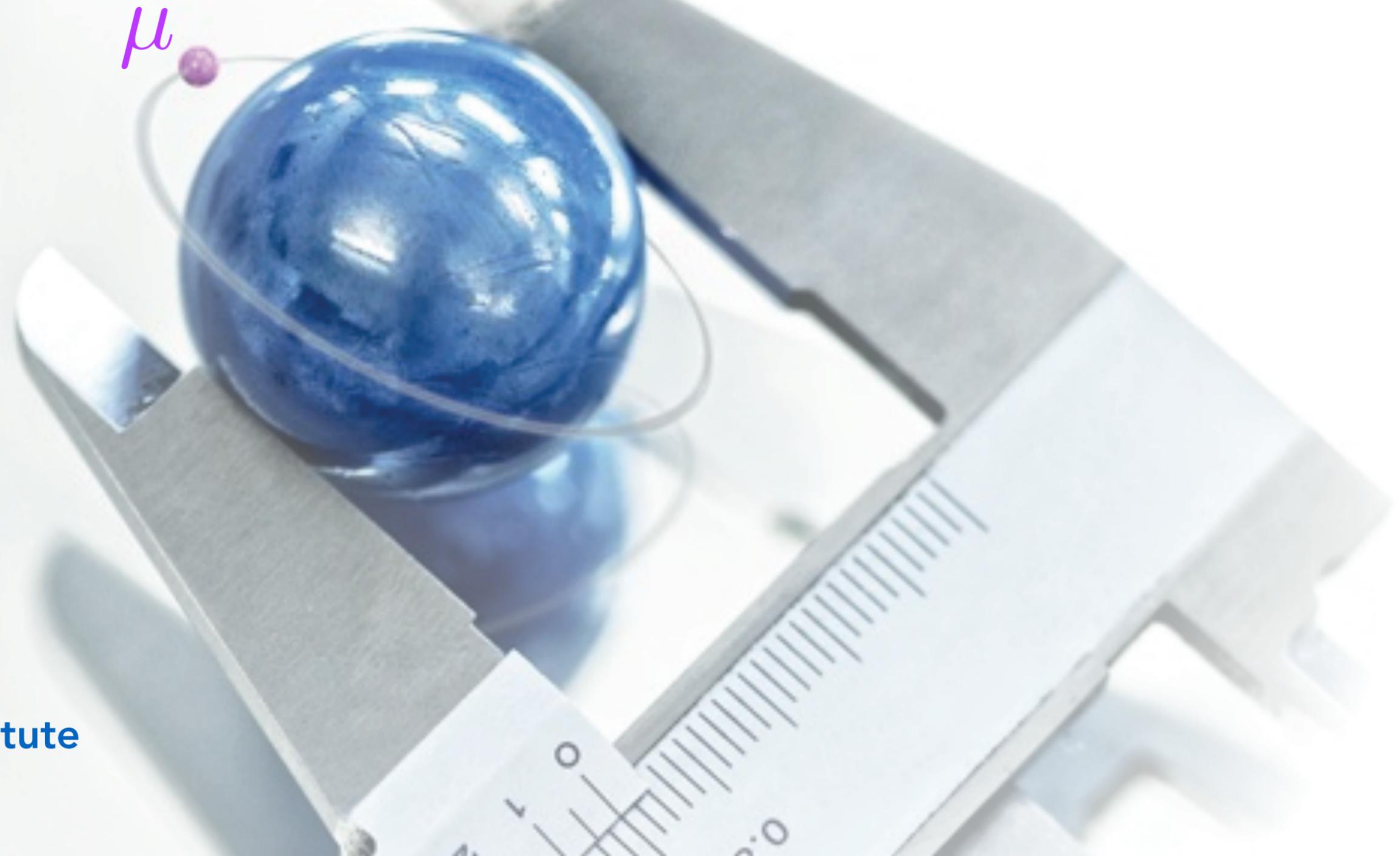


Laser spectroscopy of muonic helium ions

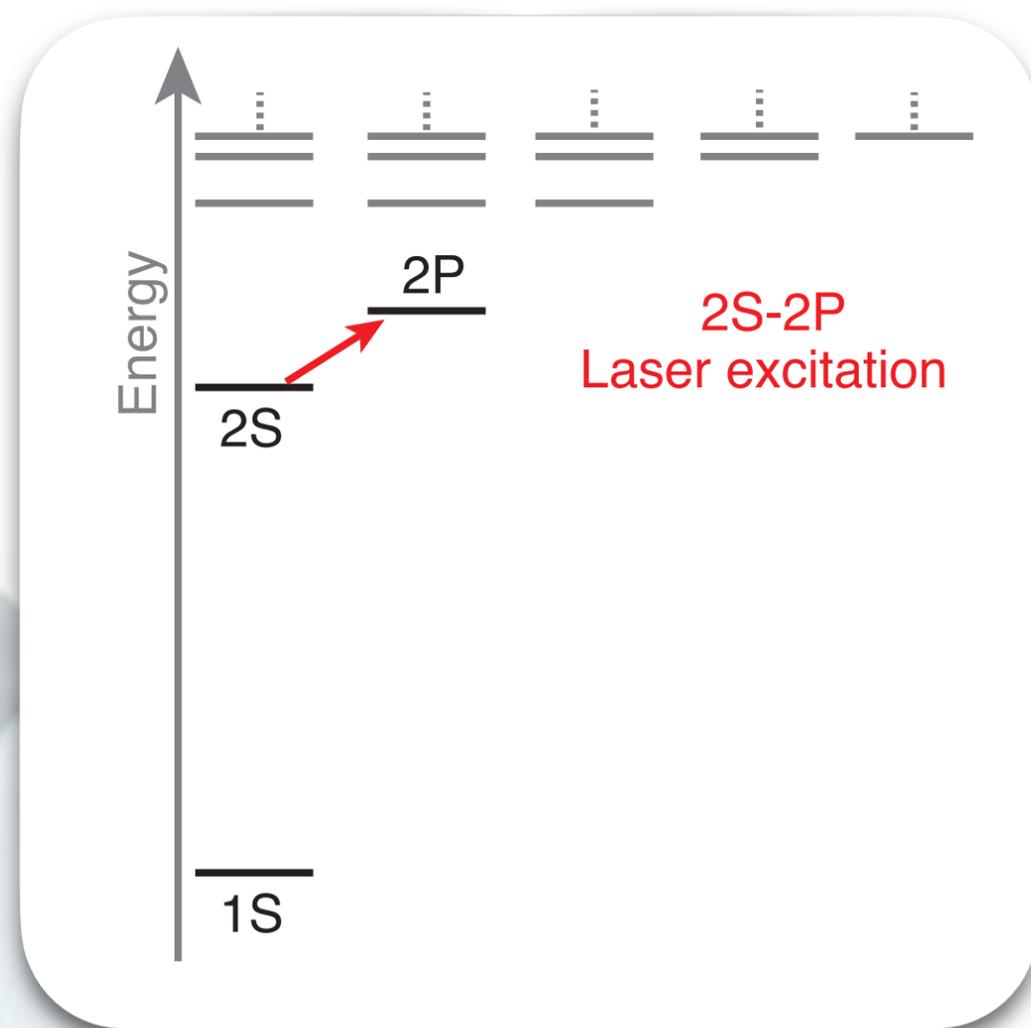
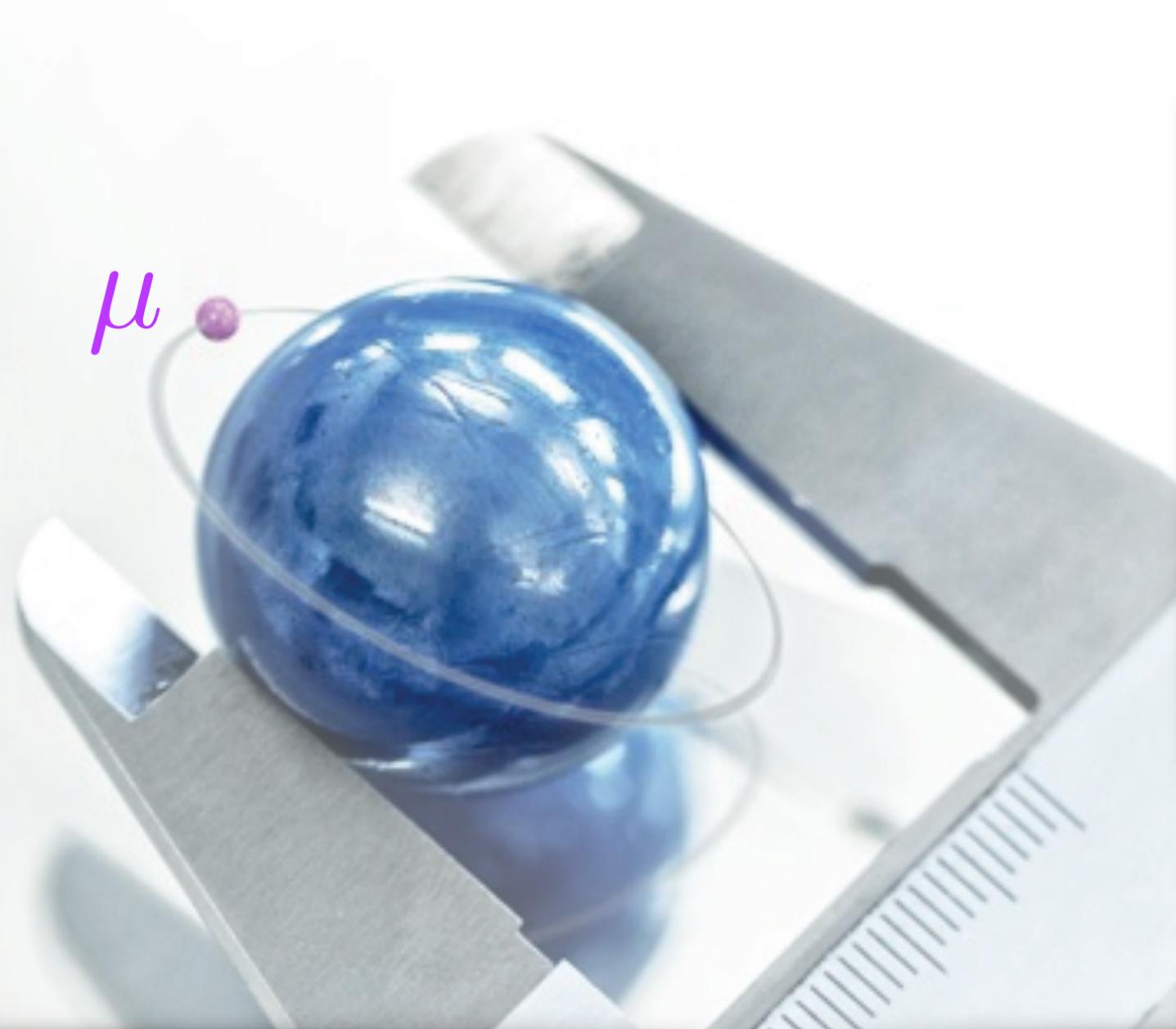
CREMA collaboration



Aldo Antognini

Paul Scherrer Institute
ETH, Zurich

Laser spectroscopy of light muonic atoms



We measured 10
2S-2P transitions in
 μp , μd , $\mu^3\text{He}^+$, $\mu^4\text{He}^+$

+

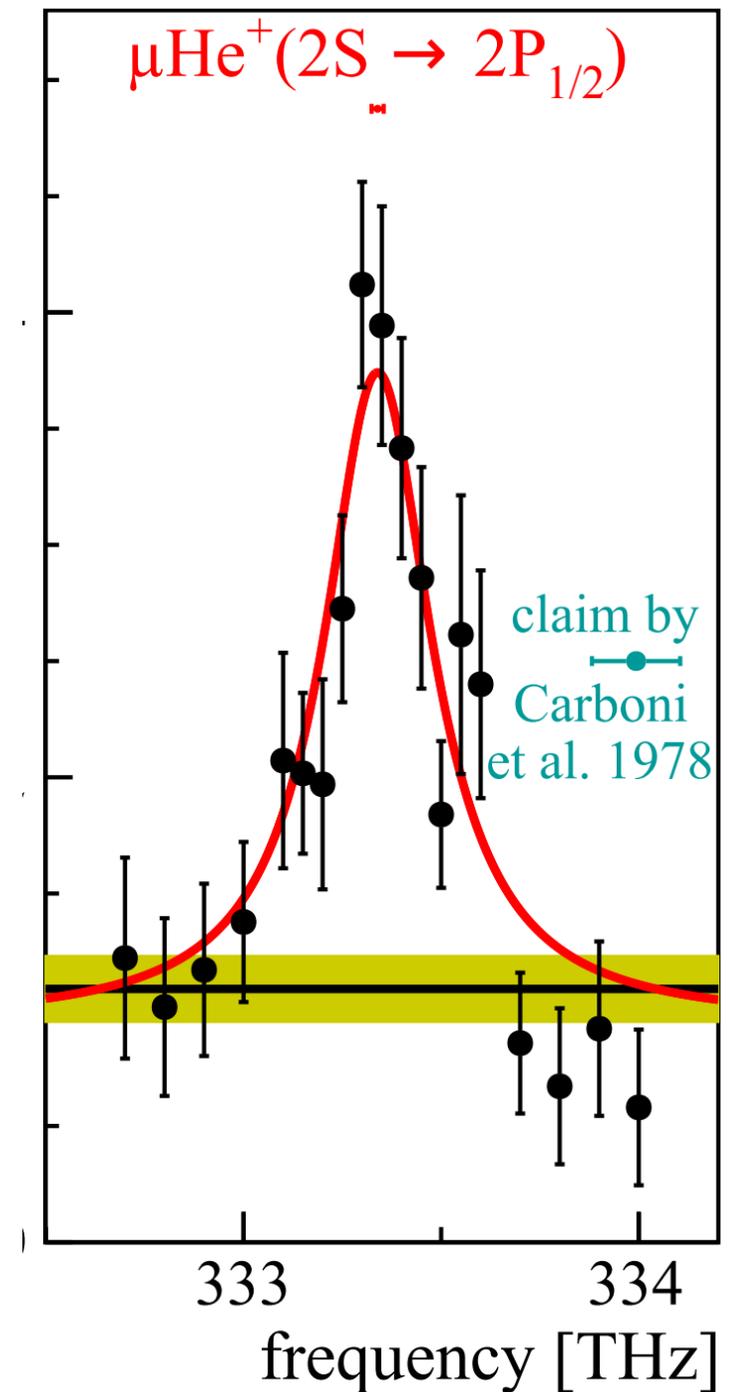
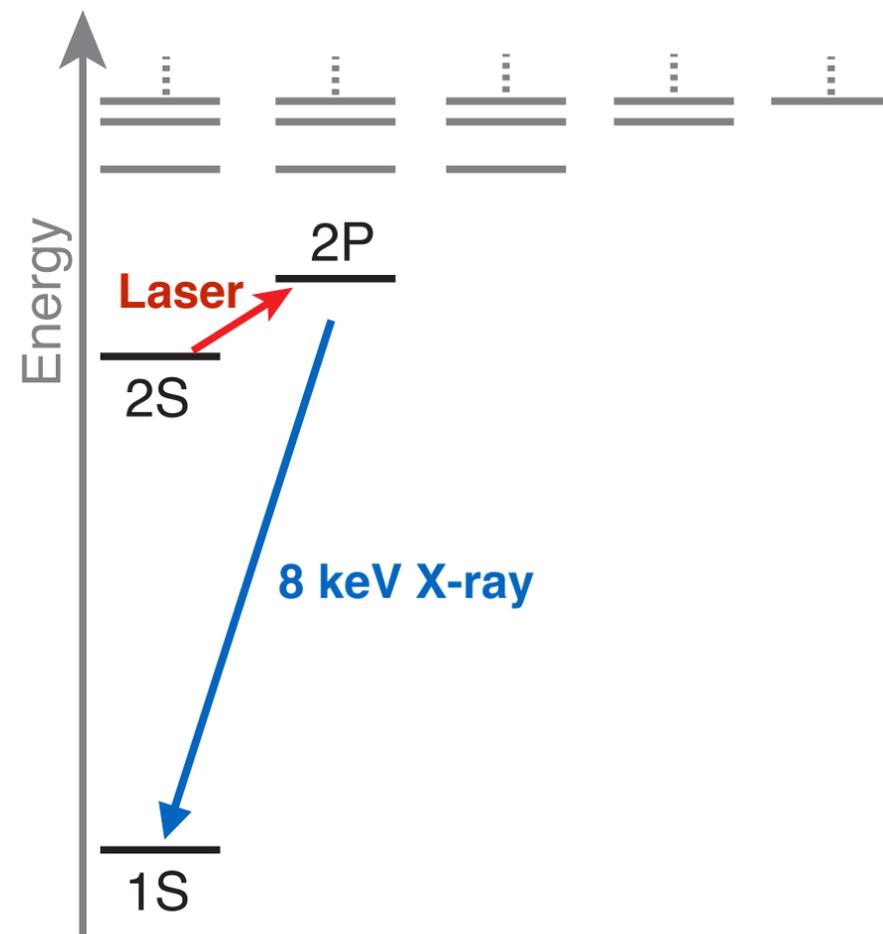
Theoretical predictions:
QED + Nuclear structure



p , d , ^3He , ^4He
charge radii

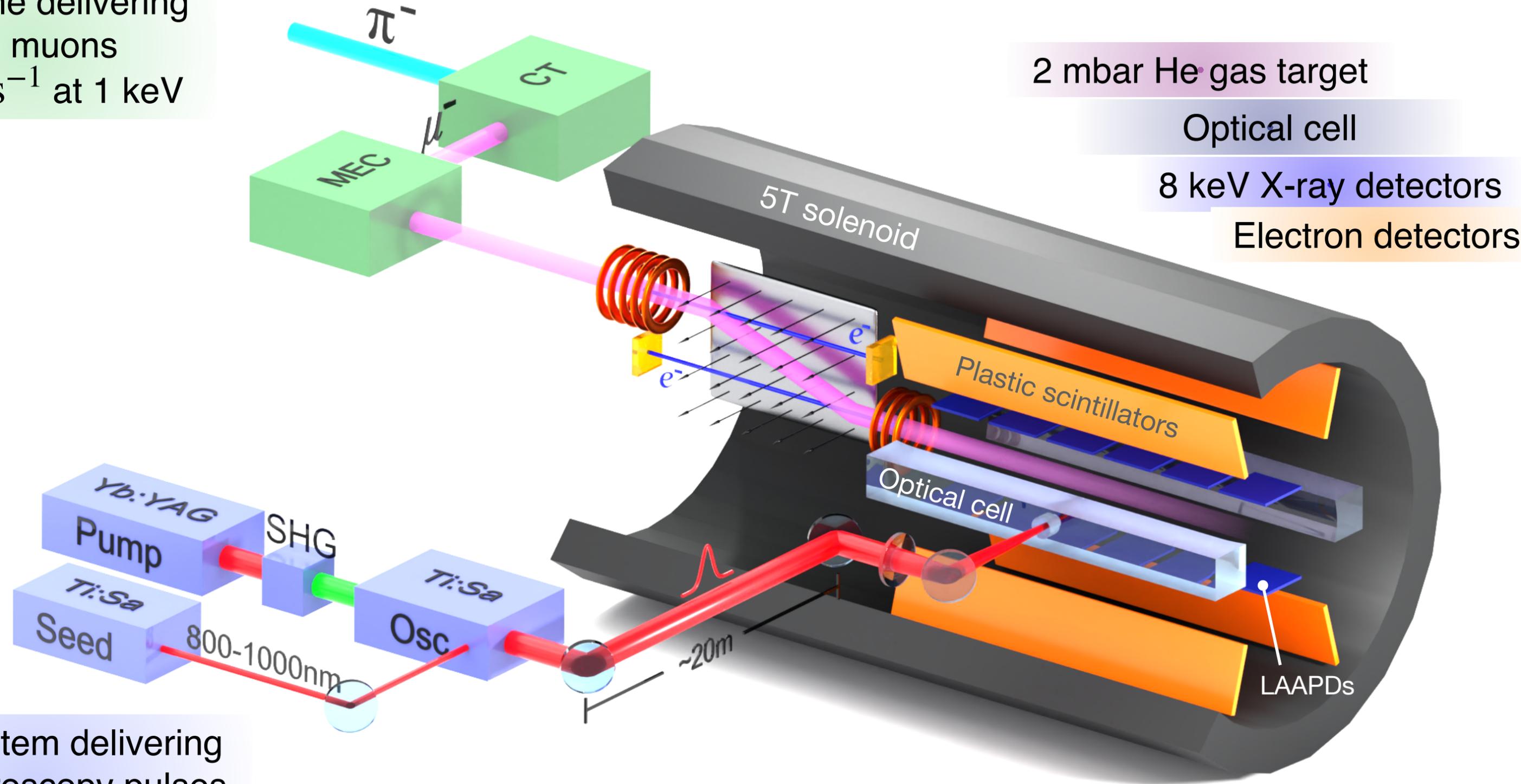
The μHe^+ 2S-2P experiment

- ▶ Stop low-energy muons in 2 mbar Helium gas
- ▶ μHe^+ are formed (1% in the 2S-state)
- ▶ Excite 2S-2P transition with laser
- ▶ Detect X-ray from 2P-1S de-excitation
- ▶ Plot number of X-rays vs. laser frequency



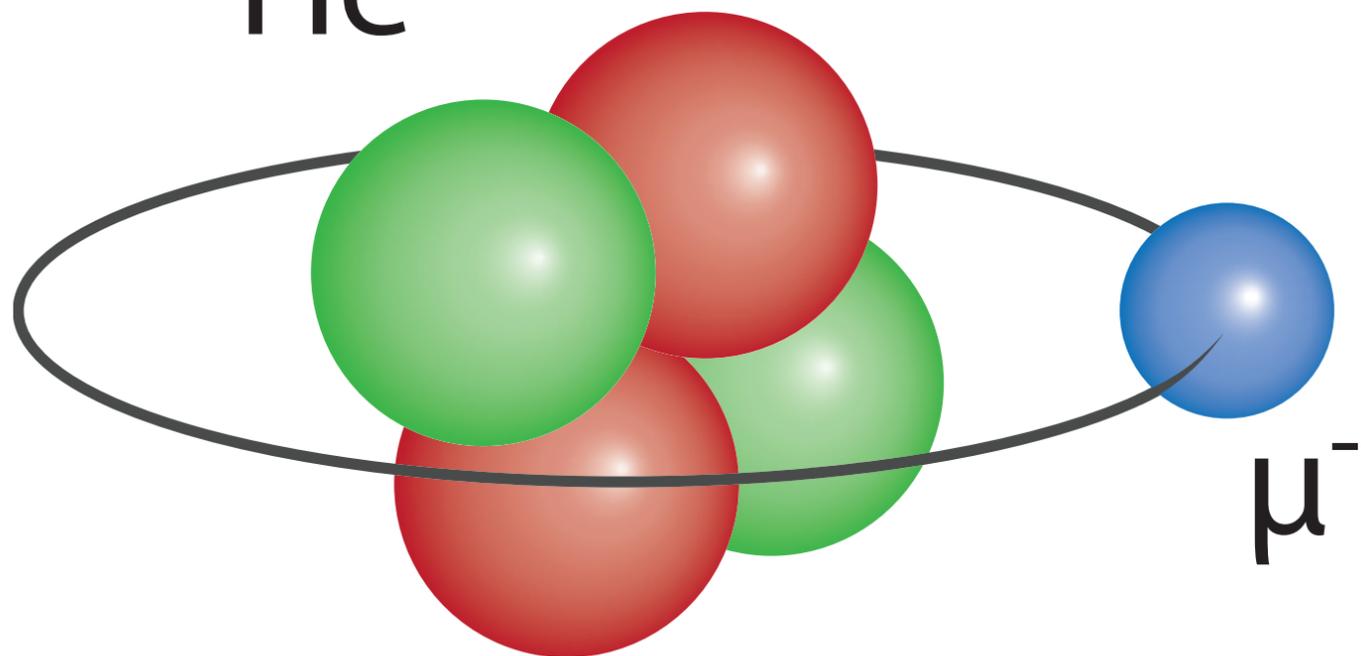
The setup

Beam line delivering
slow muons
 $500 \mu^{-} s^{-1}$ at 1 keV

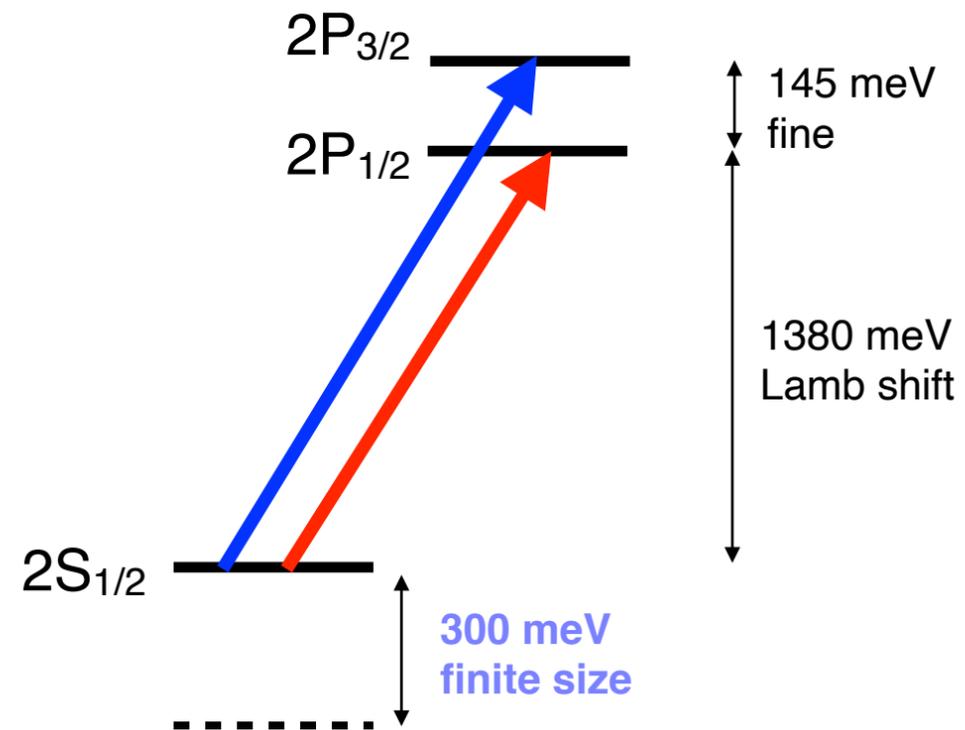


Laser system delivering
the spectroscopy pulses
within $1 \mu s$

${}^4\text{He}^{++}$

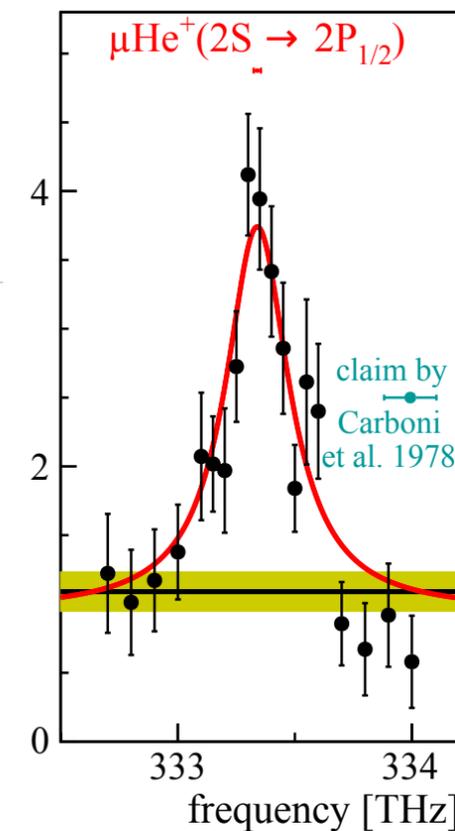
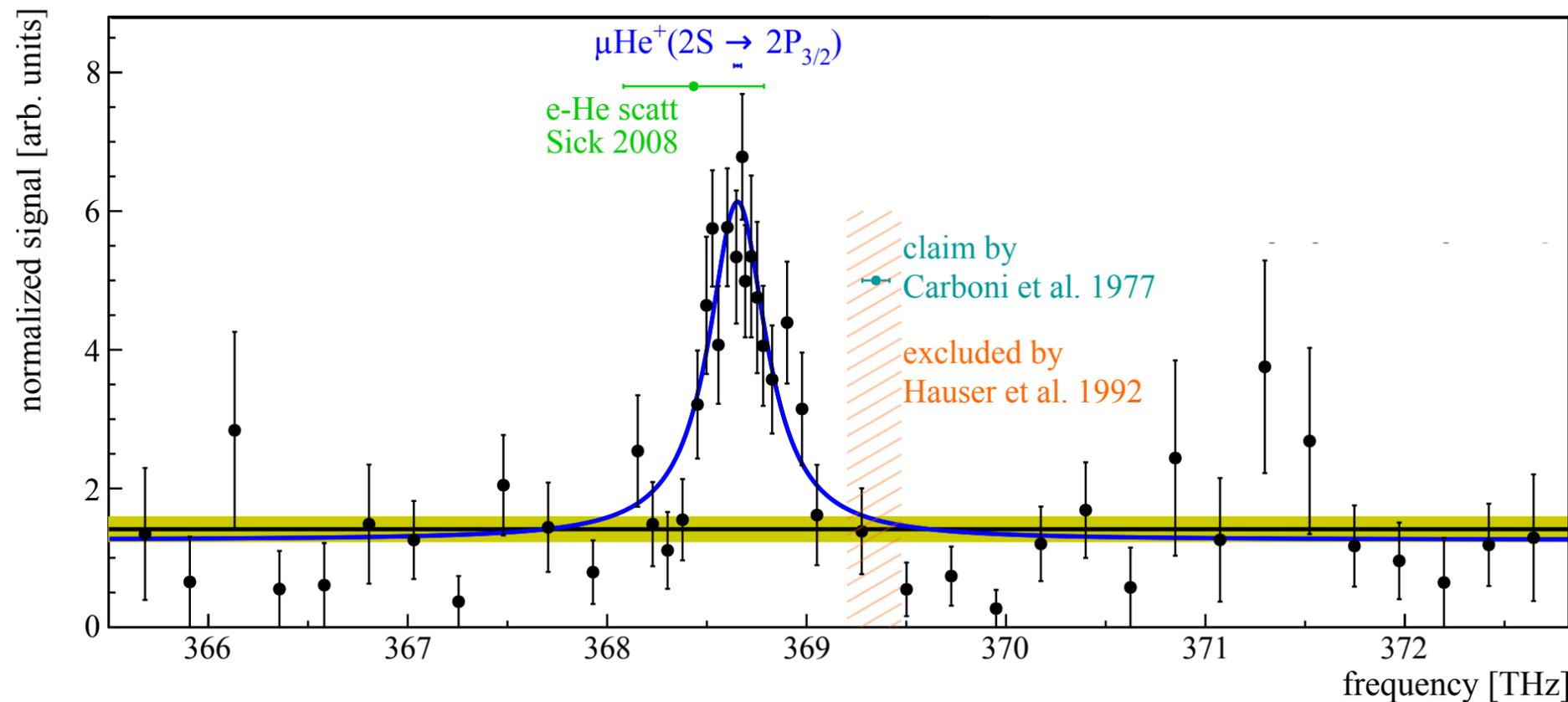


The measured transitions in $\mu^4\text{He}^+$



$$\nu^{\text{exp}}(2S \rightarrow 2P_{3/2}) = 368,653 \pm 18 \text{ GHz}$$

$$\nu^{\text{exp}}(2S \rightarrow 2P_{1/2}) = 333,339 \pm 15 \text{ GHz.}$$



Numerous laser-off events used to determine the background

High-sensitivity low-precision laser spectroscopy

High sensitivity to finite size

Vacuum polarisation

$$\Delta E_{2S-2P} \sim Z^3$$

Finite-size effect

$$\Delta E_{\text{FNS}} = \frac{2}{3n^3} Z^4 \alpha^4 m_r^3 r^2$$

Low precision laser spectroscopy

2S-2P line width

$$\Gamma_{2P} \sim Z^4 m$$

	μH	$\mu^4\text{He}^+$	
Fine-size effect /Lamb shift	2 %	20 %	
Line width	19 GHz	320 GHz	 $\frac{1}{20}$ Line width
Measurement accuracy	1 GHz	15 GHz	
Measurement rel. accuracy	10 ppm	50 ppm	

Inensitivity to systematic effects

Inensitivity to systematics

Statistical uncertainty	15 GHz
Atomic physics systematics	0.1 GHz
Laser frequency calibration	0.1 GHz
Laser energy fluctuations	3 GHz

Resonance fit accounts for energy fluctuations. Yet the energy was measured regularly but not for each pulse

Binding energy

$$E_n = -\frac{m}{m_e} Z^2 \frac{R_\infty}{n^2}$$

Atomic size

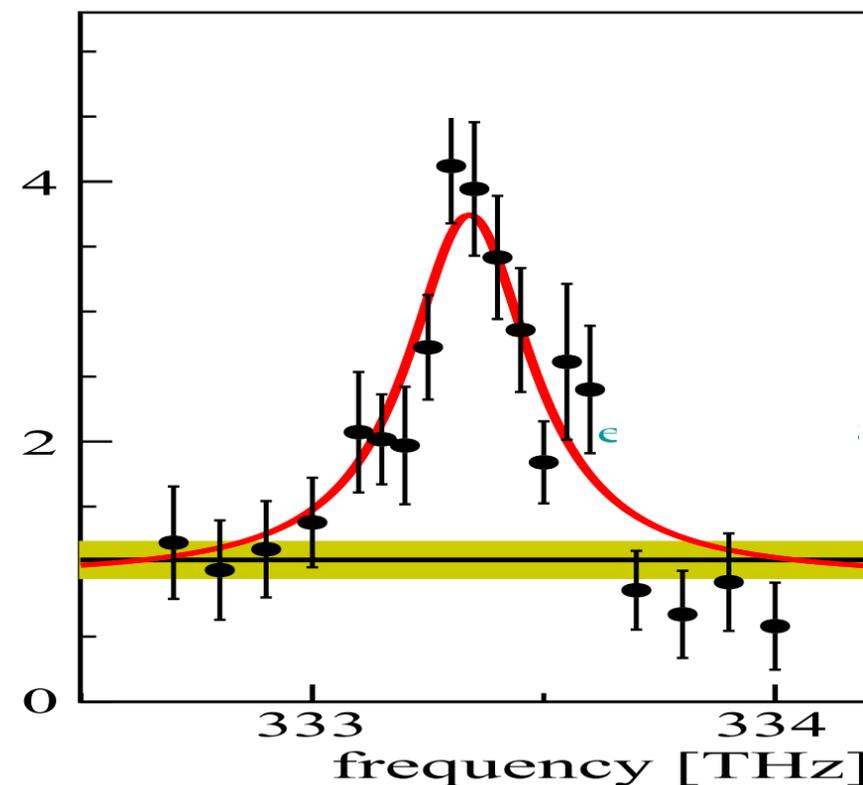
$$\langle r \rangle = \frac{h}{Z\alpha c} \frac{n^2}{m}$$

Matrix elements for perturbations

$$\Delta E = \langle \bar{\Psi} | H_1 | \Psi \rangle$$

$$H_1 = -\vec{\mu} \cdot \vec{B} \sim 1/m$$

$$H_1 = -\vec{d} \cdot \vec{E}$$



The 4He charge radius

QED

Finite size

nucl. str.

Theory

$$E_{\text{LS}}^{\text{th}} = 1668.491(7) - 106.209 r_{\alpha}^2 + 9.276(433) \text{ meV}$$

Pachucki et al., arXiv:2212.13782

Experiment

$$E_{\text{LS}}^{\text{exp}} = 1378.521(48) \text{ meV}$$

Nature 589 (2021) 7843, 527-531

$$r_{\alpha} = 1.6786(12) \text{ fm}$$

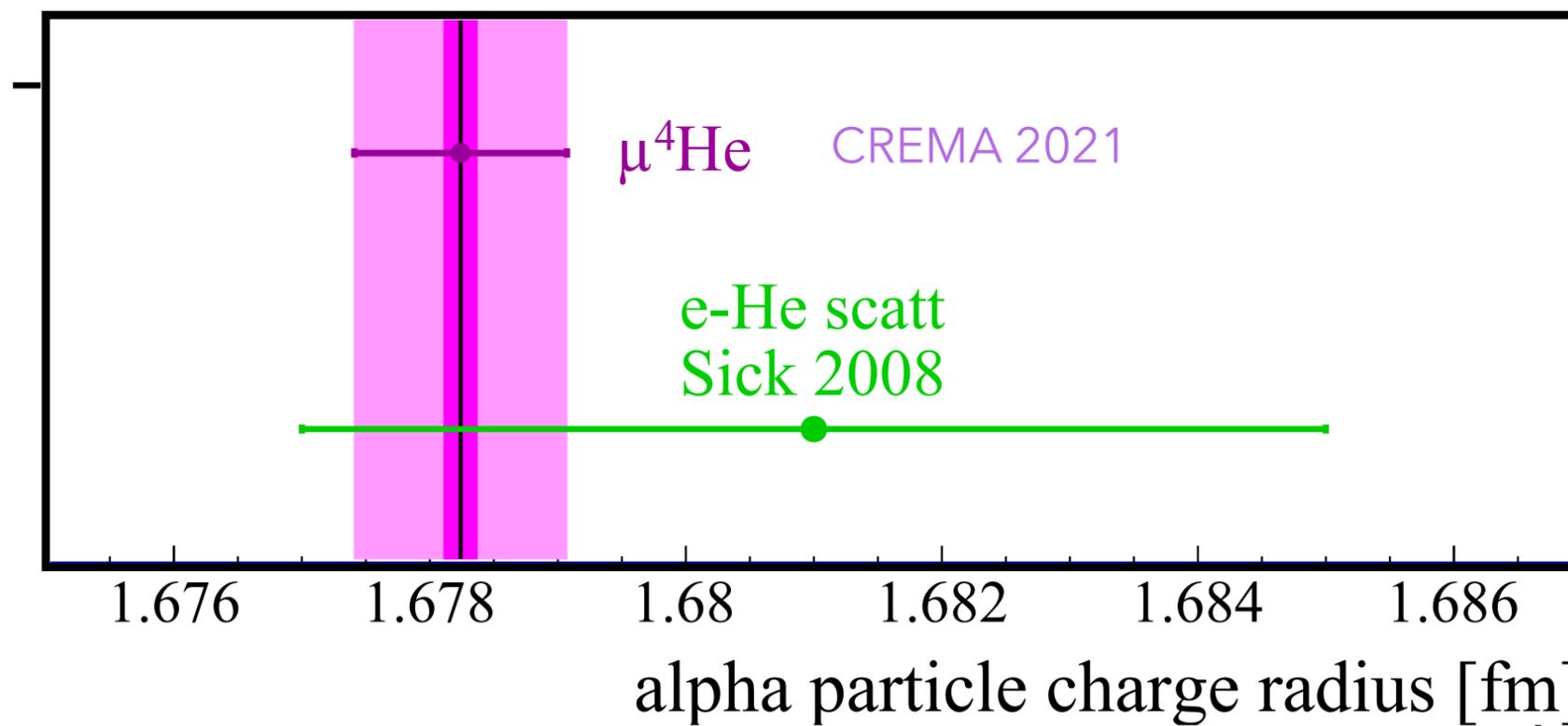
Using 2023 theory

$$r_{\alpha} = 1.67824(13)_{\text{exp}}(82)_{\text{th}} \text{ fm}$$

CREMA 2021

Uncertainty 2PE increased by 45%.
No elastic/inelastic separation.
More consistent treatment.

Bacca, Li Muli, Acharya, Ji,
Hernandez, Barnea,



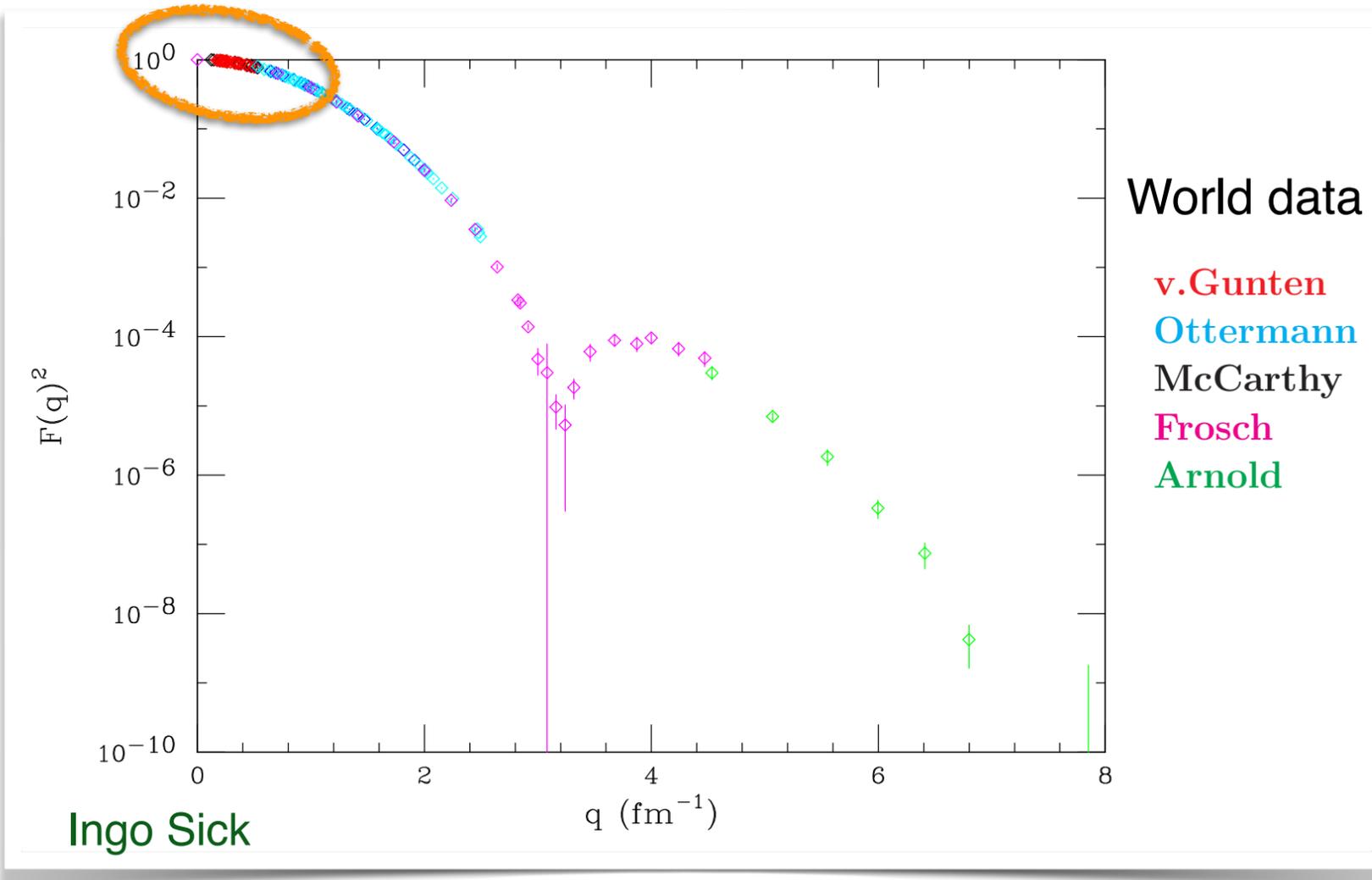
4He charge radius from electron scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} F^2(Q^2)$$

$$r_\alpha^2 = -6 \left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0}$$

$$r_\alpha = 1.681(4) \text{ fm}$$

PRC 77, 041302R (2008)



Good data

Only one form factor

He-nucleous has smaller tail compared to proton

The procedure used by Ingo Sick

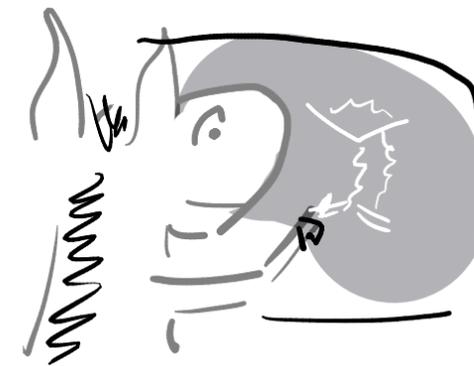
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} F^2(Q^2)$$

- ▶ Parametrize $\rho(r)$ with sum of Gaussians
including information on large- r behaviour of the proton
and the neutron wave function
- ▶ Calculate the cross section integrating the Dirac
equation with the static potential given by $\rho(r)$
- ▶ Fit the calculated cross sections to data
(in Born approx. after subtracting radiative corrections)
- ▶ Fit the form factor matching better the data and get the
slope

$$r_\alpha = -6\hbar^2 \left. \frac{dF(Q^2)}{dQ^2} \right|_{Q^2=0}$$

Assumption: The form factor is the Fourier
transform of the charge distribution

After the talk of **A. Signer** we should be very
worried about old radiative corrections



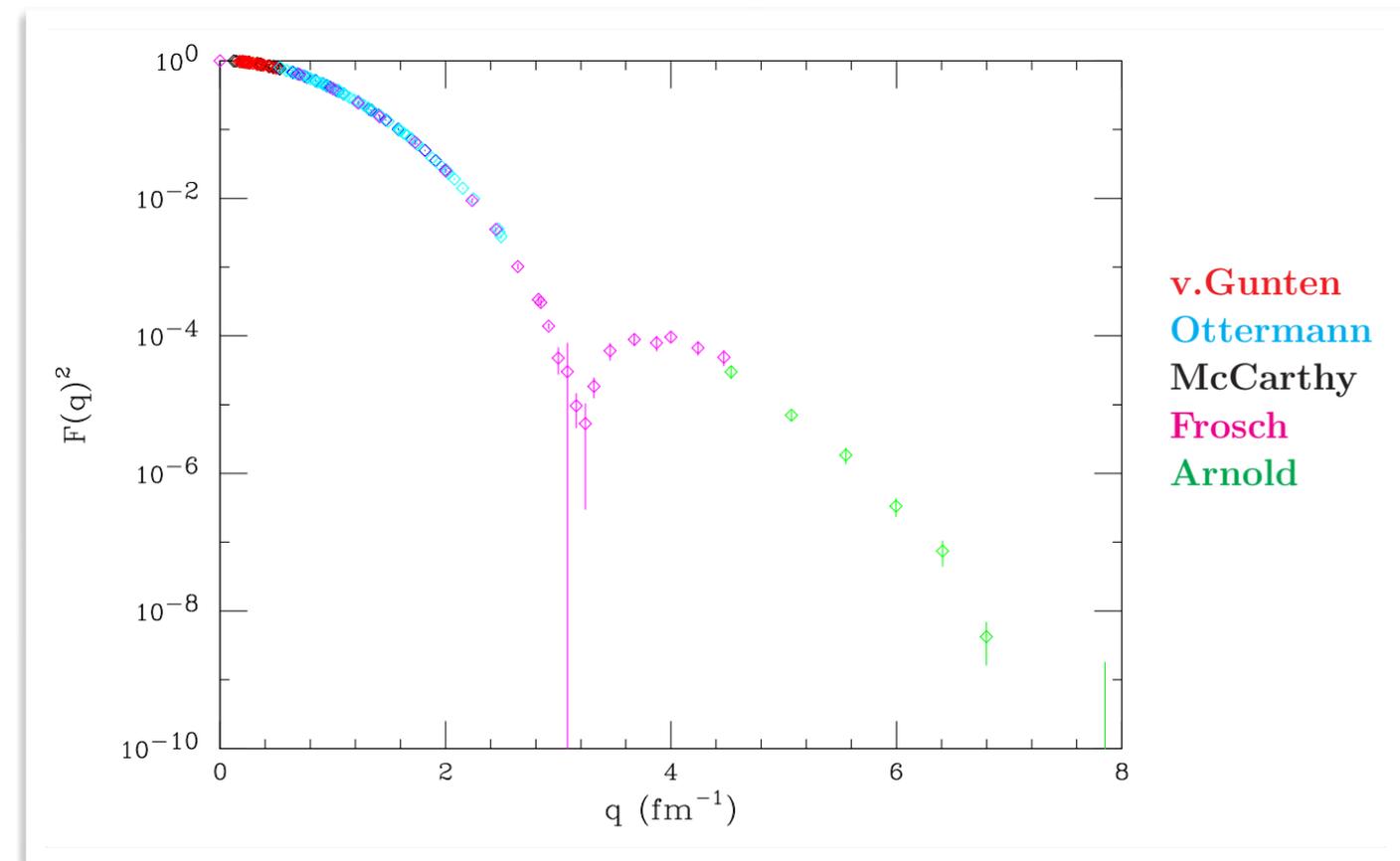
Few per mille radiative
correction of the NNLO for
"total" cross section at the
MUSE cinematic

Some proton information is used in
this analysis

Residual correlation between proton and ^4He charge radii?

It is probably small but some expert could take a look

- ▶ The proton radius is needed to obtain the charge density at large- r from the proton wave function and point density
- ▶ Otterman et al., measured the cross-section **ratio e-He/e-p**. To obtain absolute e-He cross a modern fit of the e-p data has been used

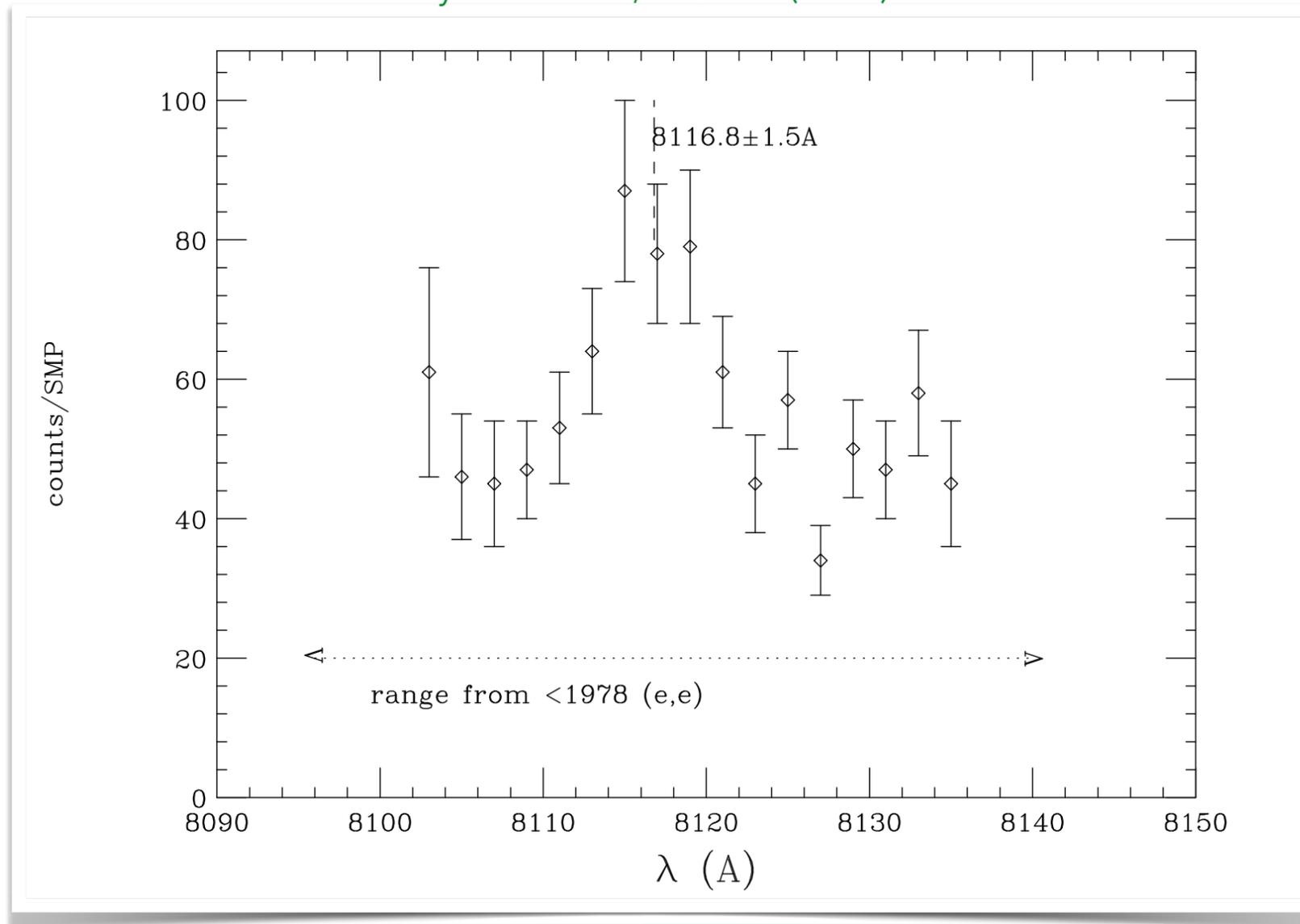


For the 60+

The $\mu^4\text{He}^+$ resonance measured at CERN (1978)

Nucl. Phys. A **278**, 381-386 (1977)

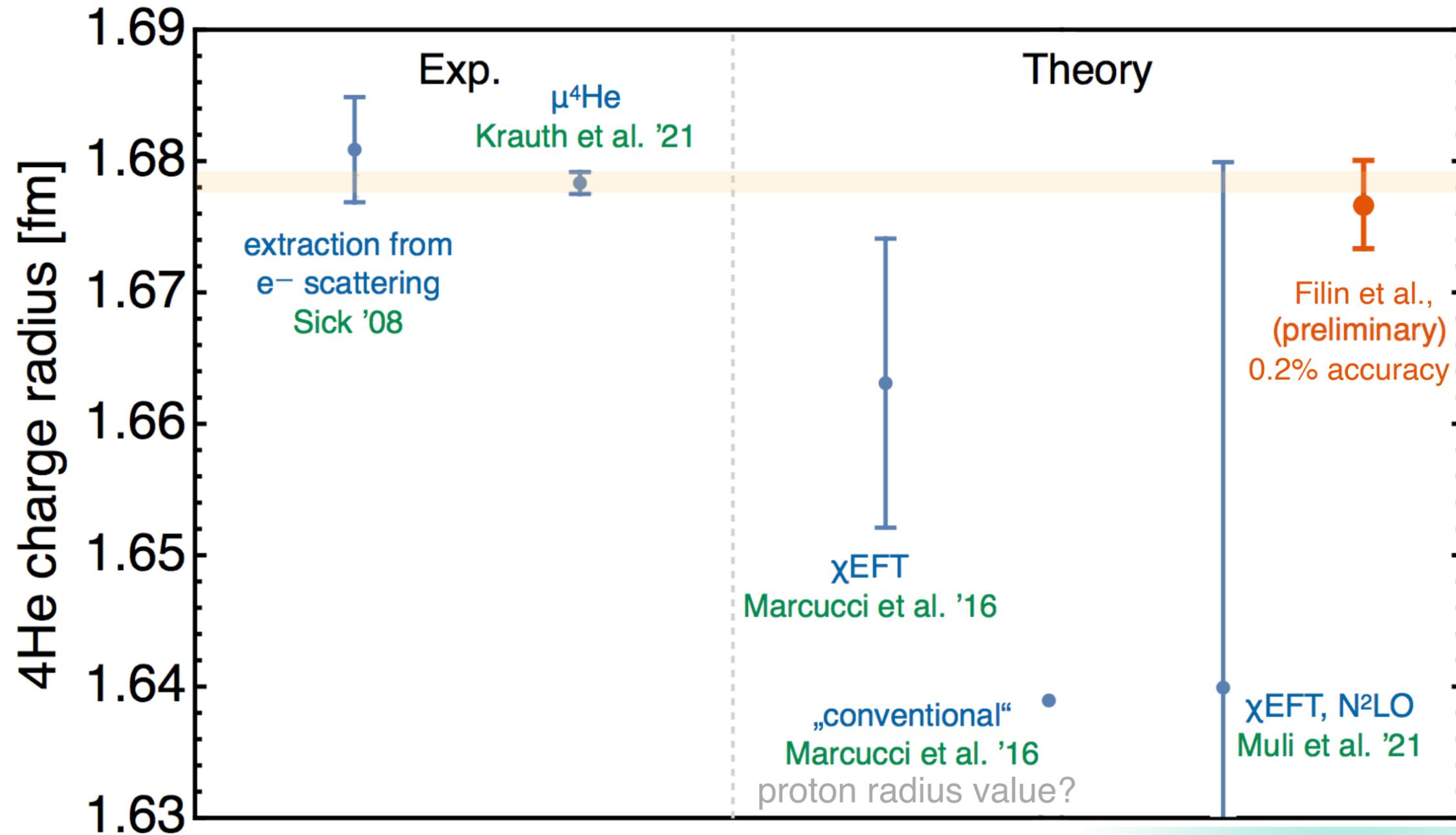
Phys. Lett. B **73**, 229-231 (1978)



- ▶ 1/200 cases get peak of similar significance
- ▶ Peak found after peak-optimising cuts
- ▶ Measurement performed at a He pressure of 40 bar, our experiment at 0.002 bar
- ⇒ collisional quenching of 2S state (17 ps lifetime of the 2S state)

The obtained radius 1.673(1) fm is not far away from our value due to an awkward combination of wrong measurement and incomplete 2S-2P theory.

Radius as a benchmark for *ab initio* few-nucleon theories

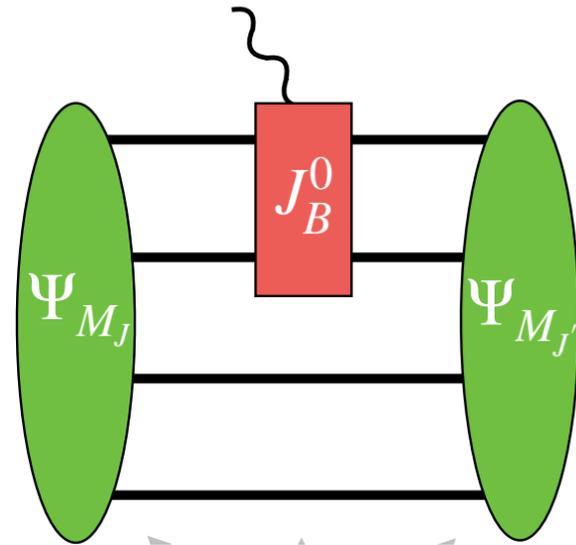


Plot from Filin

Towards consistent treatment of the nuclear structure: TPE and radii

Radius as benchmark for *ab initio* few-nucleon predictions

Filin

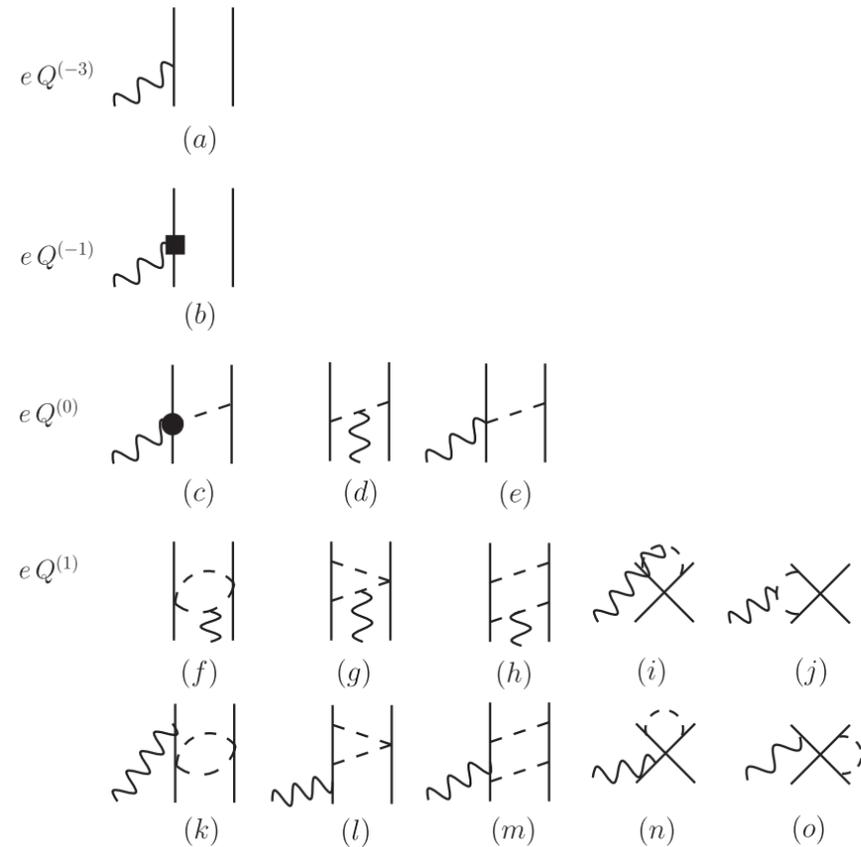


$$F_C(Q^2) = \frac{1}{2J+1} \sum_{M_J} \langle P', M_J | J_B^0 | P, M_J \rangle$$

In Breit frame

$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2} \right) + \frac{A-Z}{Z} r_n^2$$

Current operator



Li Muli

Consistent derivation and regularization of many body forces and nuclear current operators + Rel. Dynamics+ isospin breaking+...

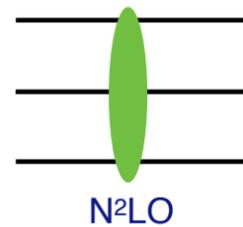
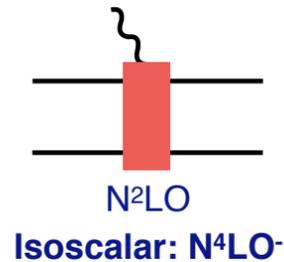
Bochum group: Epelbaum

Mainz group: Bacca

Nuclear forces

	2N force	3N force	4N force
LO			
NLO			
N2LO			
N3LO			

Consistent treatment



Taken from
Filin

- ▶ Advances in the consistent derivation and regularization of the NN and NNN forces. Various optimisation schemes used to define the LEC
- ▶ Consistent derivation and regularization of many body forces and nuclear current operators.
- ▶ Effort to include the relativistic dynamics of the nucleons ongoing
- ▶ Significant advances in uncertainty estimation: EFT, Bayesian approaches, etc

H. Krebs, EPJA 56 (2020) 240

Reinert et al. PRL 126, 092501 (2021)

E. Epelbaum, H. Krebs, and P. Reinert, arXiv:2206.07072.

P. Maris et al., PRC **106**, 064002 (2022)

H.-W. Hammer et al., REVIEWS OF MODERN PHYSICS, 92, (2020)

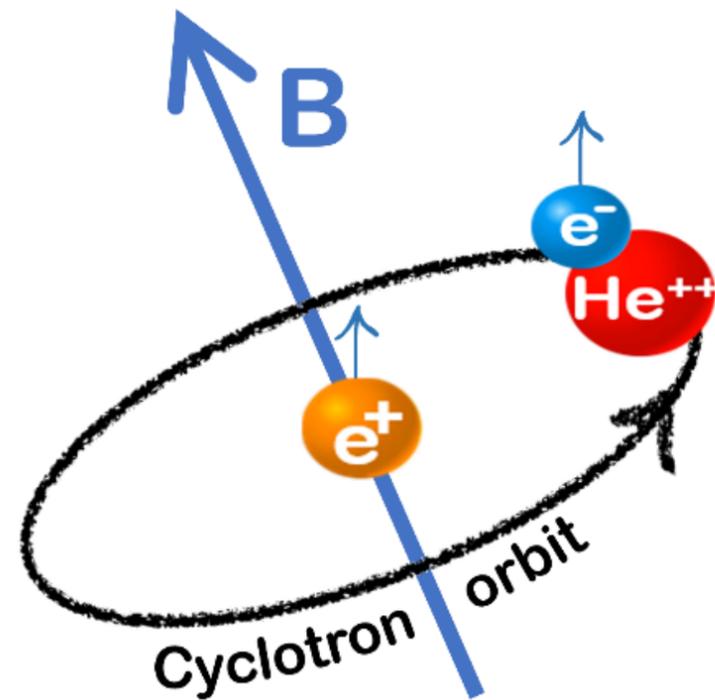
Ekström et al., Phys. Rev. C **91**, 051301(R) (2015)

Lynn et al., A 2017 Phys. Rev. C **96** 0540 (2017)

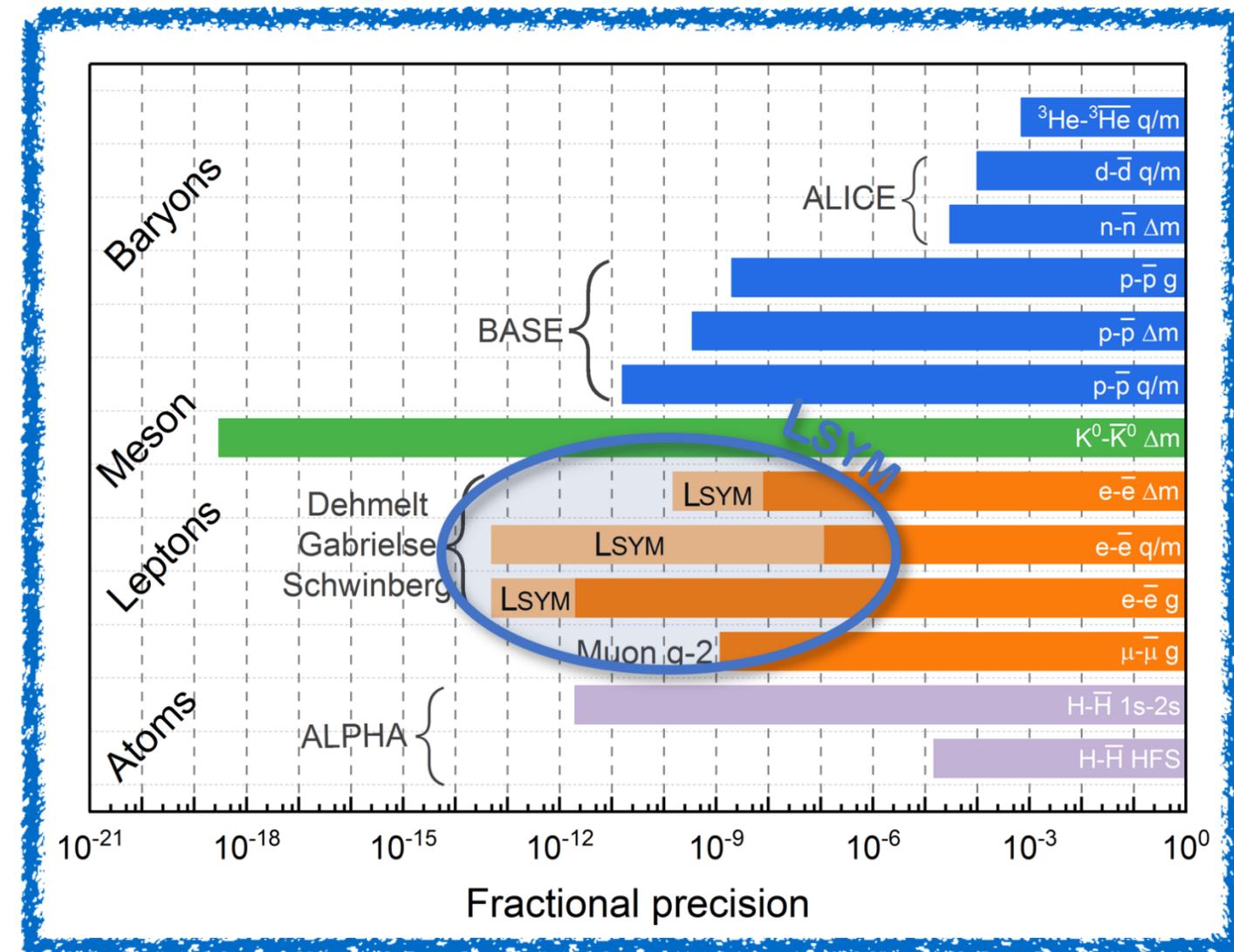
Li Muli et al., J. Phys. G: Nucl. Part. Phys. **49** 105101 (2022)

CPT tests (Lsym-project)

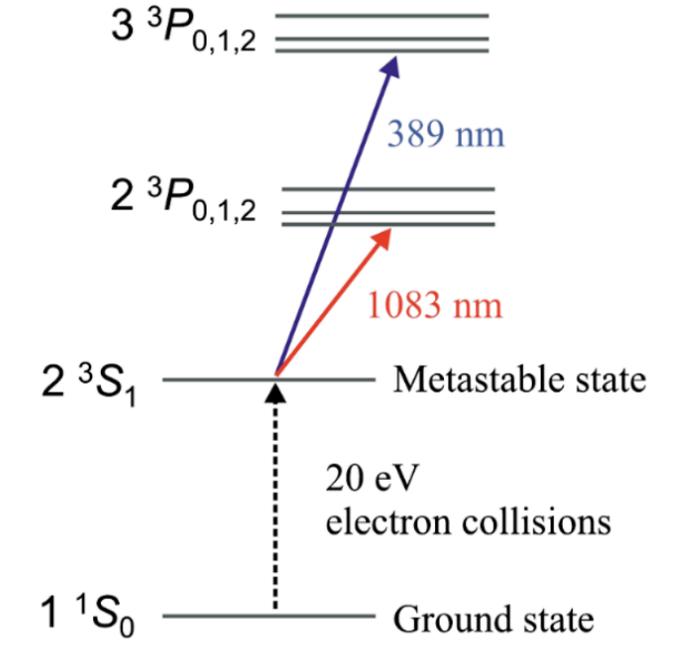
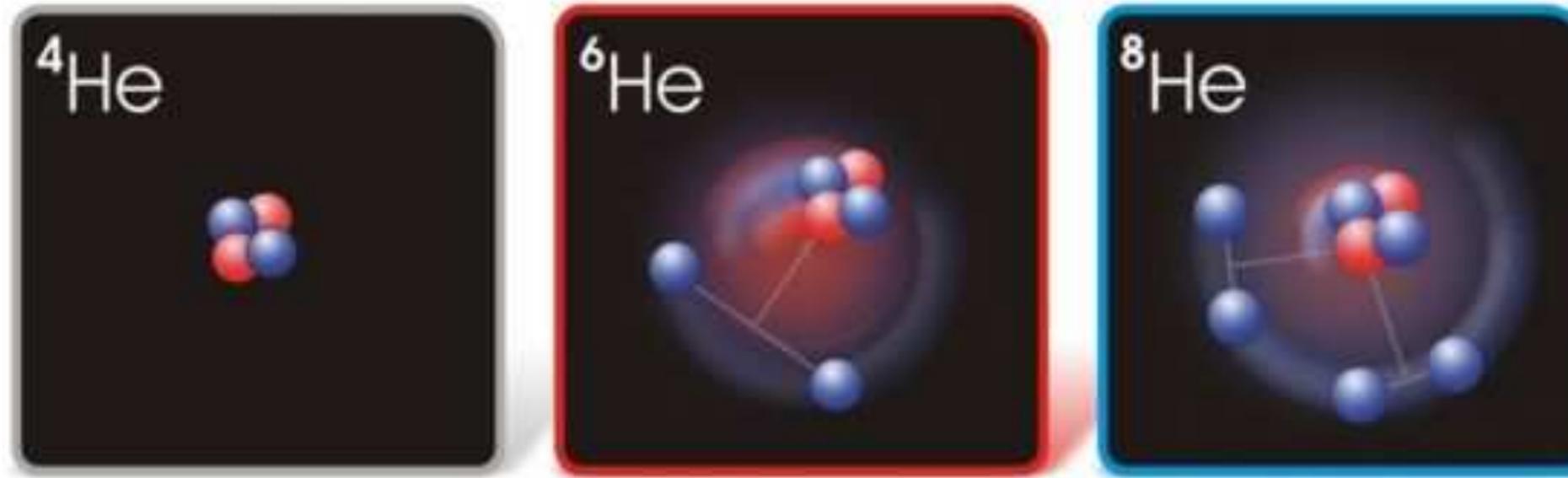
S. Sturm



He charge radius and He structure needed to advance CPT tests



r_α : anchor point for isotopic shift measurements



Measured

Measured

Mass shift

Field shift

$$\nu(^A\text{He}) - \nu(^4\text{He}) = \Delta\nu_{\text{mass}} + k(r_A^2\text{He} - r_\alpha^2)$$

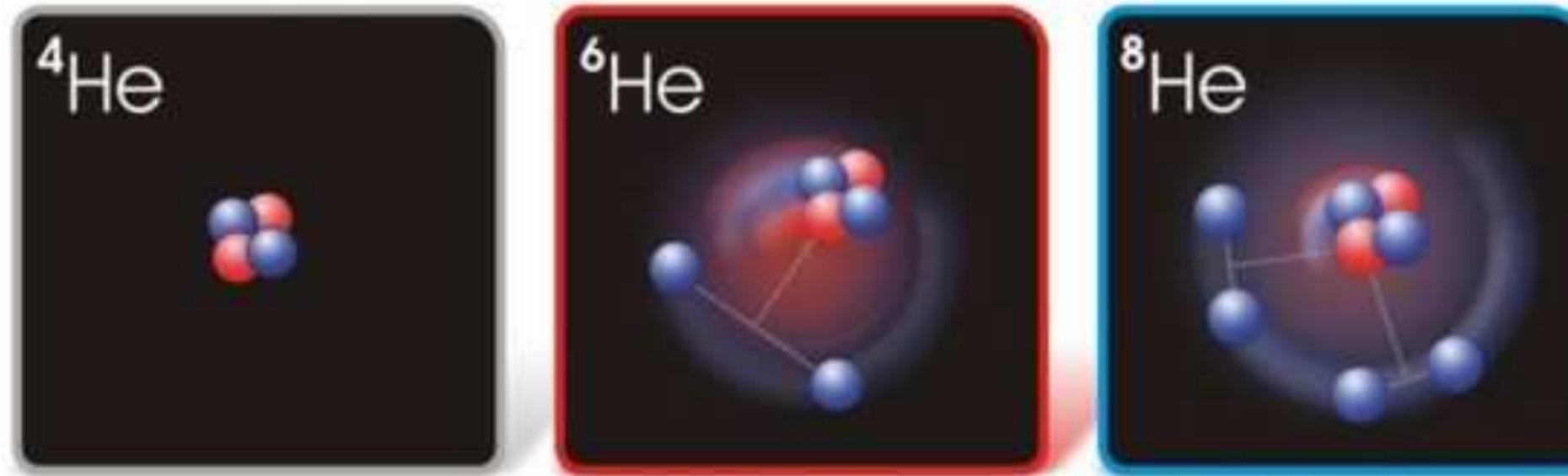
anchor point:
 r_α from μHe^+

$$r(^6\text{He}) = 2.0571 (7)_{r_\alpha} (75)_{\text{iso}} \text{ fm}$$

$$r(^8\text{He}) = 1.9559 (7)_{r_\alpha} (158)_{\text{iso}} \text{ fm},$$

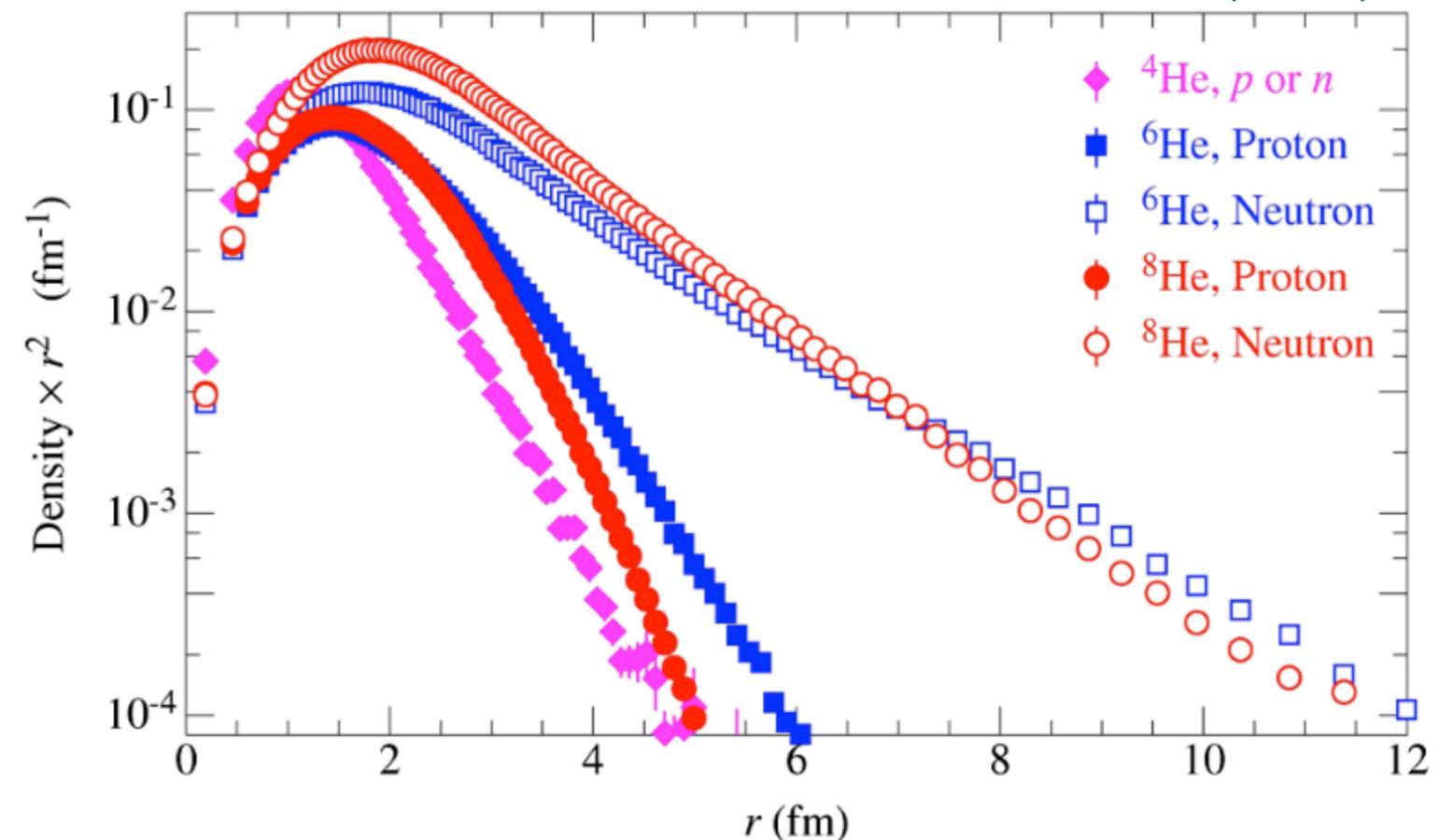
RMP 85 1383 (2013)

${}^6\text{He}$ and ${}^8\text{He}$ halo nuclei



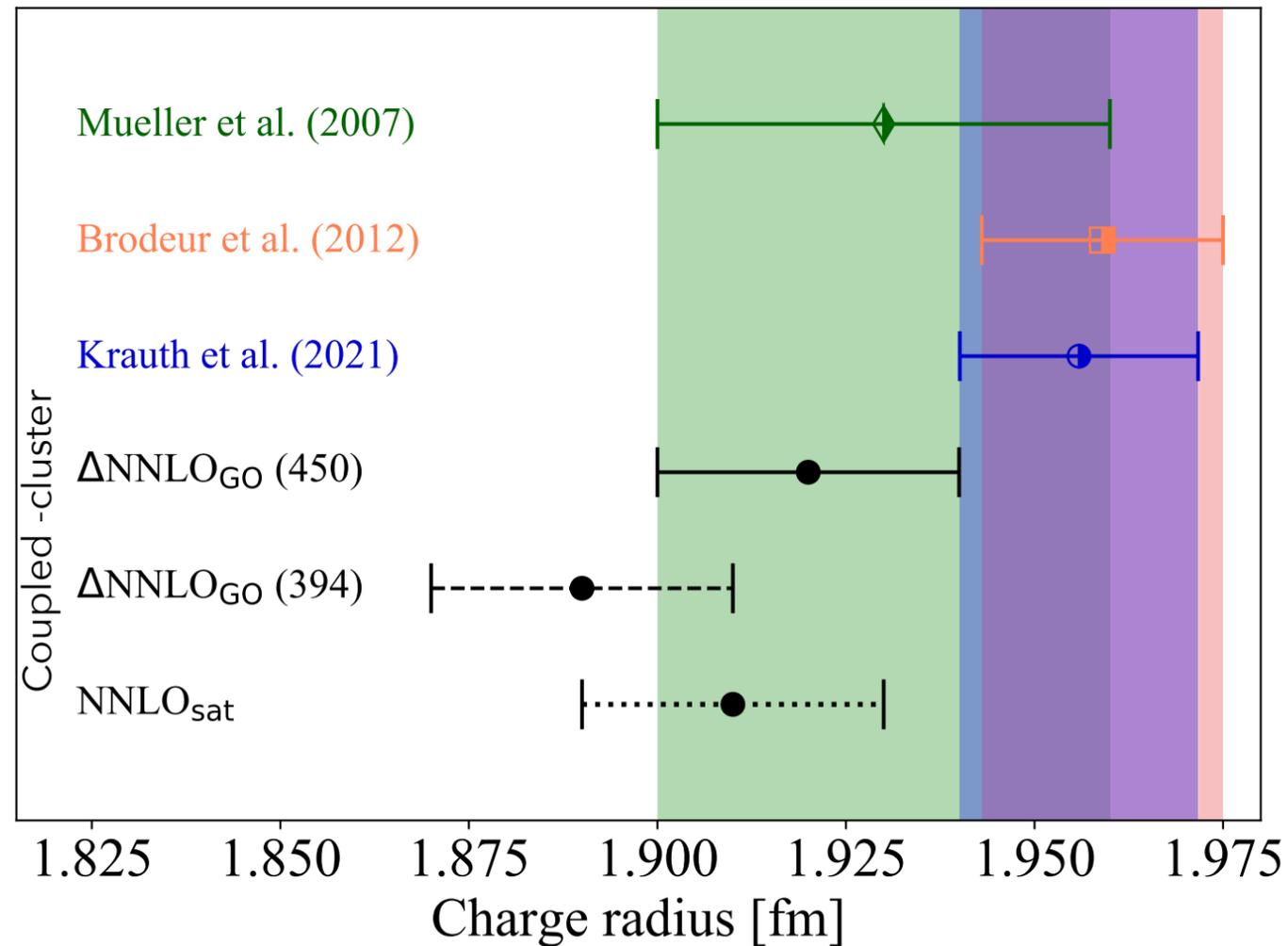
- ▶ Large matter radius and small neutron separation energy
- ▶ The size of the core can be associated to the rms charge radius. Its swelling results from polarisation effects due to the strong interaction

RMP 85 1383 (2013)

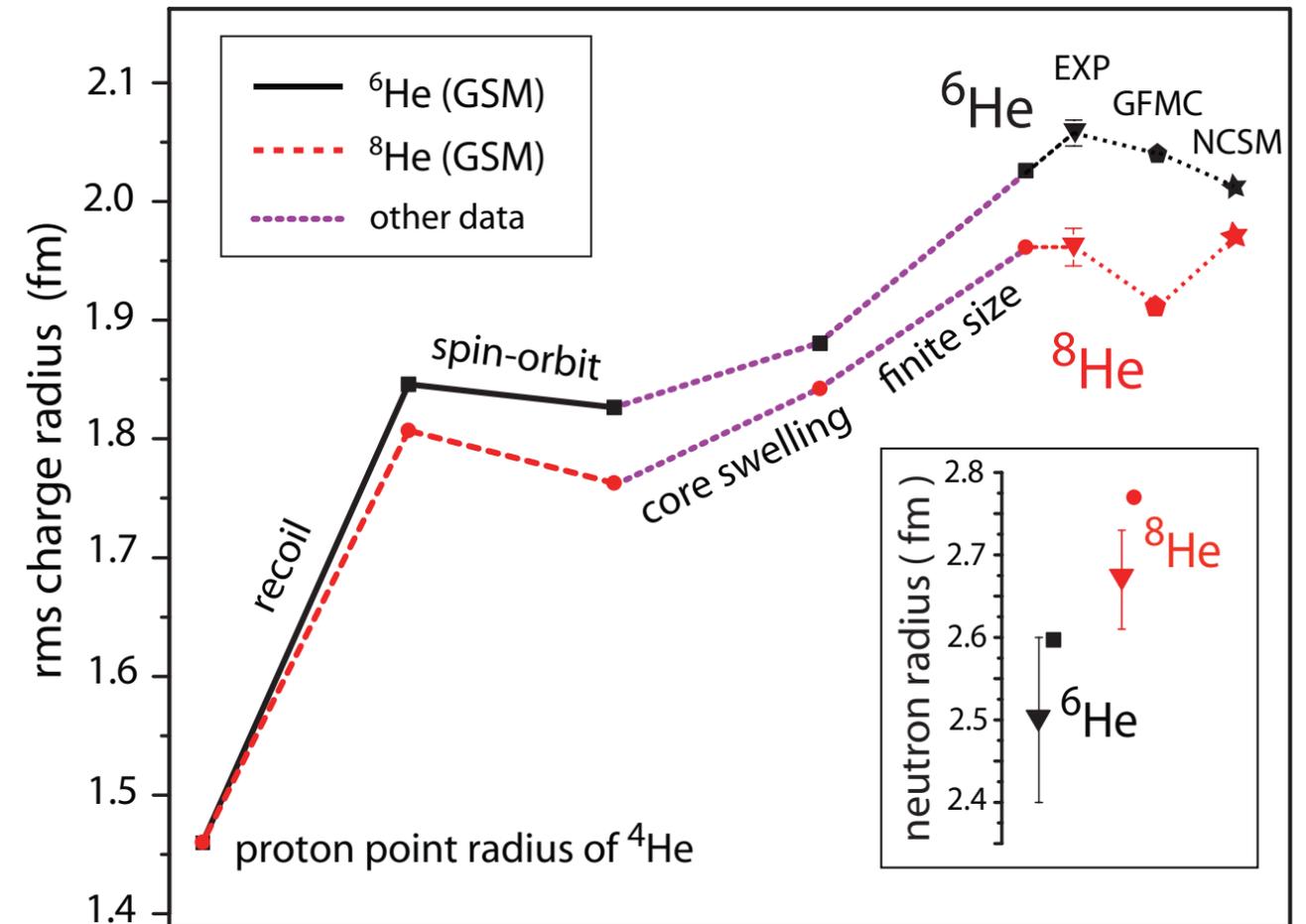


Comparison theory-experiment for ^8He and ^6He charge radii

F. Bonaiti et al., PRC **105**, 034313 (2022)



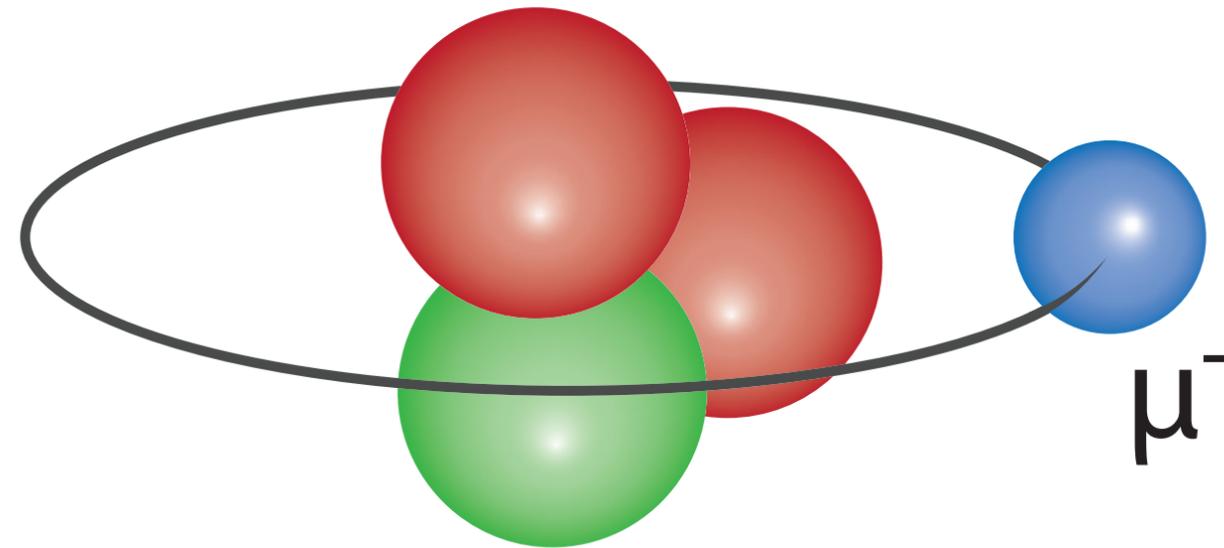
G. Papadimitriou, PRC **84**, 051304(R) (2011)



$$\langle R_{\text{ch}}^2 \rangle = \langle R_{pp}^2 \rangle + R_p^2 + \frac{N}{Z} R_n^2 + \frac{3}{4M_p^2} + R_{\text{SO}}^2,$$

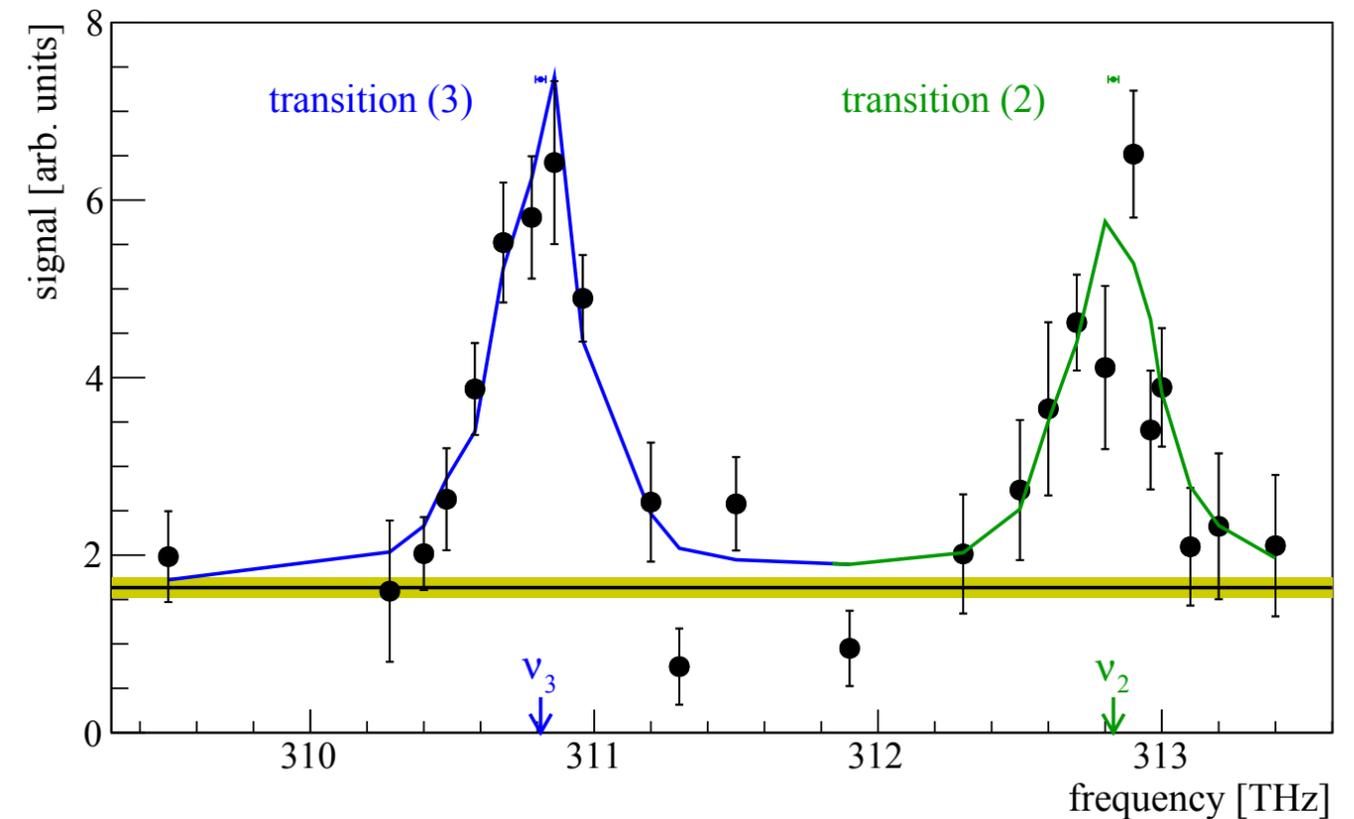
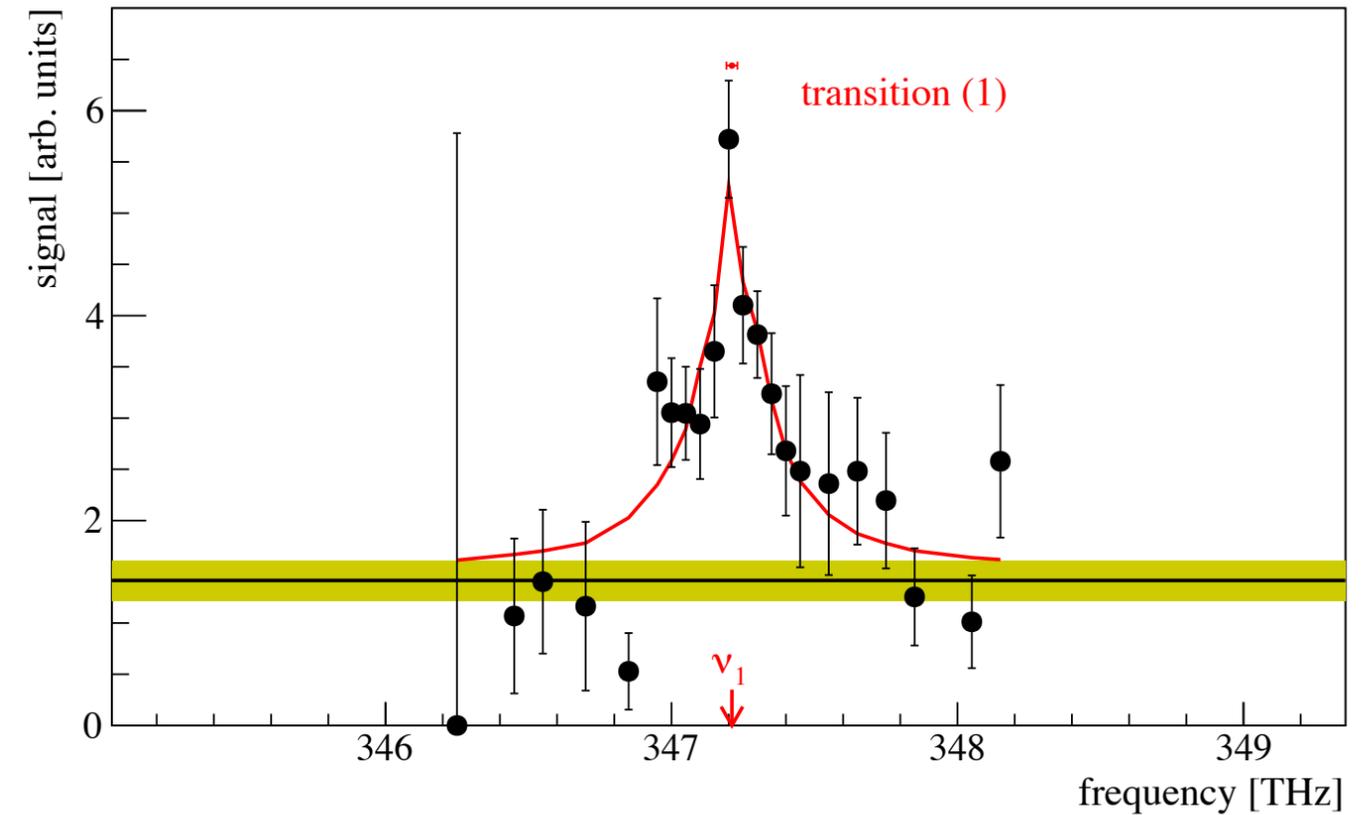
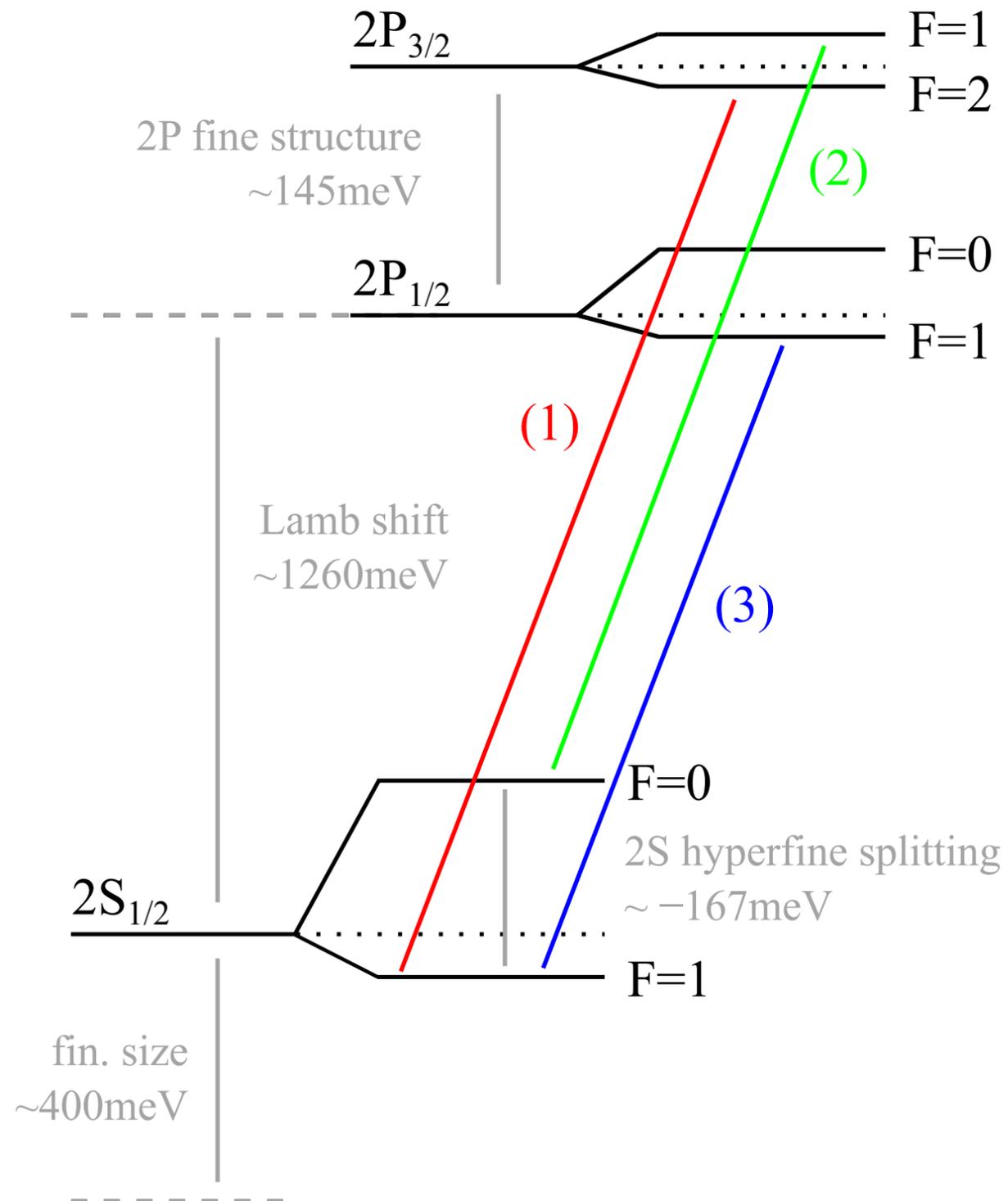
The *ab initio* GFMC calculations predict a 4.58% and 6.66% increase of the α -particle proton point radius in ^6He and ^8He , respectively

${}^3\text{He}^{++}$

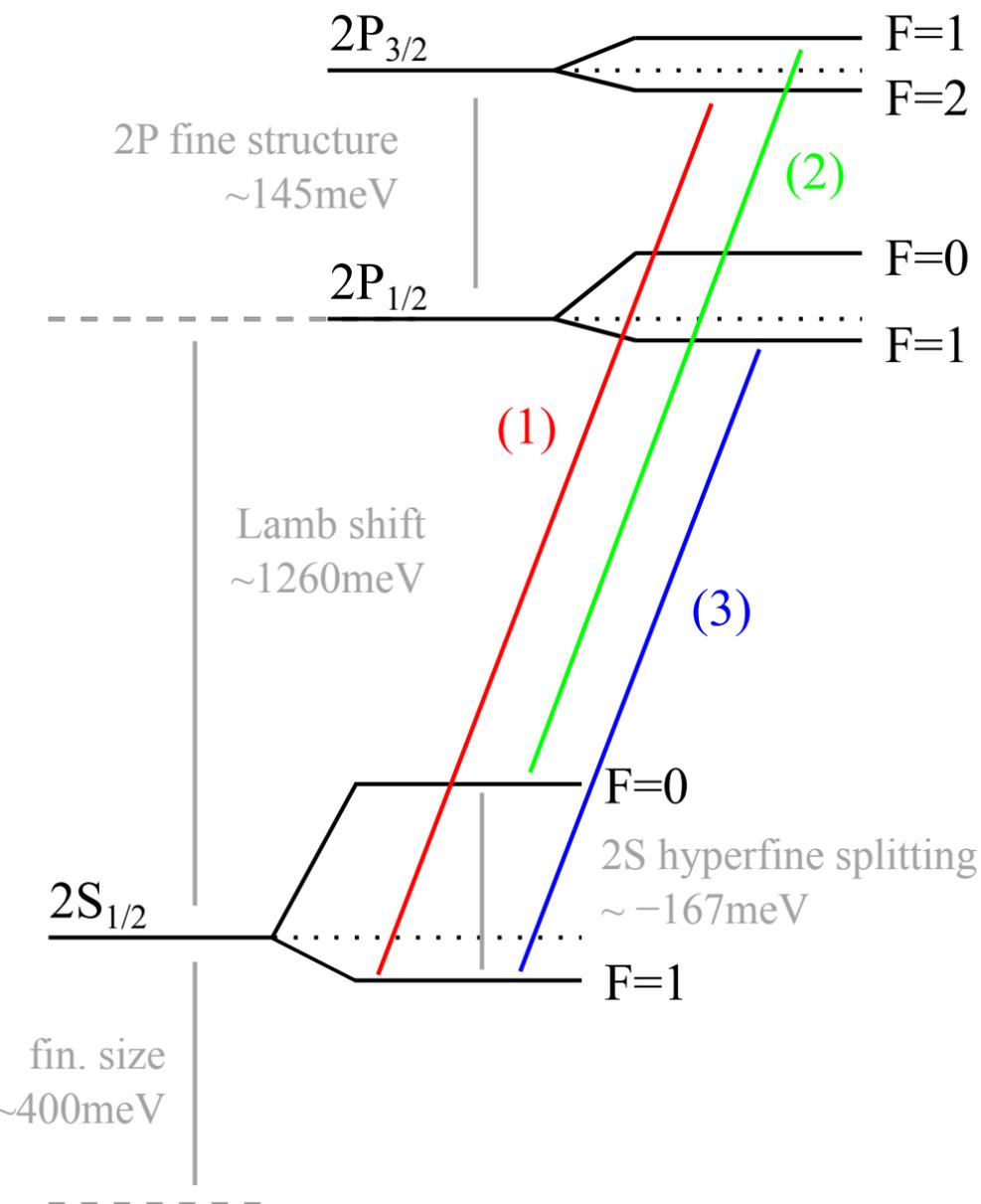


Schuhmann et al., arXiv 2305.11679

The measured transitions



The measured transitions between fine and hyperfine sub-levels



For laser physics

$$\nu_{\text{exp}}^{(1)} = 347.212(20)^{\text{stat}}(1)^{\text{sys}} \text{ THz}$$

$$\nu_{\text{exp}}^{(2)} = 312.830(21)^{\text{stat}}(1)^{\text{sys}} \text{ THz}$$

$$\nu_{\text{exp}}^{(3)} = 310.814(20)^{\text{stat}}(1)^{\text{sys}} \text{ THz}$$

Statistical unc. \gg Systematic

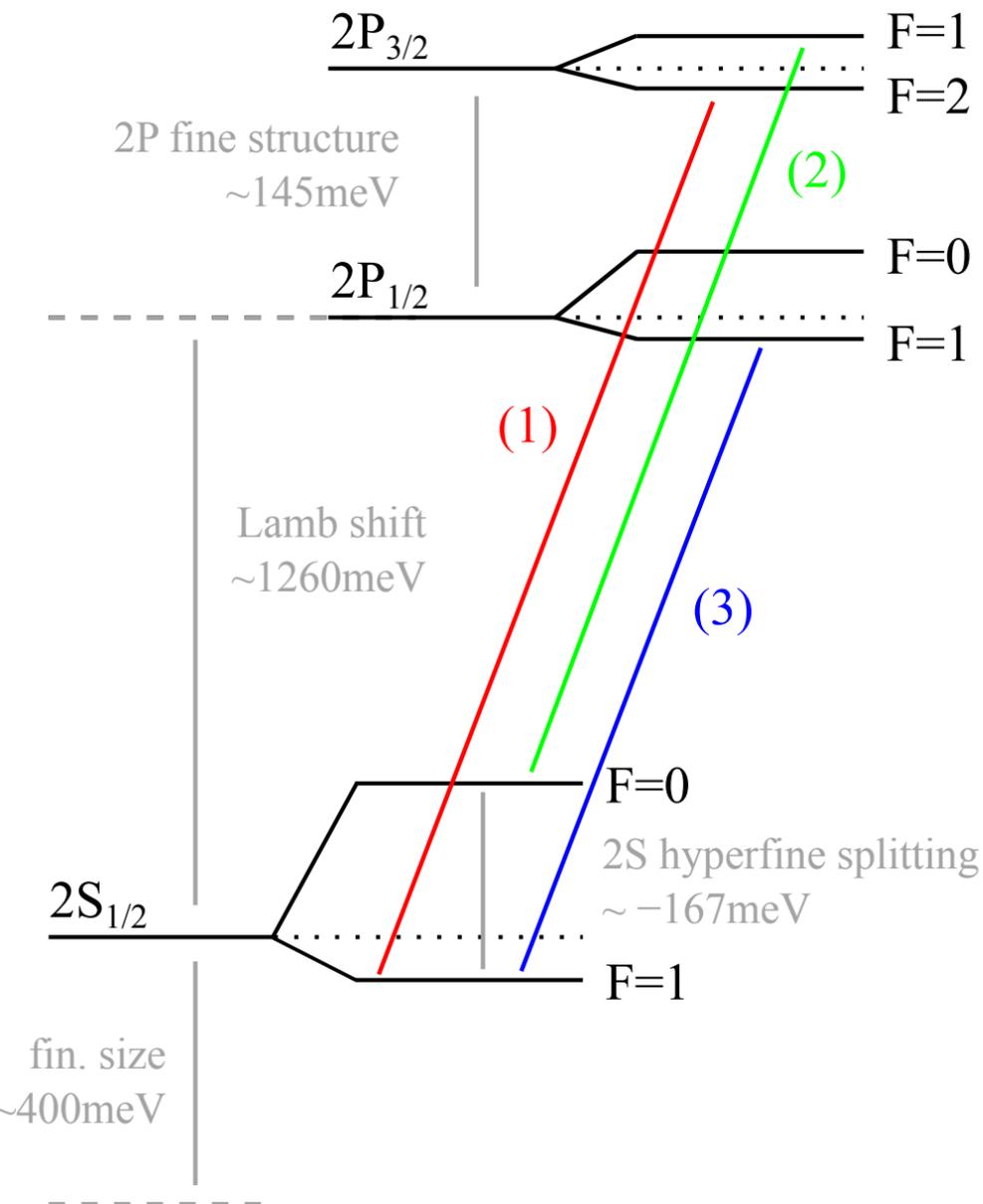
For theorist

$$\Delta E_{\text{exp}}^{(1)} = 1435.951(81) \text{ meV}$$

$$\Delta E_{\text{exp}}^{(2)} = 1293.759(86) \text{ meV}$$

$$\Delta E_{\text{exp}}^{(3)} = 1285.425(81) \text{ meV}$$

From the measured transitions to the quantities of interest



Paying attention at the various definitions of the Lamb shift and centroid positions....

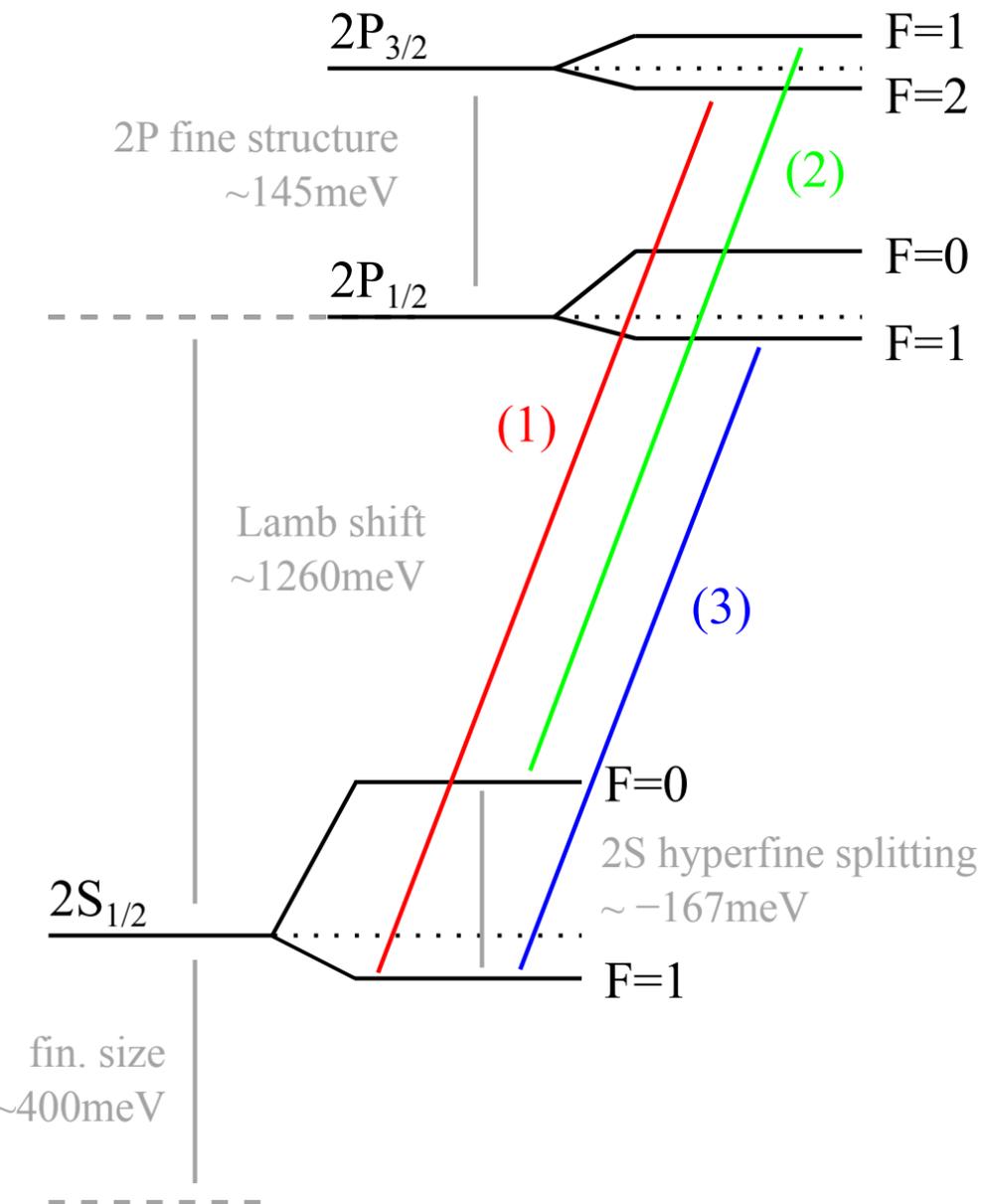
$$\Delta E_{\text{exp}}^{(1)} = E_{\text{LS}} - \frac{1}{4} E_{\text{HFS}} + E_{\text{FS}} - 9.23945(26) \text{ meV}$$

$$\Delta E_{\text{exp}}^{(2)} = E_{\text{LS}} + \frac{3}{4} E_{\text{HFS}} + E_{\text{FS}} + 15.05305(44) \text{ meV}$$

$$\Delta E_{\text{exp}}^{(3)} = E_{\text{LS}} - \frac{1}{4} E_{\text{HFS}} - 14.80851(18) \text{ meV}$$

↑
2P hyperfine theory
Karshenboim, et al.
PRA 96, 022505 (2017).

Lamb shift, 2S-hyperfine splitting, 2P fine splitting



Extracted quantities

$$E_{\text{LS}}^{\text{exp}} = 1258.612(86) \text{ meV}$$

$$E_{\text{HFS}}^{\text{exp}} = -166.485(118) \text{ meV}$$

$$E_{\text{FS}}^{\text{exp}} = 144.958(114) \text{ meV.}$$

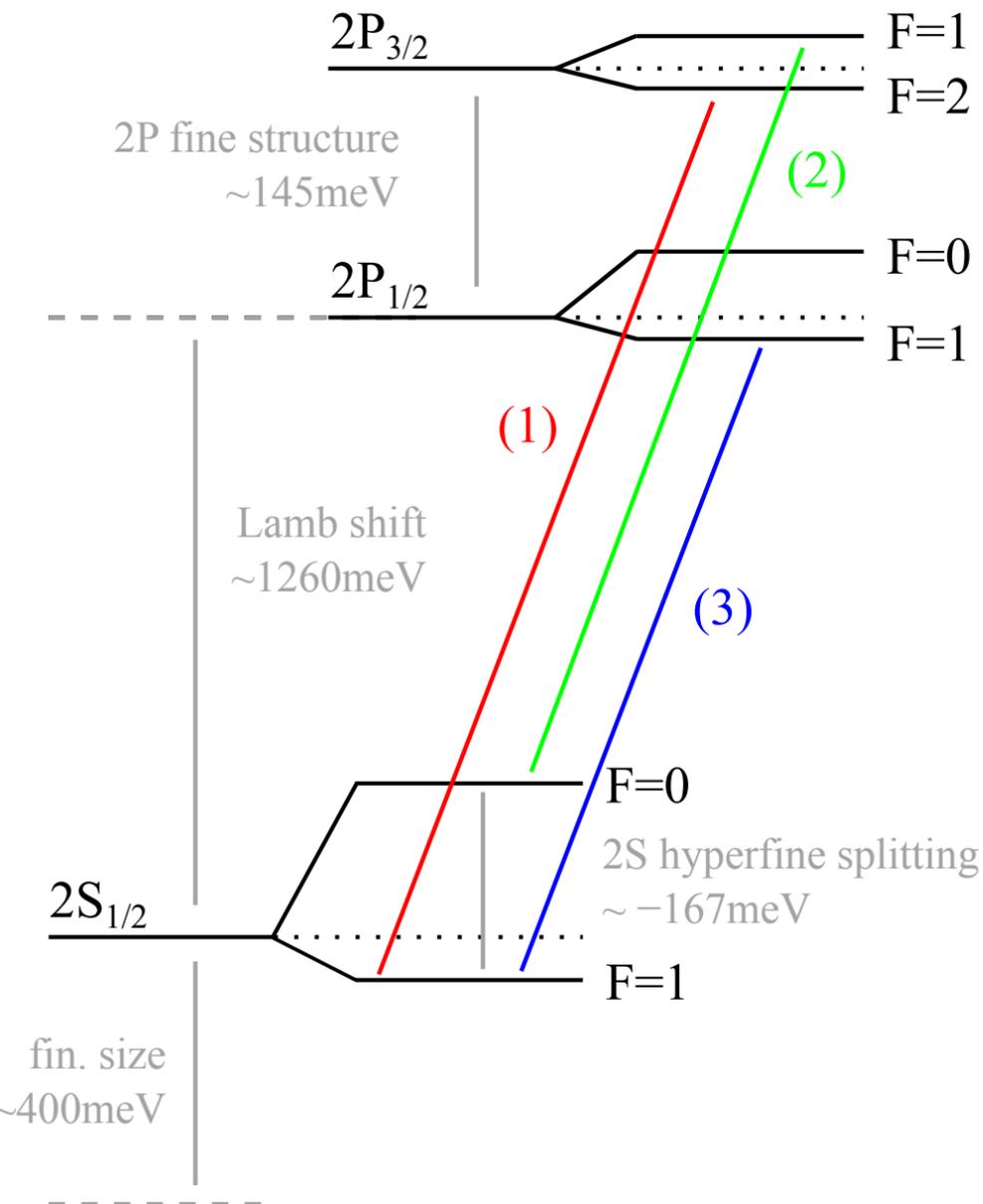
$$E_{\text{FS}}^{\text{th}} = 144.979(5) \text{ meV}$$

Karshenboim, et al.
PRA 96, 022505 (2017).

Agreement
th-exp

Consistency check

Lamb shift and 2S-hyperfine splitting



Use theory for the 2P fine and hyperfine splittings

$$\Delta E_{\text{exp}}^{(1)} = E_{\text{LS}} - \frac{1}{4} E_{\text{HFS}} + E_{\text{FS}} - 9.23945(26) \text{ meV}$$

$$\Delta E_{\text{exp}}^{(2)} = E_{\text{LS}} + \frac{3}{4} E_{\text{HFS}} + E_{\text{FS}} + 15.05305(44) \text{ meV}$$

$$\Delta E_{\text{exp}}^{(3)} = E_{\text{LS}} - \frac{1}{4} E_{\text{HFS}} - 14.80851(18) \text{ meV}$$

Improve on the Lamb shift and the 2S-hyperfine splitting

$$E_{\text{LS}}^{\text{exp}} = 1258.598(48)^{\text{exp}} (3)^{\text{theo}} \text{ meV}$$

$$E_{\text{HFS}}^{\text{exp}} = -166.496(104)^{\text{exp}} (3)^{\text{theo}} \text{ meV}.$$

The helion charge radius

QED

Finite size

nucl. str.

Theory

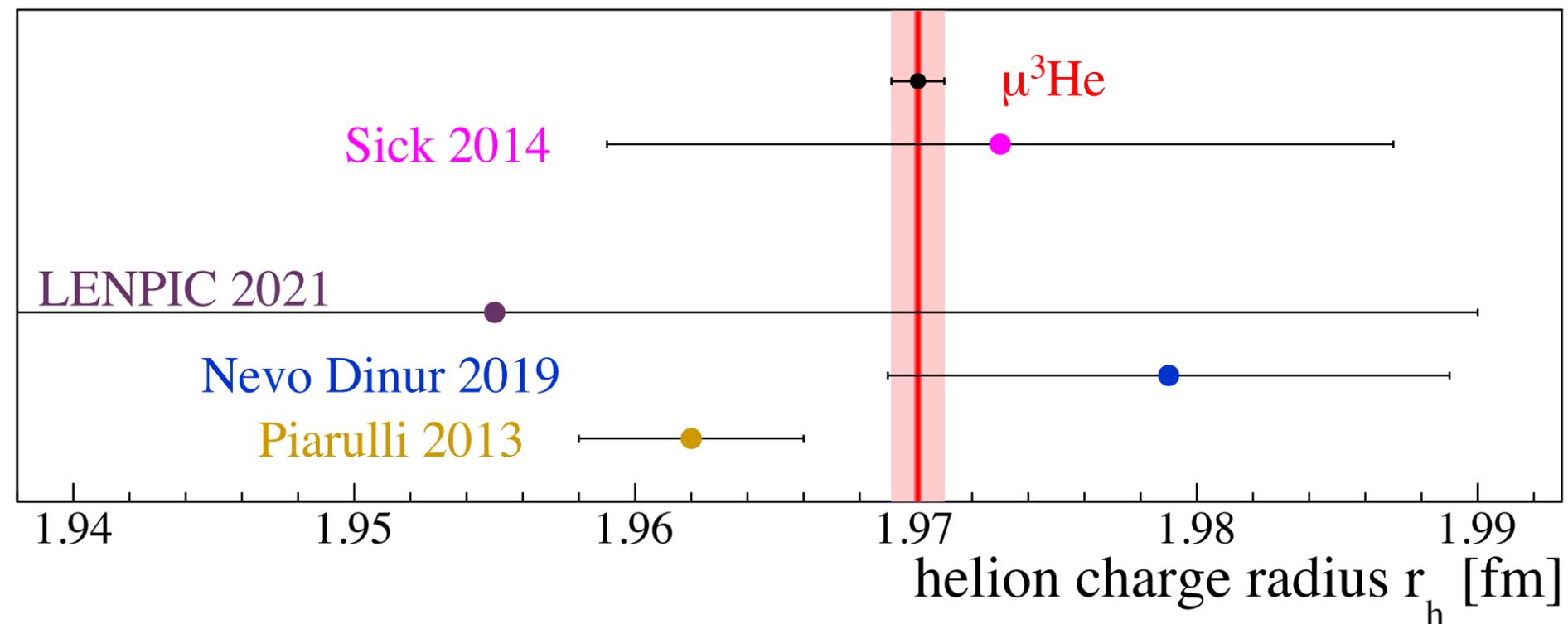
$$E_{\text{LS}}^{\text{th}} = 1644.348(8) - 103.383 r_h^2 + 15.499(378) \text{ meV} \quad \text{Pachucki et al., arXiv:2212.13782}$$

Experiment

$$E_{\text{LS}}^{\text{exp}} = 1258.598(48)^{\text{exp}}(3)^{\text{theo}} \text{ meV}$$

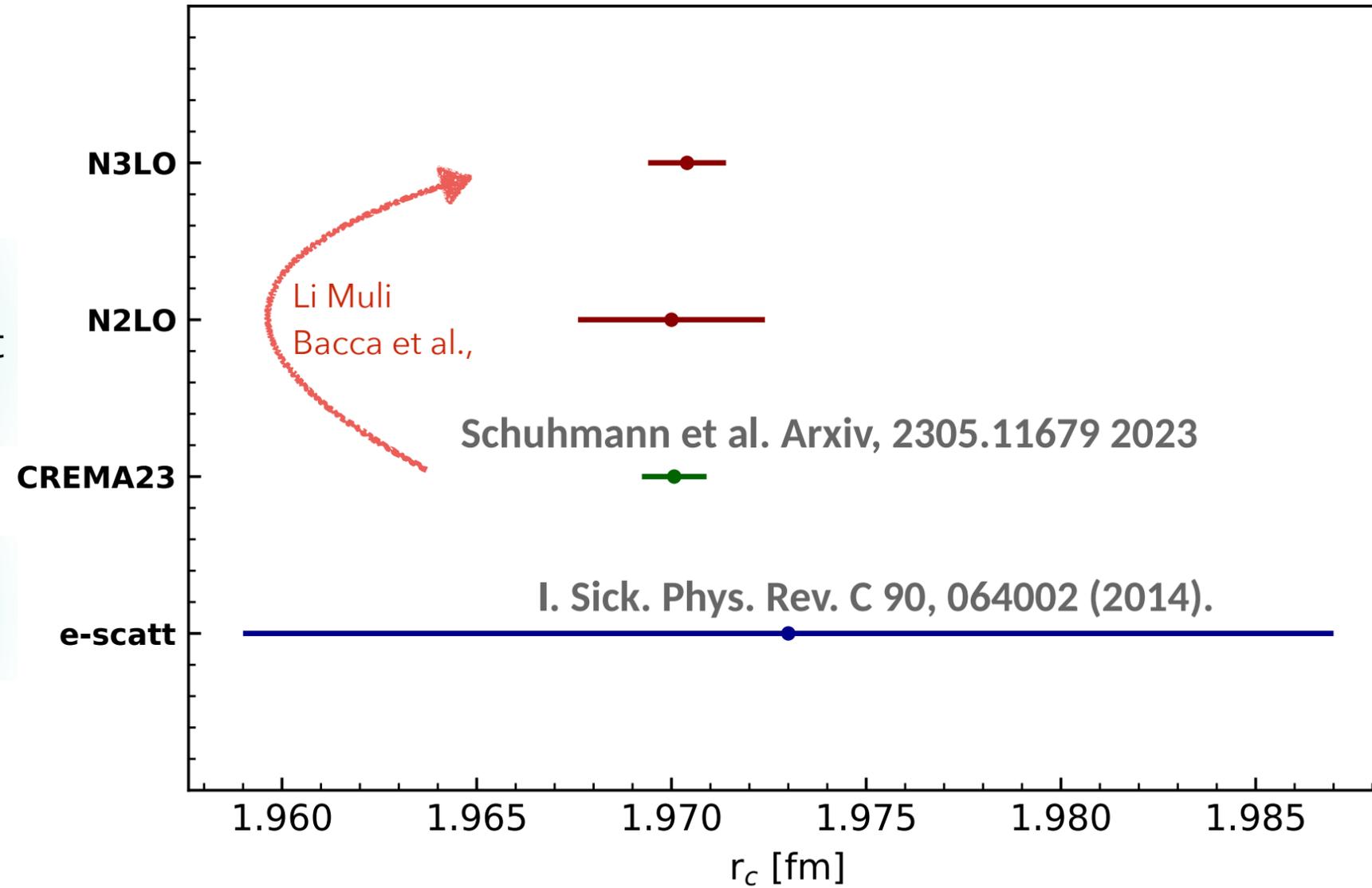
Schuhmann et al., arXiv 2305.11679

$$r_h = 1.97007(12)^{\text{exp}}(93)^{\text{theo}} \text{ fm} = 1.97007(94) \text{ fm}.$$



I have a personal complaint

S.S.LM, et al. In preparation for 2023



The error bars will slightly increase in next future

But they will decrease in next-to-next future

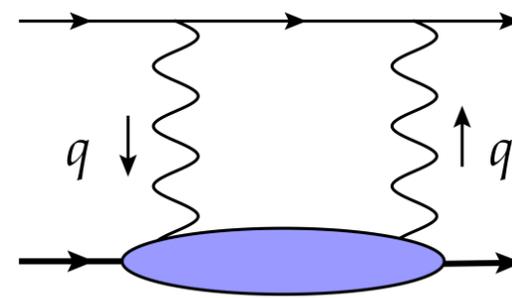
Not yet published and already corrected



Two ways to the two-photon exchange

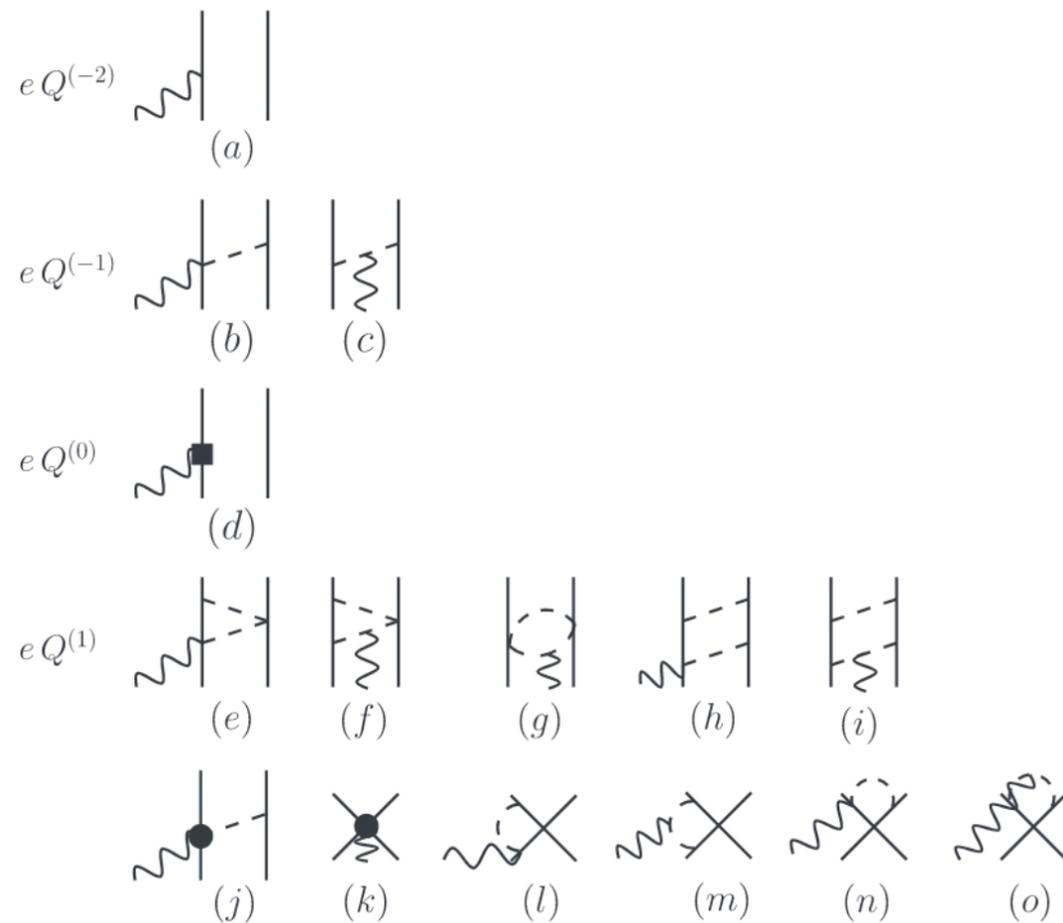
Dinur, Ji, Barnea,
Bacca, Hernandez, Li Muli ...

few-nucleon ab initio
theory
with chiral-inspired
potentials



Dispersion relation
and data

Carlson, Gorchtein,
Vanderhaeghen



Agreement

For $\mu^3\text{He}$

Dispersion: 15.14 (49) meV
Few-nucleon th.: 16.38 (31) meV

Another benchmark for nuclear theories: 2S HFS

Fermi+QED

nucl. str.

Theory

$$E_{\text{HFS}}^{\text{th}} = -172.7457(89) \text{ meV} + E_{\text{HFS}}^{\text{nucl. struct.}}$$

Franke et al., EPJD 71, 341 (2017).

Experiment

$$E_{\text{HFS}}^{\text{exp}} = -166.496(104)^{\text{exp}(3)\text{theo}} \text{ meV}$$

Schuhmann et al., arXiv 2305.11679

Nuclear structure contribution (2PE+...)

$$E_{2\text{S-HFS}}^{\text{nucl. struct.}} = 6.25(10) \text{ meV}$$

No theory predictions exists to date for the inelastic part

$$\Delta E_{2\text{PE}}^{\text{Zemach}} = 6.53(4) \text{ meV}$$

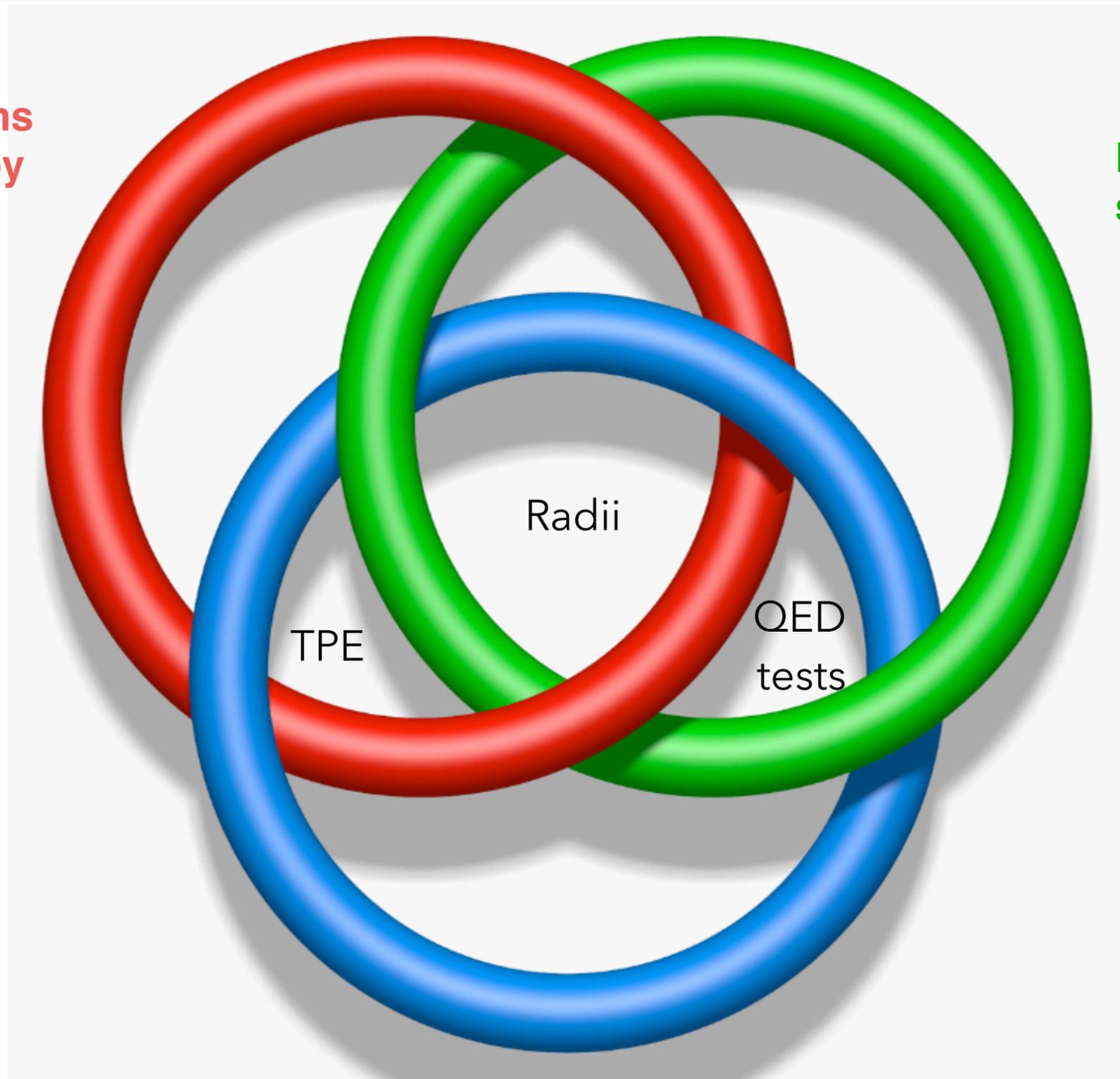
using $r_Z = 2.528(16) \text{ fm}$ Sick. PRC 90, 064002 (2014).

Compare with μD HFS

Kalinowski, Pachucki

Muonic atoms
spectroscopy

High-precision laser
spectroscopy in He and He⁺



Nuclear and hadron theories

The hydrogen-like He⁺

Karshenboim et al., PLB795:432(2019)

Yerokhin et al., *Ann.Phys.*531(5):1800324(2019)

$$f_{2S-1S}(\text{He}^+) \approx \frac{3Z^2 c R_\infty}{4} \frac{1}{1 + \frac{m_e}{M_\alpha}} + \text{QED}_{\text{He}^+} (Z^{3.7}, Z^{5\dots7}) - \frac{7(Z\alpha)c^4}{24\pi a_B^3 \hbar^3} r_\alpha^2 + \text{nucl. struct.}$$

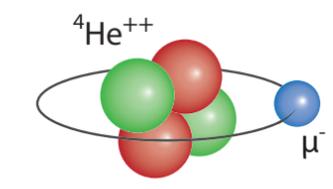
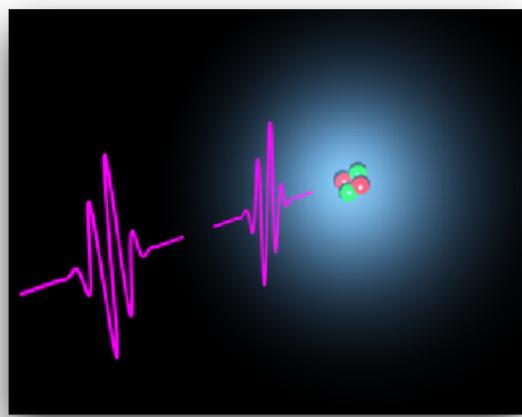
Theory sensitive to higher order contributions

Uncertainty	(1 kHz)	(9 kHz)	(40 kHz)	(61 kHz)	(3 kHz)
-------------	---------	---------	----------	----------	---------

Prospected experimental accuracy
 $u_r = 1 \times 10^{-13}$

Using R_∞ from H+ μ H

Using r_α from $\mu^4\text{He}^+$



Pachucki et al., arXiv:2212.13782 90 kHz
 Li Muli, Bacca (in preparation) >90 kHz

Moreno, et al. EPJD 77, 67 (2023).
 Krauth et al. PoS FFK2019, 49 (2019).

Combining $\mu^4\text{He}^+$ with He^+

$\mu\text{H} + \text{H}$

He+

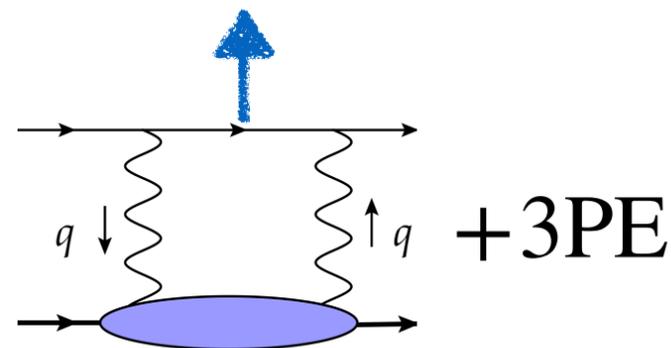
$$E_{1S-2S}(\text{He}^+) \approx \frac{3}{4} Z^2 R_\infty + \text{QED}^*(Z^{3.7}, Z^{5\dots7}) + k' Z^4 r_\alpha^2$$

muonic He+

$$E_{2S-2P}(\mu^4\text{He}^+) \approx \text{QED} + k'' r_\alpha^2 + \text{NS}$$

Determine
 r_α
 $\delta \sim 5 \cdot 10^{-4}$

Consistency check



Combining $\mu^4\text{He}^+$ with He^+

He+

$$E_{1S-2S}(\text{He}^+) \approx \frac{3}{4} Z^2 R_\infty + \text{QED}^*(Z^{3.7}, Z^{5\dots7}) + k' Z^4 r_\alpha^2$$

$\mu\text{H} + \text{H}$

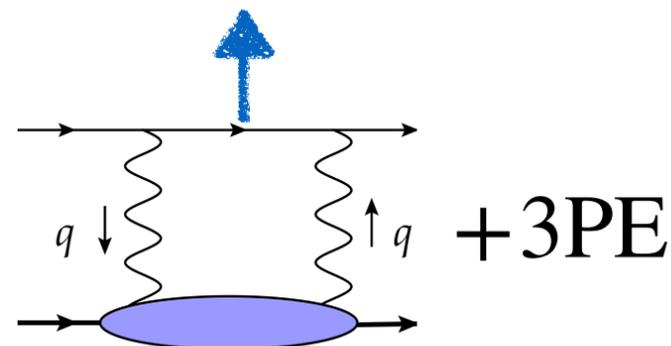
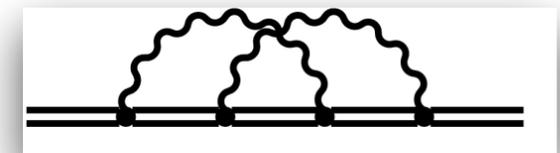
muonic He+

$$E_{2S-2P}(\mu^4\text{He}^+) \approx \text{QED} + k'' r_\alpha^2 + \text{NS}$$

Test Bound-
state QED

$$\delta \sim 5 \cdot 10^{-12}$$

Sensitive to
interesting higher-
order corrections



Combining $\mu^4\text{He}^+$ with He^+

$\mu\text{H} + \text{H}$

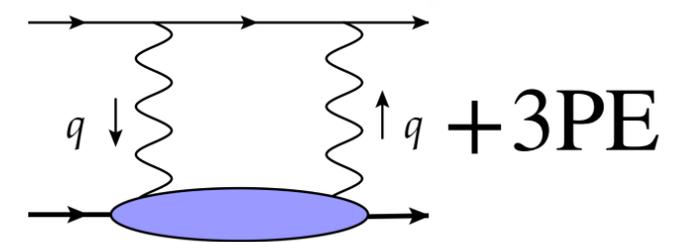
He^+

$$E_{1S-2S}(\text{He}^+) \approx \frac{3}{4} Z^2 R_\infty + \text{QED}^*(Z^{3.7}, Z^{5\dots7}) + k' Z^4 r_\alpha^2$$

muonic He^+

$$E_{2S-2P}(\mu^4\text{He}^+) \approx \text{QED} + k'' r_\alpha^2 + \text{NS}$$

Benchmark
for 2PE+3PE

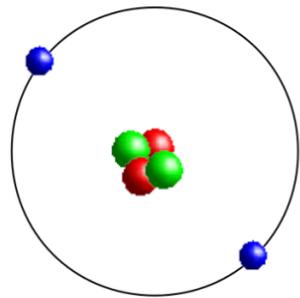


$$\delta \sim 1 \cdot 10^{-2}$$

Isotopic shift in He

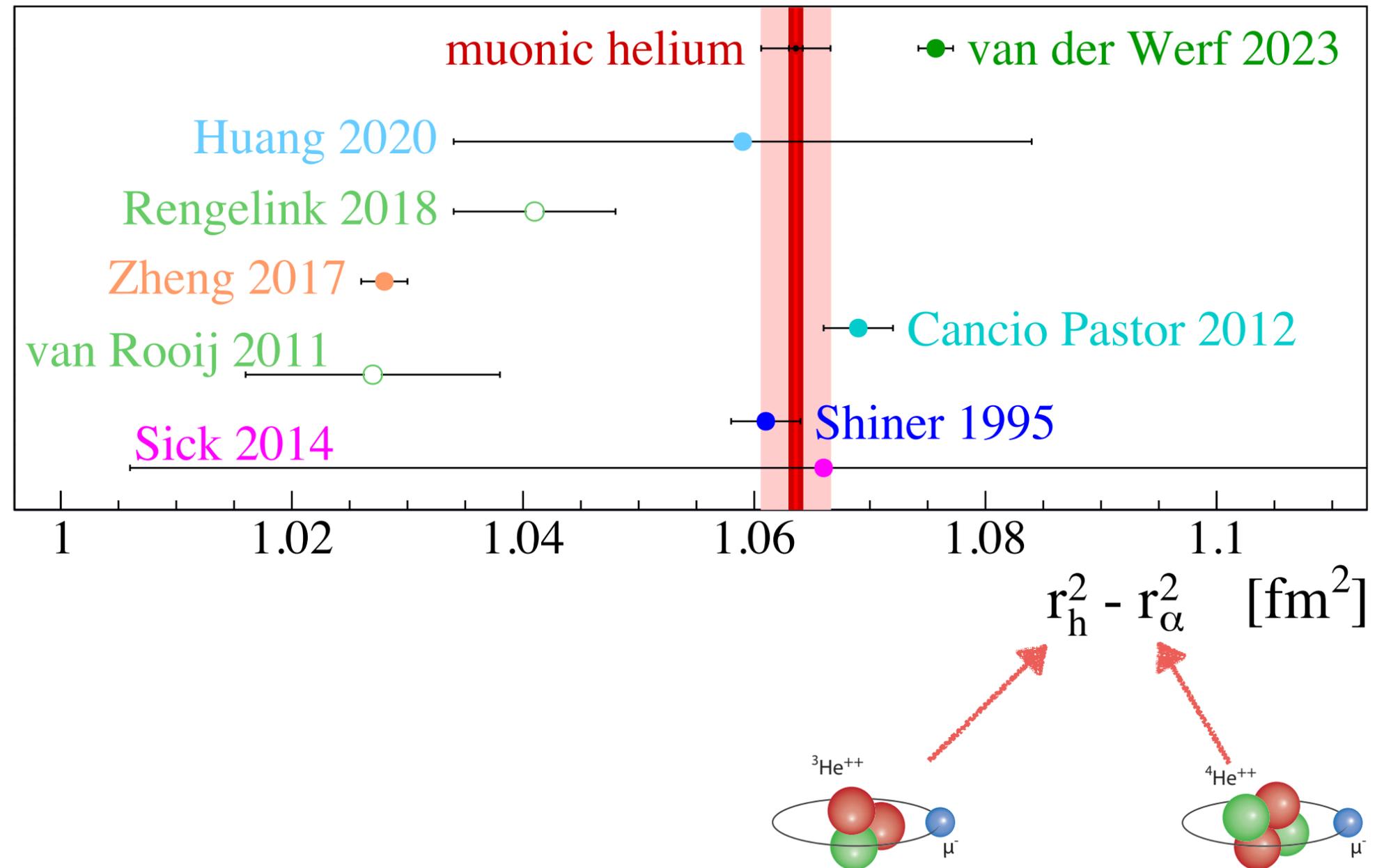
Two electrons are much more than one electron

Helium



But **large** theory cancellations are taking place in regular He in the isotopic shift

Minor theory cancellations are taking place in μHe^+



Comments

- ▶ Van Rooij results corrected to include a differential ac-stark shift effect. The van der Werf result makes use of the magic wavelength to eliminate this systematic

arXiv:2306.02333 (2023)

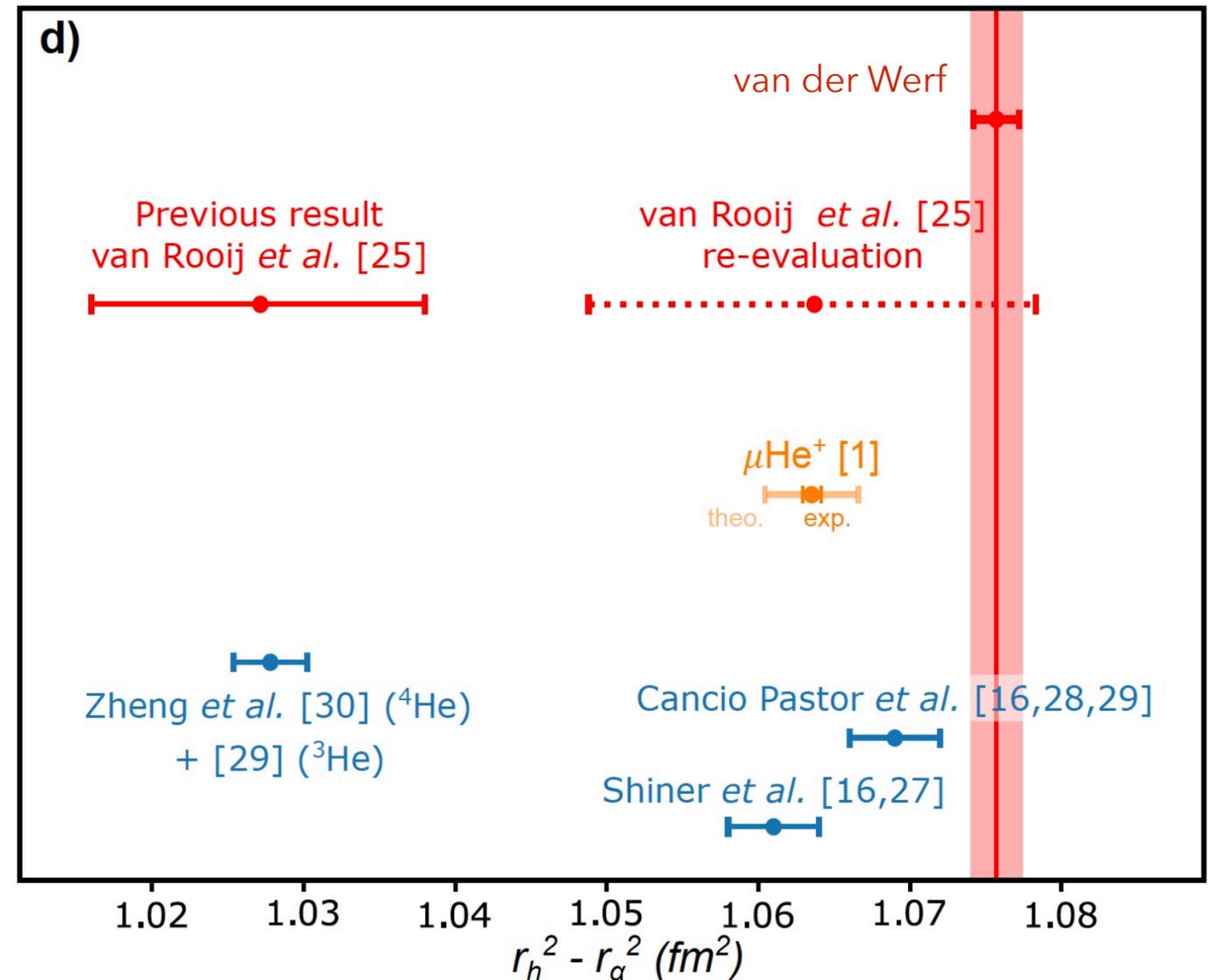
- ▶ Between μHe^+ and Amsterdam result there is a 3.6σ tension

- ▶ The values of Shiner and Pastor may lack a systematic correction due to quantum interference effects

Marsman et al., PRA 89, 043403 (2014)

- ▶ The value of Zheng may have to be corrected for a systematic Doppler shift
Wen, et al. PRA 107, 042811 (2023).

Van der Werf et al., arXiv:2306.02333





Fundamental constants

Radiative corrections

High-precision laser spectroscopy

Scattering experiments and accelerator

Two-body QED tests in H-like systems

Three-body QED tests in He-like systems

QED test in simple molecules

Muonic atom spectroscopy

BSM searches

Hadron structure

Nuclear structure

Penning traps program

The CREMA collaboration



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