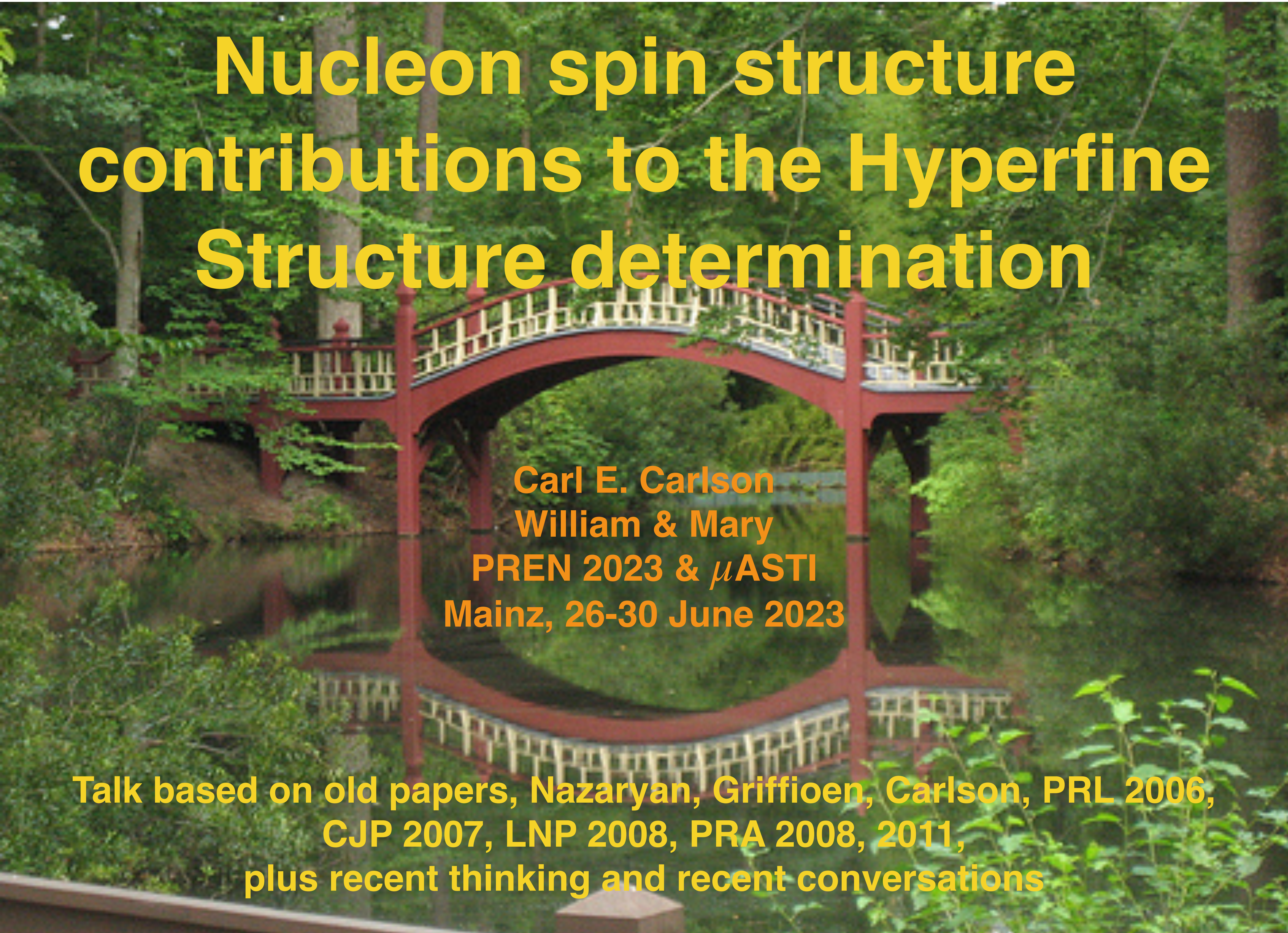


Nucleon spin structure contributions to the Hyperfine Structure determination

Carl E. Carlson
William & Mary

PREN 2023 & μASTI
Mainz, 26-30 June 2023

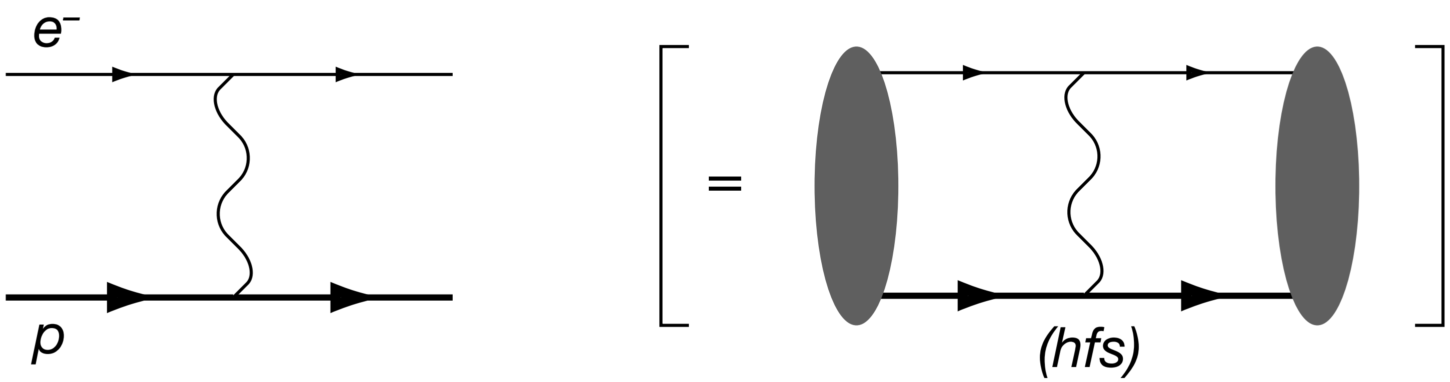
Talk based on old papers, Nazaryan, Griffioen, Carlson, PRL 2006, CJP 2007, LNP 2008, PRA 2008, 2011, plus recent thinking and recent conversations

In this talk

- Discussion of accurate calculation of hyperfine splitting (HFS) in hydrogen atom, both eH & μH
- Newly motivated by coming experiments
- Lowest order calculation gives the "Fermi energy" and we will discuss corrections to this

Lowest order (easy)

- UG textbook calculation!



- Get $$E_F^p = \frac{8\pi}{3}(m_r\alpha)^3\mu_B\mu_p$$

- $\mu_B = e/(2m_e) =$ Bohr magneton
  $\mu_p =$ exact magnetic moment for proton

- "Fermi energy"

- Can evaluate to about 10-figure accuracy

Next need corrections

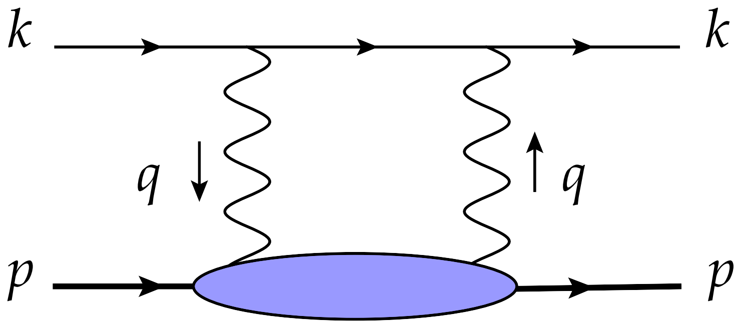
- Write as

$$E^p_{HFS} = E^p_F \left(1 + \Delta_{QED} + \Delta_S + \text{some smaller corrections}\right)$$

- Δ_{QED} well calculated, won't discuss here

- "some smaller corrections" won't be mentioned again

- Δ_S = structure dependent corrections,
  here meaning corrections from 2-γ exchange,



- Conventionally separate as

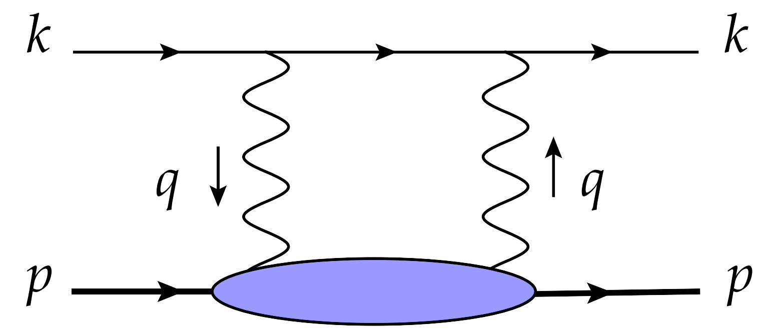
$$\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$$

Δ_Z	Δ_R	Δ_{pol}
NR elastic "Zemach"	Rel. elastic Corrections	Polarizability corrections

To be discussed

- How do we get the 2γ corrections from ep scattering data? (General answer: dispersion relations)
- Can we use unsubtracted dispersion relation?
- Comparison with another method: BχPT results
- Effect of new data—saw some already in Karl Slifer's talk, and defer further discussion to next talk (David Ruth).

2γ corrections



- Not calculable *ab initio*.
  But lower part is forward Compton scattering of off-shell photons, algebraically gotten from

$$T_{\mu\nu}(q,p,S) = \frac{i}{2\pi m_p} \int d^4\xi \; e^{iq\cdot\xi} \langle pS \big| \; T j_\mu(\xi) j_\nu(0) \big| pS \rangle$$

- Spin dependence is in the antisymmetric part

$$T^A_{\mu\nu} = \frac{i}{m_p} \epsilon_{\mu\nu\alpha\beta} q^\alpha \left[H_1(\nu,Q^2)\, S^\beta + H_2(\nu,Q^2) \frac{p\cdot q\, S^\beta - S\cdot q\, p^\beta}{p\cdot q} \right]$$

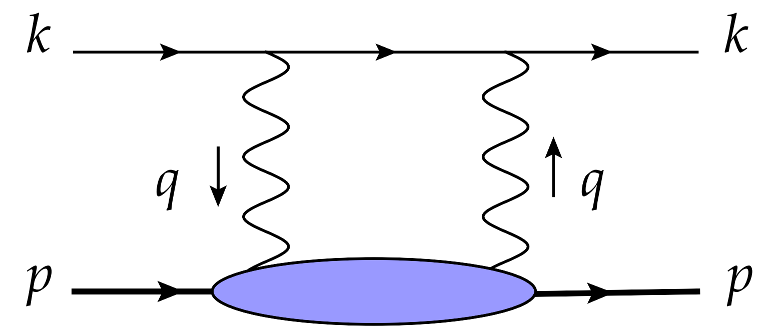
Some use $S_{1,2} = 4\pi^2 \alpha H_{1,2}$

- Imaginary part of above is related to polarized inelastic ep scattering, with

$$\mathrm{Im}\, H_1(\nu,Q^2) = \frac{1}{\nu}\, g_1(\nu,Q^2) \qquad \text{and} \qquad \mathrm{Im}\, H_2(\nu,Q^2) = \frac{m_p}{\nu^2}\, g_2(\nu,Q^2)$$

- Emphasize: g_1 and g_2 are measured at SLAC, HERMES, JLab,…

2γ corrections



- Combine electron part of diagram with Compton bottom, and energy from 2γ exchange

$$\Delta_{\rm pol} = \left.\frac{E_{2\gamma}}{E_F}\right|_{\rm inel} = \frac{2\alpha m_e}{(1+\kappa_p)\pi^3 m_p}$$
$$\times \int \frac{d^4Q}{(Q^4 + 4m_e^2 Q_0^2)Q^2} \left\{ (2Q^2 + Q_0^2) H_1^{\rm inel}(iQ_0, Q^2) - 3Q^2 Q_0^2 H_2^{\rm inel}(iQ_0, Q^2) \right\}$$

- (Wick rotated). Great, but don't know $H_{1,2}$ from data.

- But do know Im parts, and if no subtraction, simple Cauchy (dispersion relation) gives

$$H_1^{\rm inel}(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{th}^2}^{\infty} d\nu'^2 \, \frac{\mathrm{Im}\, H_1(\nu', Q^2)}{\nu'^2 - \nu^2}$$

and similarly for H_2.

Do some integrals analytically, getting

- $$\Delta_{\text{pol}} = \frac{\alpha m_e}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$
- $$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{16 m_p}{9}\int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2}\beta_1\left(Q^2/\nu^2\right) g_1(\nu, Q^2)\right\}$$
- $$\Delta_2 = -12 m_p \int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2}\beta_2\left(Q^2/\nu^2\right) g_2(\nu, Q^2)$$
  - $\beta_1(\tau) = -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}$ (for $m_e = 0$)
  - $\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$

Comments

- Early history: begun by Iddings (1965),
  finalized by Drell and Sullivan (1967),
  put in present notation by de Rafael (1971).
  No spin-dependent data existed,
  no nonzero evaluation for > 30 years,
  until Faustov and Martynenko (2002),
  then modern era starts

- Someone added something: the F_2^2 term. Not inelastic.
  (Put in here, taken out somewhere else.)
  Thought convenient in 1967, still there.

- Term as written finite in $m_e \to 0$ limit, because of known sum rule, $4m_p \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2} g_1(\nu,0) = -\kappa_p^2$ (DHGHY)

Get results

- Use data, modeling regions where data is scarce

- From CNG 2008, mostly using JLab 2003 data
  $\Delta_{pol}(eH,2S) = \quad 1.88\,(0.07)\,(0.60)\,(0.20)$ ppm
  $\Delta_{pol}(\mu H,2S) = 351.0\,(12.0)\,(107.0)\,(36.0)$ ppm

- Improved by Tomalak and by Peset and Pineda (2018). They realized that the experimental $E^p_{HFS}(eH)$ is known to 13 figures and the bulk of the μH calculation just scales with the m_μ/m_e mass ratio, known to 10 figures. Just need to calculate the smaller pieces that don't scale this way, leading to a final result with smaller overall uncertainty. Will see again soon.

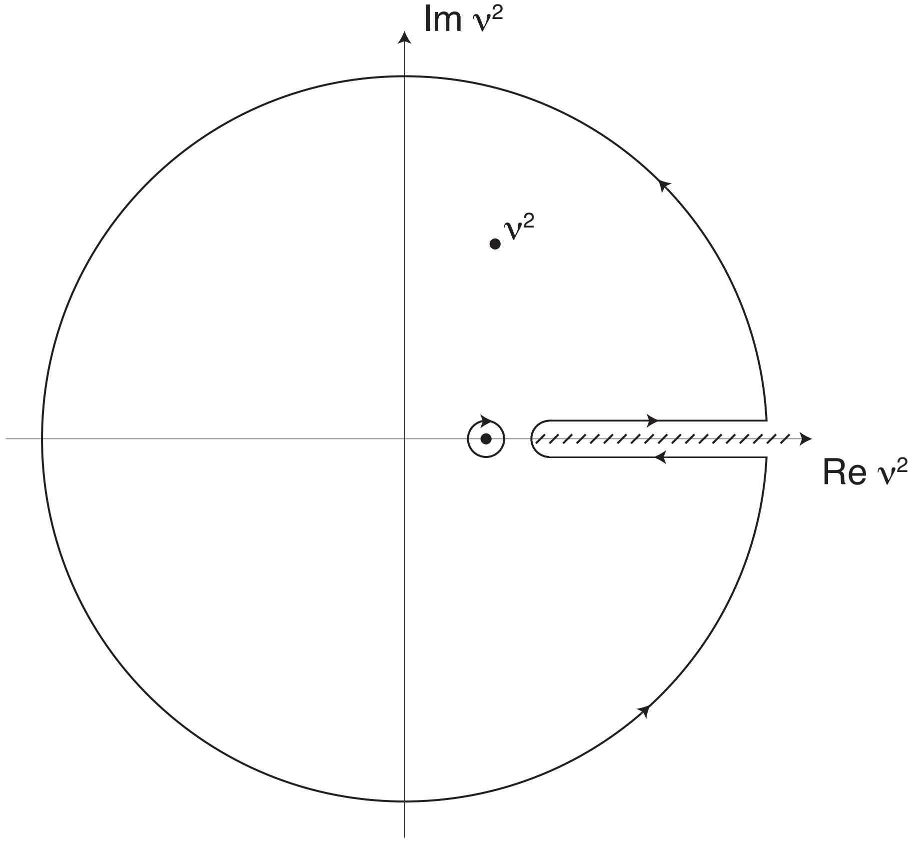
- Want to proceed to discuss subtracted or unsubtracted dispersion relation for $H_{1,2}$

Unsubtracted dispersion relation (DR)?

- Was once openly discussed (< 2006, say), now seems generally thought o.k.

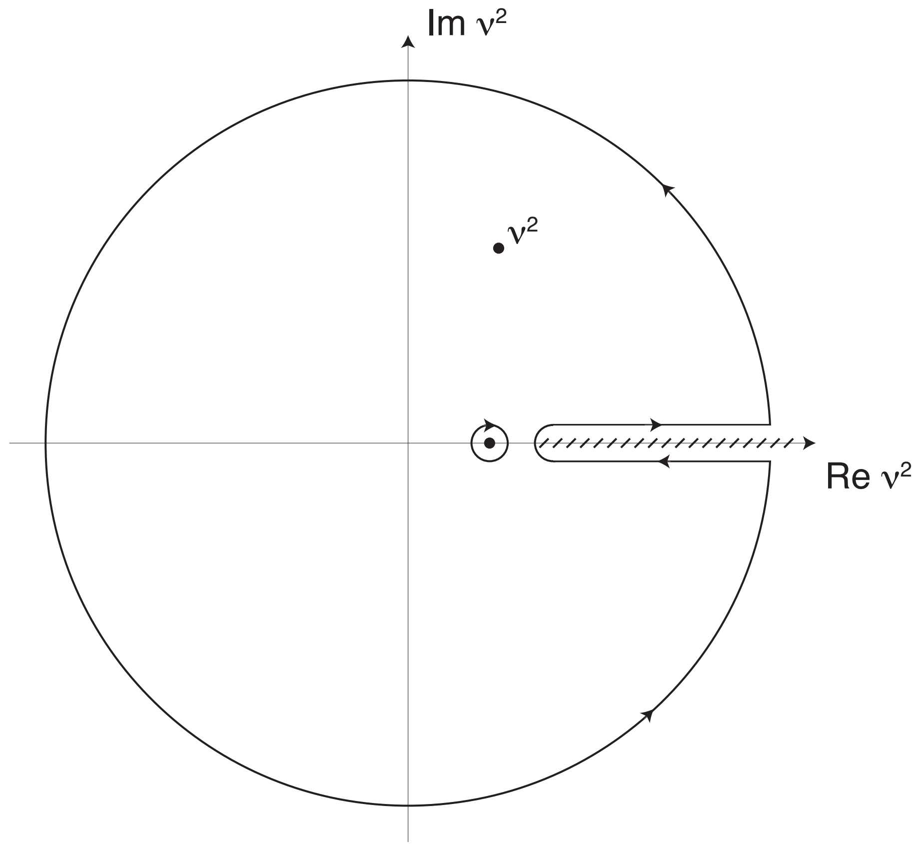
- DR comes from Cauchy integral formula applied with some contour (closed integration path)

$$H_1(\nu, Q^2) = \frac{1}{2\pi i} \oint \frac{H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2$$



- (DR in ν (or ν^2) with Q^2 fixed)

Dispersion relation



- Work into

$$H_1(\nu,Q^2) = \frac{\text{Res } H_1(\nu,Q^2)\Big|_{el}}{\nu_{el}^2-\nu^2} + \frac{1}{\pi}\int_{cut}\frac{\text{Im}\, H_1(\nu',Q^2)}{\nu'^2-\nu^2}d\nu'^2 + \frac{1}{2\pi i}\int_{|\nu'|=\infty}\frac{H_1(\nu',Q^2)}{\nu'^2-\nu^2}d\nu'^2$$

- Drop the $|\nu| = \infty$ term. O.k. if H_1 falls at high ν.

- Can view as standard or as dramatic assumption.

H_1

- The elastic term can be worked out, sticking on-shell form factors at the γp vertices,

$$H_1^{el} = \frac{2m_p}{\pi}\left(\frac{Q^2 F_1(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2\nu^2} - \frac{F_2^2(Q^2)}{4m_p^2}\right)$$

- The second term does not fall with ν at fixed Q^2.

- Unsubtracted DR fails for H_1^{el} alone. Overall success requires exact cancelation between elastic and inelastic contributions.

- (In case of interest: $H_2^{el} = -\frac{2m_p}{\pi}\frac{m_p \nu F_2(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2\nu^2}$.)

But then,

- Free quarks if there is at least one large momentum scale. So at high ν, Compton amplitude for proton should be sum of Compton amplitudes for free quarks, which have zero F_2 .

- Regge theory suggests H_1 must fall with ν. See Abarbanel and Nussinov (1967), who show $H_1 \sim \nu^{\alpha-1}$ with $\alpha < 1$.*

- Very similar DR derivation gives GDH sum rule, which is checked experimentally and works, within current experimental uncertainty.

- GDH sum rule also checked in LO and NLO order perturbation theory in QED. Appears to work.

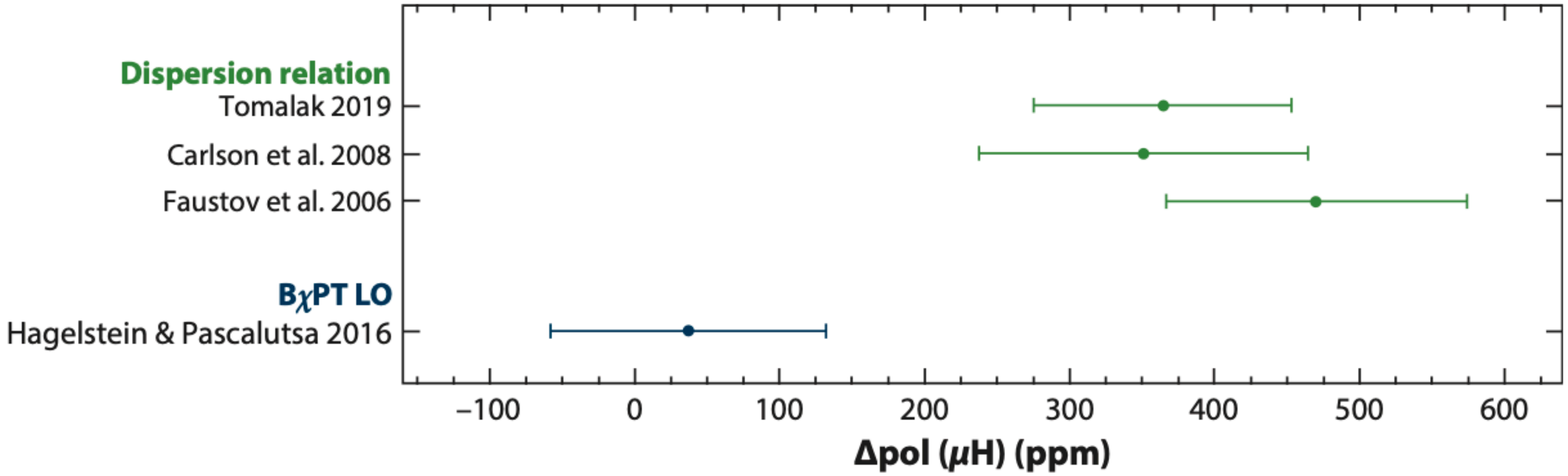
*Footnote 16 of this paper reads: For an indication of the lengthy details involved one may see M. Gell-Mann, M. L. Goldberger, and F. .E Low, Rev. Mod. Phys. 36, 640 (1964).

Resolution?

- In modern times, authors who use experimental scattering data and DR to calculate the 2γ corrections assume an unsubtracted DR works for all of H_1 .
- Reevaluation always possible.
- Proceed to next topic, comparison of data driven evaluations of HFS to evaluations using BχPT to obtain $H_{1,2}$.
- See if subtraction comments come into play.

Polarizability discrepancy

- Plot from Antognini, Hagelstein, Pascalutsa (2022), similar one in Hagelstein, Pascalutsa, Lensky (2022),



- Numbers explicit, $\Delta_{\text{pol}}\,(\text{Tomalak}) = 364(89)$ ppm

  $\Delta_{\text{pol}}\,(\text{H \& P}) = 29(90)$ ppm

  Difference $= 322$ ppm

- Bad: polarizability corrections calculated in different ways do not agree.

- (Happens that different authors results for total HFS are in decent agreement, because Zemach terms also different. That “agreement” seems like luck. Want individual pieces to agree.)

Side note: how good need we be?

- New measurements of HFS in μH in 1S state are planned.

- May measure to 0.1 ppm (as fraction of Fermi energy). But need theory prediction to help determine starting point of laser frequency scan.

- From 2018 conference at MITP (Mainz), want theory prediction to 25 ppm or better. Better is what we should look for.

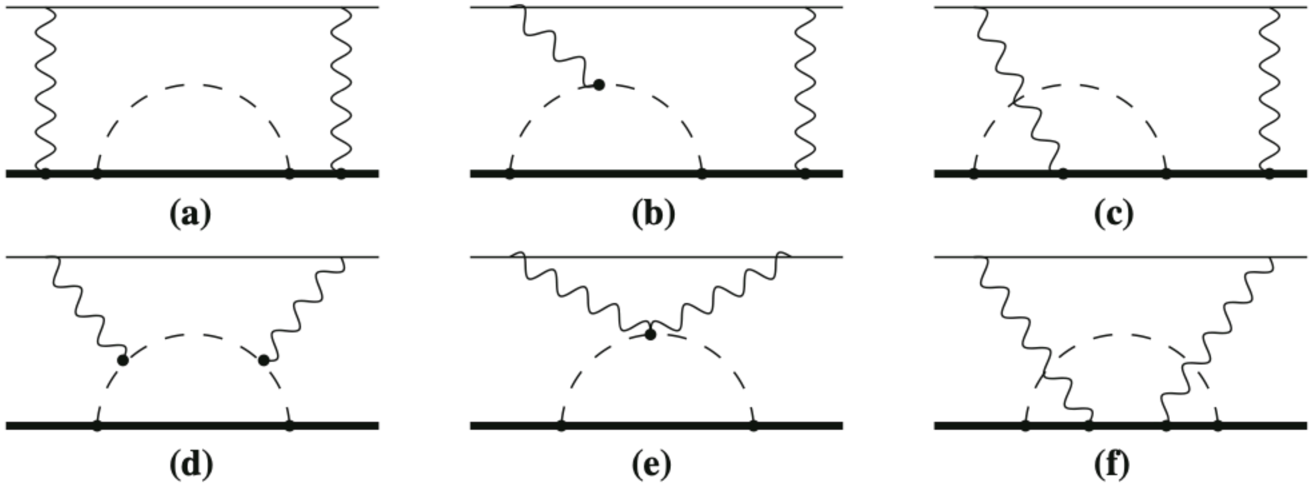
- Believe state of art for HFS in 1S μH is from Antognini, Hagelstein, Pascalutsa (2022),

$$E_{\text{HFS}}^{1\text{S}} = 182.634(8)\ \text{meV}$$

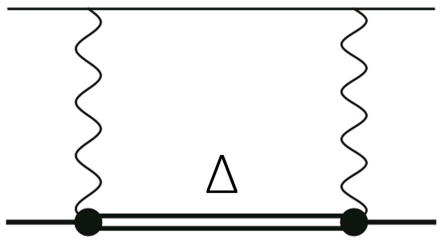
  or 44 ppm.

Application of BχPT

- Using chiral perturbation theory, one can calculate beyond the elastic case diagrams like



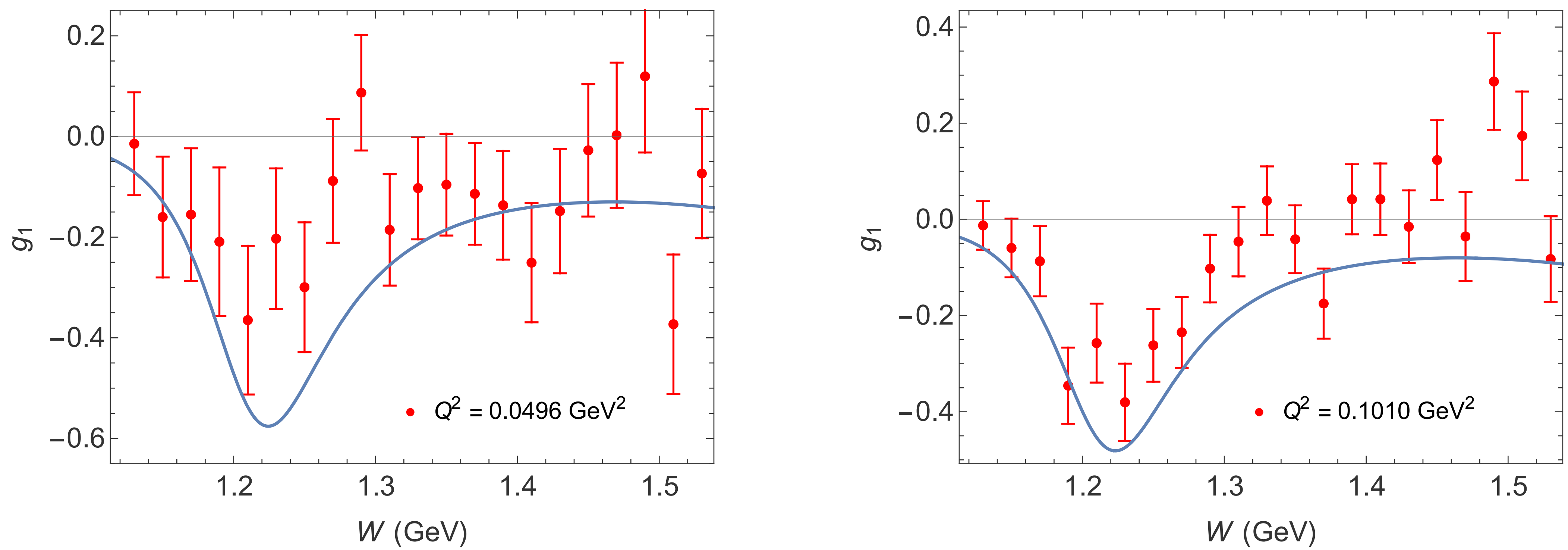
- Or diagrams where there is a Δ-baryon on the hadronic leg,



- These can be used to calculate $H_{1,2}$, at low Q^2 and CM energy W not too far from threshold. Also can get $\gamma^* N \to \pi N$ or $\gamma^* N \to \Delta$ and from them obtain $g_{1,2}$ at similarly low kinematics.

g_1 comparison

- Compare g_1 from BχPT (blue lines) to actual JLab data



- Plots are “unofficial”: Made by me* and involve spreading Δ pole out using Lorentzian of same total area.

*With greatest thanks to Pascalutsa and Hagelstein for providing code for their gamma N -> pi N

- O.k. This won’t explain difference in Δ_{pol} results.

Non-pole terms

- Non-pole means ν independent terms in $H_{1,2}$.
- Recall elastic $H_1^{el} = \frac{2m_p}{\pi}\left(\frac{Q^2 F_1(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2\nu^2} - \frac{F_2^2(Q^2)}{4m_p^2} \right)$.
- The BχPT results for H_1 with π-N and Δ intermediate states also have non-pole terms.
- To calculate energies for the non-pole terms, cannot use the DR (at least not un-subtracted ones), but can use the expressions on slide 7, which were before any Cauchy trickery was used

Pole and non-pole

- One part: The Δ contribution to μH HFS for 2S state*

$$E_{pol}^{HFS} = -\,40.69\,\mu\text{eV} \qquad \text{pole}$$
$$= \;\; 39.54\,\mu\text{eV} \qquad \text{non-pole}$$
$$= -\;\;1.15\,\mu\text{eV} \qquad \text{total}$$

- Lot of cancellation.

- But from asymptotic freedom, or from Regge analysis, or from success of DHG sum rule, expect zero non-pole term. Totality, from elastic and resonances and inelastic terms, needs to add to zero for the ν independent terms.

- Something to talk about.

*from Hagelstein (2016)

One point

- How should one deal with non-zero non-pole terms that result from partial information, when one knows that the non-pole terms are zero when one has complete information?

Δ_{pol} with newest $g_{1,2}$

- Defer to David Ruth (next after next talk).
- Except for comment on handling regions outside the data range.
- Mostly, because of the kinematic factors, the need is for data at low Q^2 and low ν (or W near threshold), and this is where the data is.
- Again, mostly, where there is no data and we use models or interpolations, the contributions to $\Delta_{1,2}$ are not great and the accruing uncertainty is not great.

Δ_{pol} with newest $g_{1,2}$

- An exception may be the very low Q^2 region, where there is no data. For the 2003 data, this was $Q^2 < 0.0492$ GeV^2.

- And there may be a problem when comparing to χPT.

- What we did: reminder

$$\Delta_1 = \frac{9}{4}\int_0^\infty \frac{dQ^2}{Q^2}\left\{F_2^2(Q^2) + \frac{8m_p^2}{Q^2}B_1(Q^2)\right\}$$

with
$$B_1(Q^2) = \frac{4}{9}\int_0^{x_{\rm th}} dx\,\beta_1(\tau)g_1(x,Q^2)\ .$$

- For very low Q^2 we used

$$B_1(Q^2) = -\frac{\kappa_p^2}{8m_p^2}Q^2 + c_{1B}Q^4 = -\frac{\kappa_p^2}{8m_p^2}Q^2 + 4.94\,Q^4/\text{GeV}^4$$

got by fitting to data $Q^2 < 0.3$ GeV^2

Δ_{pol} with newest $g_{1,2}$

- The region $Q^2 < 0.0492$ GeV^2 contributed about 15% of Δ_1 and (by our estimate) 30% of the uncertainty.

- Use standard expansion for the form factor,

$$F_2(Q^2) = \kappa_p\left(1 - \frac{1}{6}R^2_{Pauli}Q^2 + \ldots\right)$$

- Get Integrand =

$$\frac{9}{4}\frac{1}{Q^2}\left(F_2^2 + \frac{8m_p^2}{Q^2}B_1\right) = -\frac{3}{4}\kappa_p^2 R^2_{Pauli} + 8m_p^2 c_{1B}$$

- And $\Delta_1(0 \to Q^2_{low\,data}) \approx$ Integrand $\cdot\, Q^2_{low\,data} \approx 1.35$

Δ_{pol} with newest $g_{1,2}$

- χPT has knowledge of g_1 at low Q^2, and can do the integrals. Do good approximation by expanding the β_1 function for low Q^2.

- Work for a while to get Integrand =

$$-\frac{3}{4}\kappa_p^2 R_{Pauli}^2 + 8m_p^2 c_1 - \frac{5m_p^2}{4\alpha}\gamma_0 + \mathcal{O}(Q^2),$$

- Where $\gamma_0 = \frac{2\alpha}{m_p^2}\int \frac{d\nu}{\nu^4} g_1(\nu,0)$

  and c_1 came from

$$I(Q^2) \equiv 4m_p \int \frac{d\nu}{\nu^2} g_1(\nu, Q^2) = -\kappa_p^2 + c_1 Q^2 + \mathcal{O}(Q^4)$$

Δ_{pol} with newest $g_{1,2}$

- Value for known, and doing integrals to get c_1, find

  $$\Delta_1(0 \to Q^2_{low\,data}) \approx \text{Integrand} \cdot Q^2_{low\,data} \approx -0.45$$

thanks again to F. Haglestein et al.

- Not even same sign!

- Corresponding numbers for μ are ≈ 0.86 and -0.20

- Remembering $\Delta_{\text{pol}} = \dfrac{\alpha m_\mu}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$, difference gives about 50 ppm or about 15% of discrepancy.

- More to talk about!

Summary

- Dispersive calculation, assuming no subtractions are needed, is complete, well defined, and unambiguous.
- Gets value of HFS using spin-dependent ep scattering data as input.
- Really pleased about new data.
- EFT calculations should also be totally fine, but there is a “tension” that requires resolution.