PREN 2023 + μASTI 26th–30th June 2023

QED @ NNLO with McMule

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28th June 2023

QED and the mule

McMule

Monte Carlo for MUons and other LEptons

https://mule-tools.gitlab.io

P. Banerjee, A. Coutinho, T. Engel, A. Gurgone, F. Hagelstein, S. Kollatzsch, D. Moreno, L. Naterop, D. Radic, M. Rocco, N. Schalch, V. Sharkovska, A. Signer, Y. Ulrich

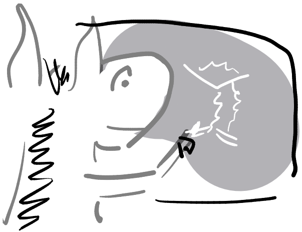
⇒ a framework for fully-differential higher-order QED calculations of scattering processes

- fixed-order NNLO QED corrections available/planned for

$\ell \to \ell' \nu \bar\nu$	$\ell \to \ell' \nu \bar\nu \gamma$	$\ell \to \ell' \nu \bar\nu (e^+ e^-)$
$e^\pm \mu \to e^\pm \mu$	$e^- e^- \to e^- e^-$	$e^+ e^- \to e^+ e^-$
$e^+ e^- \to \gamma^*$	$e^+ e^- \to \gamma\gamma$	$\ell p \to \ell p$

- full NNLO!!, toying with N^{3}LO, but no dirty protons and no parton shower/YFS (yet)
- fully differential Monte Carlo integrator ⇒ generator to follow [Ulrich]

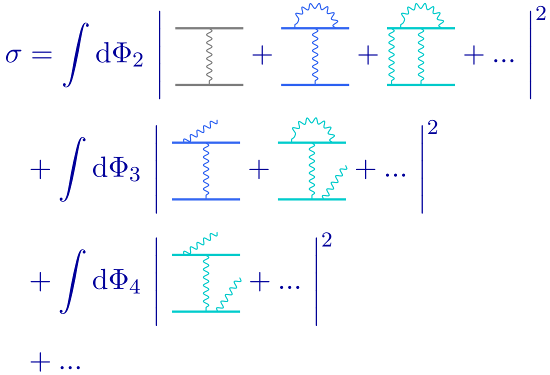
- basics and challenges of massive NNLO calculations
  - (divergent) phase-space integration
  - dealing with masses
  - numerical stability (of real-virtual corrections)
- MUonE as motivation and validation
  - muon-electron scattering @ MUonE and $(g-2)_\mu$
  - validation of NNLO results
  - NNLO corrections are crucial (and insufficient)
- lepton-proton scattering for MUSE
  - a muon of mass $938.272\,\text{MeV}$
  - pointlike vs actual proton
- outlook



basics and challenges of massive NNLO calculations

NNLO basics

physical $(2 \to 2)$ cross section (e.g. Møller)



challenges

1. fully differential phase-space integration

$\Rightarrow$ FKS$^\ell$

2. virtual amplitudes with massive particles

$\Rightarrow$ one-loop: OpenLoops

$\Rightarrow$ two-loop: massification

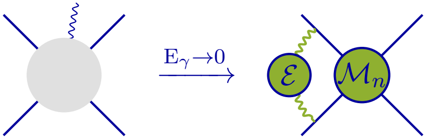
3. numerical instabilities due to pseudo-singularities

$\Rightarrow$ next-to-soft stabilisation

no approximations (a tiny cheat), no restriction on additional real photons

① FKS$^\ell$

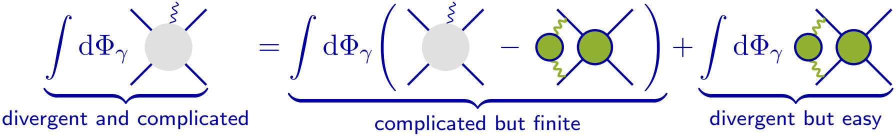
only soft singularities



$$\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\, \mathcal{M}_n^{(\ell)} \; + \; \mathcal{O}(E_\gamma^{-1})$$

eikonal $\mathcal{E} = \sum_{i,j} \frac{p_i \cdot p_j}{(p_\gamma \cdot p_i)\; (p_\gamma \cdot p_j)} \sim \mathcal{O}(E_\gamma^{-2})$

$\Rightarrow$ subtraction scheme (FKS$^\ell$)



subtraction scheme

we do not write $\sigma_n^{(1)} = \sigma_n^{(v)}(\lambda) + \sigma_n^{(s)}(\lambda,\omega) + \sigma_{n+1}^{(h)}(\omega)$ photon mass λ, resolution ω

we do write $\sigma_n^{(1)} = \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c)$ at NLO

$\sigma_n^{(2)} = \sigma_n^{(2)}(\xi_c) + \sigma_{n+1}^{(2)}(\xi_c) + \sigma_{n+2}^{(2)}(\xi_c)$ at NNLO

$$\sigma_n^{(1)}(\xi_c) = \int \mathrm{d}\Phi_n^{d=4} \Bigg(\underbrace{\mathcal{M}_n^{(1)}}_{1/\epsilon} + \underbrace{\hat{\mathcal{E}}(\xi_c)}_{1/\epsilon} \mathcal{M}_n^{(0)} \Bigg) = \int \mathrm{d}\Phi_n^{d=4} \underbrace{\mathcal{M}_n^{(1)f}(\xi_c)}_{\text{finite}}$$

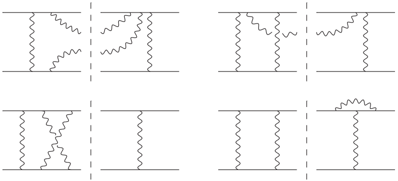
$$\sigma_{n+1}^{(1)}(\xi_c) = \int \mathrm{d}\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_1}\right)_c \left(\xi_1 \mathcal{M}_{n+1}^{(0)f}\right)$$

the ξ_c dependence cancels between the two terms (implementation/stability check)

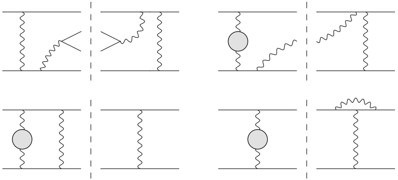
McMøller

- ready for a full (e.g. Møller) NNLO calculation photonic and fermionic contributions
- compute double-real amplitudes
- use OpenLoops [Buccioni, Pozzorini, Zoller] for real-virtual amplitudes, numerical stability → ③
- ③ → apply next-to-soft stabilisation
- massive two-loop integrals not all known → ②
- ② → massify massless two-loop amplitudes [Bern,Dixon,Ghinculov] (and one-loop squared)
- use FKS^2 (open e^+e^- production not yet included)
- let the mule trot [McMule, 2107.12311]

photonic



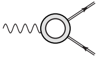
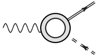
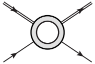
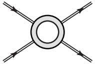
fermionic/hadronic



② loops with masses

- scales (e.g. masses) are the enemy of loop-integral calculators
- for one-loop amplitudes we use OpenLoops, remarkable numerical stability
- but massive two-loop integrals for $2 \to 2$ are not all known

[here should go a list of an army of loop-calculating theoreticians ... sorry]

$\gamma^* \to ee$		✔ full m dependence	harmonic (M)PL	~ 2004
$\mu \to e\nu\bar{\nu}$		✔ full m_1 and m_2	generalised (M)PL	2018
$\mu e \to \mu e$		✔ full m_1, $m_2 = 0$	generalised (M)PL	2021
$ee \to ee$		✗ only planar	elliptic MPL	2022

② loops with masses

simple loop integrals, one scale $z \Rightarrow$ polylogs: $\mathrm{Li}_n(z) = \int_0^z \frac{\mathrm{d}t}{t}\, \mathrm{Li}_{n-1}(z)$

more complicated loop integrals, many scales $a_1 \ldots a_n, z \Rightarrow$ multiple polylogs

$$G(a_1 \ldots a_n; z) = \int_0^z \frac{\mathrm{d}t}{t - a_1}\, G(a_2 \ldots a_n; z)$$

for $a_i \in \{-1, 0, 1\}$ HPL [Remiddi, Vermaseren]; generic a_i GPL [Goncharov]

sadly, this is not the end $\Rightarrow$ elliptic integrals ..., ouch

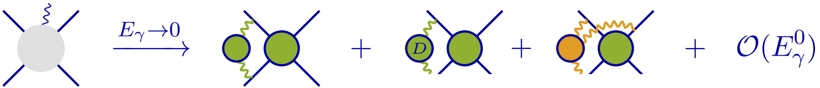
collinear factorization $\Rightarrow$ tiny cheat massification [Penin; Becher, Melnikov; Engel, Gnendiger, AS, Ulrich]

$$\mathcal{A}(m) = \mathcal{S} \times Z \times Z \times \mathcal{A}(0) + \mathcal{O}(m^2/Q^2)$$

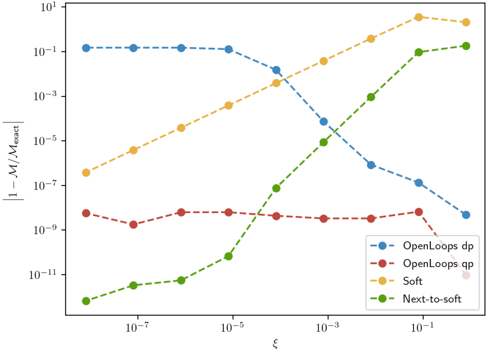
③ next-to-soft stabilisation

real-virtual corrections trivial in principle, extremely delicate numerically

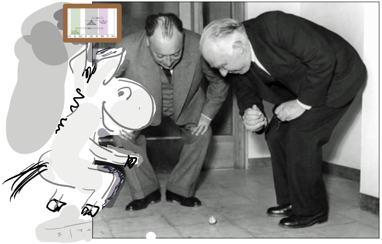
extend LBK theorem [Low 1958; Burnett, Kroll 1968] to one [Engel, AS, Ulrich, 2112.07570] any [Engel, 2304.11689] loop



- soft limit (of collinear emission)
- Bhabha scattering (as example)
  [McMule, 2106.07469]
- M_{exact} Mathematica expression
- full M vs next-to-soft limit
- stability problem solved



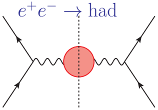
MUonE as motivation and validation

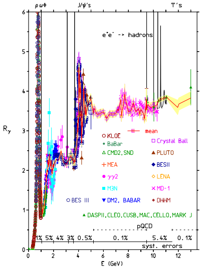


McMuone: A.Broggio, T.Engel, A.Ferroglia, M.Mandal, P.Mastrolia, M.Rocco, J.Ronca, AS, W.Torres Bobadilla, Y.Ulrich, M.Zoller

$\mu\, e$ scattering and $e^+ e^- \to$ had

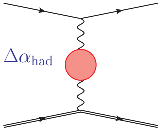
$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{\mathrm{had}}$$

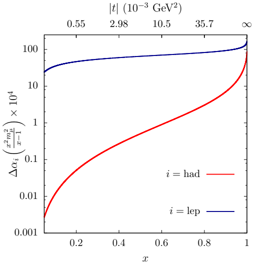




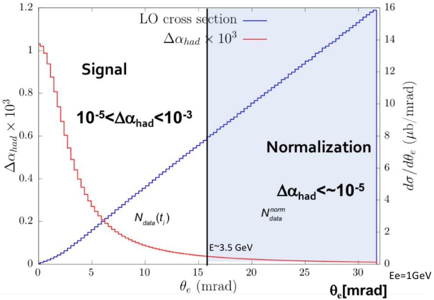
Jegerlehner:1511.04473

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\, (1-x)\, \Delta\alpha_{\mathrm{had}}$$





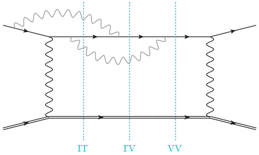
Abbiendi et al:1609.08987



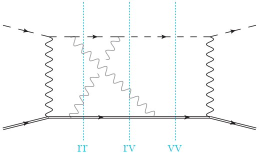
- new proposal [Abbiendi et al.]: elastic scattering $\mu e \to \mu e$ $\Rightarrow$ independent determination of HVP
- 'signal' $\sim 10^{-3}$, want $\sim 1\%$ measurement
- need ~ 10 ppm determination of shape of differential cross section
- need NNLO QED and resummation of logs
- signal region: high e energy, small angle

$\to$ "theory initiative" to provide necessary computations [2004.13663, Padua group, Pavia group, McMule . . .]

"electronic" :



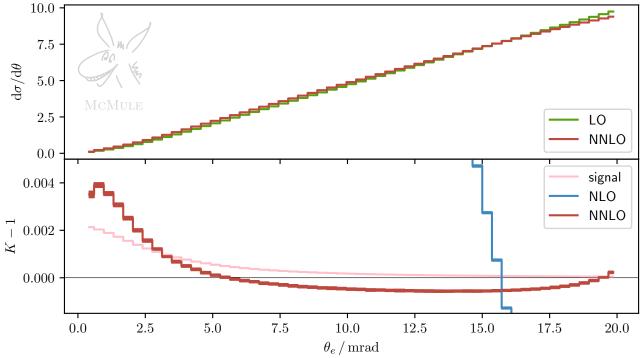
"mixed" :



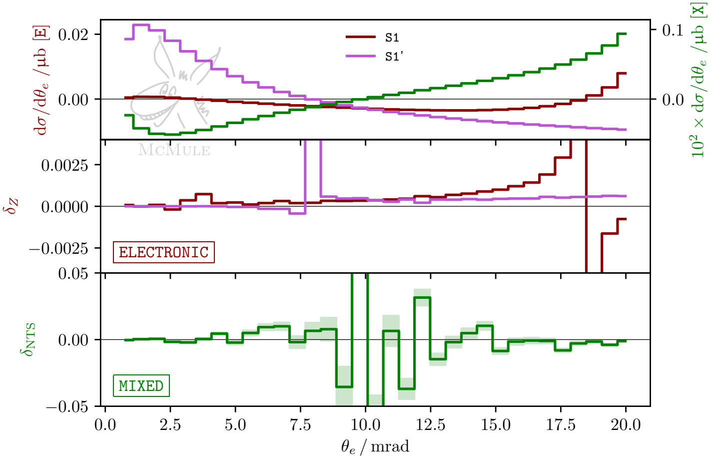
NNLO results

NNLO corrections > signal?! (can play games to suppress radiation . . .)

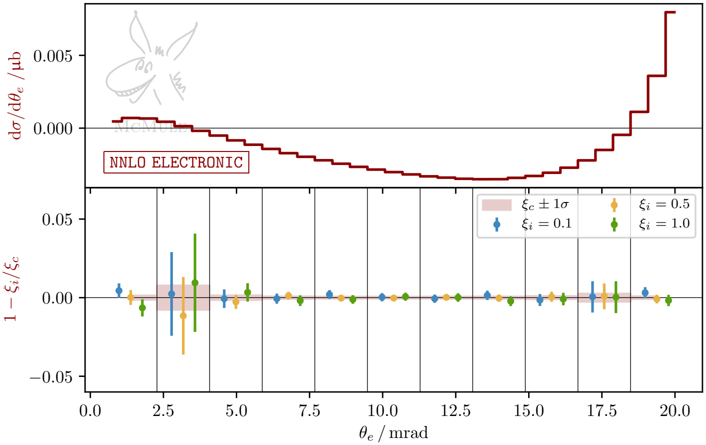
⇒ make no approximation in double-real radiation !!



[2212.06481], electronic corrections validated with MESMER validation of massification and next-to-soft stabilisation



validation of implementation through ξ_c (in)dependence



lepton-proton scattering for MUSE

McMuse: T.Engel, F.Hagelstein, M.Rocco, V.Sharkovska, AS, Y.Ulrich

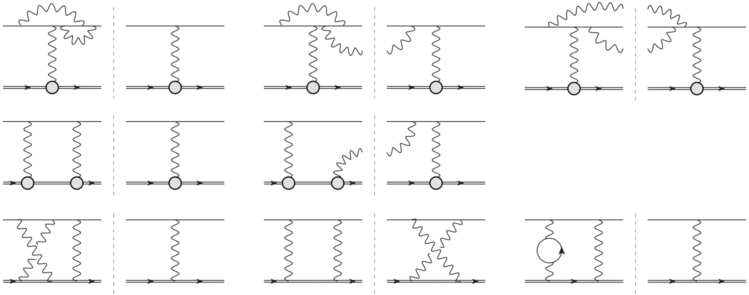
nice protons

$e^\pm \mu \to e^\pm \mu \Rightarrow \ell^\pm p \to \ell^\pm p$ with 'nice' protons (pointlike or simple form factor)

easy with any FF

T(wo)PE with simple FF

T(hree)PE pointlike



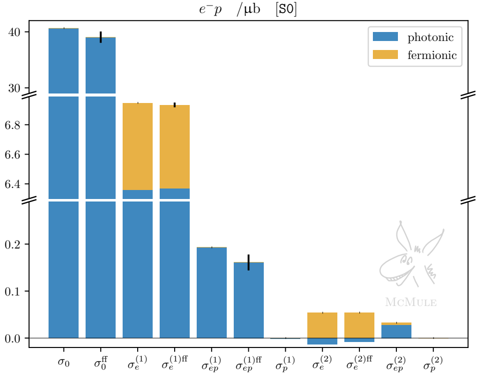
dirty stuff

not included

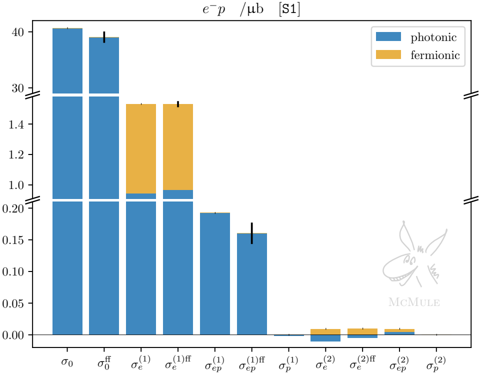
- want to assess importance of NNLO QED corrections w.r.t. 'TPE' (for MUSE)
  - starting point NLO QED with pointlike protons
  - error 1: pointlike $\to$ proper TPE
  - error 2: NLO $\to$ NNLO QED
- ballpark estimate impact of TPE through naive dipole FF
- estimate variation of TPE through $0.60\,\mathrm{GeV}^2 \le \Lambda^2 \le 0.86\,\mathrm{GeV}^2$ in dipole
- this is not meant to be a good TPE implementation
- it is a toy TPE implementation vs. a heck of a QED implementation

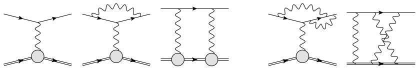
MUSE, $p = 210\,\mathrm{MeV}$

$e^- p$: no cuts on γ



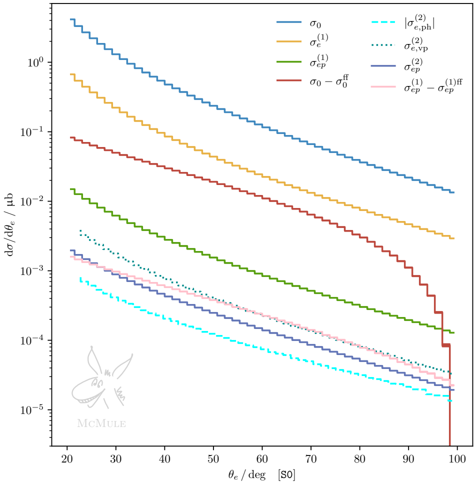
$e^- p$: cut on forward γ



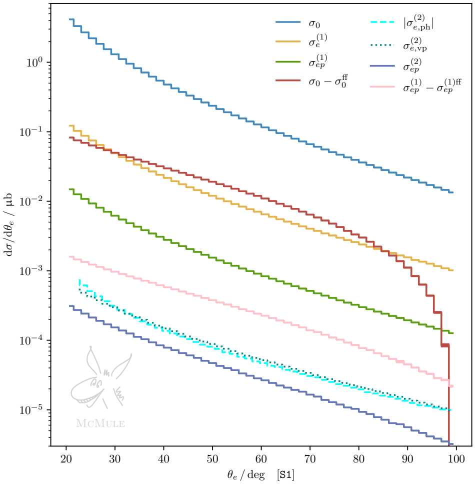


MUSE, $p = 210\,\mathrm{MeV}$

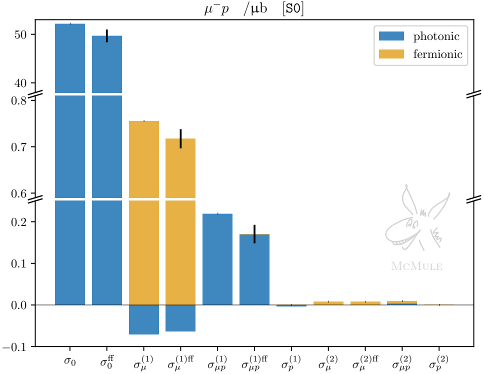
$e^- p$: no cuts on γ

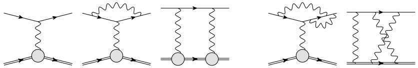


$e^- p$: cut on forward γ

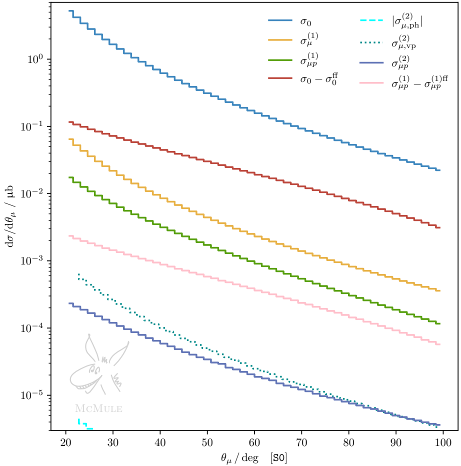


$\mu^- p$: no cut γ

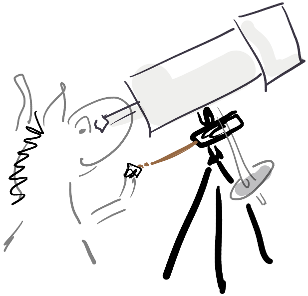




differential cross section



outlook



make use of a modern (QCD inspired) approach to QED

the NNLO era is here, not only for QCD, also for QED

$\Longrightarrow$ use it! https://mule-tools.gitlab.io

make sure bulk standard QED does not foil your analysis

future steps of the mule

- get dirty (protons and pions)
- integrator $\rightarrow$ generator
- add electroweak / polarised leptons
- NNNLO contributions for $\ell^+\ell^- \to \gamma^*$
- go beyond fixed-order QED (with YFS)
- world dominance