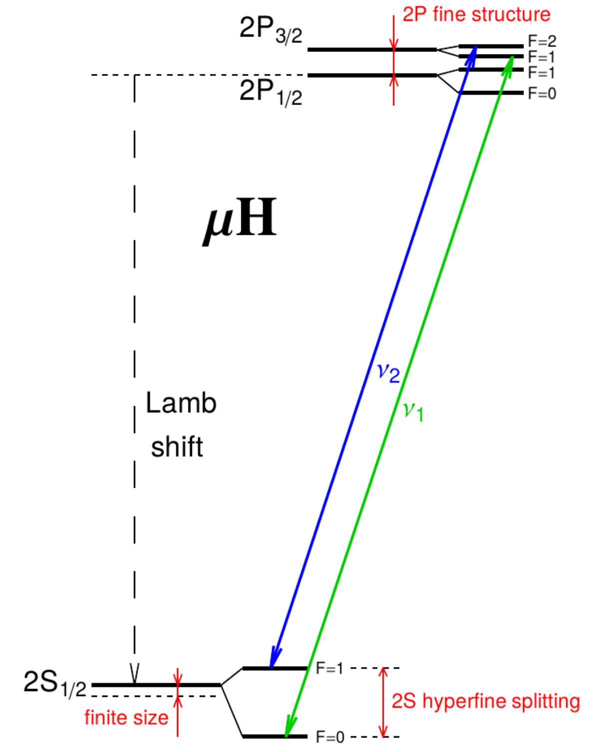
# Nuclear Structure in Light Muonic Atoms: μH

Vadim Lensky

Institut für Kernphysik, Johannes-Gutenberg Universität Mainz

PREN & μASTI Joint Meeting
Mainz, June 27, 2023

# Nuclear Structure in (Muonic) Atoms and Ions




- Atomic spectra are sensitive to nuclear properties:

  Lamb Shift: $E_{LS} = E_{QED} + \mathcal{C}\, R_E^2 + E_{NS}$

  charge radius

- Small Z: expand in $\alpha$, $Z\alpha$
  - nuclear size $\mathcal{C} \propto \alpha^4 + \ldots$
  - nuclear structure $E_{NS} \propto \alpha^5 + \ldots$

- Muonic atoms: greater sensitivity to charge radii

  Bohr radius $a = (Z\alpha m_r)^{-1}$

- But also greater sensitivity to subleading nuclear response

Lamb Shift: $$\Delta E = \frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_F^3\right] + \ldots$$

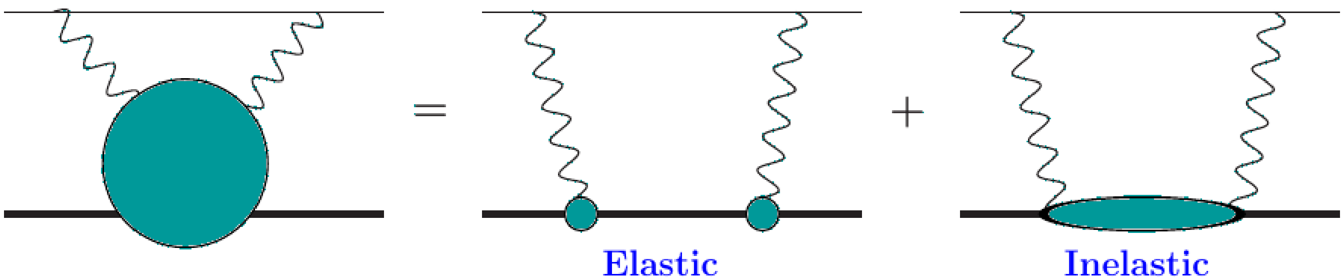
HFS: $$\Delta E = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left[1 - 2Z\alpha m_r R_Z\right] + \ldots$$

Friar radius
Zemach radius

only a part of the subleading nuclear response

# Two-Photon Exchange (TPE)

- The most important part of the nuclear response beyond finite size
- Described in terms of (doubly virtual forward) Compton scattering: VVCS
- Elastic ( $\nu = \pm Q^2/2M_{\text{target}}$ , elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)

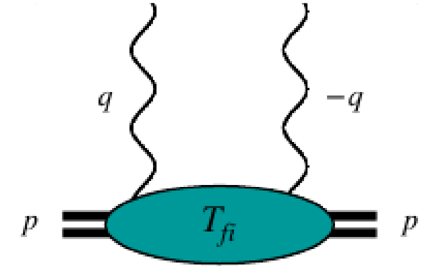



- Defines the theoretical uncertainty as of now

| | Correction | $\mu$H | $\mu$D | $\mu^3$He$^+$ | $\mu^4$He$^+$ |
|---|---|---|---|---|---|
| $E_{\text{QED}}$ | point nucleus | 206.034 4(3) | 228.774 0(3) | 1644.348(8) | 1668.491(7) |
| $\mathcal{C}\, r_C^2$ | finite size | $-5.225\,9\, r_p^2$ | $-6.107\,4\, r_d^2$ | $-103.383\, r_h^2$ | $-106.209\, r_\alpha^2$ |
| $E_{\text{NS}}$ | nuclear structure | 0.028 9(25) | 1.750 3(200) | 15.499(378) | 9.276(433) |
| $E_L(\text{exp})$ | experiment[a] | 202.370 6(23) | 202.878 5(34) | 1258.612(86) | 1378.521(48) |

Pachucki, VL, Hagelstein, Li Muli, Bacca, Pohl – theory review (2022)
[a] experiment: CREMA (2013-2023)

# VVCS and Structure Functions




- Forward spin-1/2 VVCS amplitude

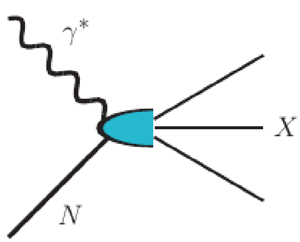
$$\alpha_{em}\, M^{\mu\nu}(\nu, Q^2) = -\left\{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\right) T_2(\nu, Q^2) + \frac{i}{M}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\nu, Q^2) + \frac{i}{M^3}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\; s_\beta - s\cdot q\; p_\beta)\, S_2(\nu, Q^2)\right\}$$

Lamb Shift: $$E^{2\gamma}_{nS} = -8 i\pi\alpha m\, [\phi_n(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2+\nu^2)\, T_2(\nu, Q^2)}{Q^4 (Q^4 - 4m^2\nu^2)}$$

Subtract point-like and finite size!

- Unitarity and analyticity, data-driven: dispersive relations

Structure functions $F_1(x, Q^2)$, $F_2(x, Q^2)$: inclusive electron scattering



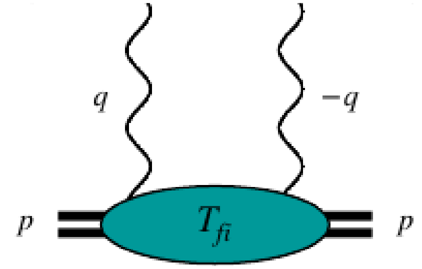

$$T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{32\pi M\nu^2}{Q^4}\int_0^1 dx\, \frac{x\, F_1(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+},$$

$$T_2(\nu, Q^2) = \frac{16\pi M}{Q^2}\int_0^1 dx\, \frac{F_2(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

$x = Q^2/(2M\nu)$

- The subtraction function $T_1(0, Q^2)$ is not directly accessible in experiment
- Data on structure functions is deficient (for anything other than proton)

# VVCS and Structure Functions



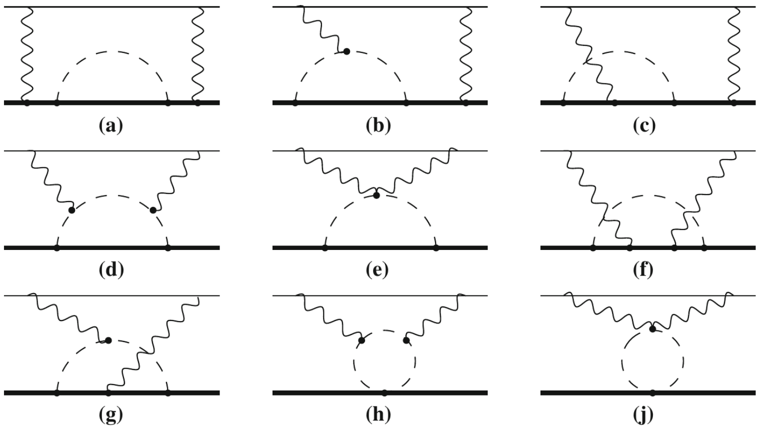

- Forward spin-1/2 VVCS amplitude

$$\alpha_{em}\, M^{\mu\nu}(\nu, Q^2) = -\left\{ \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} \left( p^\mu - \frac{p\cdot q}{q^2}\, q^\mu \right) \left( p^\nu - \frac{p\cdot q}{q^2}\, q^\nu \right) T_2(\nu, Q^2) \right.$$
$$\left. + \frac{i}{M}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\nu, Q^2) + \frac{i}{M^3}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\; s_\beta - s\cdot q\; p_\beta)\, S_2(\nu, Q^2) \right\}$$

Lamb Shift: $$E^{2\gamma}_{nS} = -8 i \pi \alpha m\, [\phi_n(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{(Q^2 - 2\nu^2)\, T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4 (Q^4 - 4 m^2 \nu^2)}$$

Subtract point-like and finite size!

- Typical energies in (muonic) atoms are small: use effective field theories
  - chiral EFT (covariant, HB, ...)
  - or even pionless EFT for nuclear effects
  - expansion in powers of a small parameter
  - order-by-order uncertainty estimate



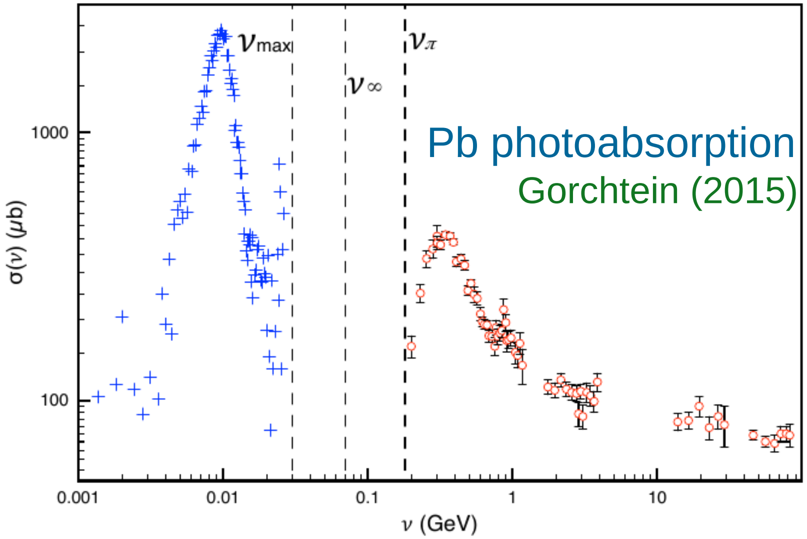

- Calculate VVCS or structure functions
- In nuclei heavier than proton: also calculate the elastic form factors

# Nuclei Heavier than Proton

- Most of the TPE correction is nuclear (as in: no nucleon polarisation)
- Nuclear part of subtraction function converges (finite energy sum rule)

Gorchtein (2015)

  - TPE integrals with nuclear response functions from χEFT will converge
  - „Most popular“ method




$$E_{\rm pol} = -\frac{4\pi\alpha^2}{3}\,\phi^2(0)\int_{E_T} dE\,\sqrt{\frac{2\mu}{E}}\,|\langle\phi_N|\vec{d}|E\rangle|^2$$

Friar, Pachucki, Wienczek, Kalinowski, Rosenfelder, Leidemann, Bacca, Ji, Hernandez, Acharya, Li Muli, VL, ...

- Single-nucleon contributions need to be accounted for separately
  - relatively more important in heavier nuclei
  - sizeable uncertainty!
  - neutron not so well constrained empirically

| | $\delta^A_{\rm Zem}$ | $\delta^A_{\rm pol}$ | $\delta^N_{\rm Zem}$ | $\delta^N_{\rm pol}$ | $\delta_{\rm TPE}$ |
|---|---|---|---|---|---|
| $\mu^2$H | −0.423(04) | −1.245(13) | −0.030(02) | −0.020(10) | −1.718(17) |
| $\mu^3$H | −0.227(06) | −0.480(11) | −0.033(02) | −0.031(17) | −0.771(22) |
| $\mu^3$He$^+$ | −10.49(23) | −4.23(18) | −0.52(03) | −0.25(13) | −15.49(33) |
| $\mu^4$He$^+$ | −6.14(31) | −2.35(13) | −0.54(03) | −0.34(20) | −9.37(44) |

nuclear | individual nucleons

Ji et al. (2018)

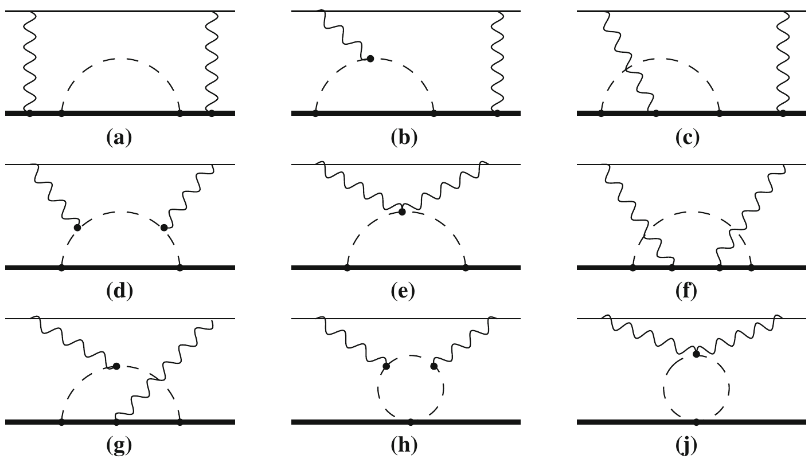
# Lamb Shift of μH in Covariant BχPT

- Delta counting: $\Delta = M_\Delta - M \gg m_\pi$ Pascalutsa, Phillips (2003)
- The contributions of the Delta isobar are suppressed by powers of $m_\pi/\Delta$
- Expansion in powers of

$$p/\Delta \sim m_\pi/\Delta \sim 0.5$$

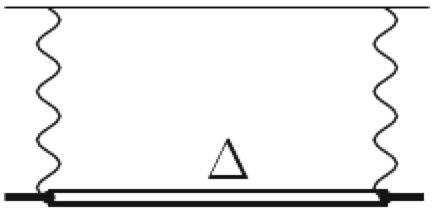
- LO BχPT: pion-nucleon loops

$$\Delta E_{2S}^{\text{LO, pol}} = -9.6^{+1.4}_{-2.9}\ \mu\text{eV}$$




Alarcon, VL, Pascalutsa (2014)

- Delta exchange:




  - suppressed in $\Delta E_{2S}^{\text{pol}}$ but affects the subtraction
  - insert transition form factors (Jones-Scadron)

$$\Delta E_{2S}^{\Delta-\text{pole}} = 0.95 \pm 0.95\ \mu\text{eV}$$

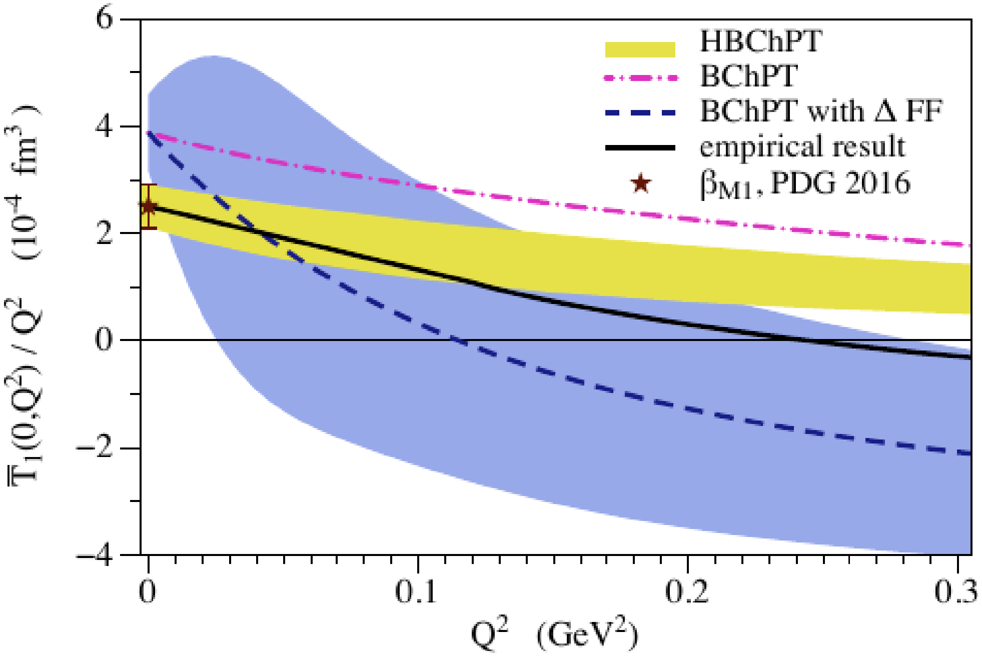
VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

# Various Subtraction Functions

- The diversity of the results for the proton subtraction function $T_1(0, Q^2)$
  - HBChPT: dipole FF, matches $\beta_{M1}$[PDG] and the slope at 0

    modification of Birse, McGovern (2012)
  - BChPT: transition FFs change the subtraction function
  - Empirical: Regge asymptotic at high energy subtracted

    Tomalak, Vanderhaeghen (2015)




VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

- Zero crossing at low $Q^2$ – is present in BChPT with FFs; established in the empirical derivation, but the position not well known (0.1..0.4 GeV$^2$)
- Big cancellations between different mechanisms (πN and πΔ loops vs. Δ pole), also cancellations in the LS integral because of the sign change
- Empirical derivation has sizeable errors towards $Q^2 = 0$ (not shown) attributed to mismatch between structure function fit in the resonance region (Christy-Bosted) and at high energies (Donnachie-Landshoff) => needs a better (combined) structure function parametrization

# Lamb Shift of μH in Various Approaches

**Table 1 Forward 2$\gamma$-exchange contributions to the 2$S$-shift in $\mu$H, in units of μeV.**

| Reference | $E_{2S}^{(\mathrm{subt})}$ | $E_{2S}^{(\mathrm{inel})}$ | $E_{2S}^{(\mathrm{pol})}$ | $E_{2S}^{(\mathrm{el})}$ | $E_{2S}^{\langle 2\gamma\rangle}$ |
|---|---|---|---|---|---|
| DATA-DRIVEN DISPERSIVE EVALUATION | | | | | |
| (75) Pachucki '99 | 1.9 | −13.9 | −12(2) | −23.2(1.0) | −35.2(2.2) |
| (76) Martynenko '06 | 2.3 | −16.1 | −13.8(2.9) | | |
| (77) Carlson *et al.* '11 | 5.3(1.9) | −12.7(5) | −7.4(2.0) | | |
| (78) Birse and McGovern '12 | 4.2(1.0) | −12.7(5) | −8.5(1.1) | −24.7(1.6) | −33(2) |
| (79) Gorchtein *et al.*'13 [a] | −2.3(4.6) | −13.0(6) | −15.3(4.6) | −24.5(1.2) | −39.8(4.8) |
| (80) Hill and Paz '16 | | | | | −30(13) |
| (81) Tomalak'18 | 2.3(1.3) | | −10.3(1.4) | −18.6(1.6) | −29.0(2.1) |
| LEADING-ORDER B$\chi$PT | | | | | |
| (82) Alarcòn *et al.* '14 | | | $-9.6^{+1.4}_{-2.9}$ | | |
| (83) Lensky *et al.* '17 [b] | $3.5^{+0.5}_{-1.9}$ | −12.1(1.8) | $-8.6^{+1.3}_{-5.2}$ | | |
| LATTICE QCD | | | | | |
| (84) Fu *et al.* '22 | | | | | −37.4(4.9) |

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the $\Delta$(1232)-isobar contribution.

Antognini, Hagelstein, Pascalutsa (2022)

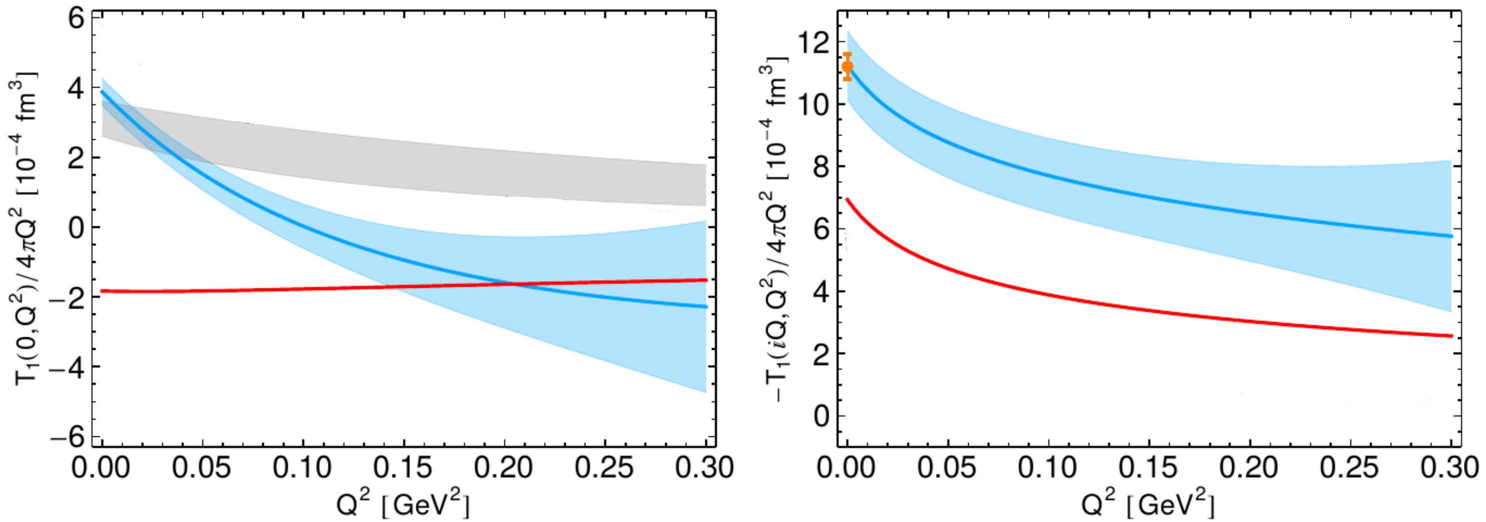
- Agreement between different approaches, also on the size of the subtraction contribution separately – despite the variation in $T_1(0, Q^2)$
- Still, the subtraction contribution has the biggest uncertainty, and needs to be further constrained

# Subtraction Function: How to Constrain it?

$$T_1(0, Q^2) \;=\; \beta_{M1}\, Q^2 + \left[\frac{1}{6}\beta_{M2} - \alpha_{\rm em}\sqrt{\frac{3}{2}}P'^{\,(M1,M1)0}(0) + \frac{1}{(2M)^2}\beta_{M1} + \alpha_{\rm em}\, b_{3,0}\right] Q^4 + \mathcal{O}(Q^6)$$

VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

- The knowledge of $b_{3,0}$ constrains the slope
- Get from dilepton electroproduction, $ep \to ep\ell^+\ell^-$ Pauk, Carlson, Vanderhaeghen (2020)
- A different subtraction point: $\nu_s = iQ$ instead of $\nu_s = 0$ Hagelstein, Pascalutsa (2021)
  - may be advantageous to use [no zero crossing at low $Q^2$, less affected by cancellations, smaller $\Delta$ contribution, inelastic contribution becomes small]




- An improvement in empirical extraction of $T_1(0, Q^2)$ [or $T_1(iQ, Q^2)$] is possible, needs better parametrizations of proton structure functions!

# HFS of μH in Covariant BχPT

$$E_{\rm hfs}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\rm QED}+\Delta_{\rm weak}+\Delta_{\rm strong}\right)$$

$$\Delta_{\rm strong} = \Delta_{\rm Zemach}+\Delta_{\rm recoil}+\Delta_{\rm pol}$$

$$\Delta_{\rm Zemach} = -2Z\alpha m_r R_Z = -2Z\alpha m_r \underbrace{\frac{-4}{\pi}\int_0^\infty \frac{{\rm d}Q}{Q^2}\left[\frac{G_M(Q^2)G_E(Q^2)}{1+\kappa}-1\right]}_{\text{Zemach radius}}$$

- Zemach radius can help one to pin down magnetic properties of the proton
- Need to know the TPE effect precisely to narrow the frequency scan range to measure the ground state HFS in μH

Planned by CREMA, FAMU, J-PARC/Riken-RAL

- Polarisability contribution connects to finer proton structure effects - higher (spin, longitudinal-transverse, …) polarisabilites
- Fine interplay between elastic (e.m. FFs) and inelastic (polarisabilities) structure properties

# HFS of μH in Covariant BχPT

$$E_{\mathrm{hfs}}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\mathrm{QED}}+\Delta_{\mathrm{weak}}+\Delta_{\mathrm{strong}}\right)$$

$$\Delta_{\mathrm{strong}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$$

$$\Delta_{\mathrm{pol.}} = \Delta_1 + \Delta_2 = \frac{Zm}{2\pi(1+\kappa)M}(\delta_1+\delta_2),$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)/Z^2 + F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4+\frac{1}{1+v_x}+\frac{1}{v_l+1}\right)\right\},$$

$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x}-\frac{1}{v_l+1}\right)$$

$$I_1(Q^2) = \frac{2M^2Z^2}{Q^2}\int_0^{x_0}\mathrm{d}x\, g_1(x,Q^2)$$

The generalised GDH integral

$$v_l = \sqrt{1+{}^1\!/\tau_l},\ \ v_x = \sqrt{1+x^2\tau^{-1}},\ \ \tau_l = {}^{Q^2}\!/{}_{4m^2},\ \ \tau = {}^{Q^2}\!/{}_{4M^2}$$

Kinematic functions

# HFS of μH in Covariant BχPT

$$E_{\rm hfs}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\rm QED}+\Delta_{\rm weak}+\Delta_{\rm strong}\right)$$

$$\Delta_{\rm strong} = \Delta_{\rm Zemach}+\Delta_{\rm recoil}+\Delta_{\rm pol}$$

$$\Delta_{\rm pol.} = \Delta_{LT}+\Delta_{TT}+\Delta_{F_2} = \frac{m}{2\pi(1+\kappa)M}\left(\delta_{LT}+\delta_{TT}+\delta_{F_2}\right),$$

$$\delta_{LT} = \frac{4M}{\alpha\pi^2}\int_0^\infty \mathrm{d}Q\int_0^{x_0} dx\,\frac{1}{v_l+v_x}\frac{1}{x^2+\tau}\left[1-\frac{1}{(1+v_l)(1+v_x)}\right]\sigma_{LT}(x,Q^2),$$

$$\delta_{TT} = \frac{4M^2}{\alpha\pi^2}\int_0^\infty \frac{\mathrm{d}Q}{Q}\int_0^{x_0}\frac{\mathrm{d}x}{x}\frac{1}{1+v_l}\left[\frac{2\tau}{x^2+\tau}+\frac{1}{(v_l+v_x)(1+v_x)}\right]\sigma_{TT}(x,Q^2),$$

$$\delta_{F_2} = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\frac{5+4v_l}{(v_l+1)^2}F_2^2(Q^2)$$

$v_l = \sqrt{1+{}^1\!/\tau_l}$, $v_x = \sqrt{1+x^2\tau^{-1}}$, $\tau_l = {}^{Q^2}\!/_{4m^2}$, $\tau = {}^{Q^2}\!/_{4M^2}$ Kinematic functions

- Rewritten in terms of scattering cross sections

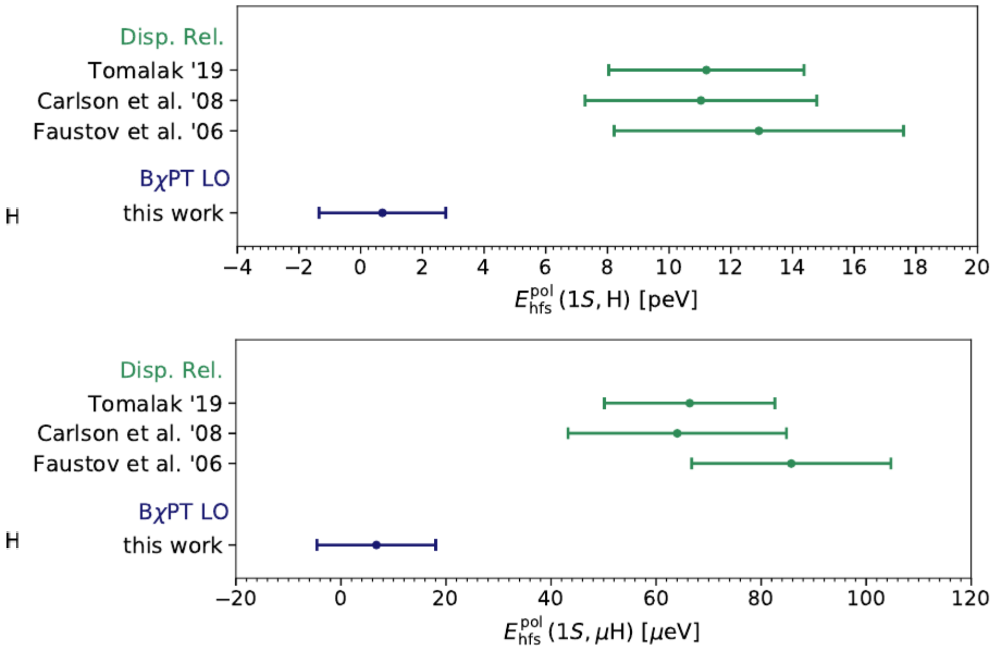
# HFS of μH in Covariant BχPT: Cancellations

- LO BχPT result

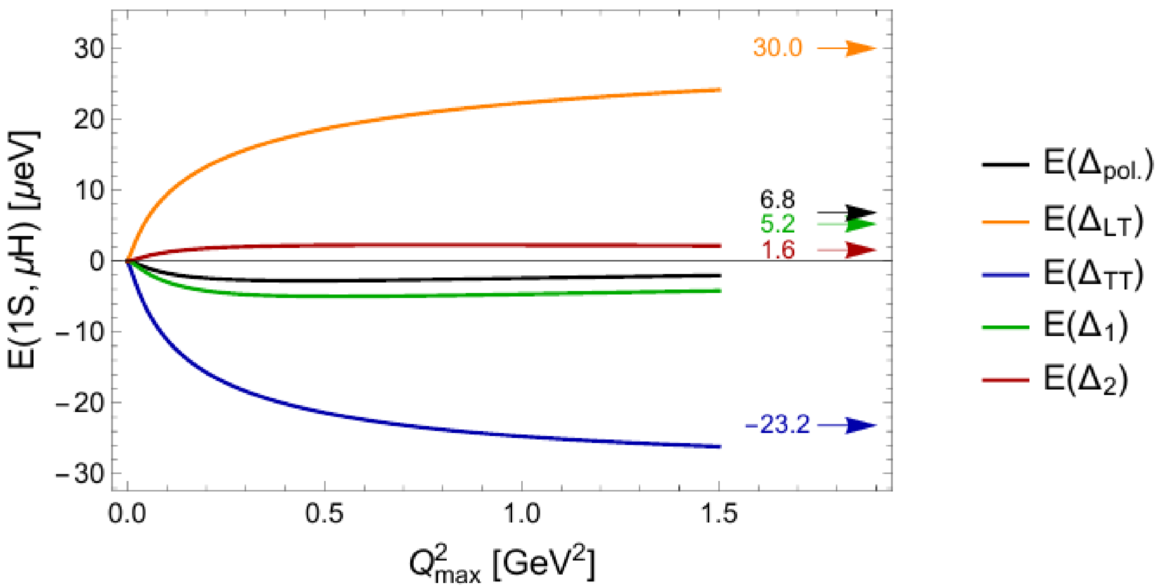
$$E_{\rm hfs}^{\langle\rm LO\rangle\ pol.}(1S,\mathrm{H}) = 0.69(2.03)\,\mathrm{peV}$$

$$E_{\rm hfs}^{\langle\rm LO\rangle\ pol.}(1S,\mu\mathrm{H}) = 6.8(11.4)\,\mu\mathrm{eV}$$

- Consistent with zero
- Cancellations!




Hagelstein, VL, Pascalutsa (2023)




- The LT and TT contributions are large and almost cancel each other
- The LO BχPT result is nearly zero
- Sizeable uncertainty

# HFS of μH in Covariant BχPT: More Cancellations

$$E_{\mathrm{hfs}}(nS) = \frac{8}{3}\frac{Z\alpha}{(an)^3}\frac{1+\kappa}{mM}\left(1+\Delta_{\mathrm{QED}}+\Delta_{\mathrm{weak}}+\Delta_{\mathrm{strong}}\right)$$

$$\Delta_{\mathrm{strong}} = \Delta_{\mathrm{Z}} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$$

$$\Delta_{\mathrm{pol.}} = \Delta_1 + \Delta_2 = \frac{Zm}{2\pi(1+\kappa)M}(\delta_1+\delta_2),$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)/Z^2+F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4+\frac{1}{1+v_x}+\frac{1}{v_l+1}\right)\right\},$$

$$\delta_2 = 96M^2\int_0^\infty\frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x}-\frac{1}{v_l+1}\right)$$

$$I_1(0) = -\frac{\kappa^2}{4} \qquad F_2(0) = \kappa$$

- Cancellation between the Pauli form factor and the intelastic contributions
- Enhanced at low $Q$!

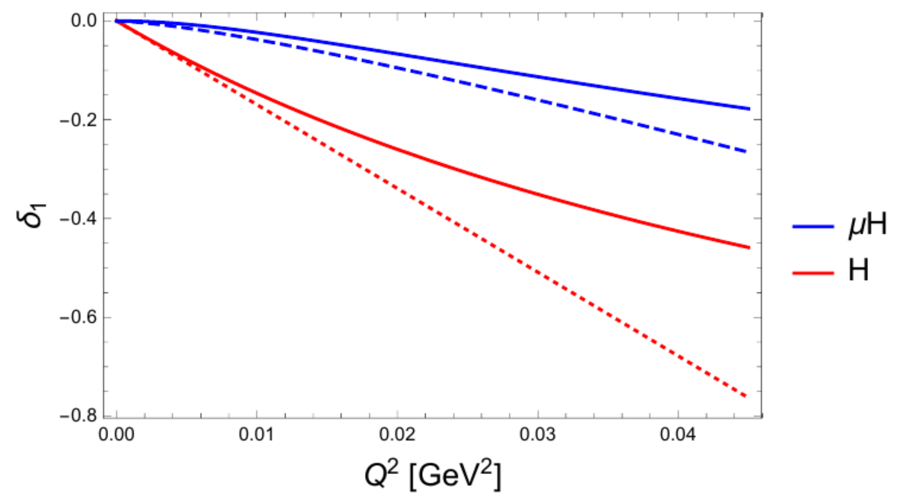
# HFS of μH in Covariant BχPT: More Cancellations

- Cancellation between the Pauli form factor and the intelastic contributions

$$\delta_1(\mathrm{H}) \sim \left(\underbrace{-\frac{3}{4}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -2.19} + \underbrace{18M^2 c_{1B}}_{\to 3.54}\right) Q_{\mathrm{max}}^2 = 1.35(90)$$

$$\delta_1(\mu\mathrm{H}) \sim \left[\underbrace{-\frac{1}{3}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -1.45} + \underbrace{8M^2 c_1}_{\to 2.13} \underbrace{-\frac{M^2}{3\alpha}\gamma_0}_{\to 0.18}\right] \int_0^{Q_{\mathrm{max}}^2} Q^2 \beta_1(\tau_\mu) = 0.86(69)$$

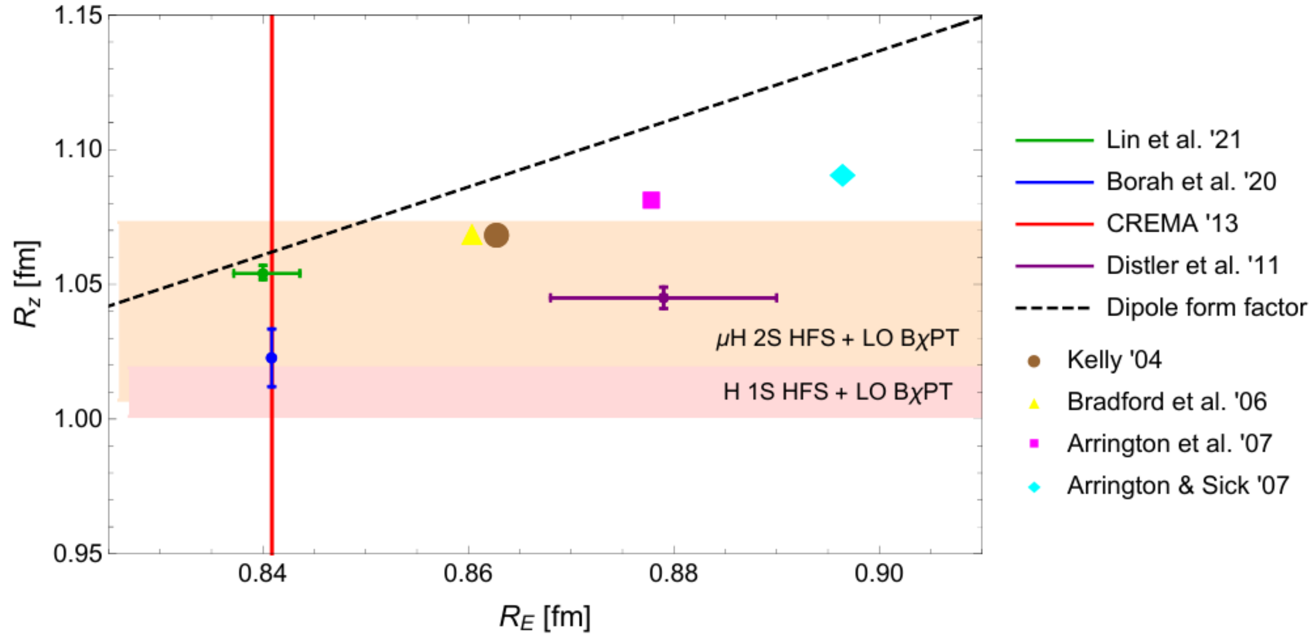
  - Potentially leads to an increased uncertainty
- No data from $Q^2 = 0$ to $Q^2 \simeq 0.045\ \mathrm{GeV}^2$
  - one invokes interpolation
  - may not work so well
- Further investigations are needed
  – new spin structure function parametrisations (cancellations with elastic parts as a constraint)
  – a careful analysis of cancellations



# HFS of μH in Covariant BχPT: Zemach Radius

- Results:

$$R_Z(\mathrm{H}) = 1.010(9)\,\mathrm{fm}$$
$$R_Z(\mu\mathrm{H}) = 1.040(33)\,\mathrm{fm}$$

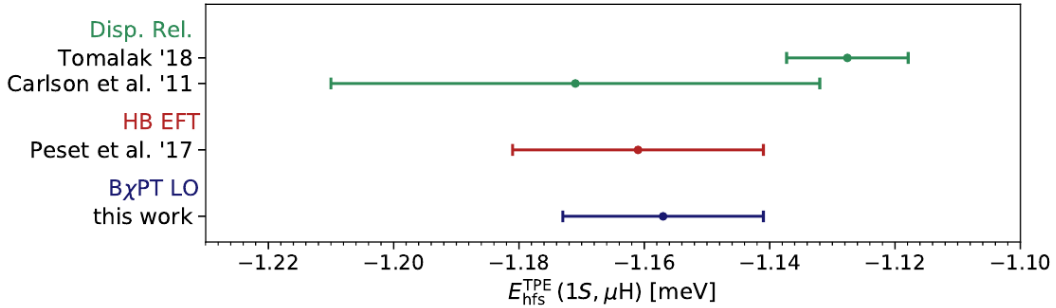



- Zemach radius extracted is smaller than in most of other works

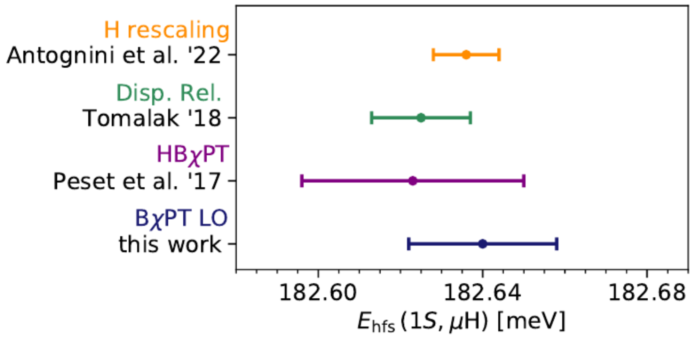
- The smallness of the Zemach radius compensates the smallness of the polarisability contribution:

$$\Delta_{\mathrm{strong}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}} \qquad \Delta_Z = -2Z\alpha m_r R_Z$$

2γ




Total HFS



# TPE Corrections: A Challenge for Theory

- The uncertainties show that TPE corrections are a challenge:

| | Correction | $\mu$H | $\mu$D | $\mu^3$He$^+$ | $\mu^4$He$^+$ |
|---|---|---|---|---|---|
| $E_{\rm QED}$ | point nucleus | 206.034 4(3) | 228.774 0(3) | 1644.348(8) | 1668.491(7) |
| $\mathcal{C}\, r_C^2$ | finite size | $-5.225\,9\, r_p^2$ | $-6.107\,4\, r_d^2$ | $-103.383\, r_h^2$ | $-106.209\, r_\alpha^2$ |
| $E_{\rm NS}$ | nuclear structure | 0.028 9(25) | 1.750 3(200) | 15.499(378) | 9.276(433) |
| $E_L$(exp) | experiment[a] | 202.370 6(23) | 202.878 5(34) | 1258.612(86) | 1378.521(48) |

Pachucki, VL, Hagelstein, Li Muli, Bacca, Pohl (2022)

- Further progress in studying the proton structure is important for matching the precision in μH (also as input for heavier nuclei)
  - Lamb shift:
    - more reliable structure function fits/parametrisations (with inputs from BχPT [polarisabilities…]) to further constrain proton subtraction function
    - subtraction: $T_1(iQ, Q^2)$ instead of $T_1(0, Q^2)$ may work better in μH
  - HFS:
    - structure function fits/parametrisations taking into account the cancellations with the elastic form factors as constraints
    - a careful analysis of uncertainties
- Test theories with new precise experimental data

# Thank You for Your Attention!