Evgeny Epelbaum, Ruhr University Bochum

PREN 2023 & μASTI Workshop, the Helmholtz-Institute Mainz

Electromagnetic form factors of light nuclei in chiral EFT

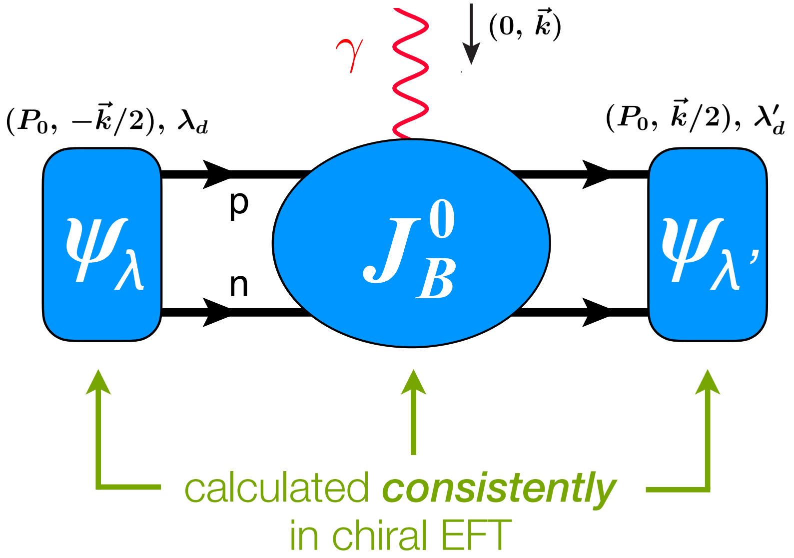
In collaboration with: Arseniy Filin, Daniel Möller, Vadim Baru, Christopher Körber, Hermann Krebs, Andreas Nogga and Patrick Reinert

State of the art, limitations, ongoing work
and future perspectives…

European Research Council
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen

Theory in a nutshell

- **Chiral EFT** for the nuclear Hamiltonian H and J^μ



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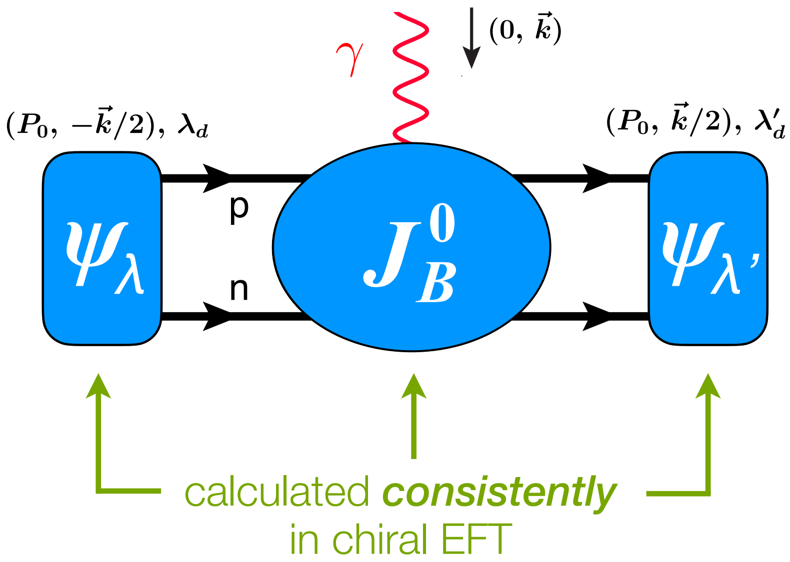

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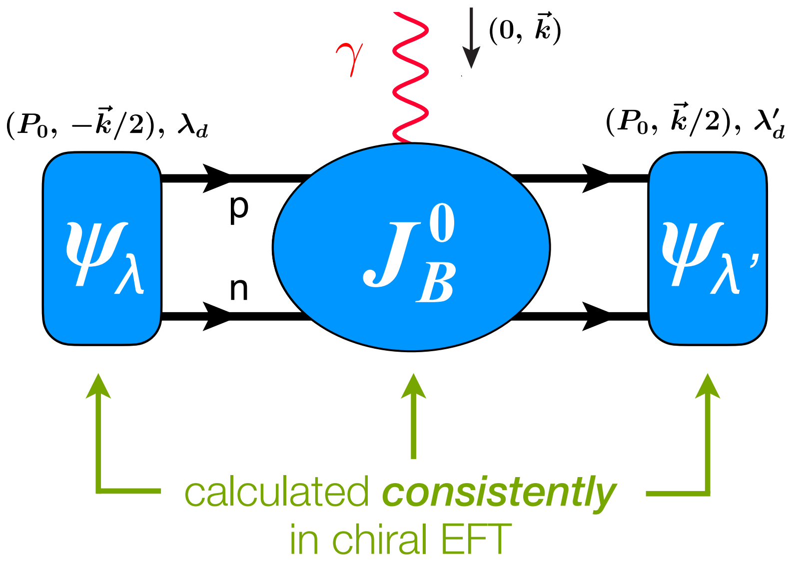

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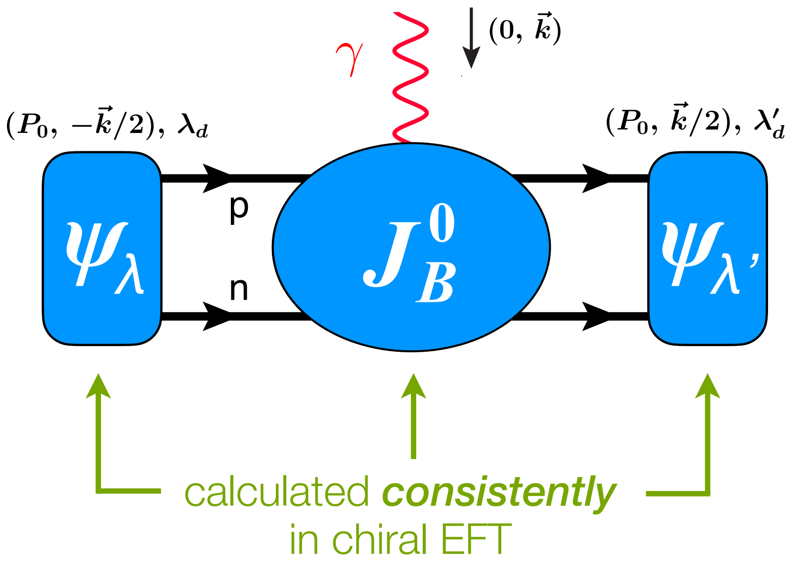

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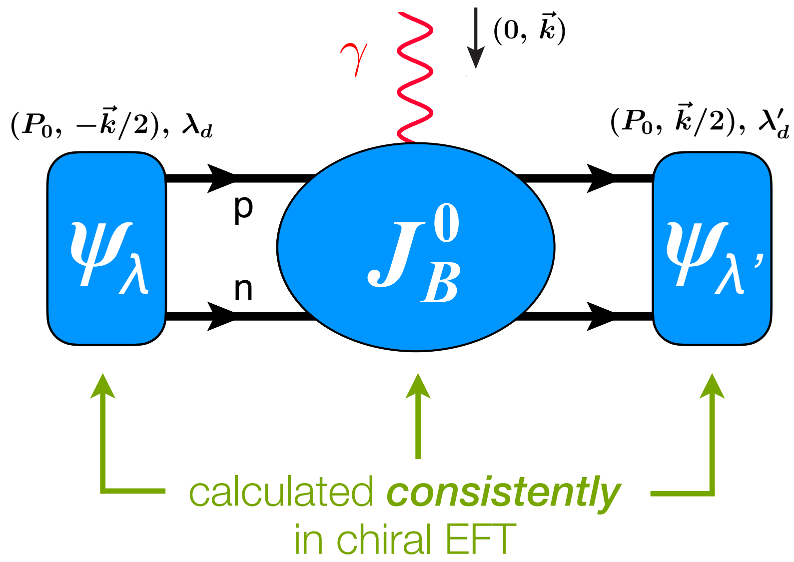
  - — relativistic corrections
  - — isospin breaking effects (including neutron-neutron interaction)
  - — electromagnetic interactions beyond Coulomb



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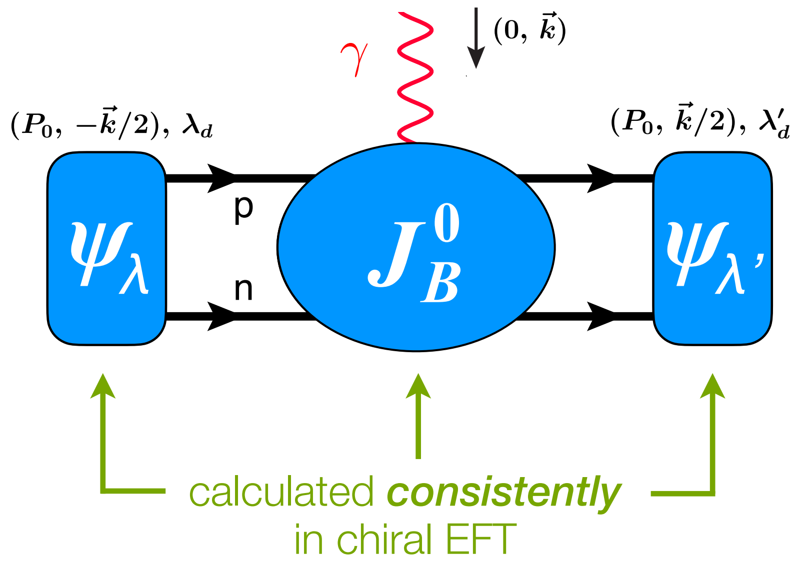

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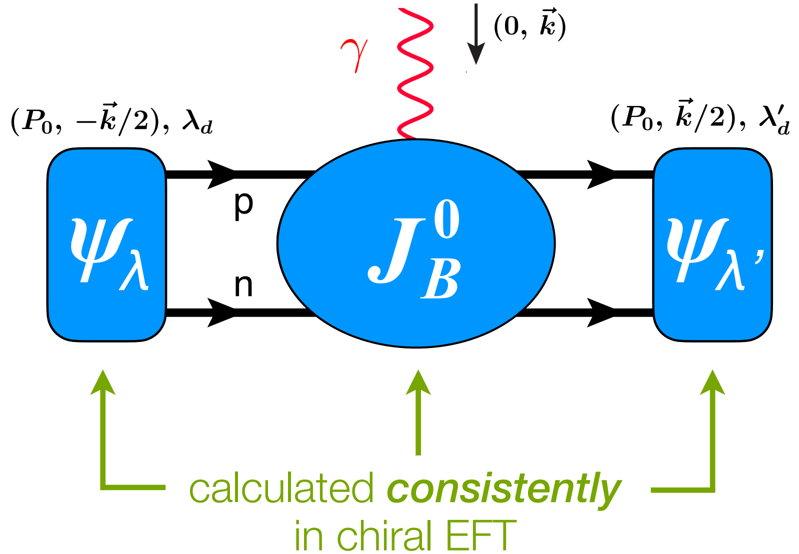
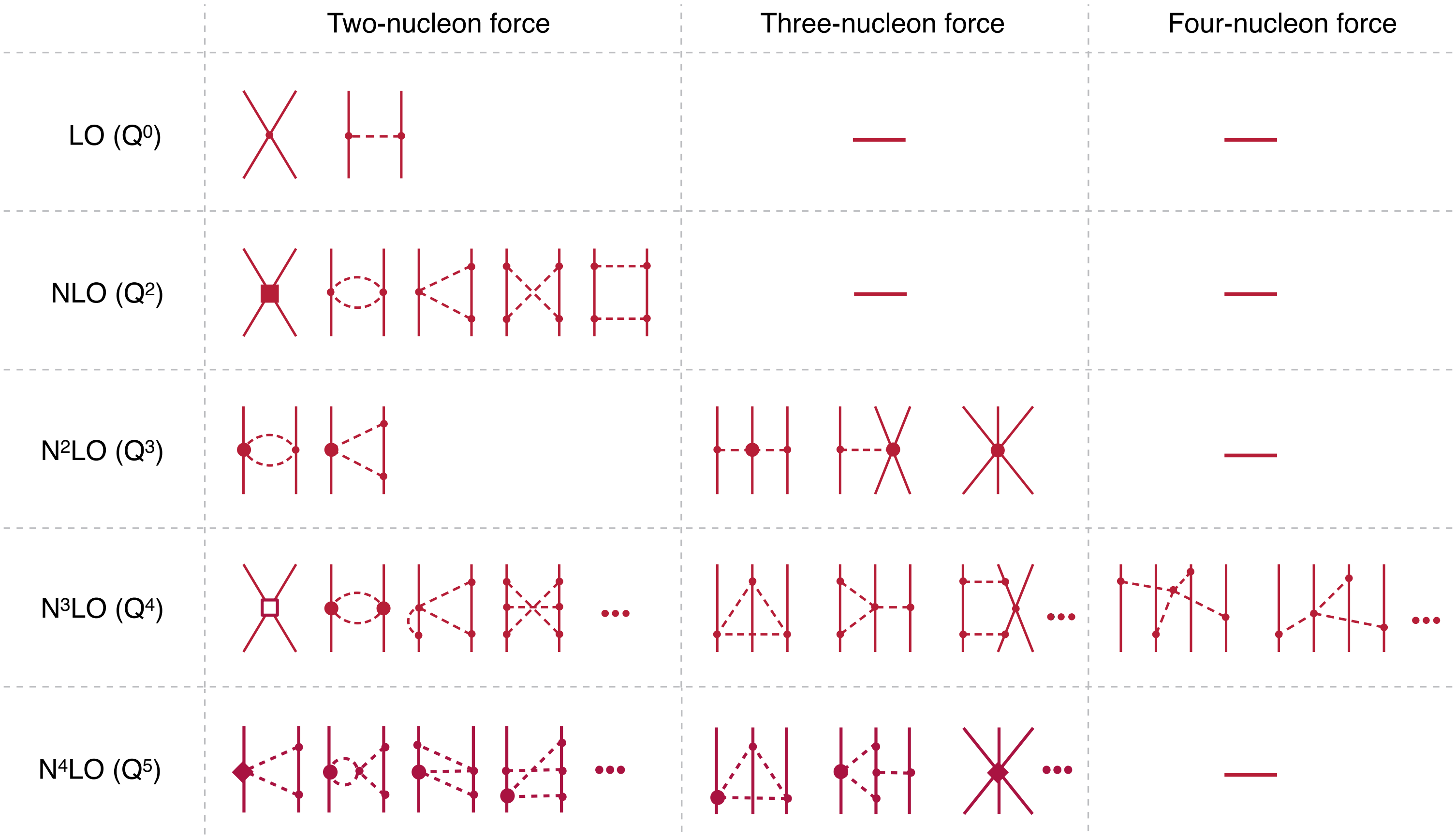
- Computational limitations for $A > 4$



The Hamiltonian



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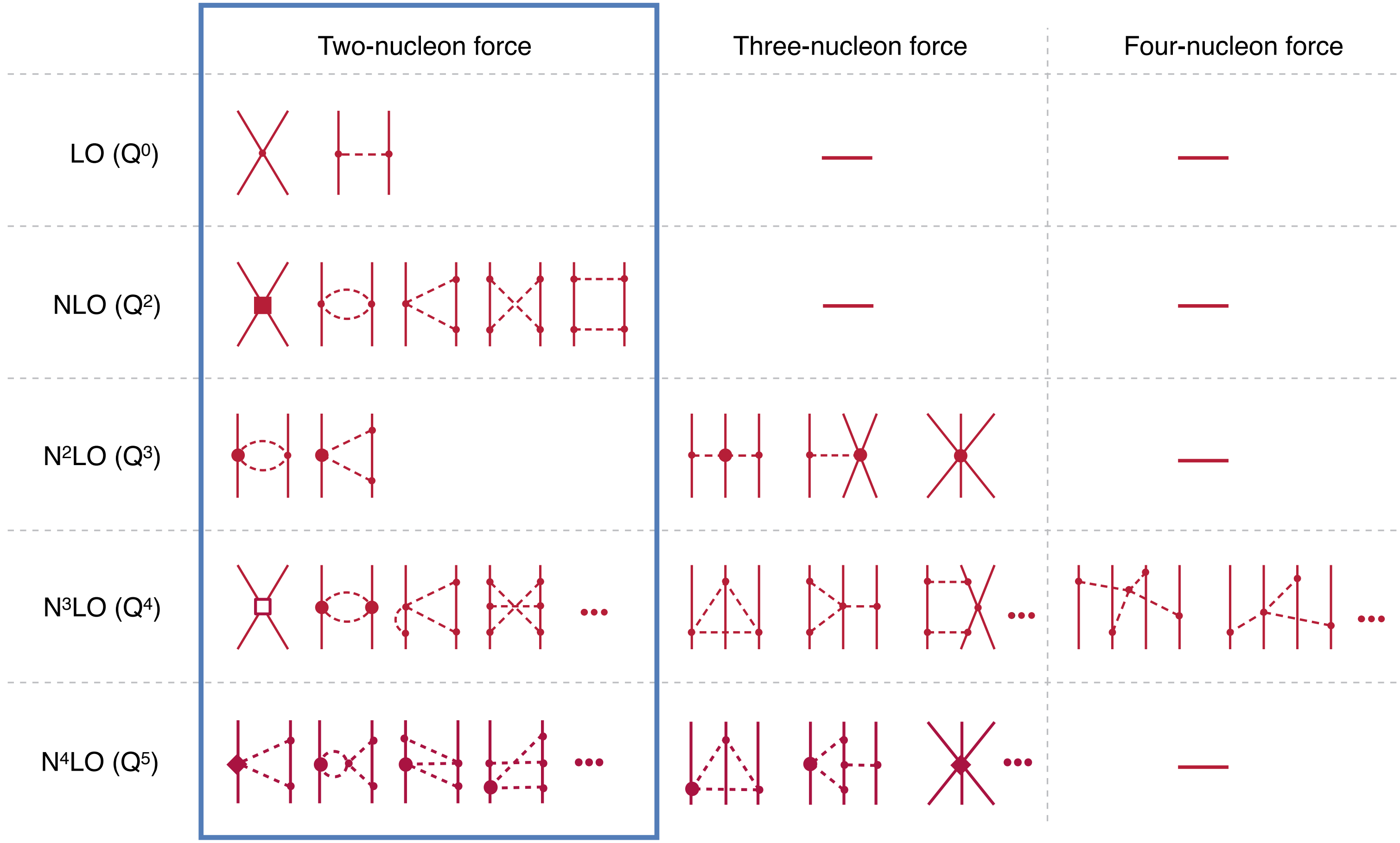


The newest Bochum NN interactions

Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\, e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2}\, e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

SMS chiral NN interactions

Reinert, Krebs, EE, EPJA 54 (18) 86; PRL 126 (21) 092501

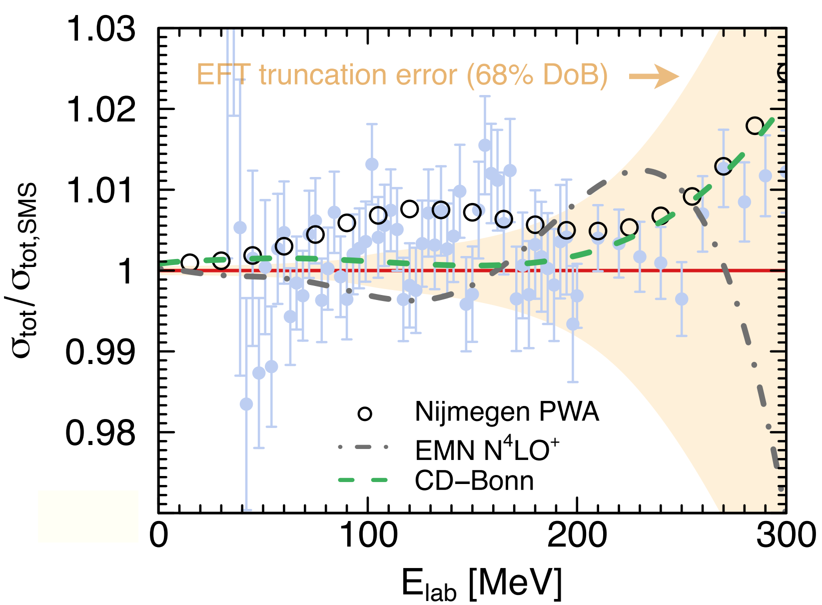
Statistically perfect description of mutually consistent NN scattering data

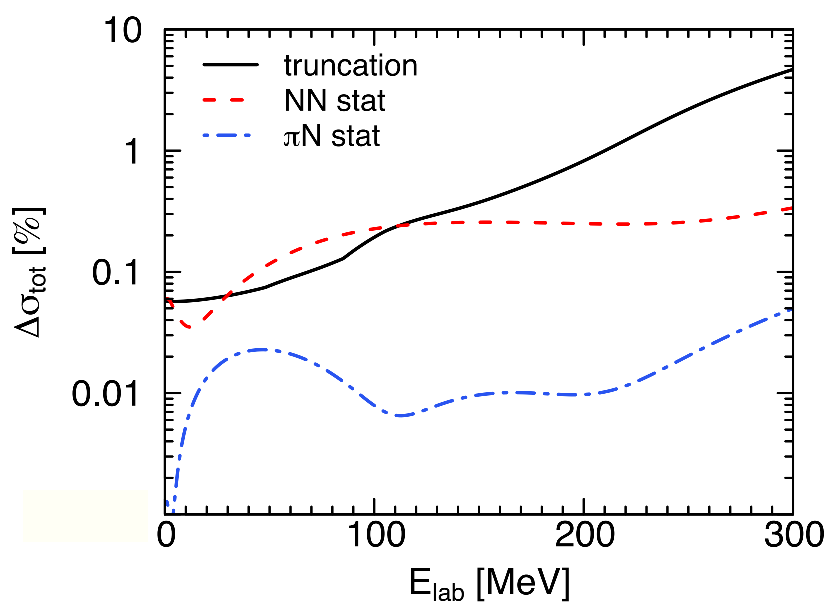
(own database of 2124 proton-proton + 2935 neutron-proton data below E_{lab} = 290 MeV)

high-precision „realistic" potentials				*Idaho χEFT*		*Bochum SMS χEFT*	
Nijm I	Nijm II	Reid93	CD Bonn	$N^4LO^+_{450}$	$N^4LO^+_{500}$	$\mathbf{N^4LO^+_{450}}$	$\mathbf{N^4LO^+_{500}}$
1.061	1.070	1.078	1.042	2.019	1.203	**1.013**	**1.015**

Reinert, Krebs, EE, 2021

Results for np total cross section and the error budget





Residual cutoff dependence

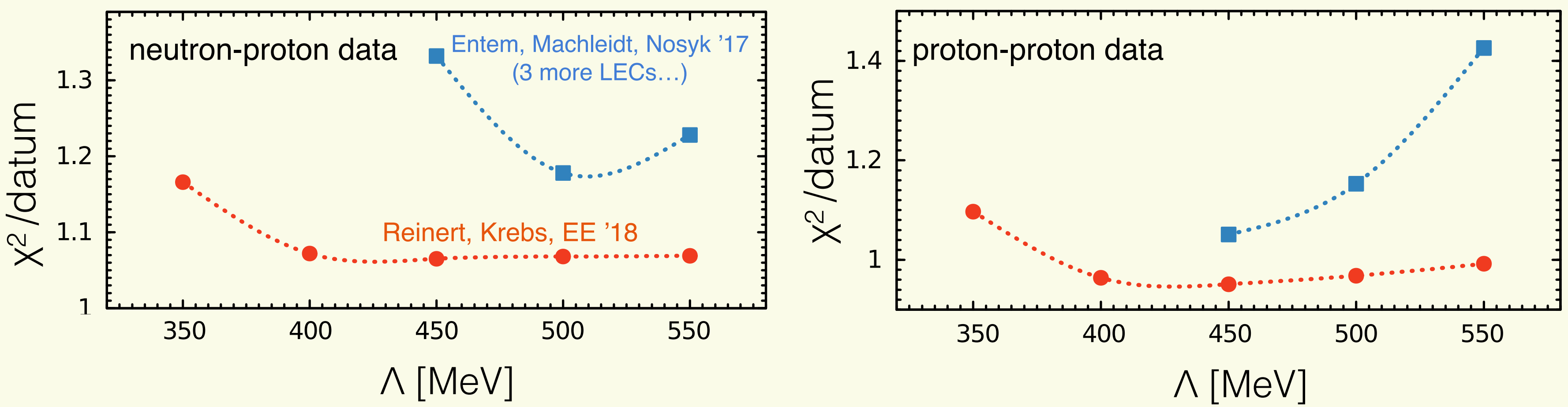
Resummation of non-renormalizable interactions (e.g., $V_{1\pi}(r) \sim 1/r^3$) in the LS equation requires keeping Λ finite, Λ ~ Λ_b Lepage 97; EE, Gegelia '09; EE, Gasparyan, Gegelia, Meißner '18

Implicit renormalization (express bare LECs $C_i(\Lambda)$ in terms of observable quantities)

Renormalizability in the EFT sense (≡ all power-counting breaking terms absorbable into the available counter terms) has been proven to NLO Gasparyan, EE, PRC 105 (2022), PRC 107 (2023)

Residual cutoff dependence at a given finite order as a tool to check consistency

$\chi^2_{\rm datum}$ for the description of NN data in the range $E_{\rm lab} = 0 - 200$ MeV at N^4LO^+

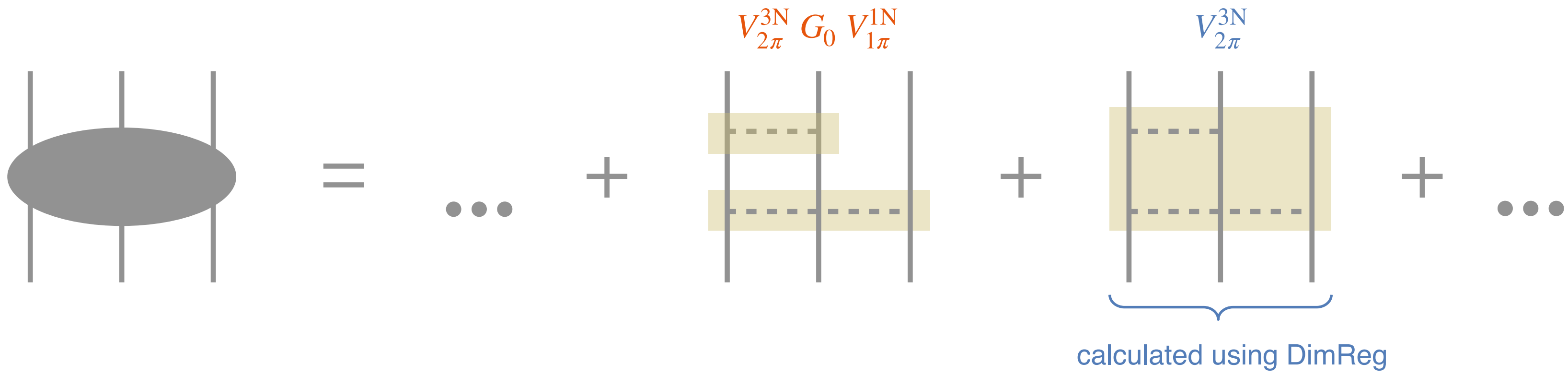


⇒ The 2N interaction is in a good shape

Regularization and symmetry

Nuclear potentials are derived using dim. reg. and supplied with an additional cutoff prior to solving the Schrödinger equation. Consistent?

Faddeev equation for 3N scattering:



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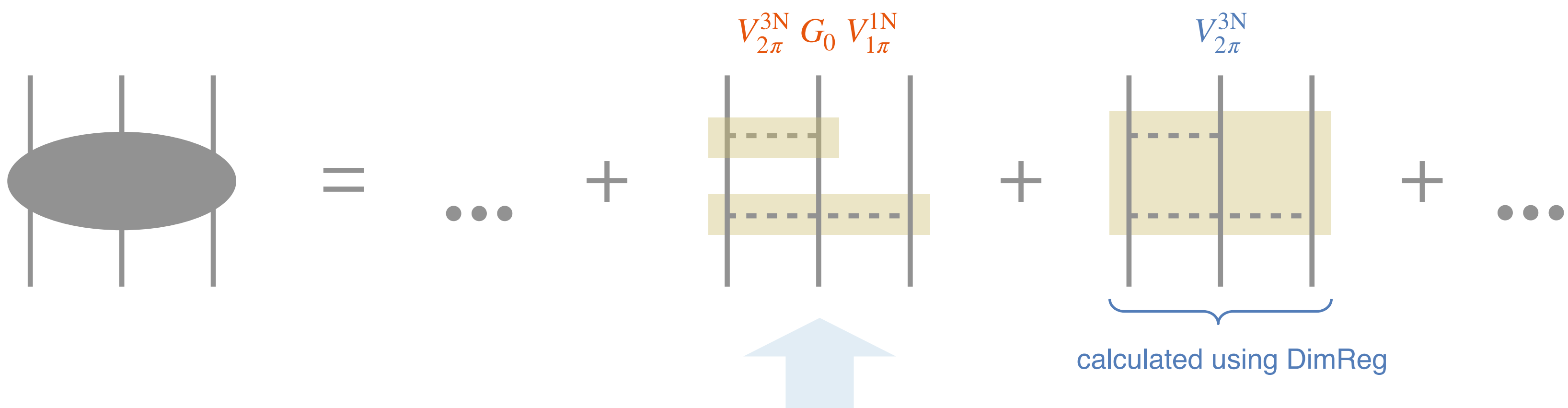

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\left[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\right]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2} + \ldots$$

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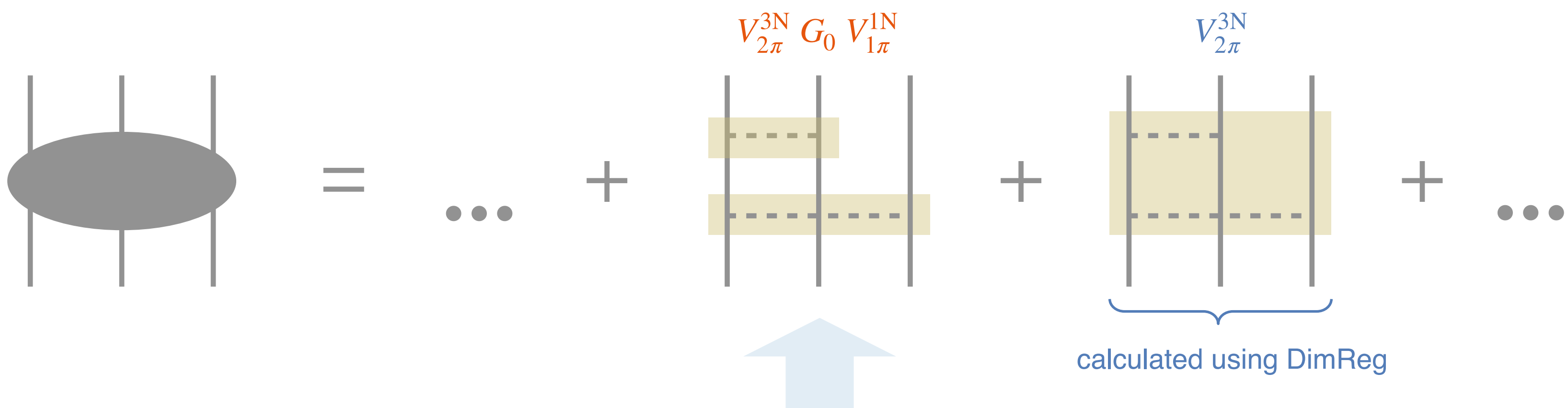

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If $V_{2\pi}^{3N}$ were calculated with a cutoff, the problematic divergence would cancel exactly. This issue affects all loop contributions beyond N²LO to 3NF and exchange currents. In contrast, NN forces are not affected (at a fixed M_π).

Gradient flow regularization

Hermann Krebs, EE, in preparation

⇒ Re-derive nuclear forces & currents using SYMMETRY PRESERVING cutoff regularization

An attractive option is the gradient flow method:

— successfully applied to Yang-Mills theories (QCD) Martin Lüscher '14

— proposed as a regulator for chiral EFT in several talks by David Kaplan

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Idea: let pion fields evolve in the flow „time" τ by replacing the pion field U by the smoothened one $W(\tau)$, $W(0) = U$, which fulfills the (covariant) gradient flow equation:

$$\partial_\tau W = i w \,\mathrm{EOM}(\tau)\, w, \quad \text{where } w = \sqrt{W} \quad \text{and} \quad \mathrm{EOM} = [D_\mu, \underbrace{w_\mu}_{w_\mu|_{\tau=0} \, = \, u_\mu}] + \frac{i}{2}\chi_i - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

The flow „time" τ acts as a regulator (smearing), the choice $\tau = (2\Lambda)^{-1}$ matches the employed regularization of the OPEP:

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}}$$

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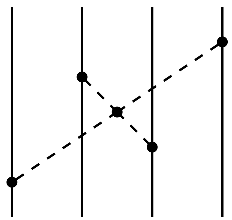
Complication: the regularized Lagrangian involves arbitrary powers of time derivatives

⇒ cannot use Hamiltonian-based methods (like MUT) to derive nuclear forces/currents

⇒ new path-integral method to derive nuclear interactions **Hermann Krebs**, EE, in preparation

Example: gradient flow reg. of the 4NF



Consider e.g. the contribution to the 4NF at N3LO involving a 4π-vertex:

Unregularized expression:

EE, PLB 639 (2006) 456; EPJA 34 (2007) 197

$$V_{4\mathrm{N}} = \frac{g_A^4}{2(2F_\pi)^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{\sigma}_3 \cdot \vec{q}_3 \; \vec{\sigma}_4 \cdot \vec{q}_4}{(\vec{q}_1^{\,2} + M_\pi^2)\,(\vec{q}_2^{\,2} + M_\pi^2)\,(\vec{q}_3^{\,2} + M_\pi^2)\,(\vec{q}_4^{\,2} + M_\pi^2)} \Big[\big(\vec{q}_1 + \vec{q}_2\big)^2 + M_\pi^2 \Big]$$

$$+ \text{ 3-pole terms} + \text{all permutations}$$

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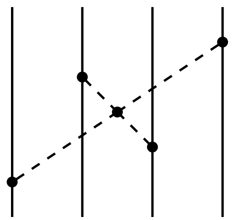

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Applying the gradient flow regularization method consistent with the 2NF yields:

Hermann Krebs, EE, preliminary

$$V_{4\text{N}} = \frac{g_A^4}{2(2F_\pi)^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \; \vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{\sigma}_3 \cdot \vec{q}_3 \; \vec{\sigma}_4 \cdot \vec{q}_4}{(\vec{q}_1^{\,2} + M_\pi^2)\,(\vec{q}_2^{\,2} + M_\pi^2)\,(\vec{q}_3^{\,2} + M_\pi^2)\,(\vec{q}_4^{\,2} + M_\pi^2)} \Big[(\vec{q}_1 + \vec{q}_2)^2 + M_\pi^2 \Big]$$

$$\times \bigg(4\, e^{-\frac{\vec{q}_2^{\,2} + M_\pi^2}{\Lambda^2}}\, e^{-\frac{\vec{q}_3^{\,2} + M_\pi^2}{\Lambda^2}}\, e^{-\frac{\vec{q}_4^{\,2} + M_\pi^2}{\Lambda^2}} - 3\, e^{-\frac{\vec{q}_1^{\,2} + M_\pi^2}{2\Lambda^2}}\, e^{-\frac{\vec{q}_2^{\,2} + M_\pi^2}{2\Lambda^2}}\, e^{-\frac{\vec{q}_3^{\,2} + M_\pi^2}{2\Lambda^2}}\, e^{-\frac{\vec{q}_4^{\,2} + M_\pi^2}{2\Lambda^2}} \bigg)$$

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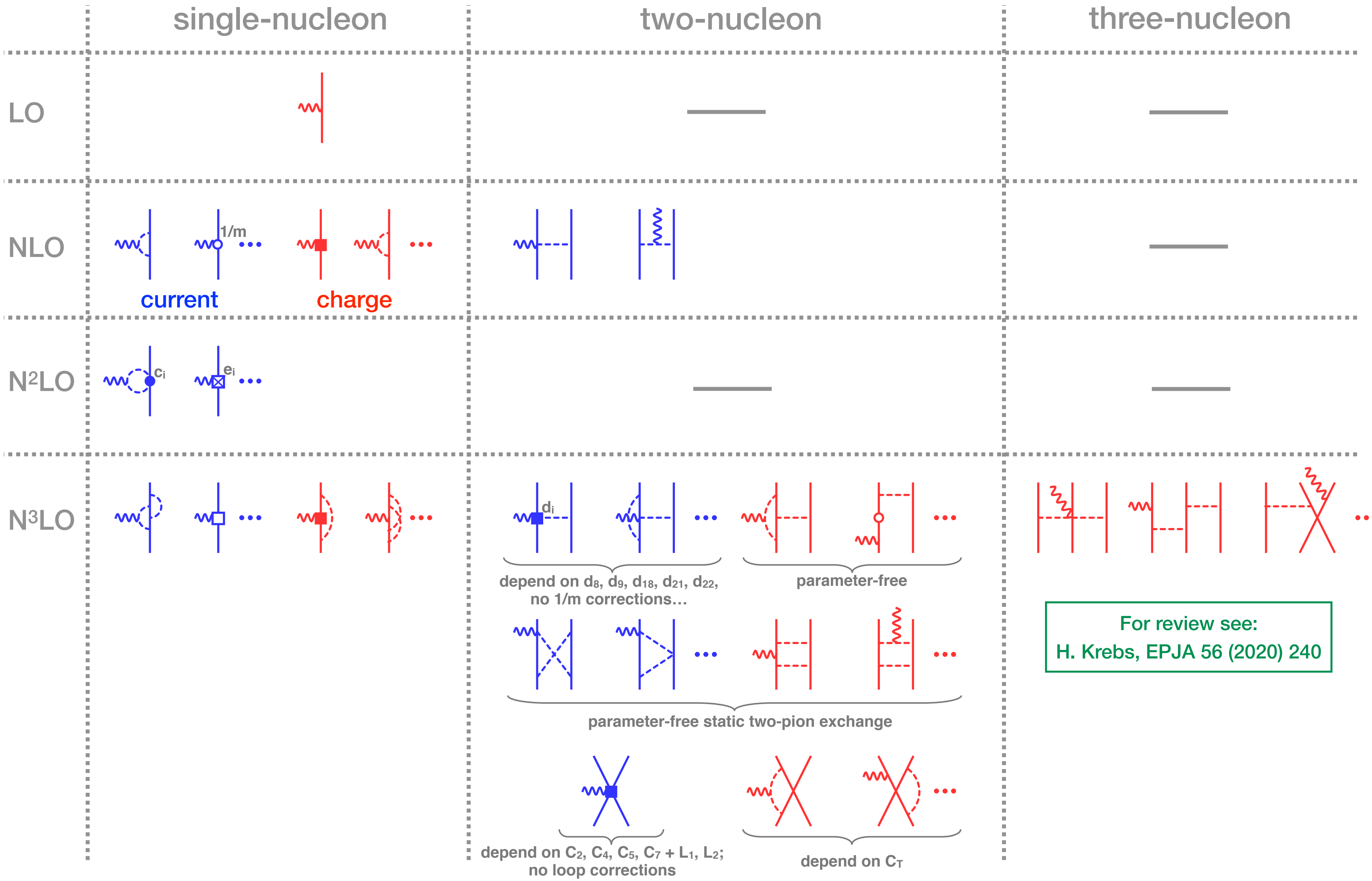
Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001; Krebs, EE, Meißner, FBS 60 (2019) 31



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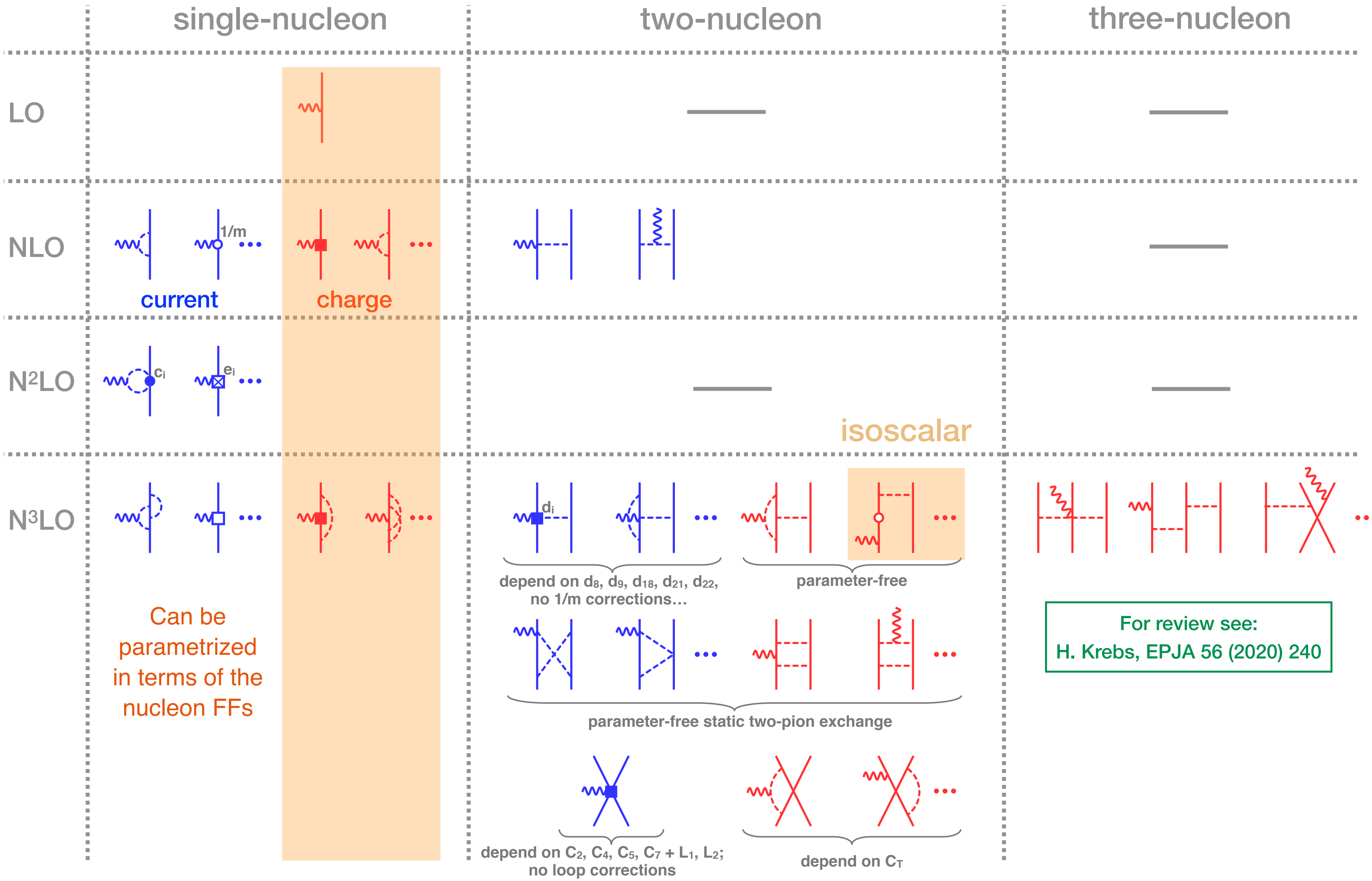

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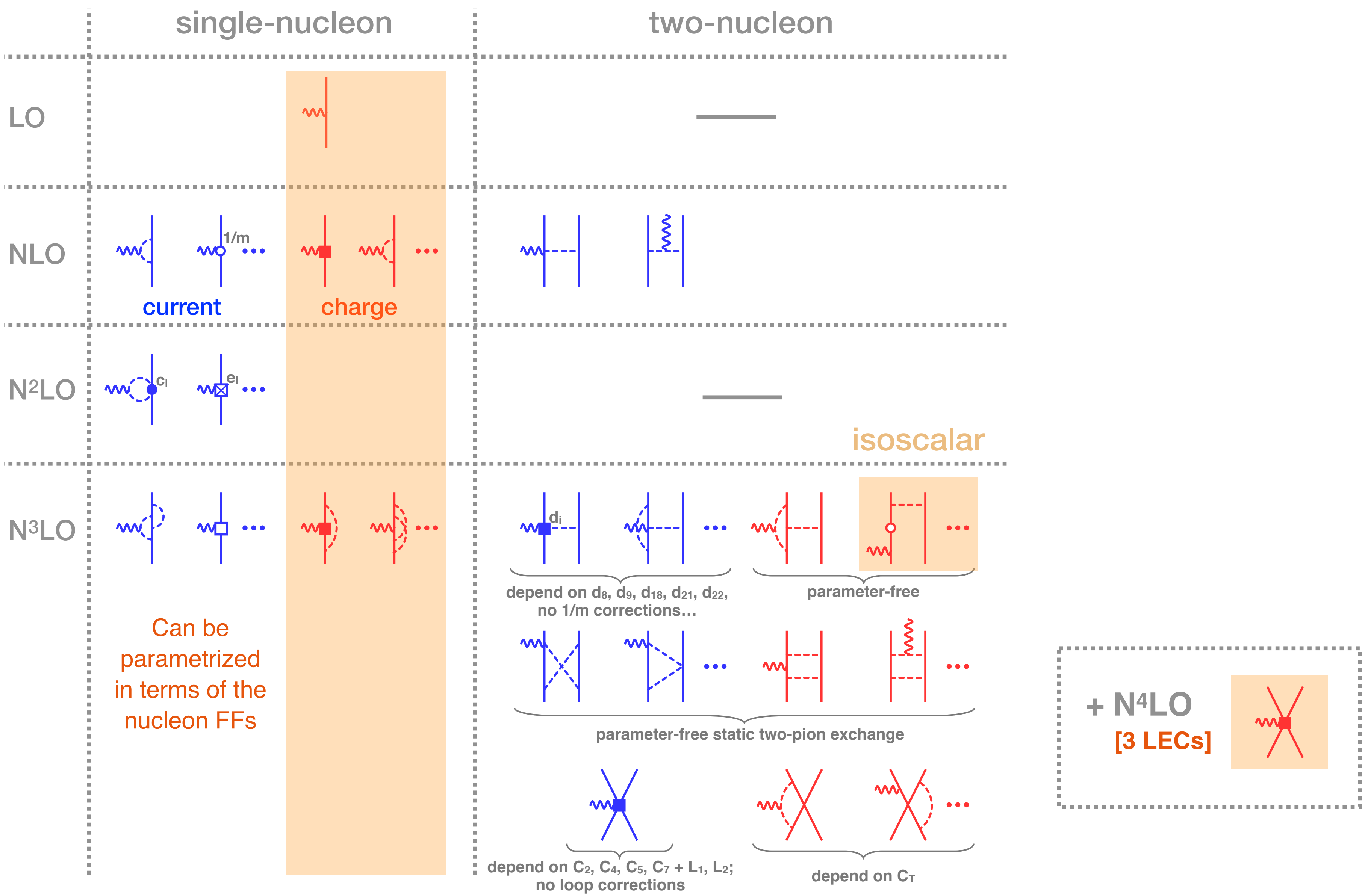

The charge and quadrupole FFs of ^{2}H

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, Phys. Rev. Lett. 124 (2020) 082501;
Phys. Rev. C103 (2021) 024313

$$\rho_{\text{1N}}^{\text{DF}} = -e\frac{\mathbf{k}^2}{8m_N^2}G_E(\mathbf{k}^2)$$

$$G(Q^2) = G^{\text{Main}}(Q^2) + G^{\text{DF}}(Q^2) + G^{\text{SO}}(Q^2) + G^{\text{Boost}}(Q^2) + G^{1\pi}(Q^2) + G^{\text{Cont}}(Q^2)$$

$$\rho_{\text{1N}}^{\text{Main}} = eG_E(\mathbf{k}^2) \qquad \rho_{\text{1N}}^{\text{SO}} = ie\frac{2G_M(\mathbf{k}^2) - G_E(\mathbf{k}^2)}{4m_N^2}\sigma\cdot\mathbf{k}\times\mathbf{p}$$

— Both the nuclear force and the 2N charge density are available to N^{4}LO

— Simple numerics

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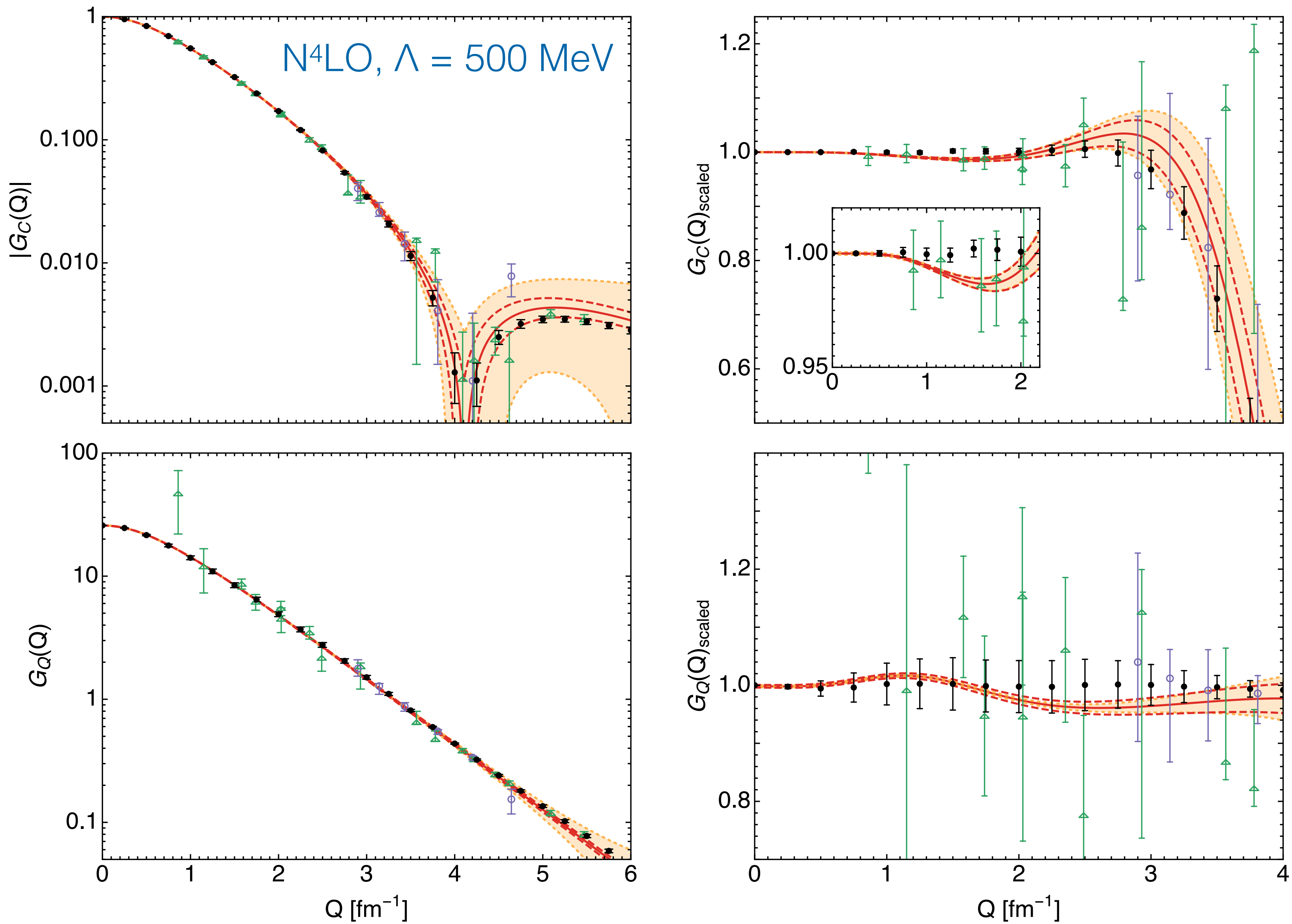

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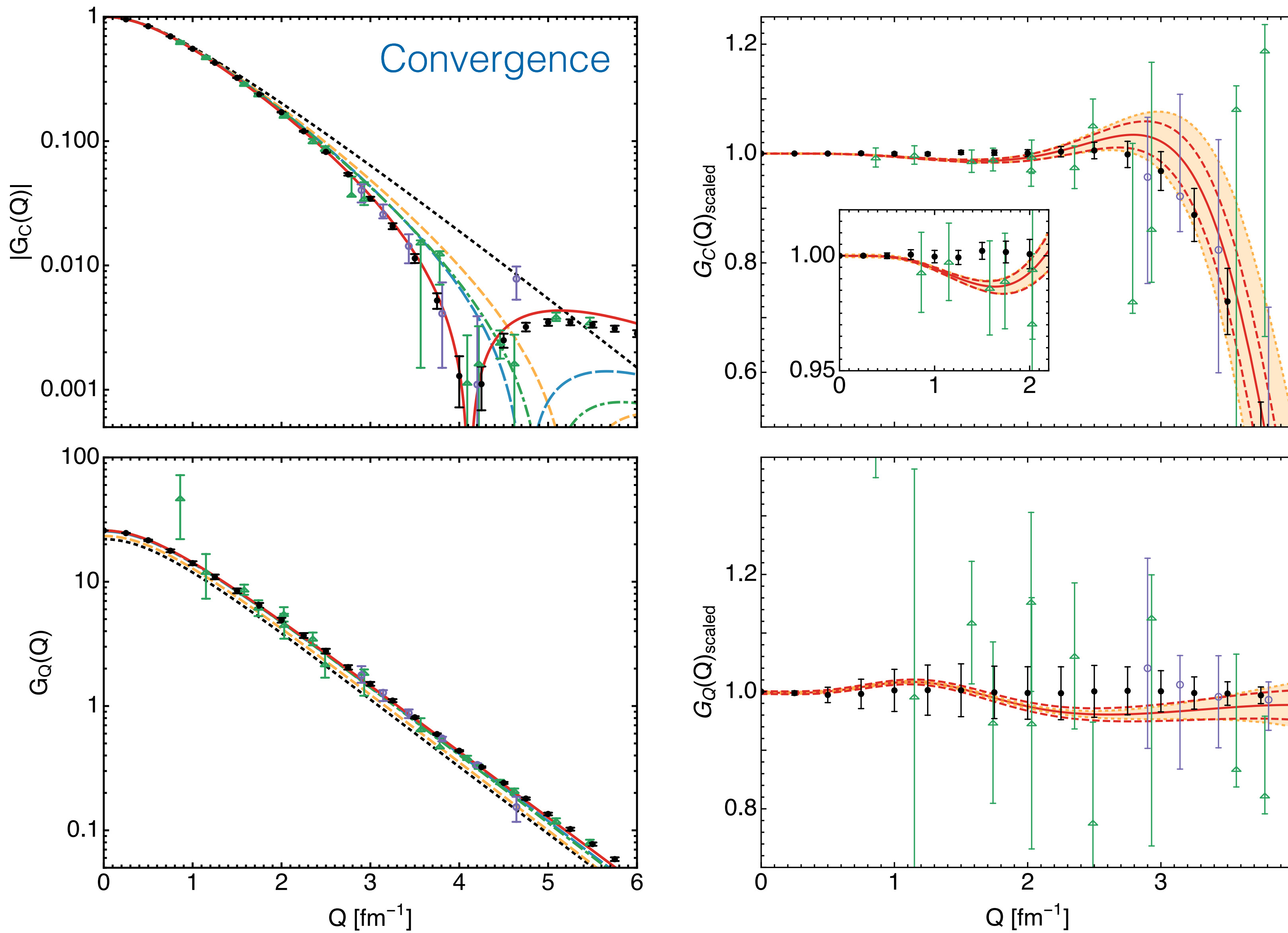

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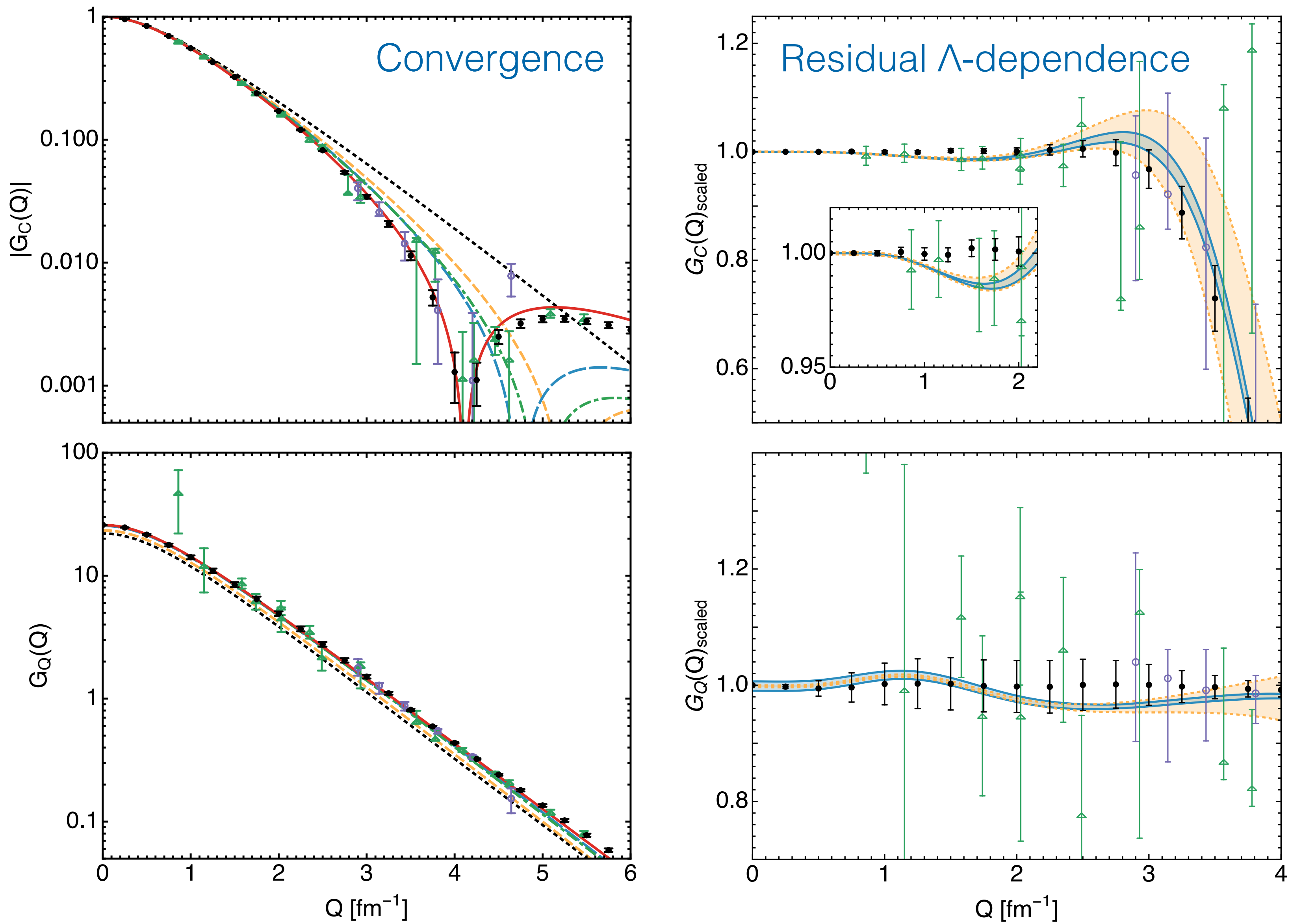

Charge radius and quadrupole moment

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Deuteron charge ***and structure*** radii: $r_d^2 = r_{\rm str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$

EFT truncation, choice of fitting range, NN, πN and γNN LECs

Our results: $r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}$ fm, $Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}$ fm^2

$Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)$ fm^2 Puchalski et al., PRL 125 (2020)

Error budget:

	central	truncation	$\rho^{\rm reg}_{\rm Cont}$	πN LECs RSA	2N LECs and f_i^2	Q-range	total
$r^2_{\rm str}$ [fm^2]	3.8925	±0.0030	±0.0024	±0.0003	±0.0025	${}^{+0.0035}_{-0.0005}$	${}^{+0.0058}_{-0.0046}$
Q_d [fm^2]	0.2854	±0.0005	±0.0007	±0.0003	±0.0016	${}^{+0.0035}_{-0.0005}$	${}^{+0.0038}_{-0.0017}$

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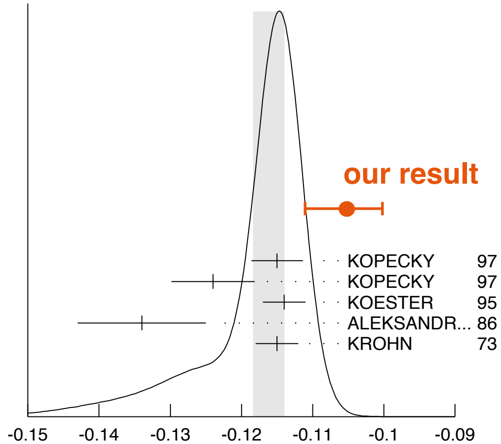
Combining our result for $r^2_{\rm str}$ with the ^{1}H-^{2}H isotope shift datum $r_d^2 - r_p^2 = 3.82007(65)$ fm^2 Jentschura et al., PRA 83 (2011) leads to the prediction for the neutron radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ \text{fm}^2$$



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EFT truncation, choice of fitting range, NN, πN and γNN LECs

Our results: $r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}$ fm, $Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}$ fm^2

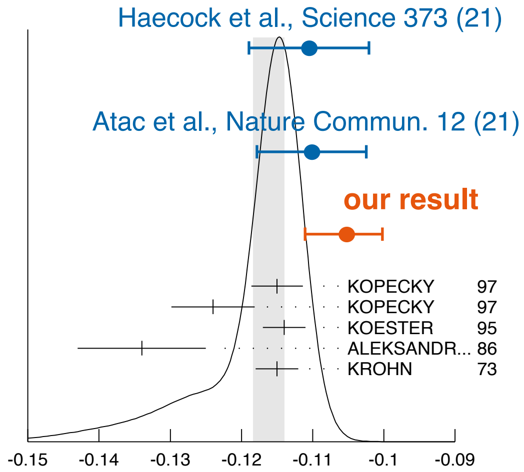
$Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)$ fm^2 Puchalski et al., PRL 125 (2020)

Error budget:

	central	truncation	$\rho^{\rm reg}_{\rm Cont}$	πN LECs RSA	2N LECs and f_i^2	Q-range	total
$r^2_{\rm str}$ [fm^2]	3.8925	± 0.0030	± 0.0024	± 0.0003	± 0.0025	$^{+0.0035}_{-0.0005}$	$^{+0.0058}_{-0.0046}$
Q_d [fm^2]	0.2854	± 0.0005	± 0.0007	± 0.0003	± 0.0016	$^{+0.0035}_{-0.0005}$	$^{+0.0038}_{-0.0017}$

Combining our result for $r_{\rm str}^2$ with the [1]H-[2]H isotope shift datum $r_d^2 - r_p^2 = 3.82007(65)$ fm^2 Jentschura et al., PRA 83 (2011) leads to the prediction for the neutron radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ \text{fm}^2$$



4^{th} moment of the charge distribution

Arseniy Filin et al., in preparation

The fourth-order moment $\langle r_d^4 \rangle := 60\, G_C''(0)$ is being measured in the ULQ2 exp Toshimi Suda et al.

$$\langle r_d^4 \rangle = r_{\rm str}^{(4)} + \frac{10}{3} r_{\rm str}^{(2)} \left(r_n^{(2)} + r_p^{(2)} + \frac{3}{4m^2} \right) + \left(r_n^{(4)} + \frac{5}{2m^2} r_n^{(2)} \right) + \left(r_p^{(4)} + \frac{5}{2m^2} r_p^{(2)} + \frac{45}{16m^4} \right)$$

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Results for $\Lambda = 500$ MeV:
(very preliminary, likely to change)

$$r_{\text{str}}^{(4)} = \underbrace{r_{\text{matter}}^{(4)}}_{55.442} + \underbrace{r_{\text{boost}}^{(4)}}_{0.215} + \underbrace{r_{\text{SO}}^{(4)}}_{-0.007} + \underbrace{r_{\text{2N,OPE}}^{(4)}}_{0.025} + \underbrace{r_{\text{2N,CT}}^{(4)}}_{0.008} = 55.68(5)\ \text{fm}^4$$

Error budget (preliminary):

Truncation (N^4LO): ± 0.035, πN LECs: ± 0.005, NN LECs: ± 0.04, other errors negligible

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$r_p^{(4)} = 1.3 \pm 0.3\ {\rm fm}^4$

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Alternative determination of the nucleon isoscalar radius?

1% accuracy $\Rightarrow$ 8% accuracy for $r_n^{(2)} + r_p^{(2)}$ …

$$r_p^{(2)} + r_n^{(2)} = \frac{\overbrace{\left[\langle r_d^4 \rangle - r_p^{(4)} - r_n^{(4)}\right]} - r_{\text{str}}^{(4)} - \frac{15}{16m^4}}{\frac{10}{3} r_{\text{str}}^{(2)} + \frac{5}{2m^2}} - \frac{3}{4m^2}$$

Towards charge FFs of A = 3,4 nuclei

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in progress

Goal: precise determination of $r_{\text{str, }^4\text{He}}$ & prediction for $r_{\text{str, A=3}} = \sqrt{1/3 r^2_{\text{str, }^3\text{H}} + 2/3 r^2_{\text{str, }^3\text{He}}}$

Towards charge FFs of A = 3,4 nuclei

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- ✅ NN interactions available at N⁴LO
- ☑ 3NF beyond N²LO not yet available (but can use correlation between radii and BEs)
- ☑ Isovector ρ_{2N} not available beyond N²LO $\Rightarrow$ can only calculate ^{4}He & ^{3}H(e)$_{\text{isoscalar}}$; the last remaining γNN LEC ($^1S_0 \to {}^1S_0$) fixed from the ^{4}He FF data
- ✅ Isospin-breaking corrections, EM interactions (also beyond point-Coulomb)
- ✅ Relativistic corrections from boosting ^{3}H(e), ^{4}He to the Breit frame
- 🚧 Relativistic corrections to the BEs and WFs

χEFT NN potential

$$\left(2\sqrt{\hat{p}^2+m^2}+\tilde{V}\right)\Psi = 2\sqrt{k^2+m^2}\,\Psi \quad \Rightarrow \quad \left(\frac{\hat{p}^2}{m}+V\right)\Psi = \frac{k^2}{m}\Psi$$

1. Reconstruct $\tilde{V}$ from V (nonlinear eq., solved by iterations)
2. Use $\tilde{V}$ to construct few-N irreps. of the Poincaré group by employing the dynamical mass operator ($\Rightarrow$ boost effects, …) Polyzou et al., Few Body Syst. 49 (2011)
3. Solve the relativistic Faddeev / Faddeev-Yakubovsky equations

- 🚧 Propagation of uncertainties and error analysis (expect ~ 2‰ accuracy)

Summary and conclusions

- Charge & quadrupole FFs of ^{2}H are in good shape (N^{4}LO, high-precision)
- Other systems and processes are limited to N^{2}LO accuracy due to unavailability of (consistently regularized) many-body forces & exchange currents

  ⇒ symmetry-preserving gradient flow regularization Hermann Krebs, EE, in progress

- Correlations between BEs and radii can be employed to obtain precise results for the charge FFs of ^{4}He & ^{3}H(e)$_{\text{isoscalar}}$ already at this stage Arseniy Filin et al., in progress

Thanks to my collaborators!