

October 16-20, 2023 Erbacher Hof, Mainz, Germany

Electron-Ion Collider - A Giant CT Scanner for Nucleons and Nuclei - Opportunities and Challenges for Theory



- Nuclear Femtography & Hadron Properties: Need of Two-scale observables
 - Localized probe + sensitivity to structure
- Particle Nature of Quarks and Gluons
 - **3D** confined hadron structure
- Wave Nature of Quarks and Gluons
 - Hard probe at small-x is not localized
- Summary and outlook

See talks by S. Fazio & S. Dalla Torre

> Office o Science



Jian-Wei Qiu Jefferson Lab, Theory Center





Frontiers of QCD and Strong Interaction

Understanding where did we come from?



QCD at high temperature, high densities, phase transition, ... Facilities – Relativistic heavy ion collisions: SPS, RHIC, the LHC, ..

Understanding what are we made of?





- Try to understand the emergent properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?
- Understanding QCD fully is still beyond the best mind that we have!



Global Time:

Nuclear Femtography: Search for answers to these questions at a Fermi scale! Facilities – CEBAF, EIC, EICC, LHeC, ... Jefferson Lab

Nuclear Femtography: QCD Dynamics/Phenomena at the Fermi-Scale





Nuclear Femtography: QCD Dynamics/Phenomena at the Fermi-Scale



Exploring QCD & Hadron Structure needs Lepton-Hadron Facilities

Lepton-lepton colliders (e+e- collisions):



- No hadron to start with
- Hadrons are produced in the collisions
- Emergence of hadrons

□ Hadron-hadron facilities (hadrons are likely broken in the collisions):



- Partonic structure mixed with collision effects
- Emergence of hadrons
- Heavy ion beam(s) dense medium, QGP





Also the LHC, J-PARC, ...



Also future EIC, COMPASS/Amber, ... Jefferson Lab

Lepton-hadron facilities (HERA discovery: hadron stays intact 10-15% time):



- Two-scale observables are natural
- Imaging partonic structure without breaking it!
- Emergence of hadrons
- Heavy ion target or beam

Ideal facility for partonic structure of hadron/nuclei !

U.S. - based Electron-Ion Collider (EIC)



https://www.bnl.gov/eic/

See talks by S. Fazio & S. Dalla Torre

Basic Tech Requirements

- Center of Mass Energies:
 20 GeV 141 GeV
- Required Luminosity:
 10³³ 10³⁴ cm⁻²s⁻¹
- Hadron Beam Polarization:
 80%
- Electron Beam Polarization:

80%

• Ion Species Range:

p to Uranium

• Number of interaction regions:

up to two



US-EIC – can do what HERA could not do

Quantum imaging:

- ♦ HERA discovered: 10-15% of e-p events is diffractive Proton not broken!
- ♦ US-EIC: 100-1000 times luminosity Critical for 3D tomography!

Large momentum transfer without breaking the proton Luminosity!





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Quantum imaging:

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- **Quantum interference & entanglement:**
 - US-EIC: Highly polarized beams Origin of hadron property: Spin, ... Direct access to chromo-quantum interference!



Large momentum transfer without breaking the proton Luminosity!





No probability interpretation!



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Nonlinear quantum dynamics:

♦ US-EIC: Light-to-heavy nuclear beams – Origin of nuclear force, ...

Catch the transition from chromo-quantum fluctuation to chromo-condensate of gluons, ...

Emergence of hadrons (nuclei as femtometer size detectors!),

- "a new controllable knob" - Atomic weight of nuclei

Wave nature of quark/gluon field



No probability

interpretation!

Phase-Space Coverage by the Facilities around the World:



Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering (DIS)

P P xP, k_T xP, k_T $\gamma^* \frac{P_h}{z}, k'_T$ e+p/A → e'+h(p,K,p,jet)+X

□ SIDIS naturally covers both one-scale and two-scale observables:

See talk by B. Pasquini

 $\begin{array}{ll} Q_1: & P_{h_T} \sim \ell_T' & \text{In lepton-hadron frame} \\ Q_2: & |\vec{P}_{h_T} + \vec{\ell}_T'| \ll |\vec{P}_{h_T}| \sim Q_1 \end{array}$

Naturally sensitive to parton transverse motion

TMDs

See also TMD Handbook [2304.03302]



Lepton-Hadron Semi-Inclusive Deep Inelastic Scattering (DIS)



by the collision induced shower!

Jefferson Lab

- Measured k_τ is NOT the same as k_τ of the confined motion!
- Structure information is mixed with collision effects!

Transverse momentum dependent PDFs (TMDs) – Power of SIDIS

Quark TMDs with polarization:

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	υ	$f_1(x,k_T^2)$ •		$h_1^{\perp}(x,k_T^2)$ Boer-Mulders
	L		$g_1(x,k_T^2) \xrightarrow[Helicity]{} \xrightarrow{Helicity}$	$h_{1L}^{\perp}(x,k_T^2)$ \longrightarrow - \longrightarrow Long-Transversity
	т	$f_1^{\perp}(x,k_T^2)$ $f_1^{\perp}(x,k_T^2)$ $f_1^{\perp}(x,k_T^2)$ $f_1^{\perp}(x,k_T^2)$ $f_1^{\perp}(x,k_T^2)$ $f_1^{\perp}(x,k_T^2)$	$g_{1T}(x,k_T^2) \stackrel{\uparrow}{\bullet} - \stackrel{\uparrow}{\bullet}$ Trans-Helicity	$h_{1}(x,k_{T}^{2}) \begin{array}{c} & & & \\ & & & \\ \hline \\ & & \\ h_{1T}^{\perp}(x,k_{T}^{2}) \\ & & \\ & & \\ & \\ Pretzelosity \end{array} \begin{array}{c} & & \\ & & \\ \hline \\ & & \\ & \\ & \\ & \\ & \\ &$





See talk by B. Pasquini

Transverse momentum dependent PDFs (TMDs) – Power of SIDIS

Quark TMDs with polarization:





See also TMD Handbook [2304.03302]

Polarized SIDIS:



In photon-hadron frame:

 $\begin{aligned} A_{UT}^{Collins} &\propto \left\langle \sin(\phi_h + \phi_S) \right\rangle_{UT} \propto h_1 \otimes H_1^{\perp} \\ A_{UT}^{Sivers} &\propto \left\langle \sin(\phi_h - \phi_S) \right\rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1 \\ A_{UT}^{Pretzelosity} &\propto \left\langle \sin(3\phi_h - \phi_S) \right\rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp} \end{aligned}$

Angular modulation provides the best way to separate TMDs Jefferson Lab

See talk by B. Pasquini

TMDs: Correlation between Hadron Property and Parton Flavor-Spin-Motion

Quantum correlation between hadron spin and parton motion:



x=0.1 From high x=0.01 x-Bjorken Proton Spin x=0.001 K, (GeV)

Quantum correlation between parton's spin and its hadronization:



Parton's transverse polarization influences its hadronization

Fig. 2.7 NAS Report



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Polarized hadron

Collision-Induced QED Radiation to SIDIS – Major Challenge

QED radiation modifies the probe for the hadron:



Prevents a well-defined "photon-hadron" frame

• Radiation is IR sensitive as $m_e/Q \rightarrow 0$





Collision-Induced QED Radiation to SIDIS – Major Challenge



Modify the angular modulations between leptonic and hadronic planes – the separation of TMDs ?



Collision-Induced QED Radiation to SIDIS – Major Challenge

QED factorization of collision-induced radiation – treating QED and QCD equally:

Think the colliding lepton as the colliding hadron at the LHC, or RHIC

DIS = Inclusive production of a high pT lepton: $(p_T, y) \rightarrow (x_B, Q^2)$

SIDIS = Inclusive production of a high pT lepton and a high pT hadron (or a jet): $(\vec{\ell'}, \vec{P_h}) \rightarrow (x_B, Q^2, z_h, P_{h_T}, \phi)$

$$E_{\ell'}E_{P_h}\frac{\mathrm{d}^6\sigma_{\ell(\lambda_\ell)P(S)\to\ell'P_hX}}{\mathrm{d}^3\ell'\,\mathrm{d}^3P_h}\approx\sum_{ij\lambda_k}\int_{\zeta_{\min}}^1\frac{\mathrm{d}\zeta}{\zeta^2}\,D_{e/j}(\zeta)\int_{\xi_{\min}}^1\mathrm{d}\xi\,f_{i(\lambda_k)/e(\lambda_\ell)}(\xi)\left[E_{k'}E_{P_h}\frac{\mathrm{d}^6\hat{\sigma}_{k(\lambda_k)P(S)\to k'P_hX}}{\mathrm{d}^3k'\,\mathrm{d}^3P_h}\right]_{k=\xi\ell,k'=\ell'/\zeta}+\mathcal{O}(\frac{m_e^n}{Q^n})$$

- Leading power IR sensitive contribution is universal, as $m_e/Q \rightarrow 0$, and factorized into universal LDFs and LFFs [global fits of LDFs, LFFs]
- IR safe contributions are calculated order-by-order in powers of α
- □ Hybrid factorization for the TMD regime: Colinear for QED & TMD for QCD
 - **QED contribution (not correction) to SIDIS can be systematically improved order-by-order in power** α!
 - Two photon exchange is naturally included as the loop corrections to the "perturbative" hard parts no cutoff!

More works are needed for precise 3D in momentum space!



How to Explore Internal Structure of Hadron without Breaking it?



No Proton "Radius" in color charge distribution!



How to Explore Internal Structure of Hadron without Breaking it?



□ But, there is NO elastic "color" form factor!

Combine PDF and Form Factor – GPDs:

$$\begin{aligned} F^{q}(x,\xi,t) &= \int \frac{\mathrm{d}z^{-}}{4\pi} e^{-ixP^{+}z^{-}} \langle p' | \bar{q}(z^{-}/2) \gamma^{+}q(-z^{-}/2) | p \rangle \\ &= \frac{1}{2P^{+}} \left[H^{q}(x,\xi,t) \, \bar{u}(p') \, \gamma^{+}u(p) - E^{q}(x,\xi,t) \, \bar{u}(p') \, \frac{i\sigma^{+\alpha}\Delta_{\alpha}}{2m}u(p) \right], \\ \widetilde{F}^{q}(x,\xi,t) &= \int \frac{\mathrm{d}z^{-}}{4\pi} e^{-ixP^{+}z^{-}} \langle p' | \bar{q}(z^{-}/2) \gamma^{+}\gamma_{5}q(-z^{-}/2) | p \rangle \\ &= \frac{1}{2P^{+}} \left[\widetilde{H}^{q}(x,\xi,t) \, \bar{u}(p') \, \gamma^{+}\gamma_{5}u(p) - \widetilde{E}^{q}(x,\xi,t) \, \bar{u}(p') \, \frac{\gamma_{5}\Delta^{+}}{2m}u(p) \right]. \end{aligned}$$

Similar definition for gluon GPDs

No Proton "Radius" in color charge distribution!





Proton radii from quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$



□ Impact parameter dependent parton density distribution:

$$q(x, b_{\perp}, Q) = \int d^2 \Delta_{\perp} e^{-i\Delta_{\perp} \cdot b_{\perp}} H_q(x, \xi = 0, t = -\Delta_{\perp}^2, Q)$$

Quark density in $dx d^2 b_T$

Tomographic image of hadron How fast does How far does glue glue density fall? in slice of x density spread? × 0.2 0.15 0.1 Modeled by 0.05 -1 M. Burkdart, -0.5 0.5 PRD 2000 b_{\perp} (fm)



 $x + \xi / p'$ p'Measurement of p' fixes (t, ξ) x = momentum flowbetween the pair

- Should r_q(x) > r_g(x), or vice versa?
- Could $r_g(x)$ saturates as $x \to 0$
- How do they compare with known radius (EM charge radius, mass radius, ...), & why?
- How the image correlate to hadron spin, ... ?



QCD energy-momentum tensor:

$$=\sum_{i=q,g}T_{i}^{\mu\nu}\quad\text{with}\quad T_{q}^{\mu\nu}=\bar{\psi}_{q}\,i\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\,\psi_{q}-g^{\mu\nu}\bar{\psi}_{q}\left(i\gamma\cdot\overleftrightarrow{D}-m_{q}\right)\psi_{q}\quad\text{and}\quad T_{g}^{\mu\nu}=F^{a,\mu\eta}F^{a,\nu}_{\ \ \eta}+\frac{1}{4}g^{\mu\nu}\left(F^{a}_{\rho\eta}\right)^{2}$$

Gravitational" form factors:

 $T^{\mu
u}$

$$\langle p' | T_i^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^{\mu} P^{\nu}}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{4m} + m \,\bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p'|T_i^{++}|p\rangle \quad \propto \quad \bar{u}(p') \begin{bmatrix} \left(A_i + \xi^2 D_i\right) \gamma^+ + \left(B_i - \xi^2 D_i\right) \frac{i\sigma^{+\Delta}}{2m} \end{bmatrix} u(p)$$
$$\int_{-1}^{1} dx \, x \, H_i(x,\xi,t) \quad \int_{-1}^{1} dx \, x \, E_i(x,\xi,t)$$

□ Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

i = q, g

3D tomography Relation to GFF Angular Momentum $C_i(t) \leftrightarrow D_i(t)/4$

Related to pressure & stress force inside h

Ji, PRL78, 1997

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018) Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

x-dependence of GPDs!

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Need to know the x-dependence of GPDs to construct the proper moments!

Exclusive Diffractive Processes for Extracting GPDs

 \Box Hit the proton hard without breaking it \Rightarrow Diffractive scattering to keep proton intact



HERA discovery:

~ 10-15% of HERA events with the Proton stayed intact

□ Known exclusive processes for extracting GPDs:



DVCS at a Future EIC (White Paper)





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Exclusive vector meson production:



How well can we infer the (x, ξ ,t) dependence of GPDs from the EIC data?

Amplitude nature: $x \sim \text{loop momentum}$



$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}\boldsymbol{x} \, \frac{F(\boldsymbol{x},\xi,t)}{\boldsymbol{x}-\xi+i\varepsilon} \equiv "F_0(\xi,t)"$$

- also true for most other processes
- *x*-dependence is only constrained by a "moment"
- *x*-integration decouples from external Q²



NO full *x*-dependence for given t and ξ



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NO full *x*-dependence for given t and ξ

"Shadow GPDs"

PRD103 (2021) 114019

$$\begin{split} F(x,\xi,t) &\to F(x,\xi,t) + S(x,\xi,t) \\ & \text{with} \quad \int_{-1}^{1} \mathrm{d}x \, \frac{S(x,\xi,t)}{x-\xi+i\varepsilon} = 0 \end{split}$$



Blue and dashed Fit the same CFFs !

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Single-Diffractive Hard Exclusive Processes (SDHEP) – New

 \Box Two-stage diffractive $2 \rightarrow 3$ hard exclusive processes:

Single diffractive – keep the hadron intact:

Qiu & Yu, JHEP 08 (2022) 103 PRD 107 (2023) 1 PRL 131 (2023)161902



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Single-Diffractive Hard Exclusive Processes (SDHEP) – New



What kind of process/observable could be sensitive to the x-dependence?

Create an entanglement between the internal x and an externally measured variable?

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}\boldsymbol{x} \frac{F(\boldsymbol{x},\xi,t)}{x - x_p(\xi,\boldsymbol{q}) + i\varepsilon}$$

Change external *q* to sample different part of **x**.

Double DVCS (two scales):

$$x_p(\xi, q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \text{ same as DVCS if } q \to 0$$





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Production of two back-to-back high pT particles (say, two photons):

 $\pi^{-}(p_{\pi}) + P(p) \rightarrow \gamma(q_{1}) + \gamma(q_{2}) + N(p')$ Hard scale: $q_{T} \gg \Lambda_{\text{QCD}}$ Soft scale: $t \sim \Lambda_{\text{QCD}}^{2}$

Qiu & Yu JHEP 08 (2022) 103

 $x \leftrightarrow q_T$

$$\sim \Lambda^2_{\rm QCD}$$
 JHEP 08 (20

Factorization:

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$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x \, F(x,\xi,t;\mu) \cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T) \longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}t \, \mathrm{d}\xi \, \mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

$$q_T \text{ distribution is "conjugate" to x distribution}$$

Enhanced Sensitivity on x-dependence of GPDs

D Pion-photon production: $\gamma(p_{\gamma}) + h(p) \rightarrow \pi^{\pm}(q_1) + \gamma(q_2) + h'(p')$

JLab-Hall D, other Halls & EIC with a quasi-photon beam



Exclusive Photo-Production of a $\pi \gamma$ Pair – Hall D at JLab



D Polarization asymmetries

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi} \frac{d\sigma}{d|t|d\xi\,d\cos\theta} \cdot \left[1 + \lambda_N \lambda_\gamma \,A_{LL} + \zeta \,A_{UT}\cos 2\left(\phi - \phi_\gamma\right) + \lambda_N \zeta \,A_{LT}\sin 2\left(\phi - \phi_\gamma\right)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\left(\alpha_e\alpha_s\right)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2s^3}\Sigma_{UU}$$

$$\begin{split} \Sigma_{UU} &= |\mathcal{M}_{+}^{[\widetilde{H}]}|^{2} + |\mathcal{M}_{-}^{[\widetilde{H}]}|^{2} + |\widetilde{\mathcal{M}}_{+}^{[H]}|^{2} + |\widetilde{\mathcal{M}}_{-}^{[H]}|^{2}, \\ A_{LL} &= 2 \, \Sigma_{UU}^{-1} \, \mathrm{Re} \left[\mathcal{M}_{+}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{+}^{[H]*} + \mathcal{M}_{-}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{-}^{[H]*} \right], \\ A_{UT} &= 2 \, \Sigma_{UU}^{-1} \, \mathrm{Re} \left[\widetilde{\mathcal{M}}_{+}^{[H]} \, \widetilde{\mathcal{M}}_{-}^{[H]*} - \mathcal{M}_{+}^{[\widetilde{H}]} \, \mathcal{M}_{-}^{[\widetilde{H}]*} \right], \\ A_{LT} &= 2 \, \Sigma_{UU}^{-1} \, \mathrm{Im} \left[\mathcal{M}_{+}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{-}^{[H]*} + \mathcal{M}_{-}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{+}^{[H]*} \right]. \end{split}$$

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Qiu & Yu, PRL 131 (2023)161902

Exclusive Photo-Production of a $\pi \gamma$ Pair – Hall D at JLab



Exclusive Photo-Production of a $\pi \gamma$ Pair – Hall D at JLab



Internal Nuclear Landscape

A simple, but fundamental, question:

What does a nucleus look like if we only see quarks and gluons ?

Need localized hard probes – "see" more particle nature of the "glue"

But, a hard probe at small-x is NOT necessarily localized:



In c.m. frame

Longitudinal probing size

> Lorentz contracted nucleon

$$\begin{array}{c} \text{if} \quad \frac{1}{xp} > 2R\frac{m}{p} \quad \text{or} \quad x < 0.1 \end{array}$$



A hard probe at small-x can interact with multiple nucleons (partons from multiple nucleons) at the same impact parameter coherently

Another simple, and fundamental, question:

Does the color of a parton in nucleon "A" know the color of a parton in nucleon "B"?

IF YES, Nucleus could act like a bigger proton at small-x (long range of color correlation), and could reaching the saturation much sooner!

IF NOT, only short-range color correlation, and observed nuclear effect in cross-section at small-x is dominated by coherent collision effect

Saturation of gluons is a part of QCD, where to find it?

EIC can tell !



Coherent Length of the Color

□ A simple experiment to address a "simple" question:

Will the nuclear shadowing continue to fall as x decreases?



EIC White Paper





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Coherent Length of the Color



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EIC White Paper

Summary and Outlook

We have the right Theory – QCD, but, unprecedented challenges

- QCD has been very successful in describing the short-distance dynamics
- Trying to understand the emergent phenomena of QCD:
 - Hadron properties, such as the mass, spin, ..., in the most fundamental way
 - Internal structure and landscape of hadrons, such as confined motion, spatial tomography of nuclei, ...
 - Emergence of hadrons from quarks and gluons, neutralization of the color, ...
 - Emergence of jet(s) and jet substructure, ..., need femto-meter sized detectors!
 - Interplay of the particle and wave nature of quarks and gluons, ...
- □ EIC is an ultimate QCD machine and a facility, capable of discovering and exploring the emergent phenomena of QCD, and the role of color and glue, ...
- US-EIC is sitting at a sweet spot for rich QCD dynamics, capable of taking us to the next frontier of QCD and nuclear theory!

Thanks!



Inclusive Process vs. Exclusive Process



<u>Cross section</u>: Cut diagrams

$$\sigma_{\rm DIS} \simeq \int_{\boldsymbol{x}_B}^1 \mathrm{d}\boldsymbol{x} f(\boldsymbol{x}) \,\hat{\sigma}(\boldsymbol{x}/x_B)$$

- $PDF \sim probability$
- At LO: $x = x_B$
- Beyond LO: $x \in [x_B, 1]$

x-dependence: Part of measurement



Amplitude: Uncut diagrams

$$\mathcal{M}_{\mathrm{DVCS}}(\xi, t) \simeq \int_{-1}^{1} \mathrm{d}x \, F(x, \xi, t) \, \hat{\mathcal{M}}(x, \xi)$$

- GPD \sim amplitude
- $k^+ = (x + \xi) P^+$ is loop momentum
- At any order: *x* ∈ [−1, 1]

<u>*x-dependence*</u>: Hard to measure

