Cabibbo unitarity status

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Neutron beta decay review: MG, Seng, Universe **2023**, 9(9), 422, arXiv:**2307.01145**
Nuclear beta decay review: MG, Seng (for Annual Reviews Part. Nucl. Sci. - deadline Nov 2)

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Outline

Status of Cabibbo Angle Anomaly

Status of V_{ud} and V_{us}

RC to β-decays: overall setup, scale separation

Dispersion theory of γW-box

Possible BSM scenarios

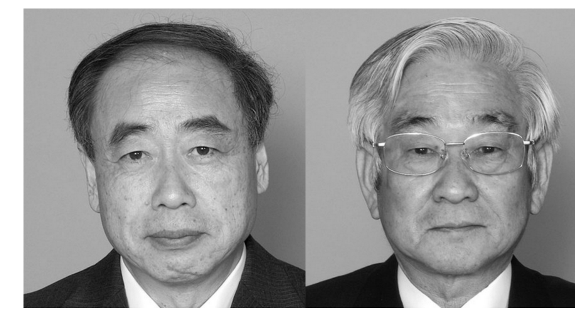
Summary & Outlook

Quark Mixing & CKM Unitarity

Cabibbo: mass and flavor eigenstates connected by Cabibbo angle θ_C
Strength is distributed among two channels

$$|G_V^{\Delta S=0}| = \cos\theta_C G_\mu$$
$$|G_V^{\Delta S=1}| = \sin\theta_C G_\mu$$

Kobayashi & Maskawa: 3 flavors, CP-violation



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM unitarity - measure of completeness of the SM: $VV^\dagger = \mathbf{1}$

Top-row unitarity constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

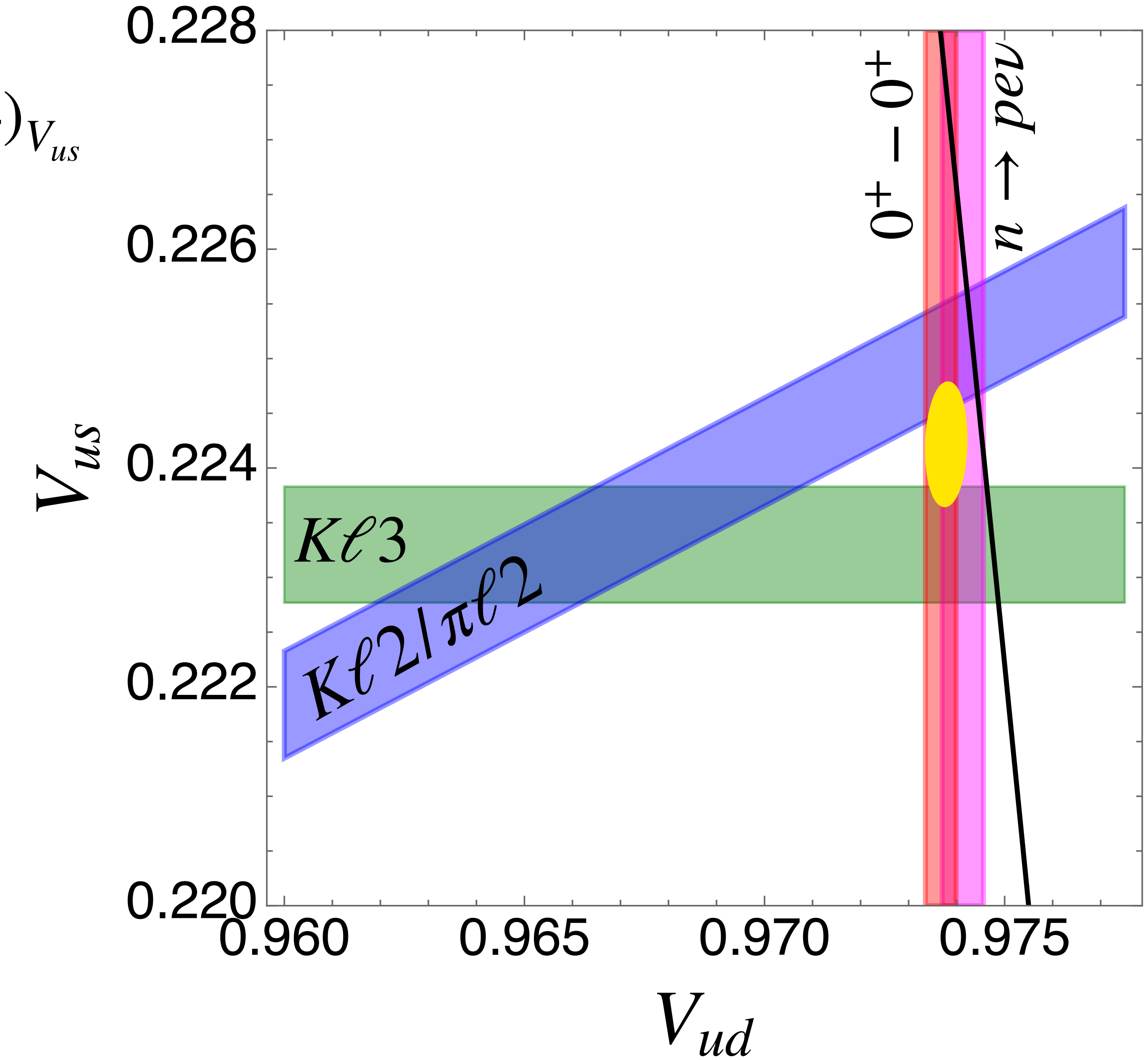
CKM unitarity is among our best precision tools to test the SM!

Status of Cabibbo unitarity

$$\underset{\sim 0.95}{|V_{ud}|^2} + \underset{\sim 0.05}{|V_{us}|^2} + \underset{\sim 10^{-5}}{|\cancel{V_{ub}}|^2} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

Inconsistencies: between measurements of V_{ud} and V_{us} and SM predictions

Main reason for Cabibbo-angle anomaly: shift in V_{ud} (and small uncertainties?)



Status of V_{us}

V_{us} / V_{ud} from Kl2 decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\mathrm{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

δ_{EM} Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

$$|V_{us}/V_{ud}| = 0.23108(23)_{\mathrm{exp}}(42)_{\mathrm{lat}}(16)_{\mathrm{IB}}$$
$$(51)_{\mathrm{tot}} = 0.22\ \%$$

$\delta_{SU(2)} + \delta_{EM}$

ChPT: $\delta_{SU(2)} + \delta_{EM} = -0.0112(21)$ *Cirigliano et al, 2011*

LQCD+EM ($N_f = 2+1+1$): $\delta_{SU(2)} + \delta_{EM} = -0.0122(16)$ *Giusti et al, 2018*

LQCD+EM ($N_f = 2+1+1$): $\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$ *Di Carlo et al, 2019*

f_K/f_π

LQCD ($N_f = 2+1$): $f_K/f_\pi = 1.1946(34)$

LQCD ($N_f = 2+1+1$): $f_K/f_\pi = 1.1978(22)$

FLAG 2021 averages

V_{us} from $K\ell 3$

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\mathrm{EW}} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\mathrm{EM}}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

S_{EW} Universal SD EW correction (1.0232)

$$K_{\ell 3}: \quad |V_{us}| = 0.22330(35)_{\mathrm{exp}}(39)_{\mathrm{lat}}(8)_{\mathrm{IB}} \quad (53)_{\mathrm{tot}} = 0.24\,\%$$

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\mathrm{EM}}$ Form-factor correction for long-distance EM effects

$f_+(0)$

LQCD ($N_f = 2 + 1$): $f_+(0) = 0.9677(27)$

LQCD ($N_f = 2 + 1 + 1$): $f_+(0) = 0.9698(17)$

FLAG 2021 averages

ChPT: $f_+(0) = 0.970(8)$ *Ecker et al 2015; Bijnens, Ecker 2014*

RC to $K\ell 3$

Until 2021: best way to compute long-distance EM RC was with ChPT

	$I^{(0)}_{K\ell}(\lambda_i)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_3)(\%)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_{4-3})(\%)$	$\delta^{K\ell}_{\rm EM}(\%)$
K^0_{e3}	0.103070	0.50	0.49	0.99 ± 0.30
$K^{\pm}_{e3}$	0.105972	-0.35	0.45	0.10 ± 0.30
$K^0_{\mu 3}$	0.068467	1.38	0.02	1.40 ± 0.30
$K^{\pm}_{\mu 3}$	0.070324	0.007	0.009	0.016 ± 0.30

Cirigliano, Gianotti, Neufeld 0807.4507

A series of works reformulated the problem as a hybrid of current algebra and ChPT, plus input from lattice QCD calculations of γW-box for $\pi e3$ and $K\ell 3$

	$\delta^{K\ell}_{\rm EM}$ $[10^{-3}]$	ChPT
$K^0 e$	$11.6(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{e^2p^4}$	$9.9(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ e$	$2.1(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(1)_{e^2p^4}$	$1.0(1.9)_{e^2p^4}(1.6)_{\rm LEC}$
$K^0 \mu$	$15.4(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$14.0(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ \mu$	$0.5(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$0.2(1.9)_{e^2p^4}(1.6)_{\rm LEC}$

Seng, Galviz, Meißner 1910.13208
Seng, Galviz, MG, Meißner 2103.04843
Seng, Galviz, MG, Meißner 2203.05217

Feng, MG, Jin, Ma, Seng 2003.09798
Ma, Feng, MG, Jin, Seng 2102.12048

Uncertainties reduced by an o.o.m.
Long-distance EM RC not responsible for the $K\ell 2$-$K\ell 3$ discrepancy!

Status of V_{ud}

V_{ud} from neutron decay

Neutron decay: 2 measurements needed

$$|V_{ud}|^2 = \frac{5024.7\ \mathrm{s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R^V)}$$

RC Δ_R^V: bottleneck since 40 years

Pre-2018: $\Delta_R^V = 0.02361(38)$ *Marciano, Sirlin PRL 2006*

Post-2018: $\Delta_R^V = 0.02479(21)$ *MG, Seng Universe 2023*

Since 2018: DR+data+pQCD+EFT+LQCD

Δ_R^V uncertainty: factor 2 reduction

C-Y Seng et al., PRL 2018; PRD 2019
A. Czarnecki, B. Marciano, A. Sirlin, PRD 2018
K. Shiells et al, PRD 2021; L. Hayen PRD 2021
P-X Ma, X. Feng, MG, L-C Jin, et al *2308.16755*

Experiment: factor 3-5 uncertainties improvement; discrepancies in τ_n and g_A

3.4σ

$g_A = -1.27641(56)$ — **PERKEO-III** *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

$g_A = -1.2677(28)$ — **aSPECT** *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506; 2308.16170*

4σ

$\tau_n = 877.75(28)^{+16}_{-12}$ — **UCNτ** *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

$\tau_n = 887.7(2.3)$ — **BL1 (NIST)** *Yue et al, PRL 111 (2013) 222501*

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9743\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

Single best measurements only

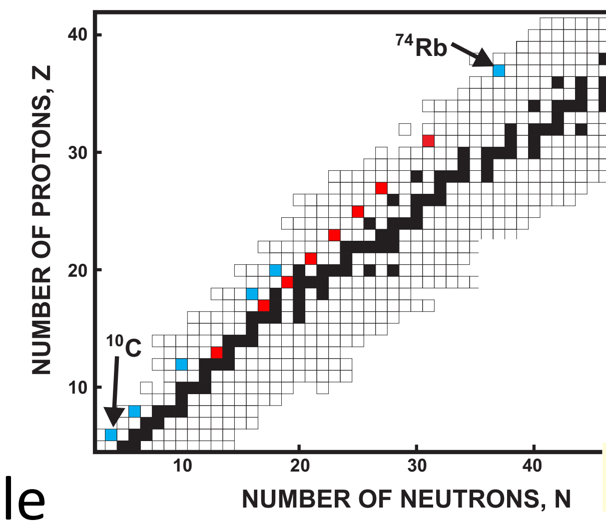
$$|V_{ud}^{\text{free n}}| = 0.9740\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

Future exp coming! RC under control

V_{ud} from superallowed decays

Advantages:

1. Only conserved vector current
2. 15 measured to better than 0.2%
3. 5 measured better than τ_n
4. Internal consistency as a check
5. SU(2) good —> corrections ~small
6. We know a lot about nuclei
7. Only scalar (or vector) BSM accessible

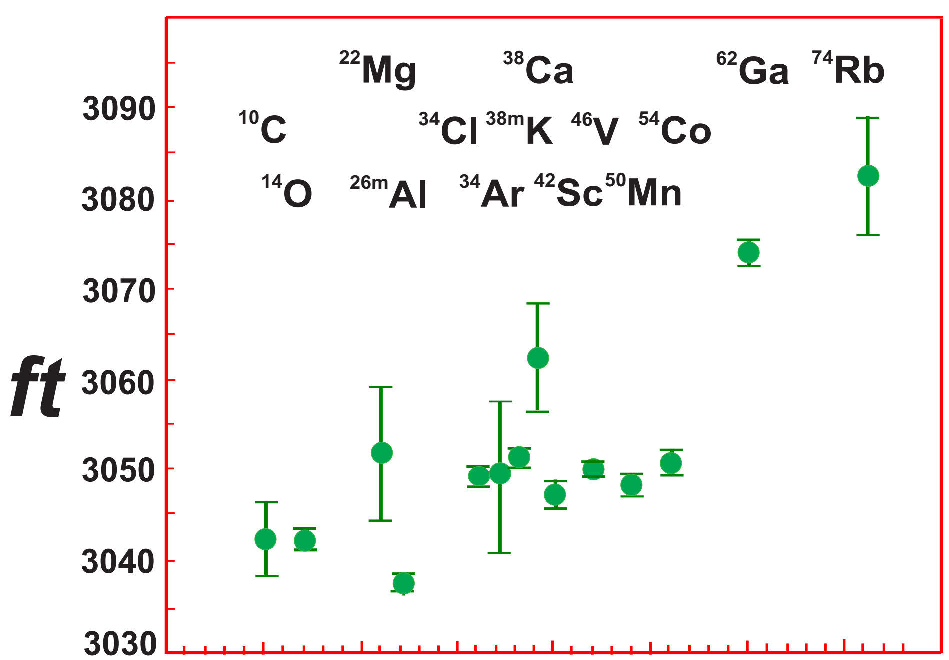


${}^{10}_{6}\mathrm{C} \to {}^{10}_{5}\mathrm{B}$
${}^{14}_{8}\mathrm{O} \to {}^{14}_{7}\mathrm{N}$
${}^{18}_{10}\mathrm{Ne} \to {}^{18}_{9}\mathrm{F}$
${}^{22}_{12}\mathrm{Mg} \to {}^{22}_{11}\mathrm{Na}$
${}^{26}_{14}\mathrm{Si} \to {}^{26}_{13}\mathrm{Al}$
${}^{30}_{16}\mathrm{S} \to {}^{30}_{15}\mathrm{P}$
${}^{34}_{18}\mathrm{Ar} \to {}^{34}_{17}\mathrm{Cl}$
${}^{38}_{20}\mathrm{Ca} \to {}^{38}_{19}\mathrm{K}$
${}^{42}_{22}\mathrm{Ti} \to {}^{42}_{21}\mathrm{Sc}$
${}^{46}_{24}\mathrm{Cr} \to {}^{46}_{23}\mathrm{V}$
${}^{50}_{26}\mathrm{Fe} \to {}^{50}_{25}\mathrm{Mn}$
${}^{54}_{28}\mathrm{Ni} \to {}^{54}_{27}\mathrm{Co}$

${}^{26m}_{13}\mathrm{Al} \to {}^{26}_{12}\mathrm{Mg}$
${}^{34}_{17}\mathrm{Cl} \to {}^{34}_{16}\mathrm{S}$
${}^{38m}_{19}\mathrm{K} \to {}^{38}_{18}\mathrm{Ar}$
${}^{42}_{21}\mathrm{Sc} \to {}^{42}_{20}\mathrm{Ca}$
${}^{46}_{23}\mathrm{V} \to {}^{46}_{22}\mathrm{Ti}$
${}^{50}_{25}\mathrm{Mn} \to {}^{50}_{24}\mathrm{Cr}$
${}^{54}_{27}\mathrm{Co} \to {}^{54}_{26}\mathrm{Fe}$
${}^{62}_{31}\mathrm{Ga} \to {}^{62}_{30}\mathrm{Zn}$
${}^{66}_{33}\mathrm{As} \to {}^{66}_{32}\mathrm{Ge}$
${}^{70}_{35}\mathrm{Br} \to {}^{70}_{34}\mathrm{Se}$
${}^{74}_{37}\mathrm{Rb} \to {}^{74}_{36}\mathrm{Kr}$

Exp.: **f** - phase space (Q value)

t - partial half-life ($t_{1/2}$, branching ratio)



ft values: same within ~2% but not exactly!

Reason: SU(2) slightly broken

a. RC (e.m. interaction does not conserve isospin)

b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_R^V) = \mathrm{ft}(1 + \delta_R')(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

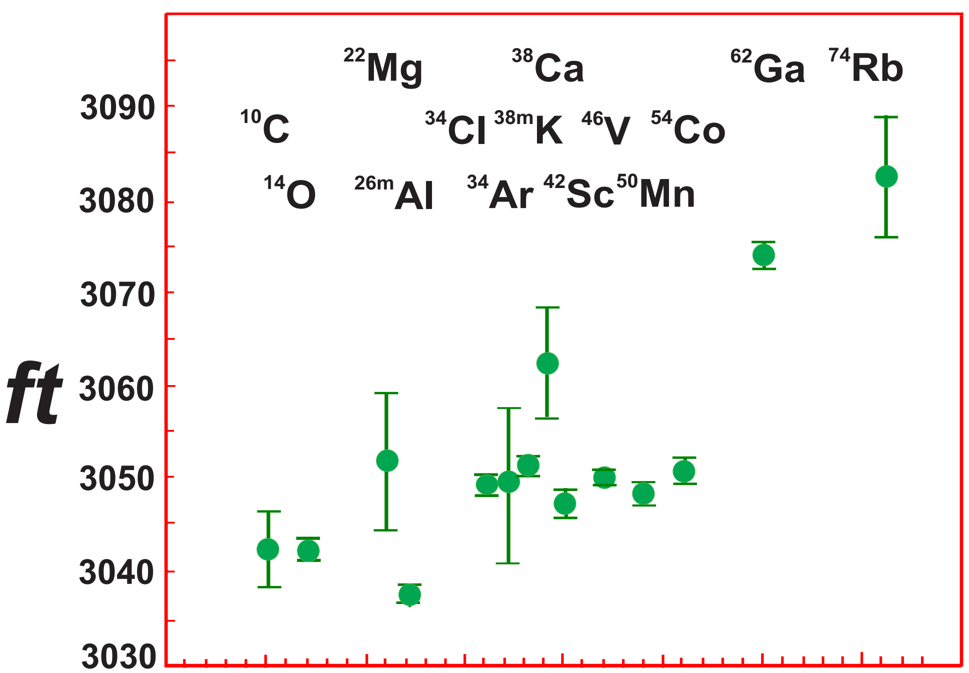
~ Measured

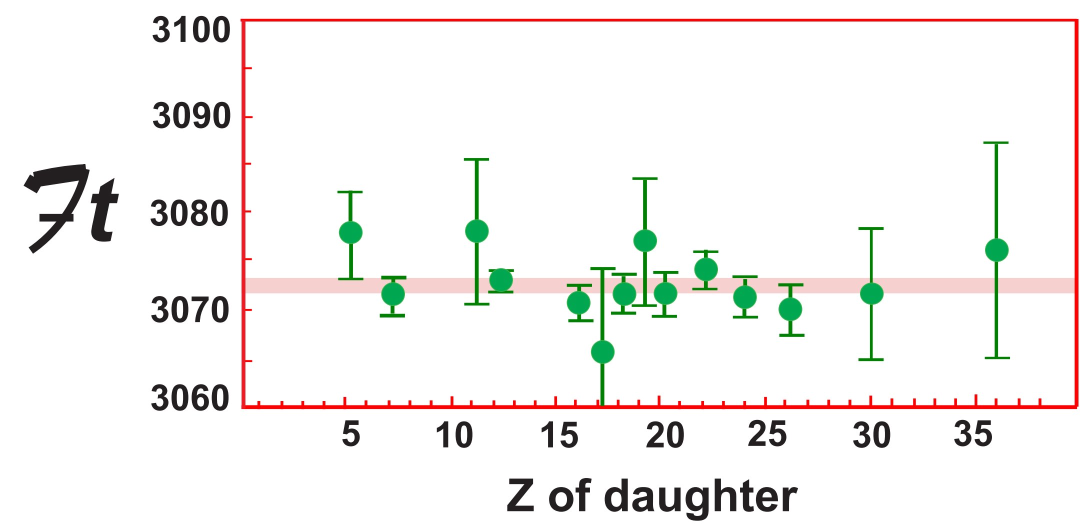
Outer: QED

Isospin-breaking

Nuclear structure

Universal inner





Average of 14 decays

Hardy, Towner 1972 - 2020

$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\, s$

PDG 2022: $\overline{\mathscr{F}t} = 3072 \pm 2\, s$

$$|V_{ud}^{0^+-0^+}| = 0.9737\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$

V_{ud} from semileptonic pion decay

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)} \frac{\Gamma_{\pi\ell 3}}{0.3988(23)\,\mathrm{s}^{-1}} \qquad |V_{ud}^{\pi\ell 3}| = 0.9739\,\boxed{(27)_{exp}}\,(1)_{RC}$$

RC to semileptonic pion decay

δ uncertainty: factor 3 reduction

ChPT: $\delta = -\,0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}}$ *Cirigliano et al, 2003; Passera et al, 2011*

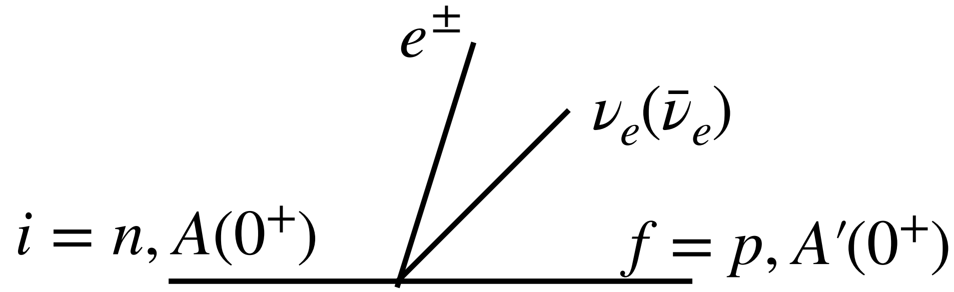
DR + LQCD + ChPT: $\delta = 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}$ *Feng et al, 2020; Yoo et al, 2023*

Future exp: 1 o.o.m. (PIONEER @ PSI)

RC to beta decay: overall setup

RC to beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

Λ

Weak boson scale

$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale

$\Lambda_{\rm had} = 300\,\mathrm{MeV}$

Universal

Nuclear scale

$\Lambda_{\rm nuc} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent (QCD)

Decay Q-value (endpoint energy)

$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass

$m_e \approx 0.5\,\mathrm{MeV}$

Nuclear structure independent (QED)

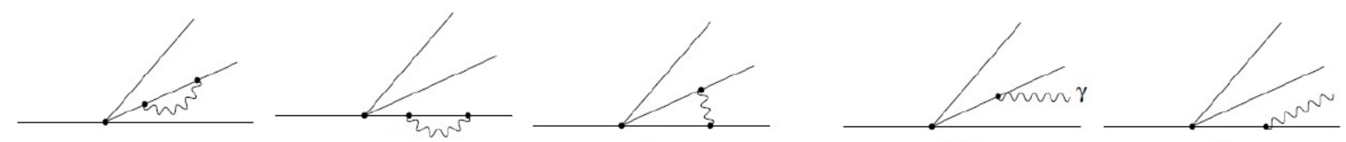
RC to beta decay: overall setup

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function + Sirlin function

Fermi function: resummation of $(Z\alpha)^n$ —> Dirac - Coulomb problem

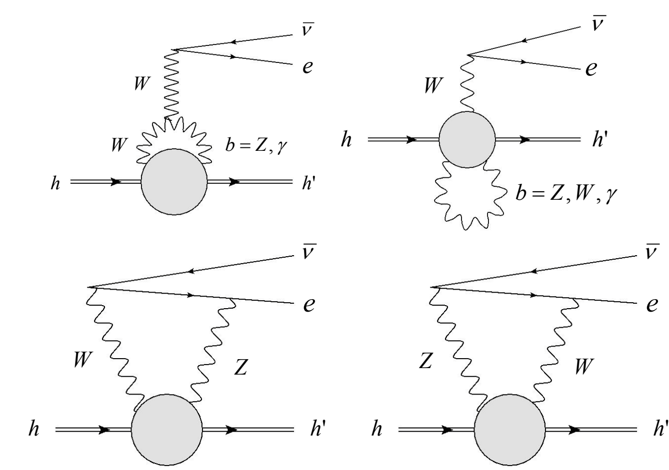
Sirlin function (outer correction):
All IR-div. pieces beyond Coulomb distortion



UV: large EW logs + pQCD corrections

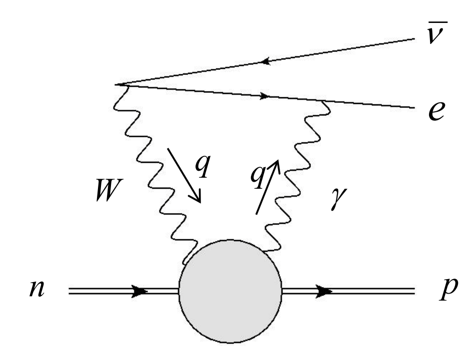
Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM



γW-box: sensitive to all scales

New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear



RC to β decay - scale separation

Fermi function (pure Coulomb + nuclear size & recoil + atomic) —> phase-space **f**

Fermi, Behrens-Bühring, Wilkinson…

Soft Bremsstrahlung: universal Sirlin's function + nucleus specific corrections —> δ_R'

All IR-sensitive pieces: recent review

Hayen et al RMP 2018

UV-sensitive RC on free neutron Δ_R^V: Sirlin, Marciano, Czarnecki 1967 - 2006

$$g_V^2 = |V_{ud}|^2 \left[1 + \frac{\alpha}{2\pi} \left\{ 3 \ln \frac{M_Z}{M_p} + \ln \frac{M_Z}{M_W} + \tilde{a}_g \right\} + \delta_{\text{QED}}^{HO} + 2 \Box_{\gamma W} \right]$$

RC on nuclei: extract the universal (free n) Δ_R^V:

Nuclear structure correction: $\delta_{\text{NS}} = 2(\Box_{\gamma W}^{\text{Nucl}} - \Box_{\gamma W}^{\text{free n}})$

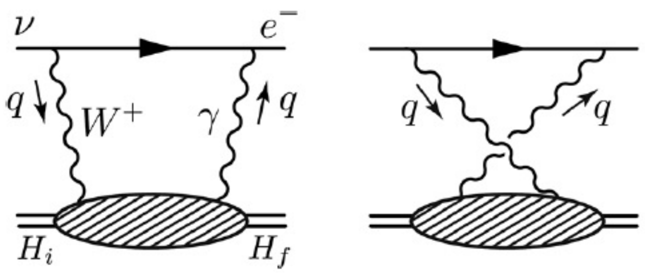
All non-enhanced terms $\sim \alpha/2\pi \sim 10^{-3}$ — only need to ~10%

Approximations are legitimate (isospin limit, …)

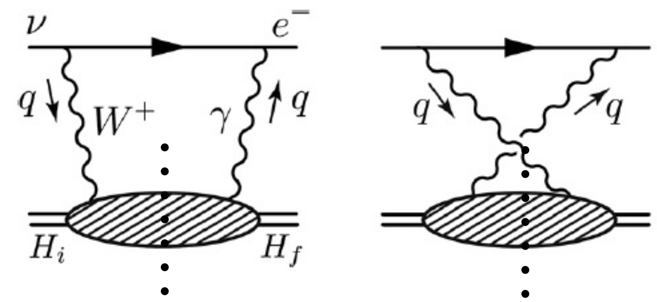
γW-box from DR

γW-box from dispersion relations

Model-dependent part or RC: γW-box



Generalized Compton tensor
time-ordered product — complicated!



Commutator (Im part) - only on-shell
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) \,|\, T\{J^{\mu}_{em}(x) J^{\nu,\pm}_{W}(0)\} \,|\, H_i(p) \rangle$$

Generalized (non-diagonal) Compton amplitudes

$$\int dx e^{iqx} \langle H_f(p) \,|\, [J^{\mu}_{em}(x), J^{\nu,\pm}_{W}(0)] \,|\, H_i(p) \rangle$$

Interference structure functions

Physics of taming model dependence with dispersion relations:

virtual photon polarizes the nucleon/nucleus;
Long- and intermediate-range part of the box sensitive to hadronic **polarizabilities**
Polarizabilities related to the excitation spectrum via dispersion relation
(Cf. Kramers-Kronig)

Universal RC from dispersion relations

Interference γW structure functions $$\mathrm{Im}T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(pq)}F_3^{\gamma W}(x,Q^2)$$

After some algebra (isospin decomposition, loop integration)

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2\frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty\frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2\int_{\nu_{\mathrm{thr}}}^\infty\frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

Advantage to previous approach (Marciano & Sirlin):
- Explicit 2-fold integral, isospin decomposition and energy dependence

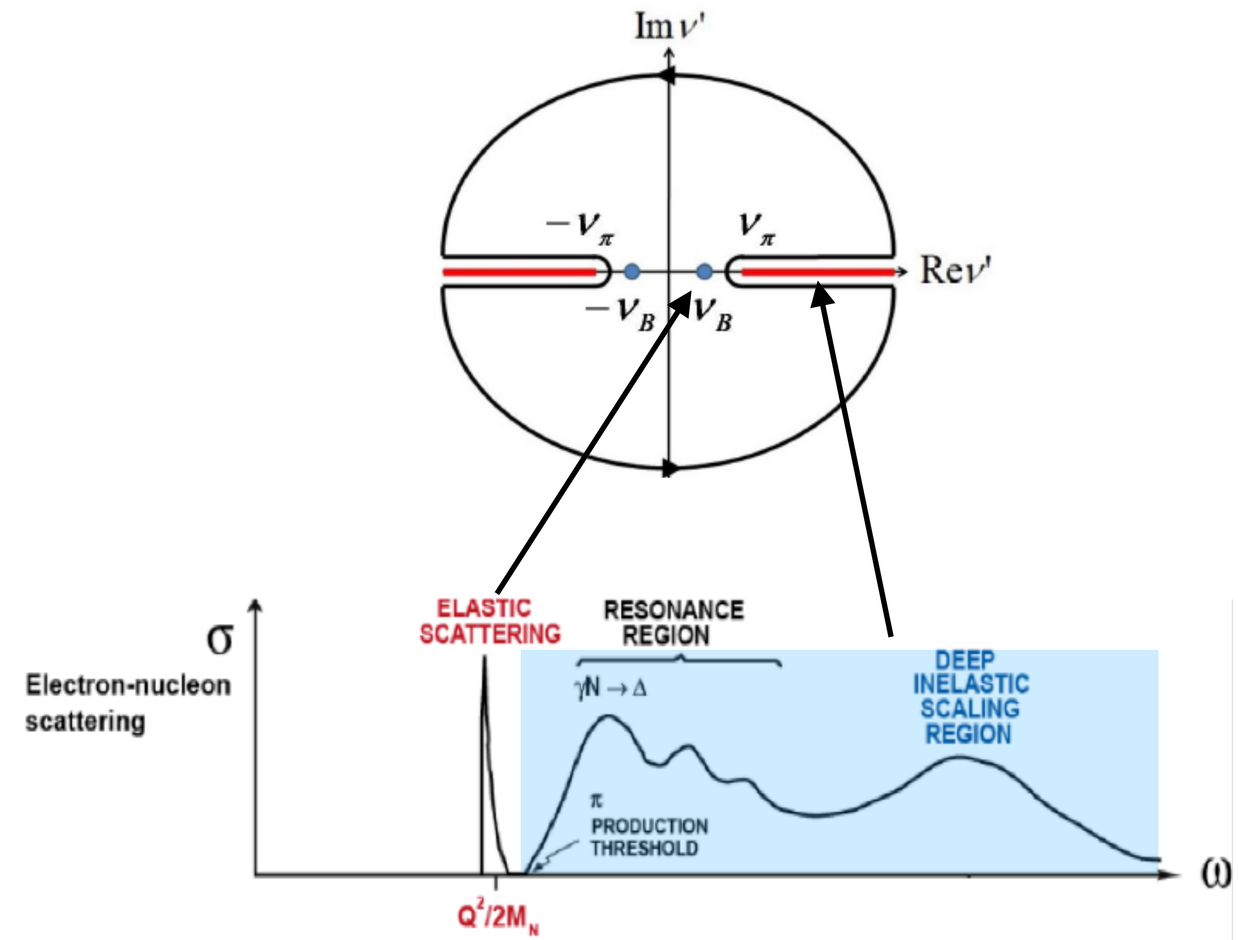
Nachtmann moments play a role in DIS $$M_3(n,Q^2) = \frac{n+1}{n+2}\int_0^1\frac{dx\xi^n}{x^2}\frac{2x(n+1)-n\xi}{n+1}F_3(x,Q^2), \qquad \xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

Hiding the nu-integration in the Nachtmann moments:

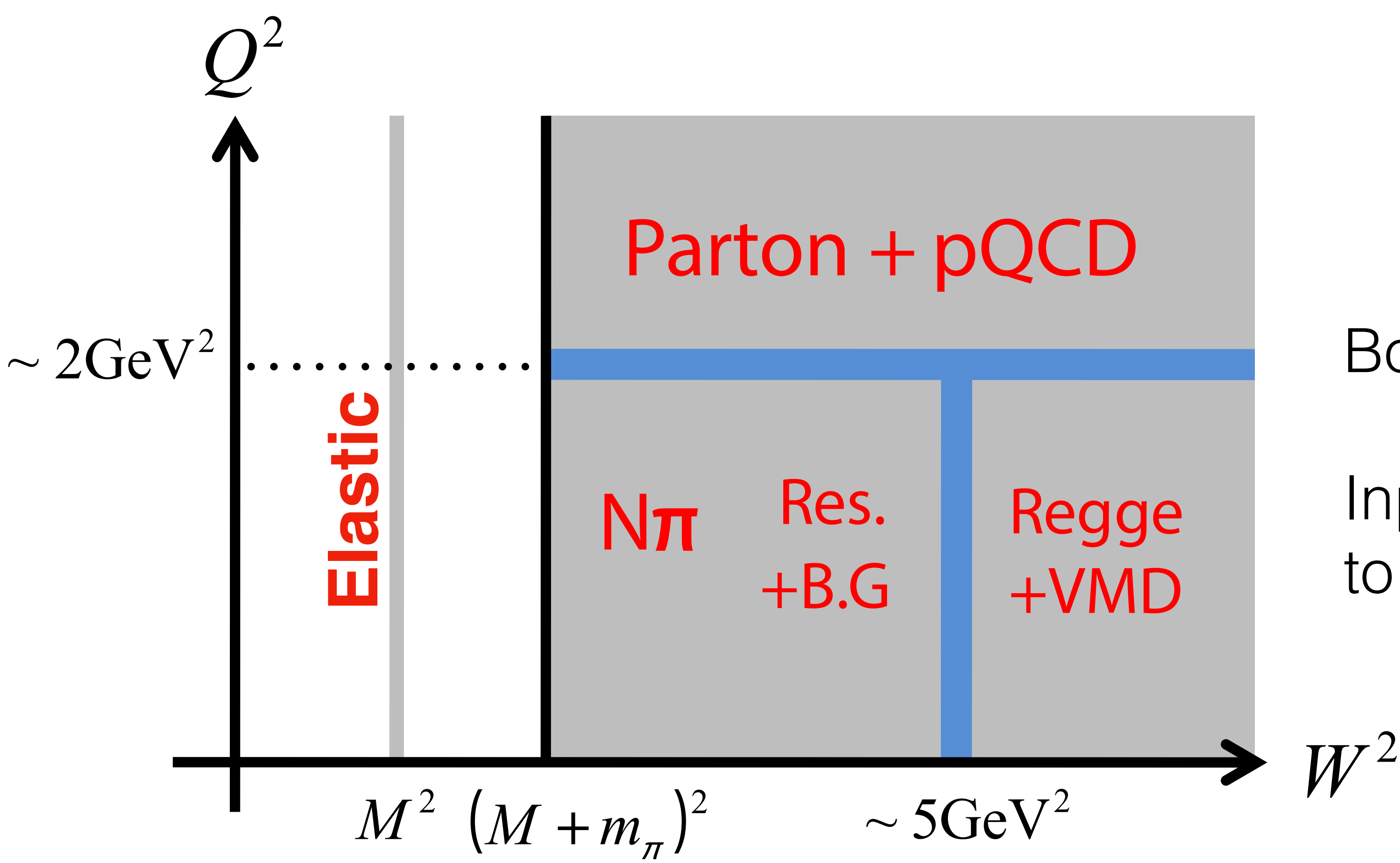
$$\Box^b_{\gamma W}(E_e) = \frac{3\alpha}{2\pi}\int_0^\infty\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\left[M_{3,-}(1,Q^2)+\frac{8E_eM}{9Q^2}M_{3,+}(2,Q^2)\right] + \mathcal{O}(E_e^2)$$

Input into dispersion integral

Dispersion in energy: $W^2 = M^2 + 2M\nu - Q^2$
scanning hadronic intermediate states



Dispersion in Q^2:
scanning dominant physics pictures



Boundaries between regions - approximate

Input in DR related (directly or indirectly)
to experimentally accessible data

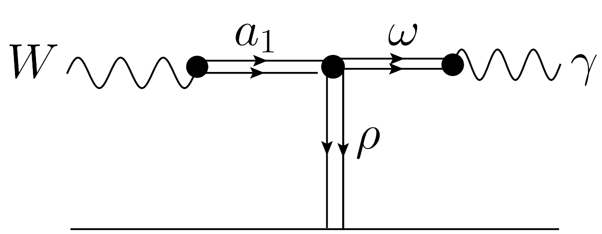
Input into dispersion integral - $\nu/\bar{\nu}$ data

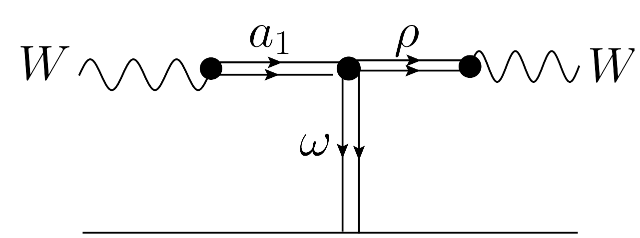
Mixed CC-NC γW SF (no data) <—> Purely CC WW SF (inclusive neutrino data)

Isospin symmetry: vector-isoscalar current related to vector-isovector current

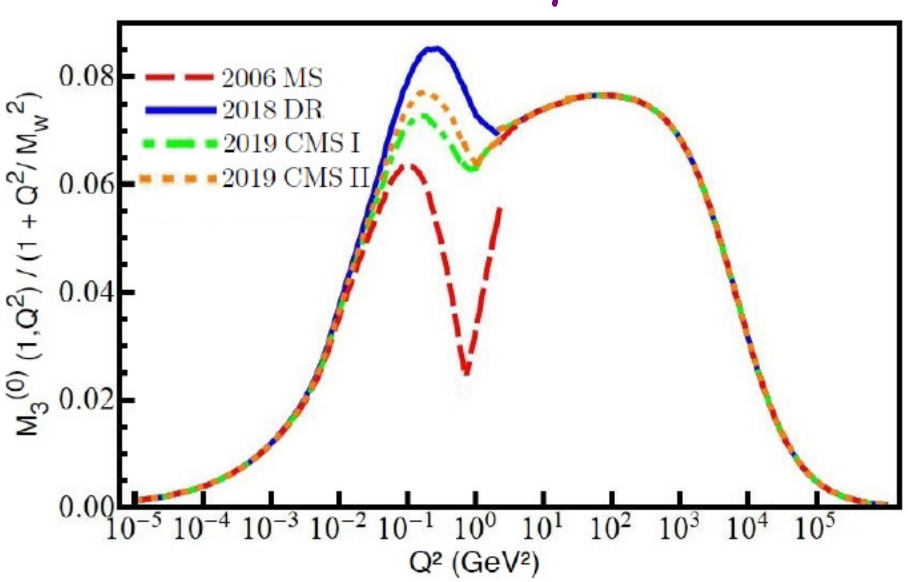
Only useful if we know the physical mechanism (Born, DIS, Regge, Resonance,…)

Were able to identify the missing part with Regge (multiparticle continuum)

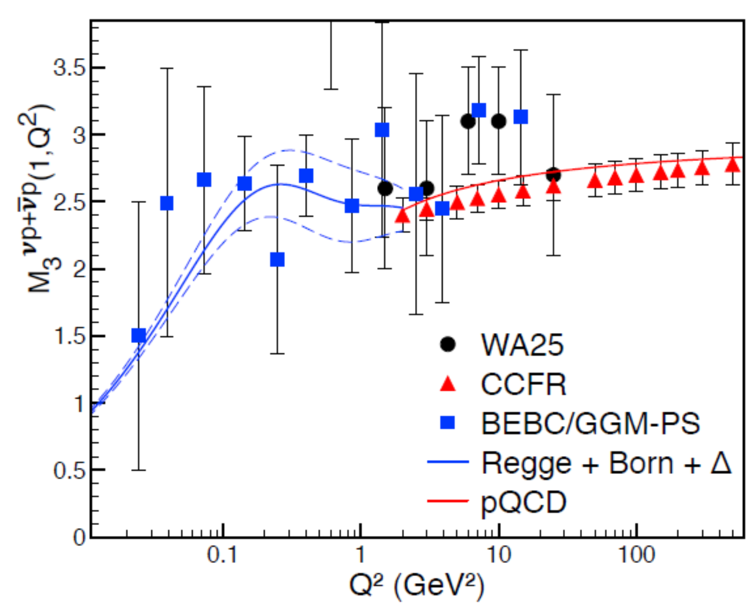




Free neutron γW box



Neutrino scattering data

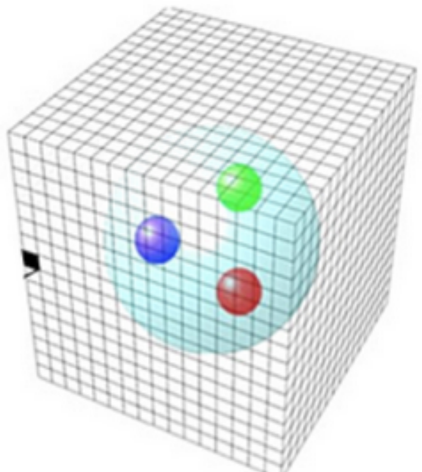


Marciano, Sirlin 2006: $\Delta_R^V = 0.02361(38)$ —> $|V_{ud}| = 0.97420(10)_{Ft}(18)_{RC}$

DR (Seng et al. 2018): $\Delta_R^V = 0.02467(22)$ —> $|V_{ud}| = 0.97370(10)_{Ft}(10)_{RC}$

γW-box from DR + Lattice QCD input

Currently available neutrino data at low Q^2 - low quality;
Look for alternative input — compute Compton amplitude on the lattice



$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p) | T[J^{\rm em}_\mu(x) J^{W,A}_\nu(0)] | \pi^-(p) \rangle$$

$$M_\pi(Q^2) = -\frac{1}{6\sqrt{2}} \frac{\sqrt{Q^2}}{m_\pi} \int d^4x \omega(Q,x) \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^{VA}_{\mu\nu}(x)$$

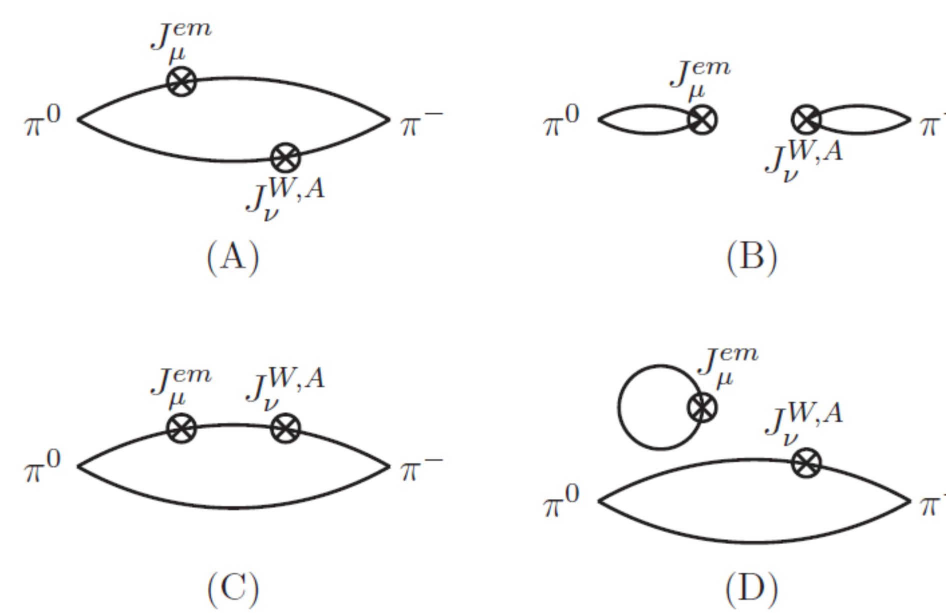
Direct LQCD computation for $\pi^- \to \pi^0 e^- \nu_e$

Feng, MG, Jin, Ma, Seng 2003.09798

5 LQCD gauge ensembles at physical pion mass
Generated by RBC/UKQCD collaboration
w. 2+1 flavor domain wall fermion

Ensemble	m_π [MeV]	L	T	a^{-1} [GeV]	$N_{\rm conf}$	N_r	$\Delta t/a$
24D	141.2(4)	24	64	1.015	46	1024	8
32D	141.4(3)	32	64	1.015	32	2048	8
32D-fine	143.0(3)	32	64	1.378	71	1024	10
48I	135.5(4)	48	96	1.730	28	1024	12
64I	135.3(2)	64	128	2.359	62	1024	18

Blue: DSDR
Red : Iwasaki



Quark contraction diagrams

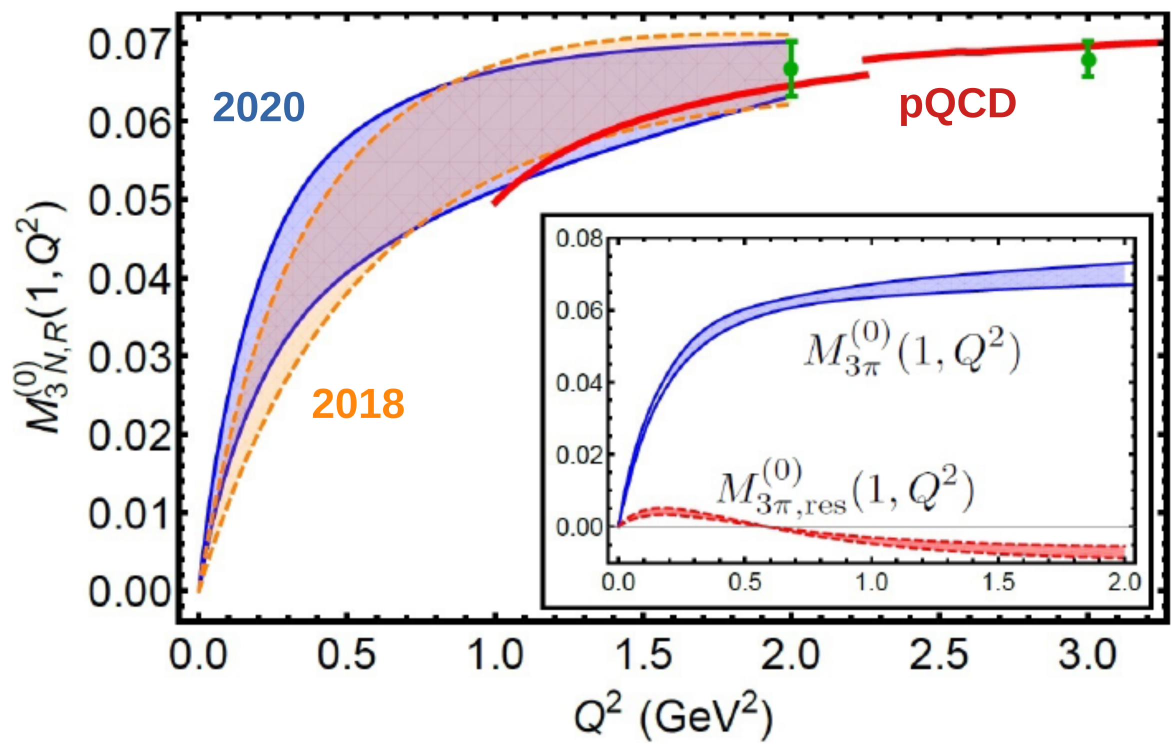
Implications for the free nucleon γW-box

Seng, MG, Feng, Jin, 2003.11264

Indirectly constrains the free neutron γW-box

Independent confirmation of the empirical DR result AND uncertainty

$$\Delta_R^V = 0.02467(22)_{\rm DR} \to 0.02477(24)_{\rm LQCD+DR}$$



Free-n RC in agreement by several groups & methods

Method	Δ_R^V
DR with neutrino data (1)	0.02467(22)
DR with neutrino data (2)	0.02471(18)
DR with indirect lattice data	0.02477(24)
Non-DR (1)	0.02426(32)
Non-DR (2)	0.02473(27)

C-Y Seng et al., Phys.Rev.Lett. 121 (2018) 24, 241804;
C-Y Seng, MG, M.J. Ramsey-Musolf, Phys.Rev. D 100 (2019)
Shiells, Blunden, Melnitchouk, Phys.Rev.D 104 (2021) 3, 033003
Seng, MG, Feng, Jin, 2003.11264
Czarnecki, Marciano, Sirlin, Phys.Rev. D 100 (2019) 7, 073008
Hayen, Phys.Rev.D 103 (2021) 11, 113001

Recently: first direct LQCD calculation on free neutron *P-X Ma, X. Feng, MG, L-C Jin,* et al *2308.16755*

$$\Delta_R^V = 0.02439(19)_{\rm LQCD}$$

Discrepancy and uncertainty to be understood!

RC to neutron decay in EFT

Cirigliano et al, 2306.03138

Effective Field Theory: in principle, the proper tool to separate scales!

SM —> LEFT —> CHPT —> $\not\pi$EFT

Formal consistency built in, RGE, transparent error estimation…

Precision limited by matching (LEC) and HO — relies on inputs (e.g. γW-box from DR!)

To improve: need to go to higher order — new LECs, still tractable?

$$\frac{\mathrm{d}\Gamma_n}{\mathrm{d}E_e} = \frac{G_F^2\,|V_{ud}|^2}{(2\pi)^5}\,(1+3\lambda^2)\;p_e E_e (E_0 - E_e)^2\,[g_V(\mu_\chi)]^2\,F_{NR}(\beta)\Big(1+\delta_{\mathrm{RC}}(E_e,\mu_\chi)\Big)\Big(1+\delta_{\mathrm{recoil}}(E_e)\Big)$$

$\lambda = g_A/g_V$ Extract from Experiment

vector coupling

$\pi^2, 1/\beta$ Enhanced

$\mathcal{O}(\alpha)$ [no logs]

$\mathcal{O}(m_e/m_N)$

Total RC: $1 + \Delta_{\mathrm{TOT}} = 1.07761(27)\,\%$

Total RC from DR: $1 + \Delta_{\mathrm{TOT}} = 1.07735(27)\,\%$

At present: order $O(\alpha, \alpha\alpha_s, \alpha^2)$ — consistent with matching input? realistic to go beyond? Nuclear structure doable?

Interpretation of Cabibbo Angle Anomaly

CAA summary - 3 anomalies!

3 observables: $|V_{us}|^{K\ell 3}$, $|V_{us}/V_{ud}|^{K\mu 2}$, V_{ud}
2 quantities to determine: V_{us}, V_{ud}
▶ **3 ways to test unitarity**

$$\Delta^{(1)}_{\rm CKM} = |V_{ud}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\rm CKM} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu 2}}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$K_{\mu 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from beta decays:

$$\Delta V_{us}(K_{\ell 3} - K_{\mu 2}) = V_{us}^{K_{\ell 3}} - V_{ud}\left(\frac{V_{us}}{V_{ud}}\right)^{K_{\mu 2}}$$ **= –0.0174(73) –2.4σ**

$\Delta^{(3)}{}_{\rm CKM}$ uses no information from β decays:

$$\Delta^{(3)}_{\rm CKM} = |V_{us}^{K_{\ell 3}}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu 2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**

CAA in presence of RH currents

Cirigliano et al.
PLB 838 (2023)

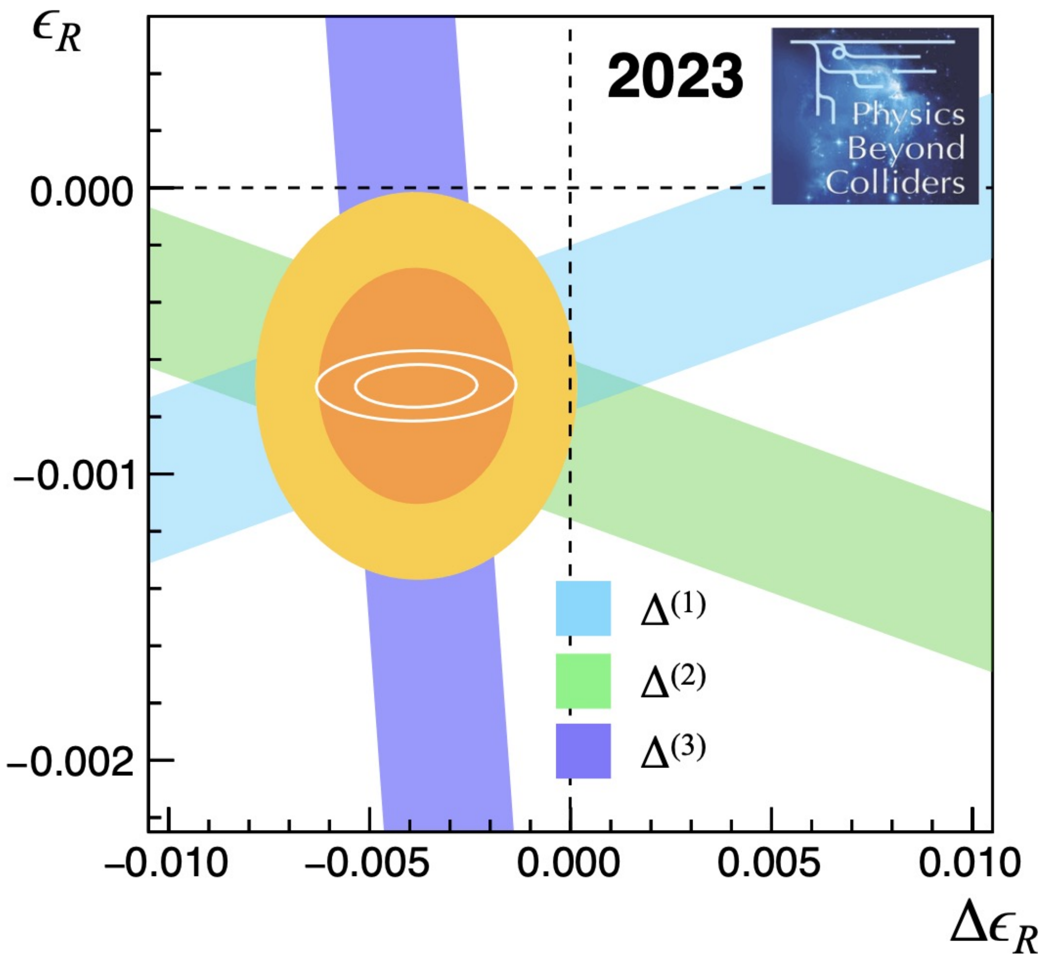
- In SM, W couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference
- Define ϵ_R = admixture of RH currents in non-strange sector
  $\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\rm CKM} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left(\frac{1+\Delta^{(2)}_{\rm CKM}}{1+\Delta^{(3)}_{\rm CKM}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)
$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ



Summary and Outlook

Summary & Outlook

Cabibbo unitarity deficit at 2-3σ observed

Great improvement in theory of RC in past 5 years
EFT formulation of free-n RC - promising avenue

Great improvement in free-n decay exp., more coming!
Barely any improvement in $0^+ - 0^+$ nuclear decays

Nuclear uncertainties under scrutiny

Meson decays: future more precise experiments

Cabibbo anomaly interpretable in terms of BSM
(e.g. flavor-dependent RH currents)

Keep tuned!