Polarizability Contribution to the Hyperfine Splitting in Muonic Deuterium

Thomas R. Richardson
Johannes-Gutenberg Universität Mainz

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ
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Cluster of Excellence
PRISMA+
Precision Physics, Fundamental Interactions and Structure of Matter

In collaboration with Bijaya Acharya, Sonia Bacca, Jose Bonilla, Chen Ji, Simone Li Muli, Lucas Platter

Why Muonic Atoms?

- Muon 200 times heavier than electron—smaller Bohr radius

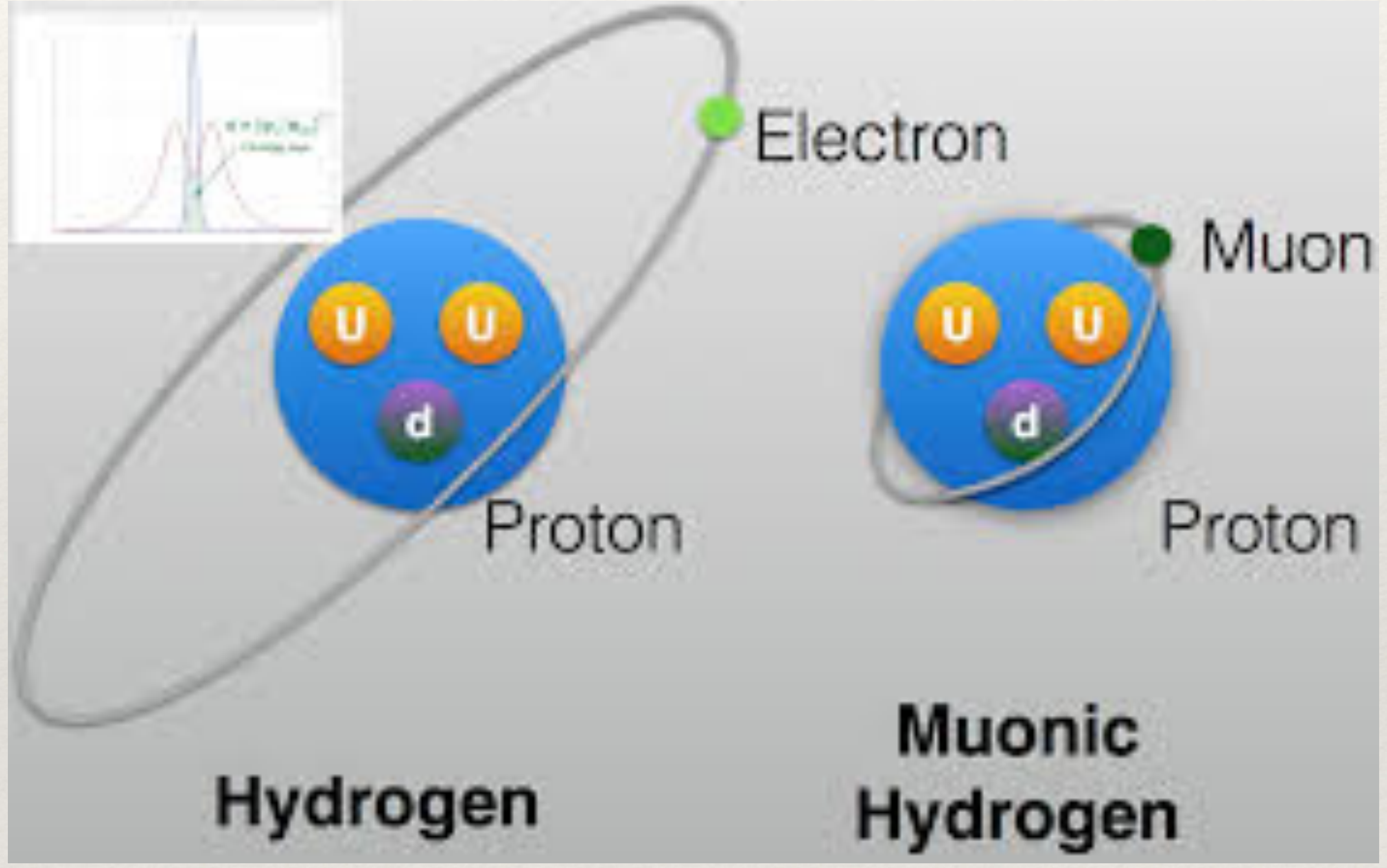


Fig: H. Gao

- Precision probe of nuclear electroweak structure
- Possible window into new physics beyond the Standard Model at the precision frontier

Experimental Effort—Hyperfine Splitting

- HFS probes the magnetic structure of the nucleus
- μH—CREMA, FAMU, J-PARC
- μD—CREMA (Pohl et al. Science 353)
- μ^3He^+—CREMA, J-PARC

PSI, Switzerland

Japan

RIKEN-RAL, UK

Uncertainties in the HFS

- Uncertainty in energy levels dominated by nuclear structure

$$\Delta E_{\mathrm{HFS}} = E_F \left(1 + \delta_{\mathrm{QED}} + \delta_{\mathrm{FS}} + \delta_{\mathrm{pol}}\right)$$

$$E_F = \frac{4\pi\alpha\mu_N}{3m_\mu} |\phi_n(0)|^2 \frac{\sigma \cdot \mathbf{J}}{J}$$

$$\Delta E_{\mathrm{FS}} = -2m_r Z\alpha E_F R_z$$

$$R_Z = \int d^3r d^3r' r \rho_E(\mathbf{r} - \mathbf{r}')\rho_M(\mathbf{r}')$$

- Polarizability contributions (inelastic nuclear excitations) start with two-photon exchange

Krauth et al., Annals 366; Pohl et al. Science 353; Kalinowski et al. PRA 98

$$\Delta E_{\mathrm{HFS}}(2S)_{\mathrm{exp}} = 6.2747(70)_{\mathrm{stat}}(20)_{\mathrm{syst}}\ \mathrm{meV}$$

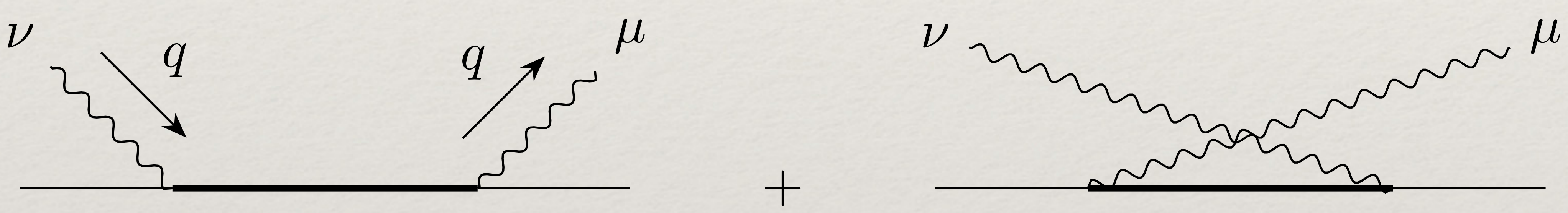
$$\Delta E_{\mathrm{HFS}}(2S)_{\mathrm{theory}} = 6.2791(50)\ \mathrm{meV}$$

TABLE II: Nuclear structure corrections for hyperfine splitting of the $1S$ and $2S$ states of muonic deuterium, in meV. Numerical results are obtained with the AV18 potential [12].

Correction	$1S$	$2S$	Source
δE_{pol1}	-1.1007	-0.1376	Eq. (22)
δE_{pol2}	-0.0823	-0.0103	Eq. (25)
δE_{pol3}	0.1513	0.0189	Eq. (26)
δE_{pol4}	-0.1979	-0.0283	Eq. (30)
δE_{pol5}	-0.0327	-0.0041	Eq. (32)
δE_{pol}	$-1.2623(631)$	$-0.1578(79)$	Eq. (33)
$\delta E_{\mathrm{1nucl}}$	$-0.9450(224)$	$-0.1181(28)$	Eq. (15)
δE_{Low}	2.566	0.3208	Eq. (14)
$\delta^{(1)} E_{\mathrm{nucl}}$	0.3587(670)	0.0448(84)	Eq. (12)
$\delta^{(2)} E_{\mathrm{nucl}}$	$-0.0547(137)$	$-0.0065(16)$	Eq. (77)
$\delta E_{\mathrm{nucl,theo}}$	0.304(68)	0.0383(86)	Eq. (8)
$\delta E_{\mathrm{nucl,exp}}$		0.0966(73)	Eq. (7)
difference		0.0583(113)	

Polarizability from Two Photon Exchange

$$\Delta E_{2\gamma} = i(4\pi\alpha)^2 \, |\phi_n(0)| \int \frac{d^4q}{(2\pi)^4} T^{\mu\nu}(q) L^{\rho\tau}(q) D_{\mu\rho}(q) D_{\nu\tau}(q)$$



$$T^{\mu\nu}(q) = \sum_{N \neq N_0} \langle N_0 | \frac{J^\mu(-\mathbf{q})|N\rangle\langle N|J^\nu(\mathbf{q})}{q_0 - \omega_N + i\epsilon} - \frac{J^\nu(\mathbf{q})|N\rangle\langle N|J^\mu(-\mathbf{q})}{q_0 + \omega_N - i\epsilon} | N_0 \rangle$$

$$L^{\mu\nu} = (ie)^2 \, \bar{u}_r(k) \gamma^\mu \frac{\not{q} + \not{k} + m_\mu}{(q+k)^2 - m_\mu^2 + i\epsilon} \gamma^\nu u_s(k)$$

Bernabéu and Jarlskog NPB 75; Rosenfelder, NPA 393; Friar and Payne, PLB 618, PRC 72

Hyperfine Splitting

Friar and Payne, PLB 618, PRC 72

- Contained in spin-dependent (antisymmetric) part of lepton tensor

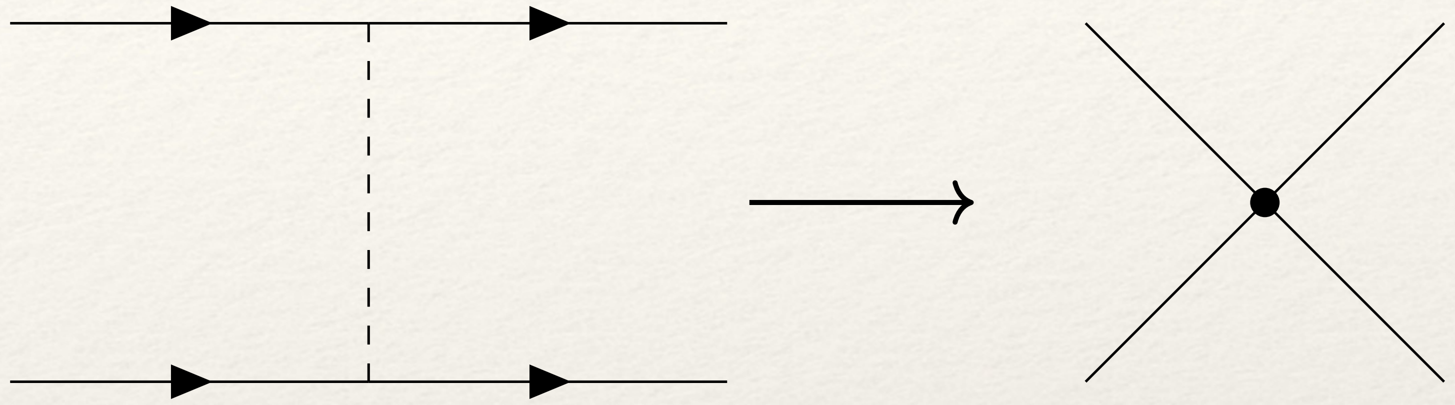
$$\Delta E_{2\gamma,\mathrm{HFS}} = -(4\pi\alpha)^2 \left|\phi_n(0)\right|^2 \epsilon^{ijk}\sigma^k \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+i\epsilon)^2} \frac{1}{q^2-2mq_0+i\epsilon} \left[\left(T^{0i}-T^{i0}\right)q^j + T^{ij}q_0\right]$$

- All of the low-energy nuclear effects are contained in the hadronic tensor

$$T^{\mu\nu}(q) = \sum_{N\neq N_0} \langle N_0| \frac{J^\mu(-\mathbf{q})|N\rangle\langle N|J^\nu(\mathbf{q})}{q_0-\omega_N+i\epsilon} - \frac{J^\nu(\mathbf{q})|N\rangle\langle N|J^\mu(-\mathbf{q})}{q_0+\omega_N-i\epsilon} |N_0\rangle$$

- Several approaches: dispersion relations, phenomenological models, ***effective field theory***

Effective Field Theory: From the Top Down



$$\frac{g^2}{p^2 - M^2} \qquad \frac{g^2}{M^2}$$

Λ

???

SMEFT — Standard Model + higher dimension operators

100 GeV

LEFT — $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{EM}}$ — RG

1 GeV

Lattice QCD

Chiral EFT

100 MeV — Hadrons

Pionless EFT

Few/many-body methods

S. Li Muli’s talk

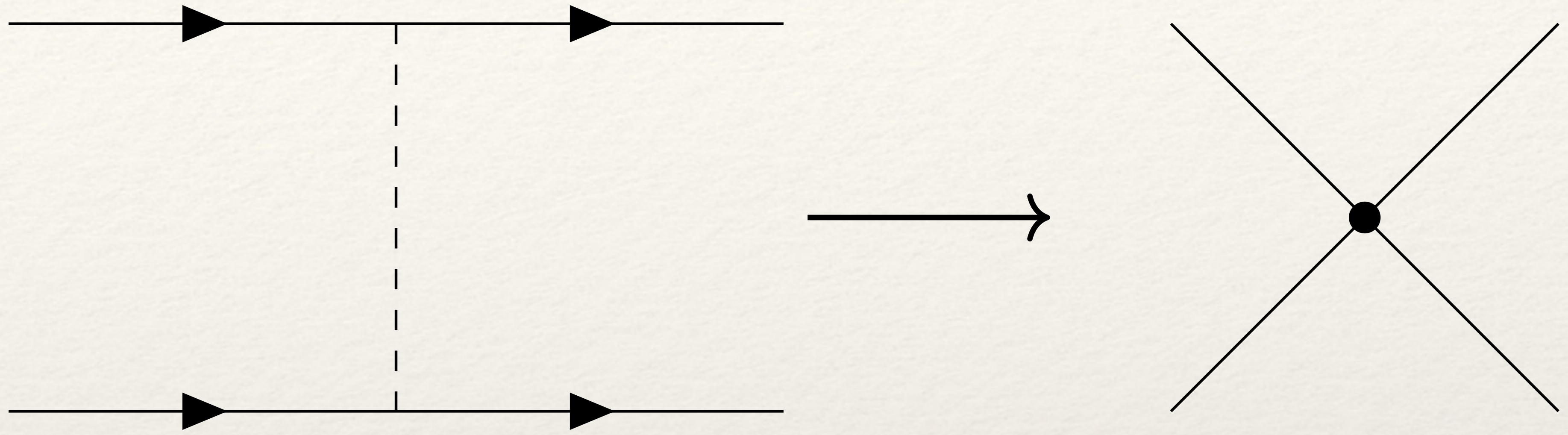
Why the EFT framework?

$$\mathcal{L}_{\text{eff}} = \sum_n \left(\frac{p}{\Lambda}\right)^n c_{\mathcal{O}} \mathcal{O}_n$$

- Model independent
- Power counting—systematically improvable
- Quantify uncertainties (S. Li Muli's talk)

Pionless EFT

Kaplan et al., PLB 424, NPB 534;
van Kolck, NPA 645



- Valid for $p \ll m_\pi$
- Only nucleon degrees of freedom

$$\mathcal{L}_{NN} = N^\dagger \left(iD_0 + \frac{1}{2m_N} D^2 \right) N - C_0 \left(N^T P N\right)^\dagger \left(N^T P N\right) + \cdots$$

- Tower of contact terms with desired symmetries, i.e. Galilean, gauge, isospin, parity, time-reversal etc.
- Lamb shift in muonic deuterium

Emmons et al., JPG 48;
Lensky et al. PRC 104, PLB 385, EPJA 58

Chiral Perturbation Theory

- Pions realize chiral symmetry of QCD nonlinearly

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$$

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right) + \cdots \qquad U = e^{\frac{i}{F_0}\pi_a\tau_a}$$

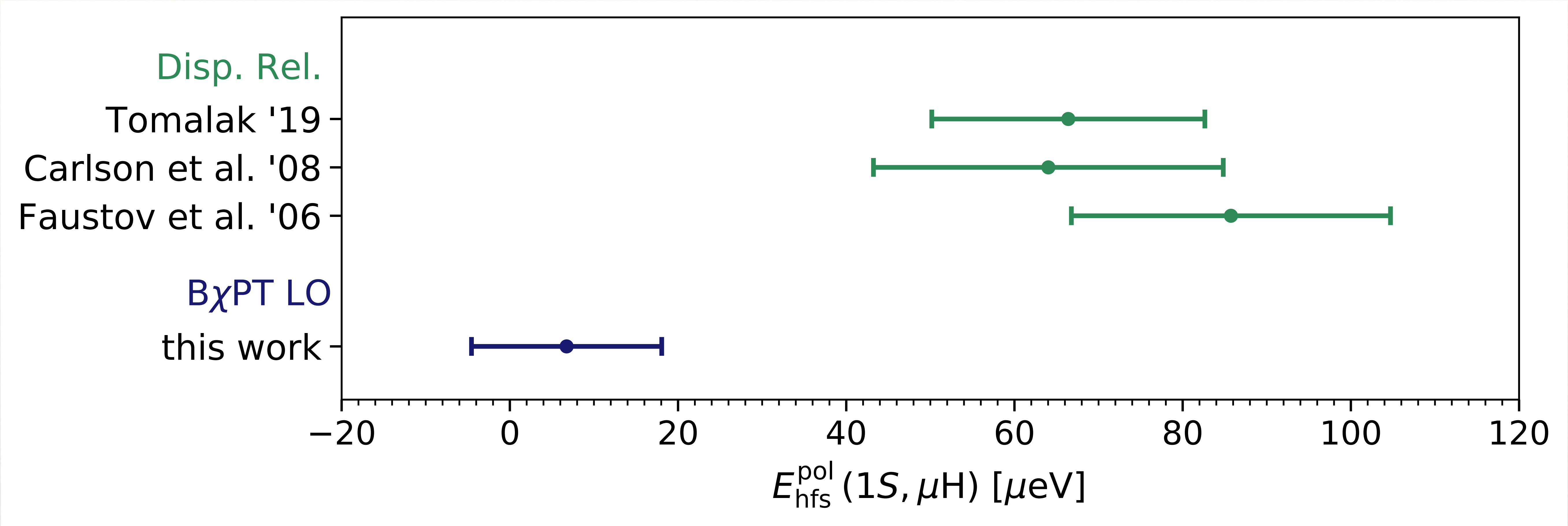
- Nucleons enter as isospin doublet

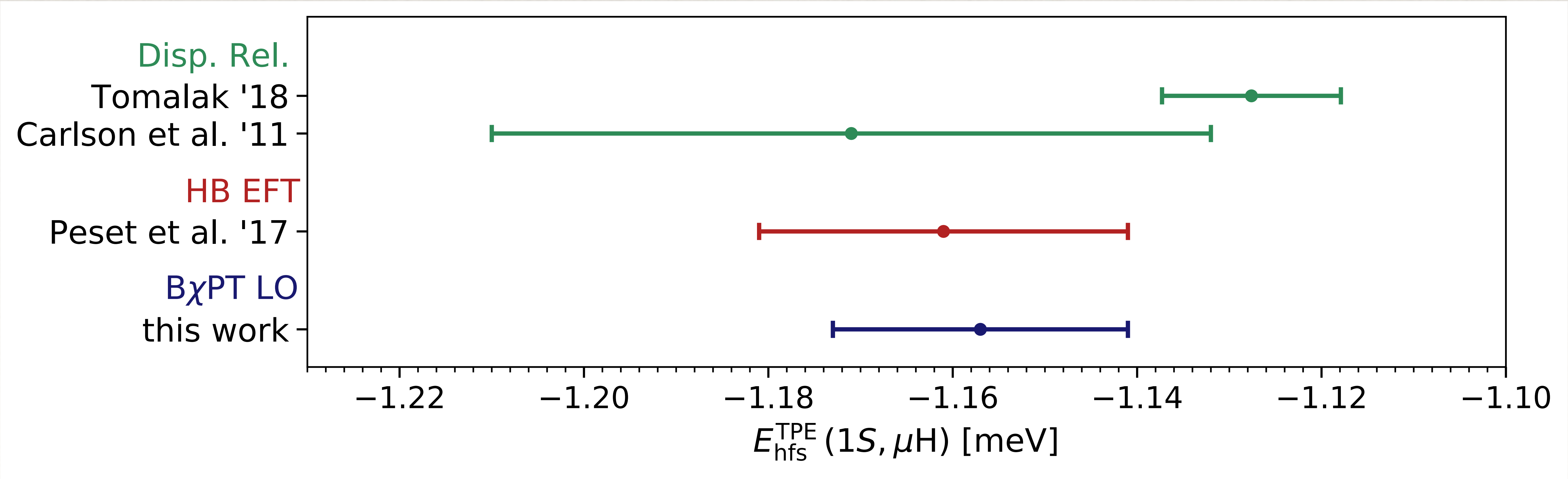
$$\mathcal{L}_{\pi N} = \bar{N}_v\left(iv\cdot D + g_A S_v \cdot u\right) N_v + O(1/m_N) + \cdots$$

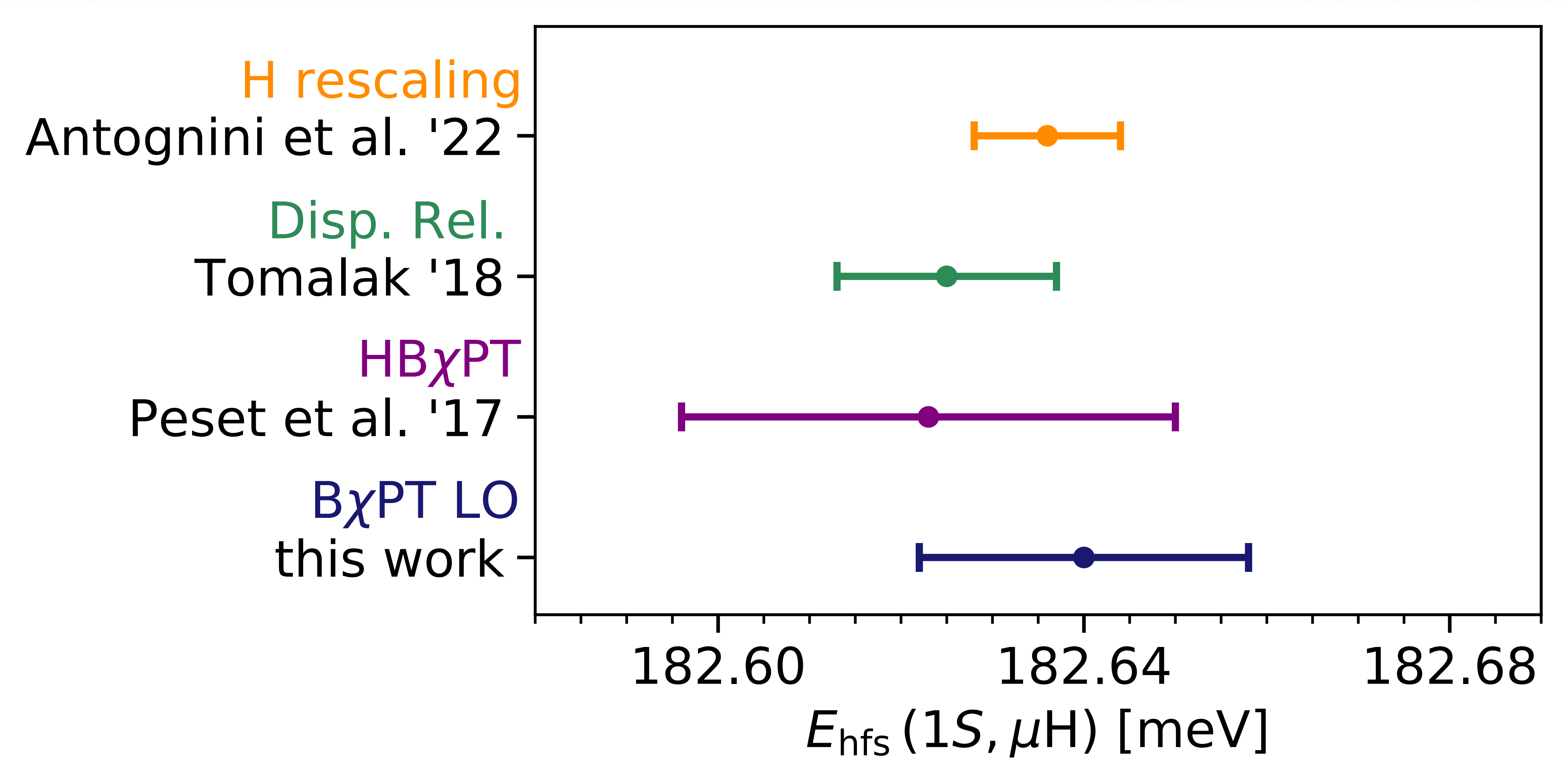
- Power counting parameter $\dfrac{(m_\pi, p)}{4\pi F_0}$

Weinberg, 1968; Gasser and Leutwyler, 1983, 1984;
Gasser et al. 1987; Jenkins and Manohar, 1991

Hyperfine Splitting of Muonic Hydrogen







Hagelstein et al. 2023

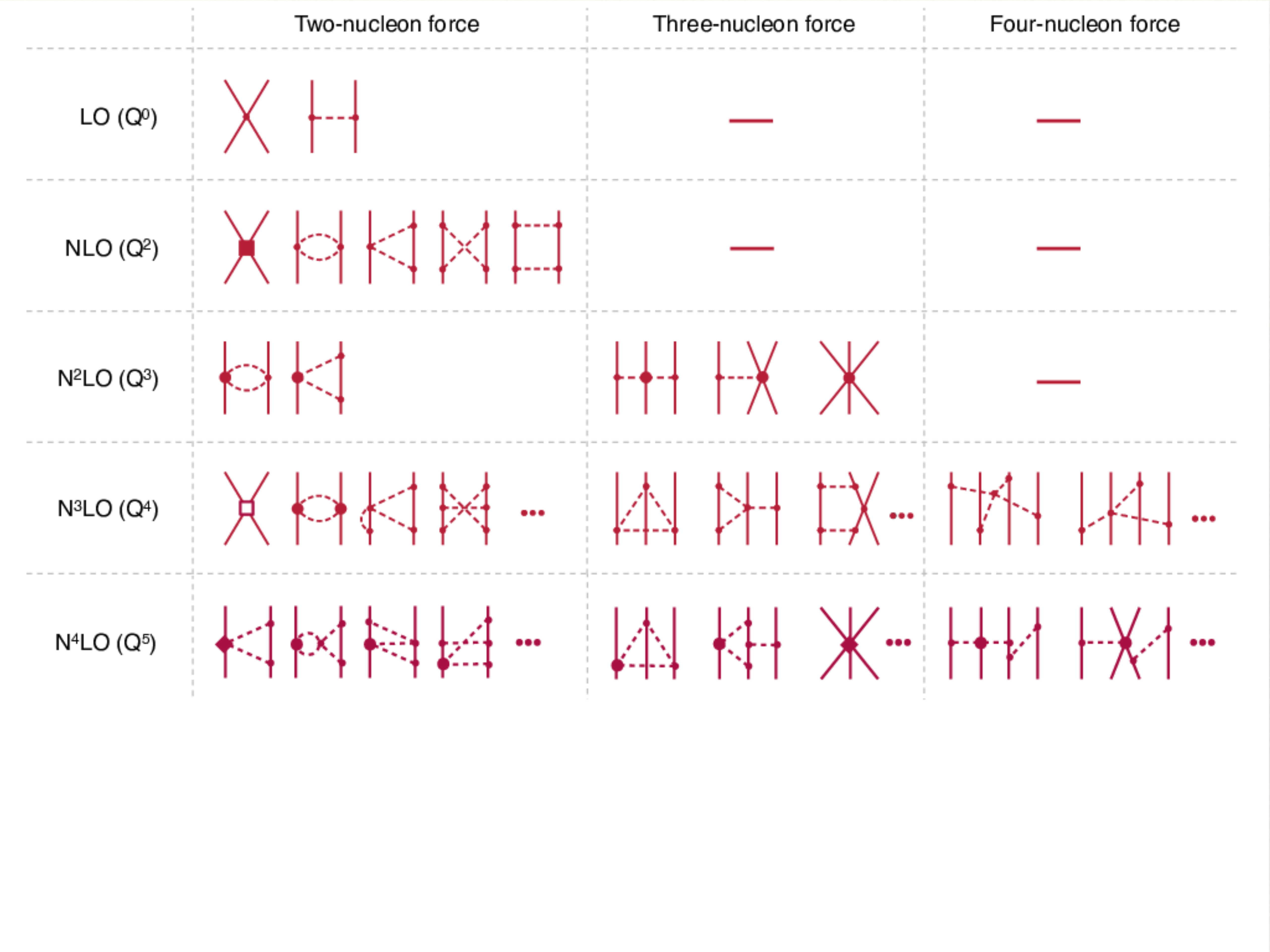
Chiral Effective Field Theory

- Extend ChPT to few-nucleon systems

$$\mathcal{L}_{\mathrm{ChEFT}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

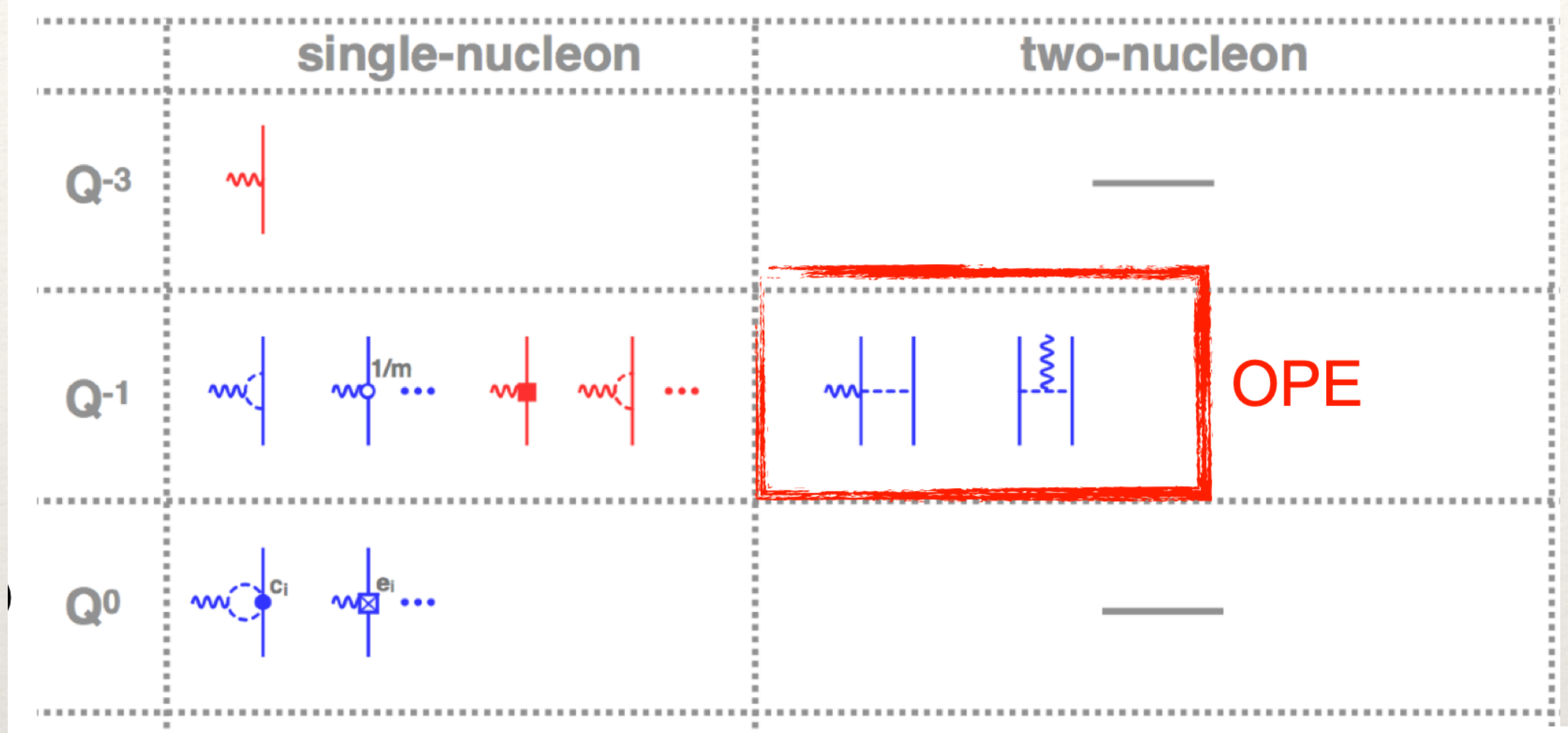
- NN potential = sum of “irreducible” diagrams

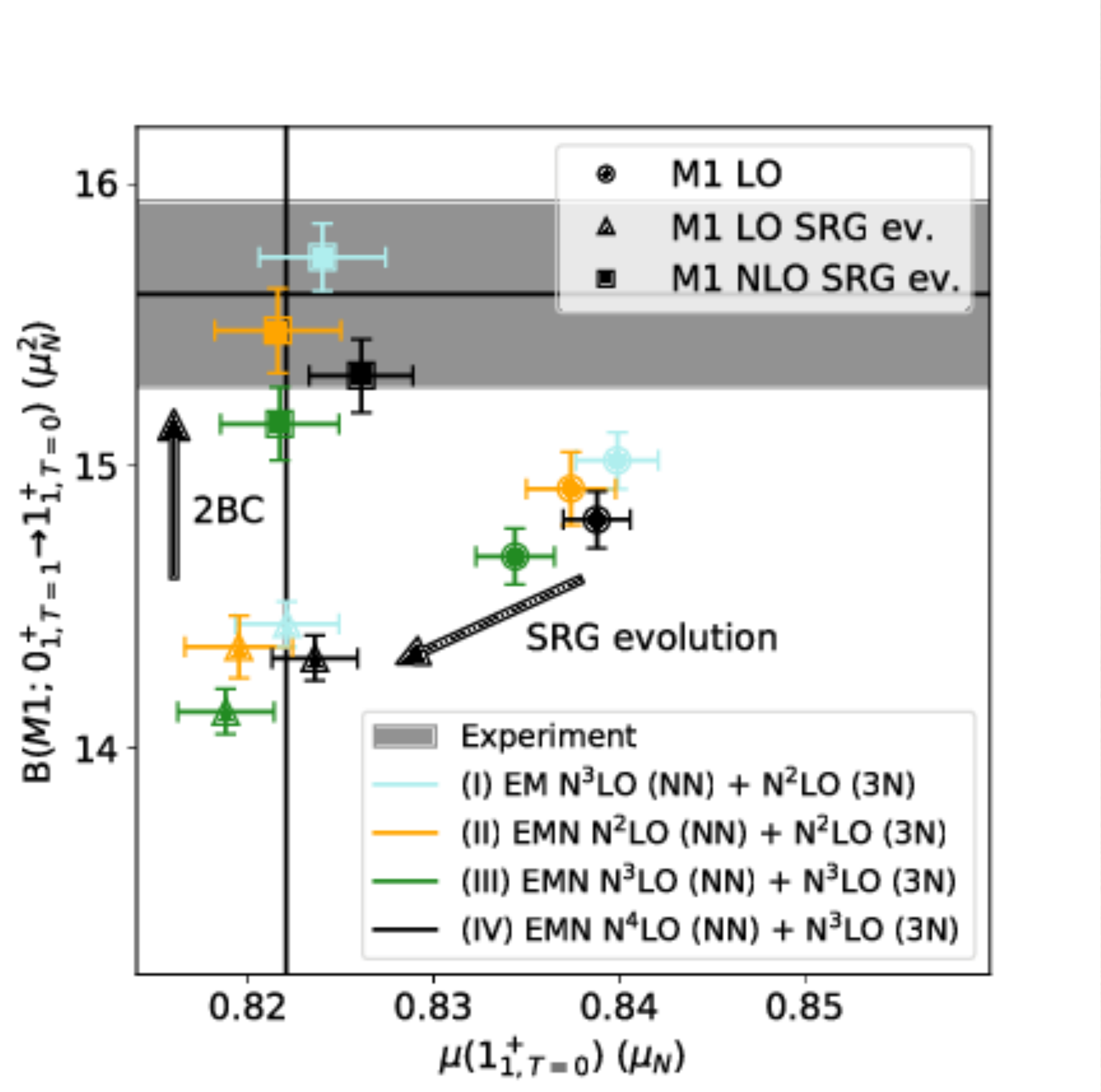
- *Ab initio* program: Construct chiral potentials and solve many-body Schrödinger equation



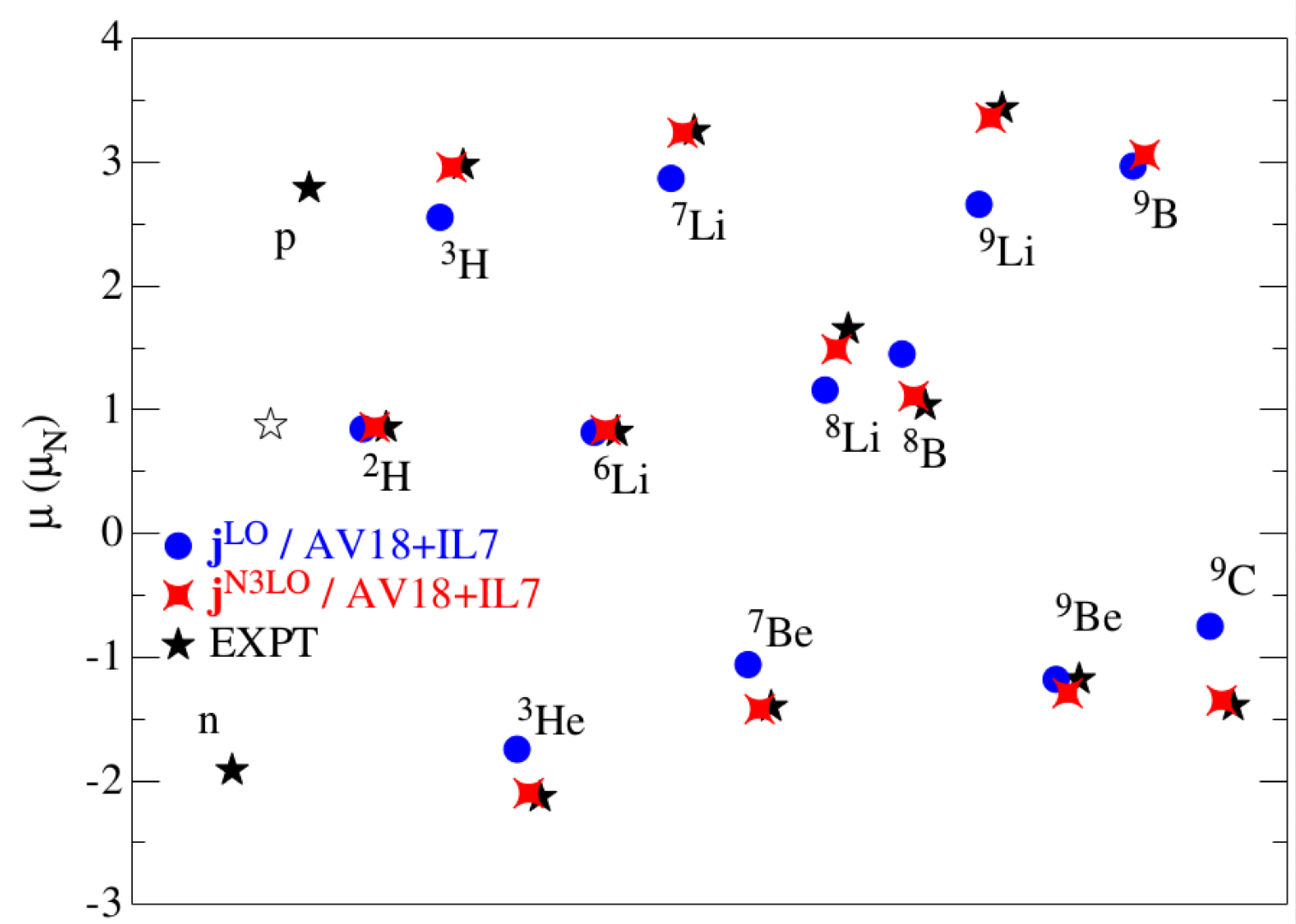
Weinberg, Entem, Machleidt, Epelbaum, Meißner…

Electromagnetic Few-body currents





Bacca et al. PRL 126



Pastore et al. PRC 90

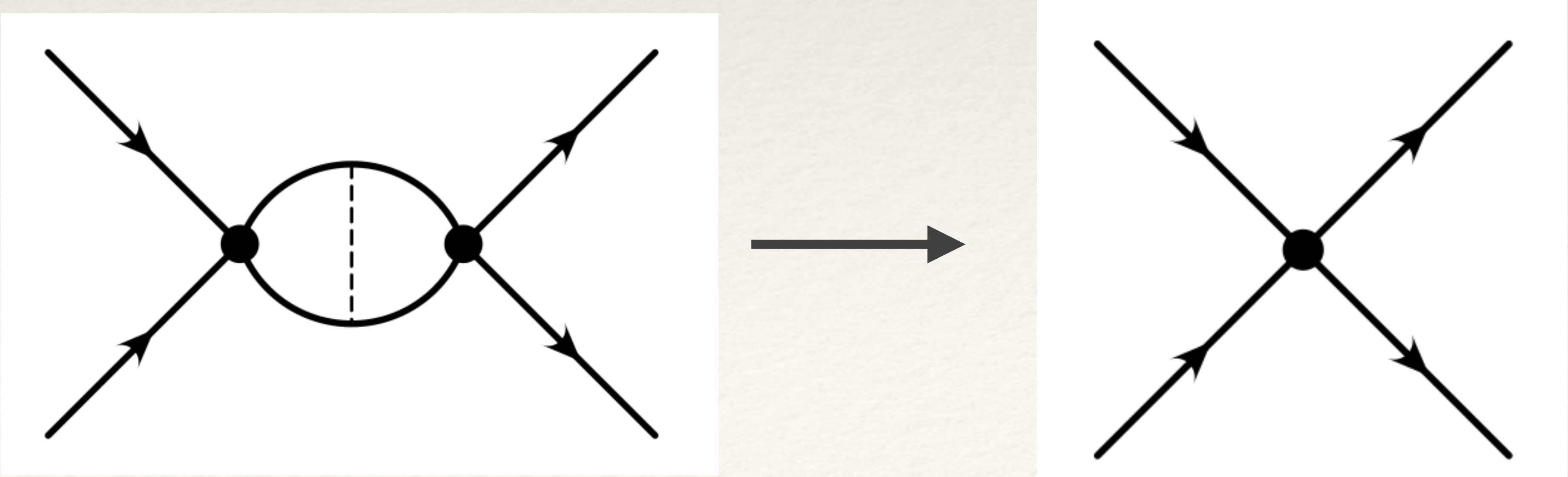
Chiral EFT—A Tale of Two Power Countings

- Weinberg power counting—interactions ordered according to naïve dimensional analysis
  - Renormalization is unambiguous
  - Solve Schrödinger equation with full potential

Epelbaum, Gegelia, Krebs, Meißner…

- Renormalization group invariant approach—additional operators may be required at leading order
  - Solve Schrödinger equation with LO potential
  - Treat the rest in perturbation theory

Kaplan, Savage, Wise, van Kolck, Long, Yang…



$\sim m_\pi^2 D_2$

Chiral EFT for Muonic Atom Spectroscopy

$$T^{\mu\nu}(q) = \langle N_0 | \frac{J^\mu(-\mathbf{q})|N\rangle\langle N|J^\nu(\mathbf{q})}{q_0 - \omega_N + i\epsilon} - \frac{J^\nu(\mathbf{q})|N\rangle\langle N|J^\mu(-\mathbf{q})}{q_0 + \omega_N - i\epsilon} | N_0 \rangle$$

- Strategy: Calculate spectrum of Hamiltonian in A-nucleon sector, calculate matrix elements of currents

- Sum includes *all* eigenstates of chiral Hamiltonian—need to truncate

$$\text{bound states},\ NN,\ NN\pi,\ \ldots$$

- Forces and currents need to be consistent (E. Epelbaum's talk)

Multipole Expansion

- Two-nucleon states and currents can be expanded in terms of partial waves and operators with good angular momentum quantum numbers

$$|\mathbf{p}, s\, m_s\rangle = 4\pi \sum_{l,m_l,j,m_j} \langle p, j\, m_j(ls)|p, l\, m_l, s\, s\, m_s\rangle Y_l^{m_l *}(\hat{p})|p, j\, m_j(ls)\rangle$$

$$\rho(\mathbf{q}) = 4\pi \sum_{\Lambda,\Lambda_z} i^\Lambda Y_\Lambda^{\Lambda_z *}(\hat{q})\mathcal{C}_\Lambda^{\Lambda_z}(q)$$

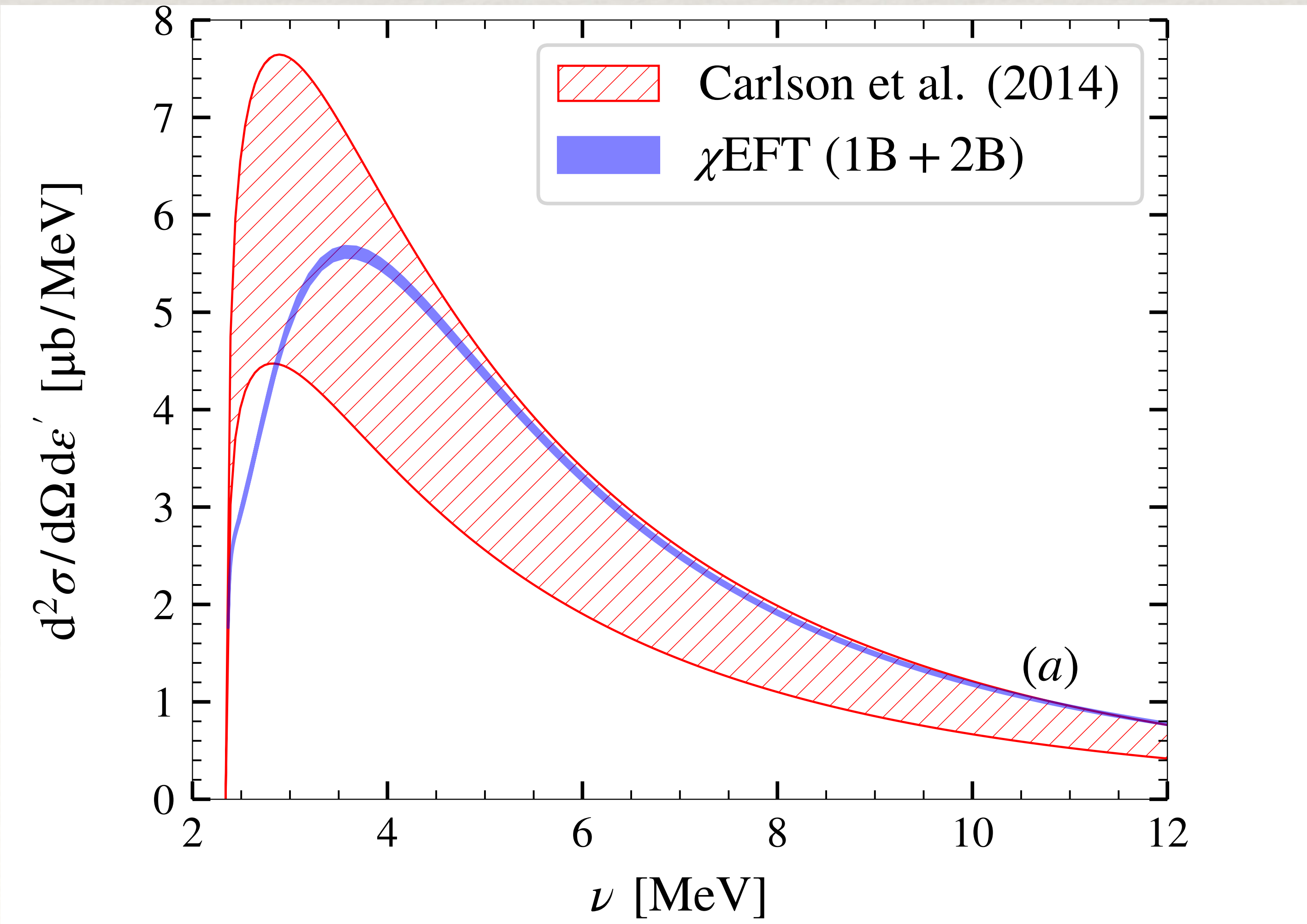
$$\mathbf{J}(\mathbf{q}) = 4\pi \sum_{\Lambda,\Lambda_z,L} i^\Lambda \mathcal{Y}_{\Lambda,L}^{\Lambda_z *}(\hat{q})\mathcal{D}_\Lambda^{\Lambda_z}(q)$$

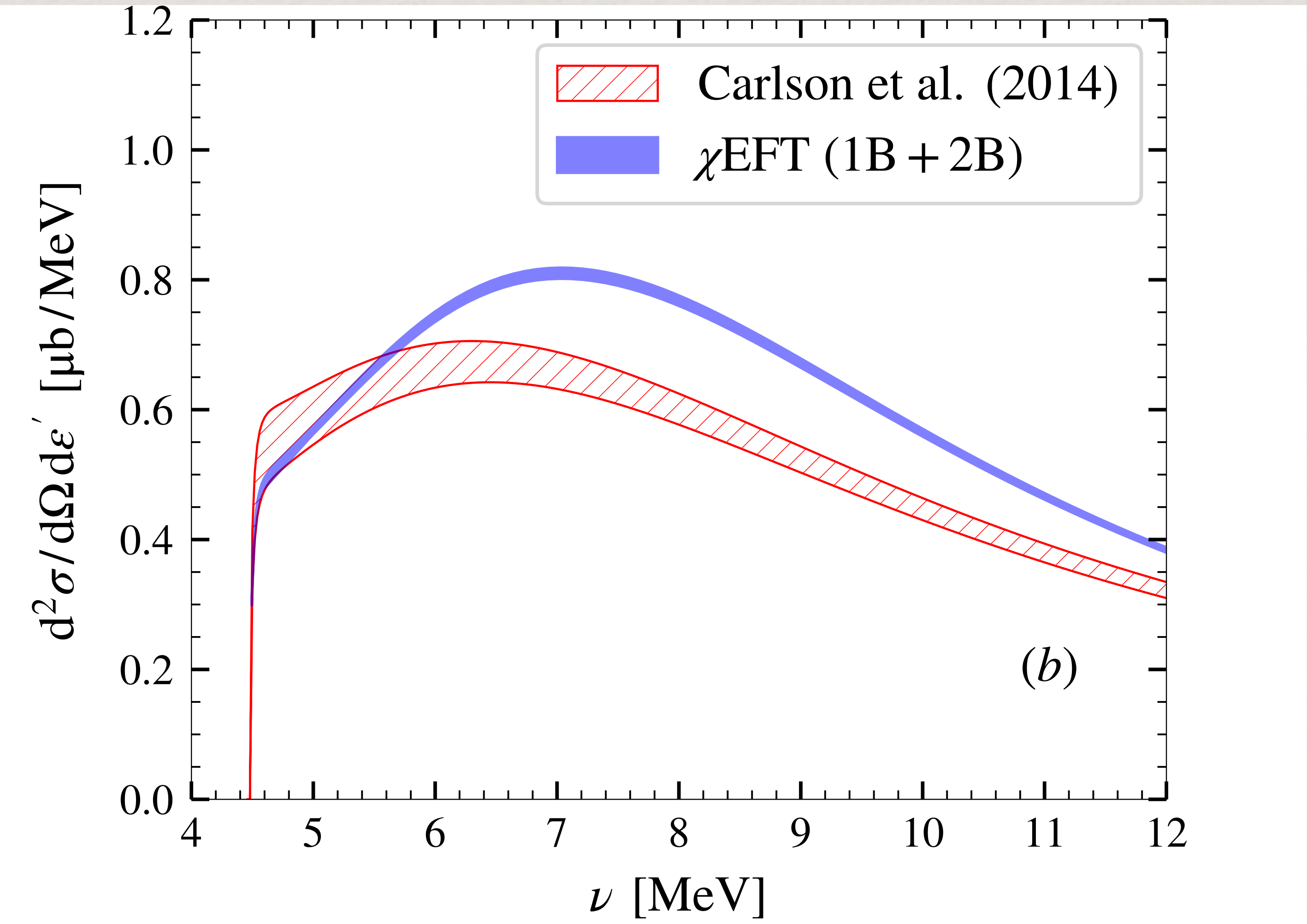
Deuterium Response Functions

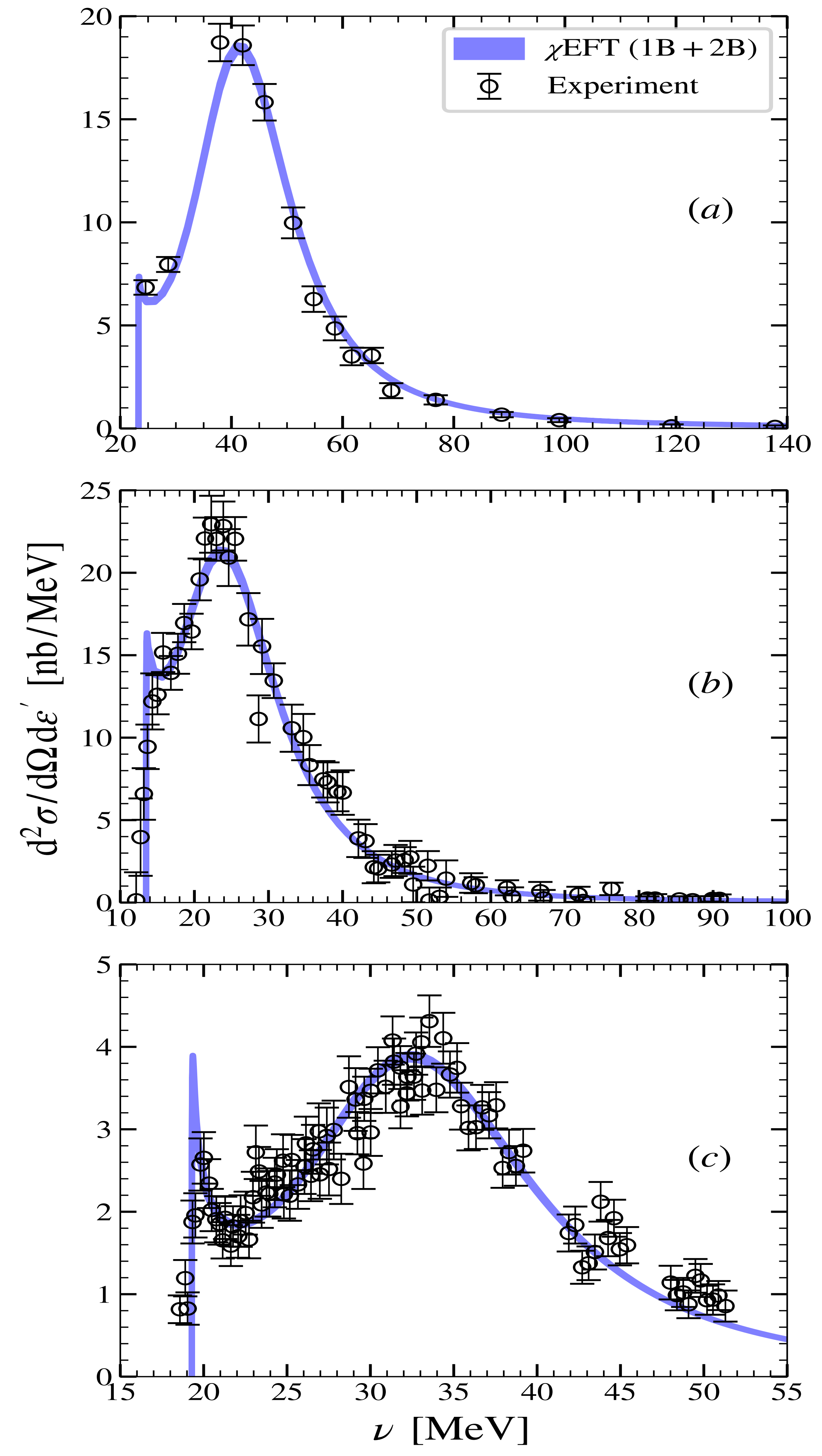
- Input from chiral EFT for dispersion relations

$$R_L(\nu, q) = \frac{1}{3}\sum_{m_d}\sum_{s,m_s}\sum_{t}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\delta(\nu + M_d - E_+ - E_-)\,|\langle \mathbf{p}, sm_s, t0|\rho|\psi_d m_d\rangle|^2$$

$$R_T(\nu, q) = \frac{1}{3}\sum_{m_d}\sum_{s,m_s}\sum_{t}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\delta(\nu + M_d - E_+ - E_-)\sum_{\lambda=\pm 1}|\langle \mathbf{p}, sm_s, t0|J_\lambda|\psi_d m_d\rangle|^2$$







Carlson, et al., PRA 89; Acharya, PRC 103

Two Photon Exchange Contribution to the Lamb Shift

- Polarizability contribution to the Lamb shift

$$\Delta E_{2S}^{\rm 2PE} = \Delta E_{2S}^{\rm inel+subt} + \Delta E_{2S}^{\rm el} + \Delta E_{2S}^{\rm hadr} + \Delta E_{2S}^{\rm Coul}$$

$$\Delta E_{2S}^{\rm inel+subt} = -1.511(12)\ {\rm meV}$$

$$\Delta E_{2S}^{\rm el} = -0.417(2)\ {\rm meV}$$

$$\Delta E_{2S}^{\rm hadr} = -0.028(2)\ {\rm meV}$$

$$\Delta E_{2S}^{\rm Coul} = 0.262(2)\ {\rm meV}$$

$$\Delta E_{2S}^{2PE} = -1.695(13)\ {\rm meV}$$

		$\Delta E_{2S}^{\rm TPE}$ [meV]
This work		
	— 1B+2B	-1.695(13)
	— Siegert	-1.703(15)
Ref. [8]		-1.680(16)
Ref. [9]		-1.717(20)
Ref. [11]		-1.690(20)
Ref. [12]		-1.712(21)
Ref. [13]		-1.703
Ref. [14]		-2.011(740)

Acharya, PRC 103

Towards Two Photon Exchange for the Hyperfine Splitting

- Lamb shift requires

$$|\langle \mathbf{p}, sm_s, t0|\rho|\psi_d m_d\rangle|^2 \qquad \sum_\lambda |\langle \mathbf{p}, sm_s, t0|J_\lambda|\psi_d m_d\rangle|^2$$

- Need *polarized* response functions and different matrix element combinations

$$T^{0i} \to \langle N_0|\rho(-\mathbf{q})|N\rangle\langle N|\mathbf{q}\times\mathbf{J}(\mathbf{q})|N_0\rangle \qquad T^{ij} \to \epsilon^{ijk}\langle N_0|J^j(-\mathbf{q})|N\rangle\langle N|J^k(\mathbf{q})|N_0\rangle$$

Ordinary atoms + closure approximation (Low term)

Friar and Payne

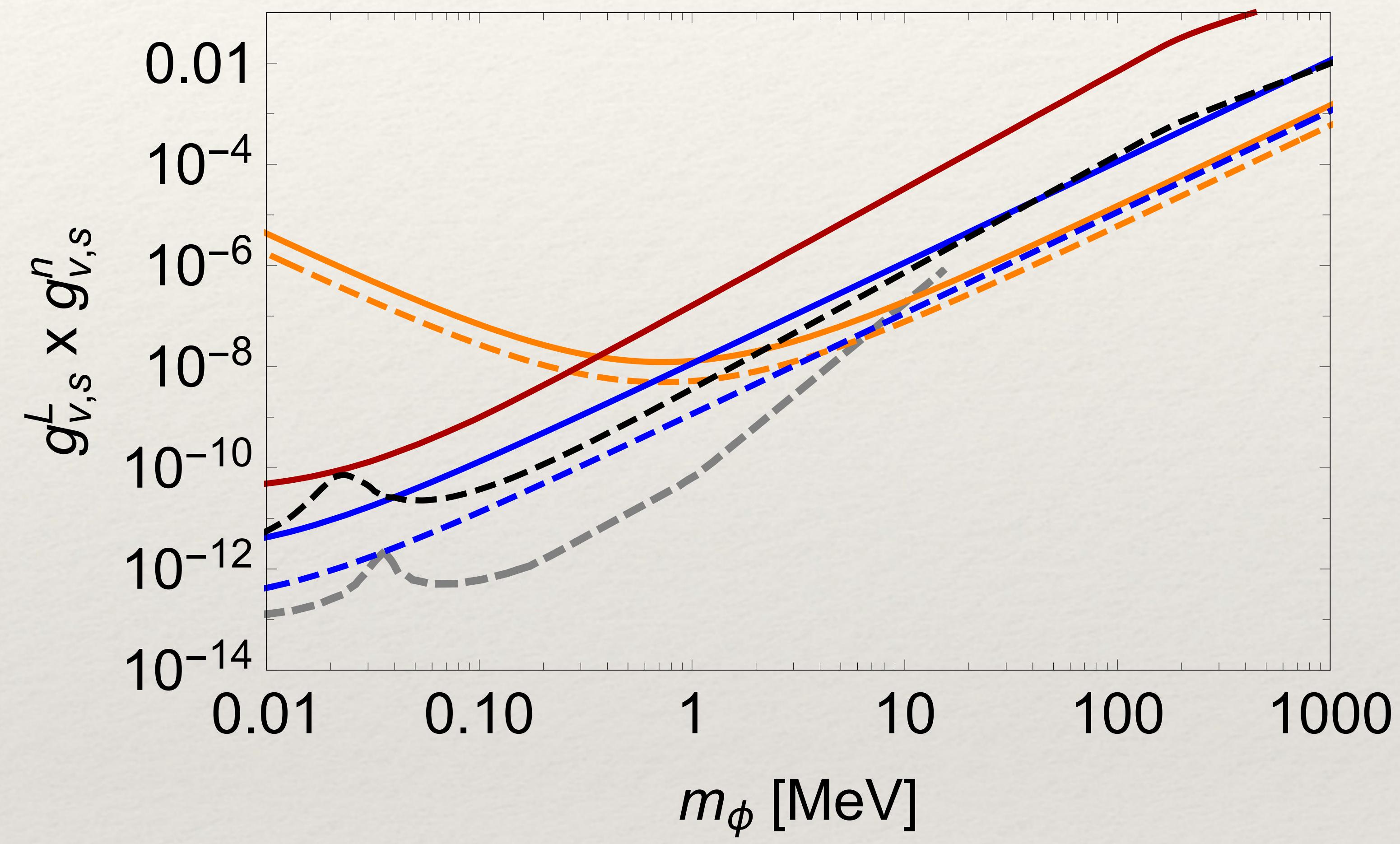
Meson exchange currents, suppressed?

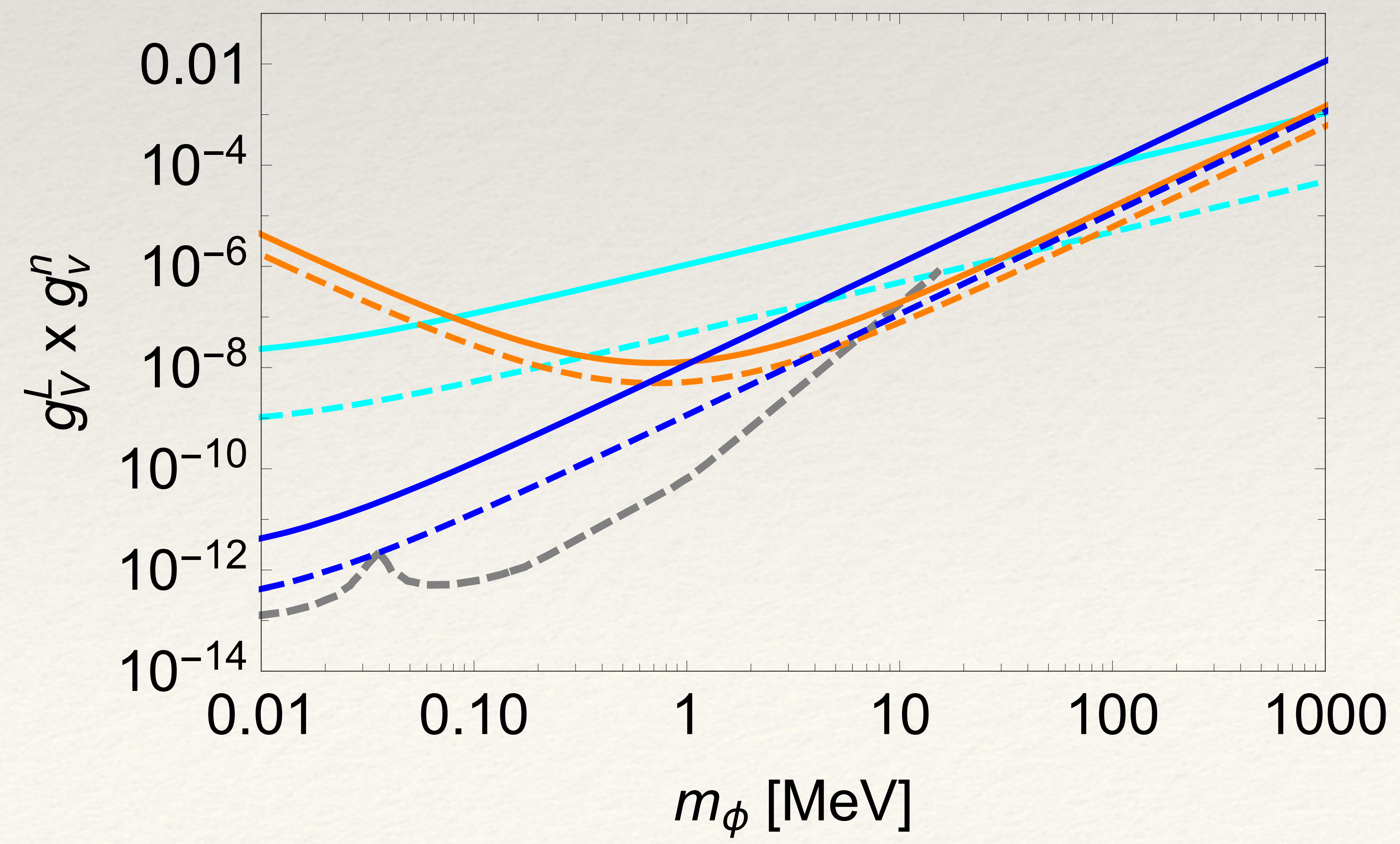
Kalinowski et al.

- Technology is *mostly* the same as that used in the Lamb shift

Connections to BSM

- EFT framework naturally facilitates connection between BSM physics and low-energy phenomena
- Precision electroweak physics (M. Gorchtein talk)
- Search for light new particles

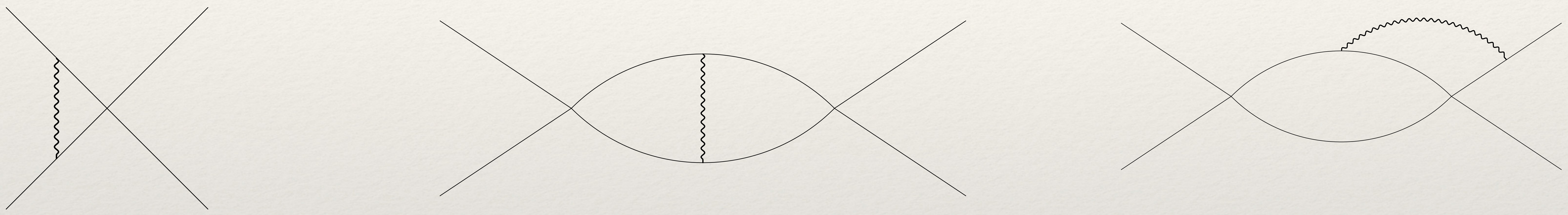




Frugiuele and Peset, JHEP 05

Renormalization Group Approach

- Couple NRQED to pionless/chiral EFT



- Renormalization group improvement in hydrogen
  - $\alpha^8 \log^3 \alpha$ Lamb shift
  - $\alpha^7 \log^2 \alpha$ Hyperfine splitting

Manohar and Stewart, PRL 85

Summary

- Effective field theory techniques connect (beyond) Standard Model to low-energy observables
- Chiral EFT for the Lamb shift in muonic deuterium is encouraging
- Apply the same toolbox to the hyperfine splitting
- Additional avenues: BSM physics, renormalization group