How precisely can we predict the muon g-2 in the standard model?

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16th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon
MENU2023, Mainz, Oct. 15-20, 2023

INTRODUCTION

FNAL-E989 Run-2 and Run-3 data

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,057(25) \cdot 10^{-11} \quad [0.21\,\mathrm{ppm}]$$

D. P. Aguillard et al. [Muon g-2 Coll.], Phys. Rev. Lett. 131, 161802 (2023)

$\longrightarrow$ talk by A. Driutti

FNAL-E989 Run-1 data

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

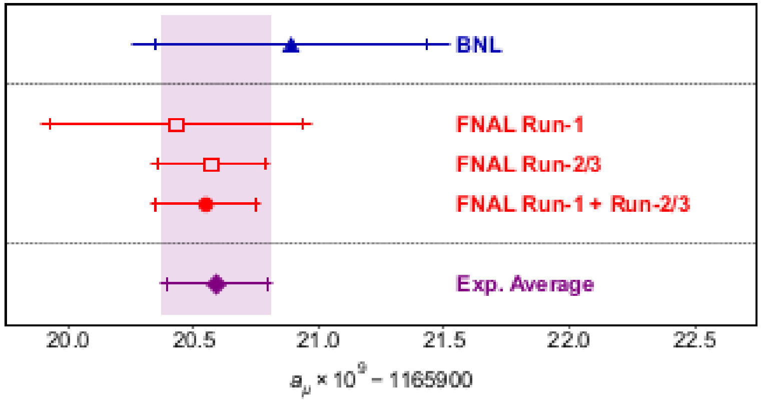
B. Abi et al. [Muon g-2 Coll.], Phys. Rev. Lett. 126, 120801 (2021)

BNL-E821 final result

$$a_{\mu^\pm}^{\mathrm{E821}} = 116\,592\,080(54)(33) \cdot 10^{-11} \quad [0.54\,\mathrm{ppm}]$$

G. W. Bennett et al. [Muon g-2 Coll.], Phys. Rev. D 73, 072003 (2006)

INTRODUCTION



The world-average value

$$a_\mu^{\mathrm{exp;WA}} = 116\,592\,059(22) \cdot 10^{-11} \quad [0.19\,\mathrm{ppm}]$$

leads to a discrepancy at the level of 5.2σ

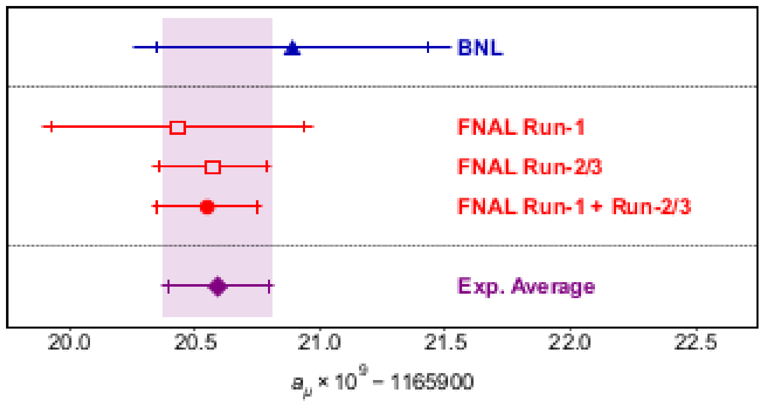
$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{th;WP}} = 249(48) \cdot 10^{-11} \quad [5.2\sigma]$$

when compared to the SM predicted value

$$a_\mu^{\mathrm{th;WP}} = 116\,591\,810(43) \cdot 10^{-11} \quad [0.35\,\mathrm{ppm}]$$

as reported in the White Paper T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

INTRODUCTION



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Uncertainty in $a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{th;WP}}$ dominated by theory uncertainty!

INTRODUCTION

This discrepancy

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{th;WP}} \;=\; 249(48) \cdot 10^{-11} \;\; [5.2\sigma]$$

raises a certain number of questions

- Can we compute a_μ^{SM} to the required level of precision?

- To which extent does $a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{th;WP}}$ hold today?

- If the discrepancy is real, what explains it?

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- If the discrepancy is real, what explains it?

$\longrightarrow$ hundreds of papers on arXiv since April 7, 2021

OUTLINE

- Theory aspects: a_μ in the SM
  - QED contributions
  - Contributions from the weak contributions
  - Contributions from the strong interactions
  - The situation today: the post White Paper era

- Summary

Theory aspects

One wants to probe the response of a charged lepton to an external (and static) electromagnetic field

$$\langle \ell; p' | \mathcal{J}_\rho(0) | \ell; p \rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p',p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\Big[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\Big]\mathrm{u}(p)$$

(uses only the conservation of the electromanetic current $\mathcal{J}_\rho$, $k_\mu \equiv p'_\mu - p_\mu$)

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow \not{P}, \not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell / q_\ell$

$F_4(k^2) \rightarrow \not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\ G_M(k^2) = F_1(k^2) + F_2(k^2)$$

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops $\longrightarrow$ calculable!

[tree-level contributions would correspond to terms with dim > 4]

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At $k^2 = 0$, G_M corresponds to a gyromagnetic factor, $G_M(0) = g_\ell/2$

$$\boldsymbol{\mu_\ell} = g_\ell \left(\frac{q_\ell}{2m_\ell c}\right)\mathbf{S}, \quad \mathbf{S} = \hbar\,\frac{\boldsymbol{\sigma}}{2}$$

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At tree level in the SM, $g_\ell = g_\ell^{\mathrm{Dirac}} \equiv 2$

The *anomalous* magnetic moment is induced at loop level $\quad a_\ell \equiv \dfrac{g_\ell - g_\ell^{\mathrm{Dirac}}}{g_\ell^{\mathrm{Dirac}}} \ (\equiv F_2(0))$

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a_ℓ probes all the degrees of freedom of the standard model, *and possibly beyond...*

Considering SM contributions only, one has, by order of importance

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

a_μ^{QED}: loops with only photons and leptons

a_μ^{had}: loops with photons and leptons and at least one quark loop dressed with gluons

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For a full and detailed account [up to June 15, 2020], see the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

Theory I: QED

QED contribution :

$\longrightarrow$ loops with only photons and leptons

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$$(\alpha/\pi)^4 = 2.91 \ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76 \ldots \cdot 10^{-14}$$

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$\longrightarrow$ becomes technically challenging $(1, 6, 72, 891, 12\,672, \ldots)$

$$a_\mu^{\text{QED}} = C_\mu^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\mu^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\mu^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\mu^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\mu^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

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$\longrightarrow$ multiflavour QED

$$C_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\mu/m_e) + A_2^{(2n)}(m_\mu/m_\tau) + A_3^{(2n)}(m_\mu/m_e, m_\mu/m_\tau)$$

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$$\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$$

A few comments about the QED contributions

- expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

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- $A_1^{(8)}$ has also been evaluated! ($\longrightarrow a_e$) S. Laporta, Phys. Lett. B 772, 232 (2017)
- Mass dependent parts $A_2^{(8)}(m_\mu/m_e)$, $A_2^{(8)}(m_\mu/m_\tau)$, $A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau)$

evaluated numerically

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

and crossed-checked by independent QFT methods A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

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A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

- $(\alpha/\pi)^5$: 6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results available

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops ($\longrightarrow a_e$)

QED contribution :

$C_\mu^{(2)}$	0.5
$C_\mu^{(4)}$	$0.765\,857\,425(17)$
$C_\mu^{(6)}$	$24.050\,509\,96(32)$
$C_\mu^{(8)}$	$130.878\,0(61)$
$C_\mu^{(10)}$	$750.72(93)$

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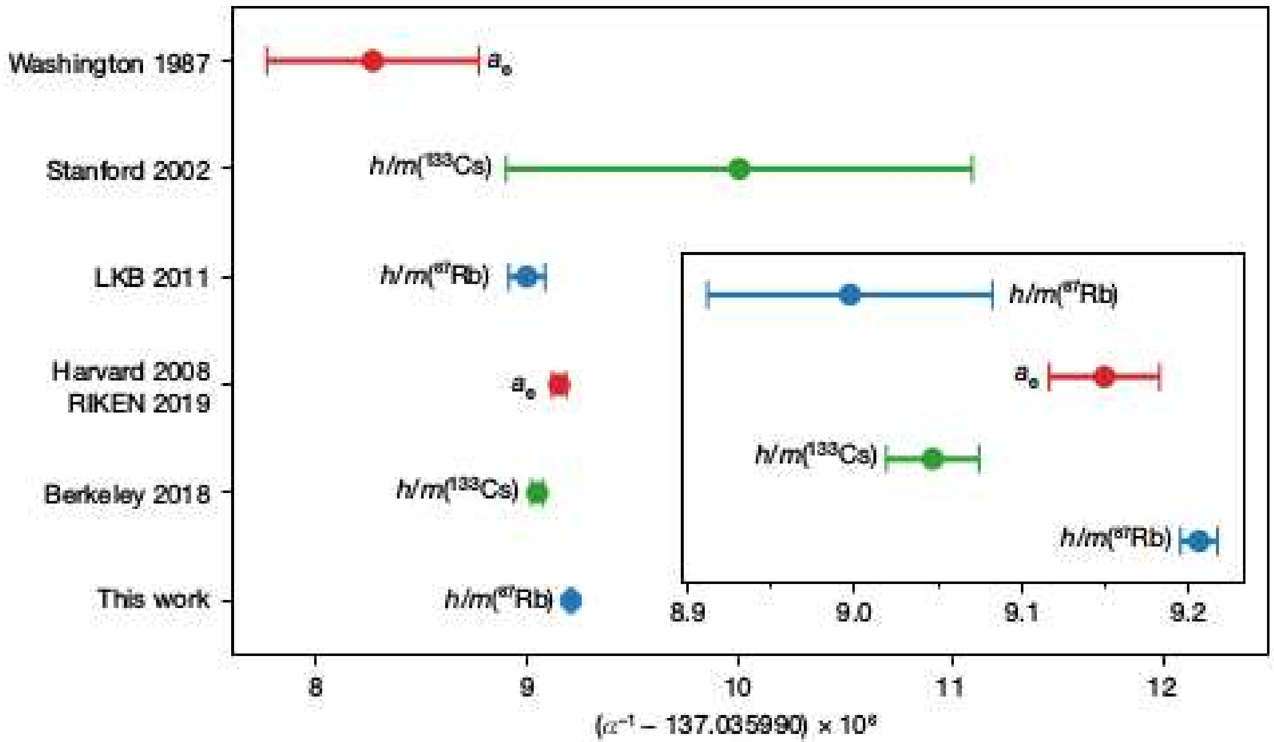
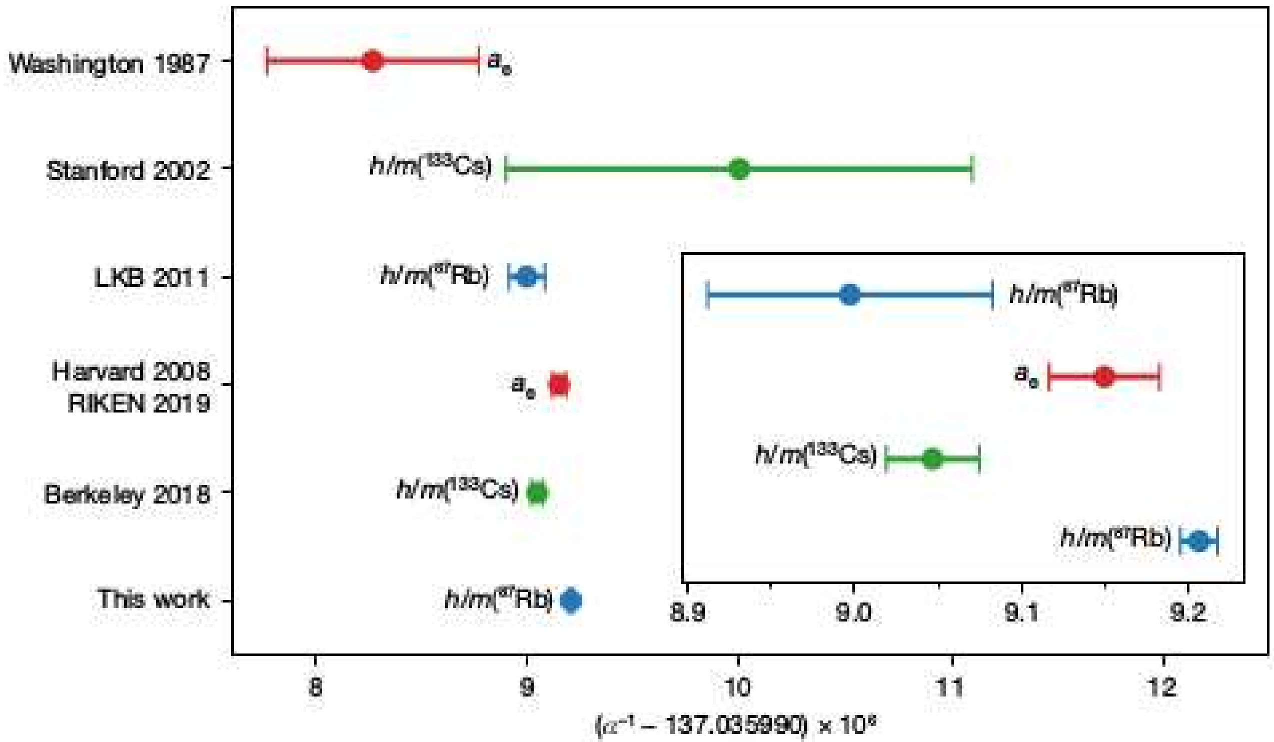


A. Wicht, J. M. Hensley, E. Sarajilic, S. Chu, Phys. Scr. T102, 82 (2002)

R. Bouchendira, P. Clade, S. Guellati-Khelifa, F. Nez and F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Müller, Science 360, 191 (2018)

L. Morel, Z. Yao, P. Cladé, S. Guellati-Khélifa, Nature 588, 61 (2020)



$\longrightarrow$ existing tension/discrepancy between $\alpha(\mathrm{Cs18})$ and $\alpha(\mathrm{Rb20})$ (but also between $\alpha(\mathrm{Rb11})$ and $\alpha(\mathrm{Rb20})$) of no concern for a_μ

$\longrightarrow$ for a_μ the value of α could be provided by the qH effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \quad [19.7\mathrm{ppb}]$$

P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)

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n	$C_\mu^{2n}(\alpha/\pi)^n \cdot 10^{11}$
1	$116\,140\,973.321(23)$
2	$413\,217.6258(70)$
3	$30\,141.90233(33)$
4	$381.004(17)$
5	$5.0783(59)$
$a_\mu^{\mathrm{QED}}(\mathrm{Cs18})$	$116\,584\,718.931(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs18)} \cdot 10^{-11}$

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Requires an experimental determination of α with

$$\frac{\Delta\alpha}{\alpha} \sim \frac{\Delta a_\mu}{a_\mu} \sim 0.14\text{ppm}$$

- $a_\mu^{\text{QED}}(\text{Cs18}) = 116\,584\,718.931(7)_{\text{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs18)} \cdot 10^{-11}$

- $$a_\mu^{\text{exp;WA}} - a_\mu^{\text{QED}}(\text{Cs18}) = 7341(22) \cdot 10^{-11}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

- The missing part has to be provided by weak and strong interactions (or else, new physics...)

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 22 \cdot 10^{-11}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \longrightarrow \quad \sim 14 \cdot 10^{-11}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 380 \cdot 10^{-11} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 5 \cdot 10^{-11}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

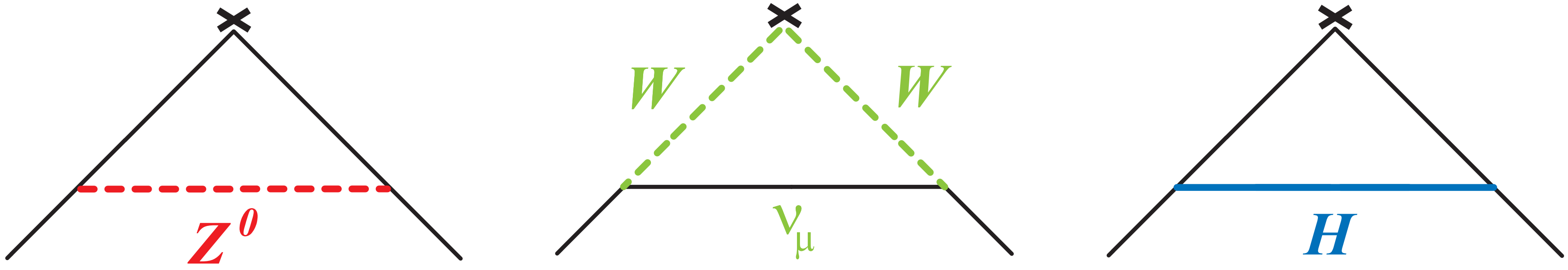
$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e;\text{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 5.4 \cdot 10^3 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 0.08 \cdot 10^{-11}$$

- No sign of substantial contribution to a_μ from higher order QED

$$\left(\frac{\alpha}{\pi}\right)^6 = 1.56\ldots \cdot 10^{-16} \quad ==> \quad \text{would require} \quad C_\mu^{(12)} \sim 10^5 ?!$$

Theory II: weak interactions

- Weak contributions : W, Z,... loops



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$

$$= 194.8\cdot 10^{-11}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)

M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)

A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

$$a_\mu^{\text{weak}} = (154 \pm 1) \cdot 10^{-11}$$

$$a_e^{\text{weak}} = (0.0297 \pm 0.0005) \cdot 10^{-12}$$

Updated a decade ago: $a_\mu^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

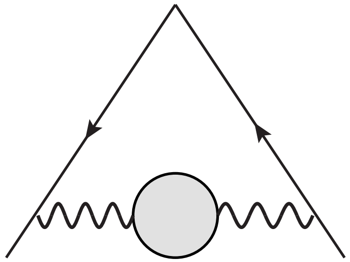
Numerical evaluation: $a_\mu^{\text{weak}} = (152.9 \pm 1.0) \cdot 10^{-11}$

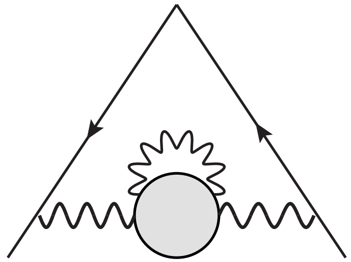
T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathsf{weak}} = 7187(22) \cdot 10^{-11}$$

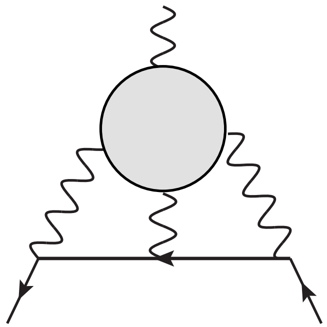
Theory III: strong interactions

- hadronic vacuum polarization





- (virtual) hadronic light-by-light (HLxL



⟶ non-perturbative regime of QCD

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

$$a_\ell^{\text{HVP-LO}} = 4\alpha^2 \int_0^\infty \frac{ds}{s} K(s) \frac{1}{\pi} \text{Im}\Pi(s) \qquad K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}$$

$$= \frac{1}{4\pi^3} \int_{4M_\pi^2}^\infty ds\, K(s)\, \sigma^{e^+e^- \to \text{had}}(s) \qquad \text{optical theorem}$$

$$= \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^\infty \frac{ds}{s} K(s) R^{\text{had}}(s)$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)

L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)

M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\text{had}}(s) \quad K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}$$

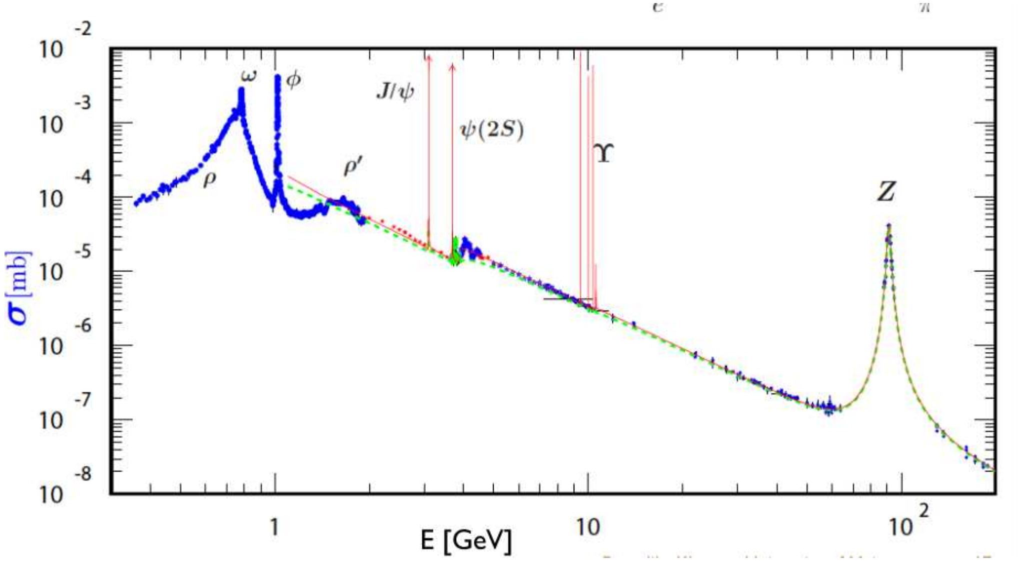
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

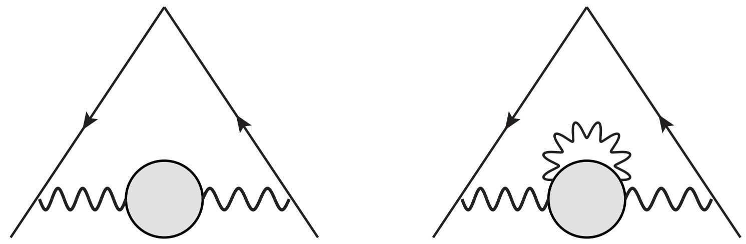
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



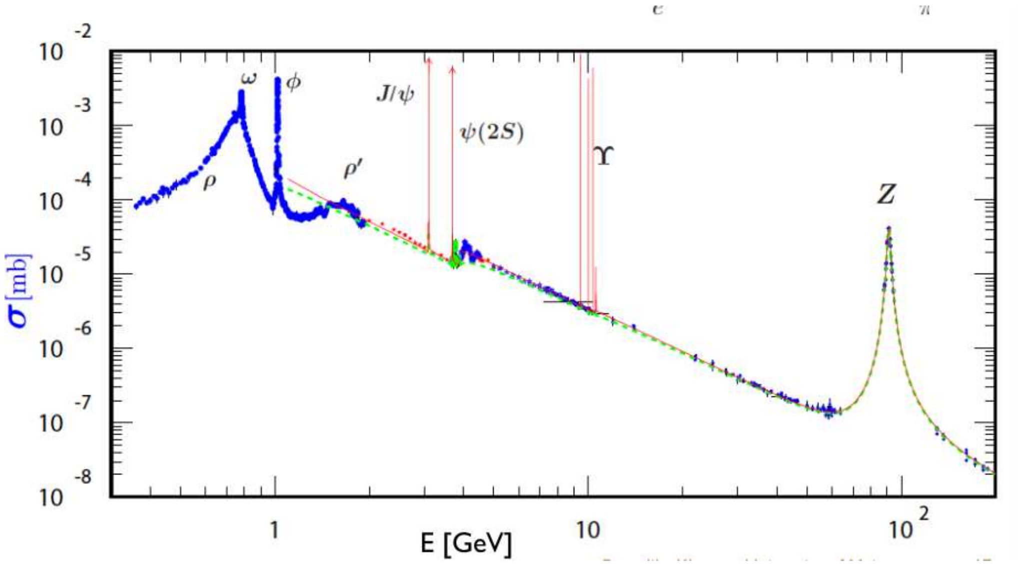
- Combination of ~ 39 exclusive channels

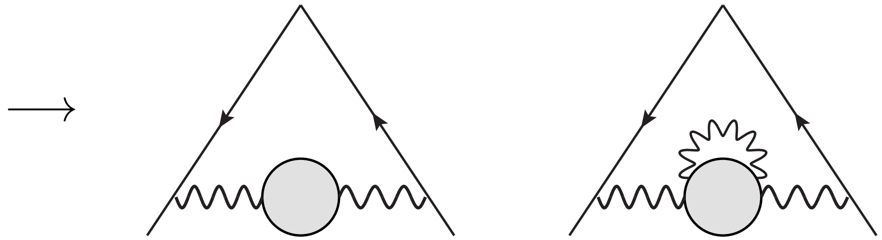
$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Hadronic vacuum polarization

- Can be evaluated using available experimental data





- Some tension, for instance, in the $\pi\pi$ channel (region of the ρ resonance)

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi} \cdot 10^{10}$
CMD2	366.5(3.4)
SND	364.7(4.9)
KLOE	360.6(2.1)
BaBar	370.1(2.7)
BESIII	361.8(3.6)
SND2k	366.7(3.2)

Hadronic vacuum polarization

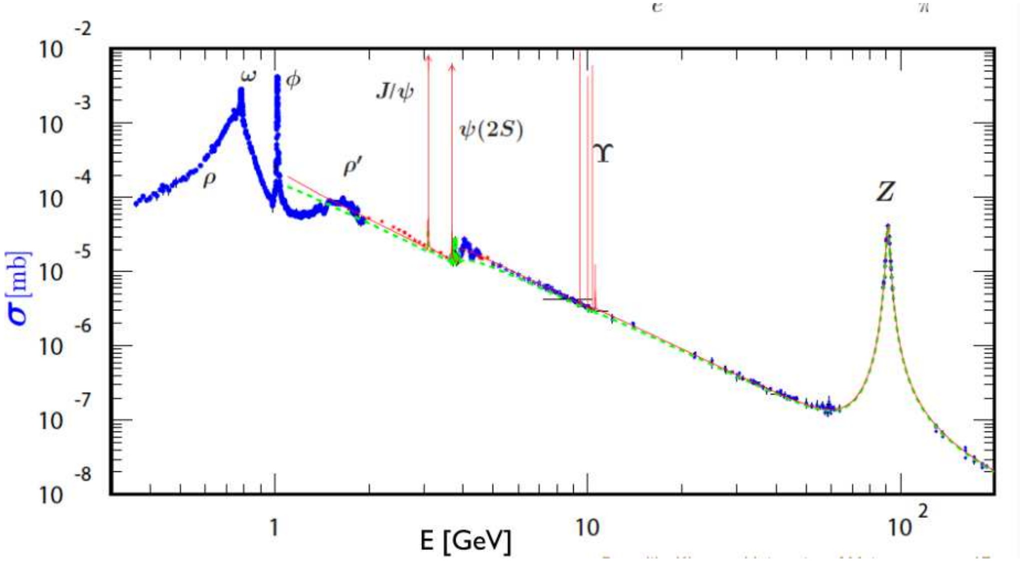
$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$, e^+e^-	
$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$694.0(4.0)$	M. Davier et al., Eur. Phys. J. C 80, 341 (2020); Err. C 80, 410 (2020)
$692.78(2.42)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$, e^+e^-	
$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$-9.82(4)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$-9.83(4)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

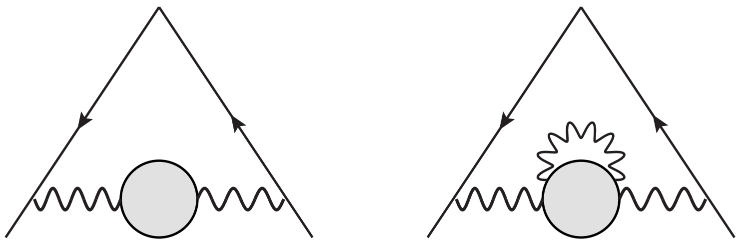
$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$, e^+e^-	
$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- Lattice results (for the time being, stick to WP)

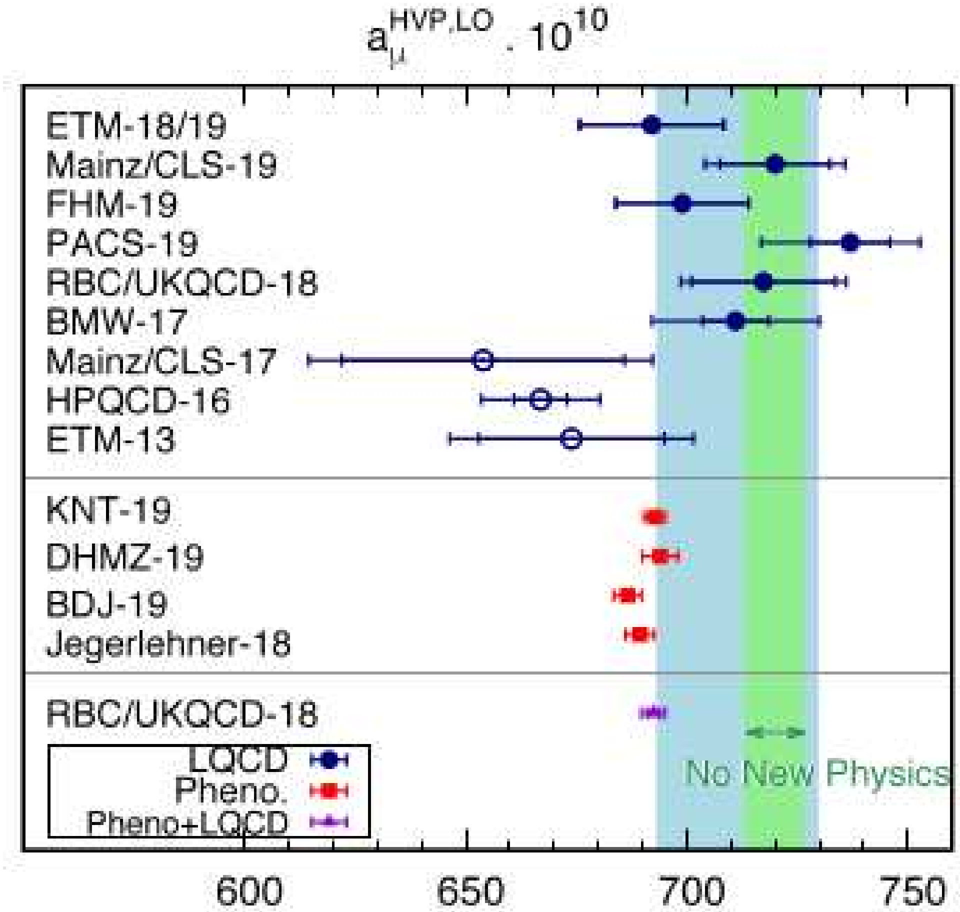
A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)
T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)
S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

White Paper summary

- Data evaluation:

$$a_\mu^{\mathrm{HVP;LO}} = 6931(40)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NLO}} = -98.3(7)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NNLO}} = 12.4(1)\cdot 10^{-11}$$

- Lattice WA: $a_\mu^{\mathrm{HVP;LO}} = 7043(150)\cdot 10^{-11}$

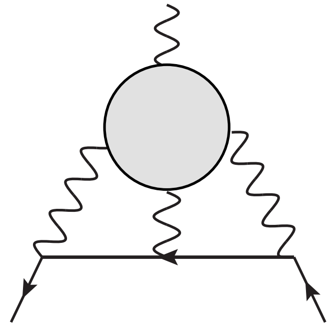


Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$

- Not related, as a whole, to an experimental observable...

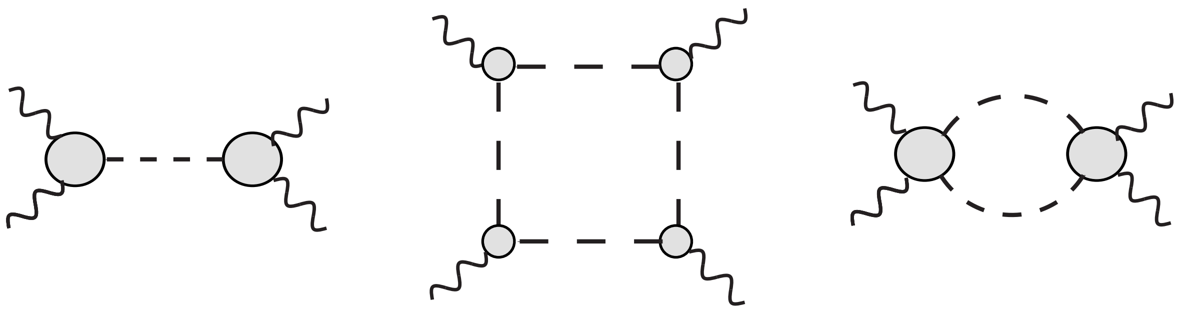
? $\longrightarrow$



- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) \quad q_1 + q_2 + q_3 + q_4 = 0$$

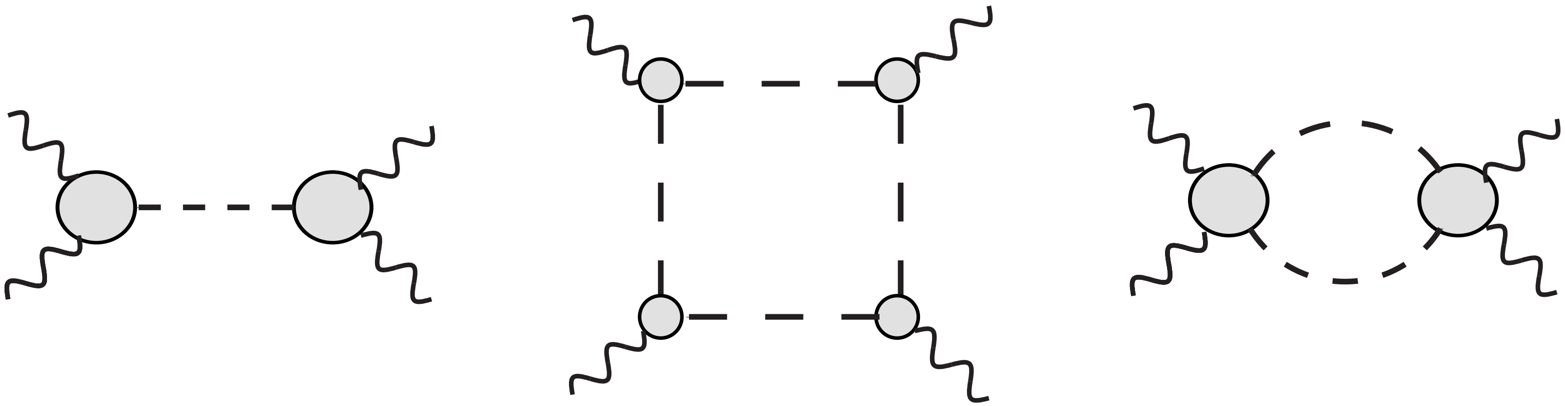
- Many individual contributions have been identified...



Hadronic light-by-light

- More recently: dispersive approaches

$-\ \Pi_{\mu\nu\rho\sigma} \longrightarrow$



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Needs input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

A. Nyffeler, arXiv:1602.03398 [hep-ph]

Hadronic light-by-light

- Lattice QCD calculations

T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)
T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)
N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

Hadronic light-by-light

- Lattice QCD calculations

T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)
T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)
N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

White Paper summary

$$a_\mu^{\rm HLxL} = 92(19) \cdot 10^{-11}$$

$$a_\mu^{\rm exp;WA} - a_\mu^{\rm th;WP} \quad = \quad 249(48) \cdot 10^{-11} \;\; [5.2\sigma]$$

Theory IV: strong interactions

the post-WP era

Post-WP results

- New lattice QCD result for HLxL at slightly less than 15% accuracy

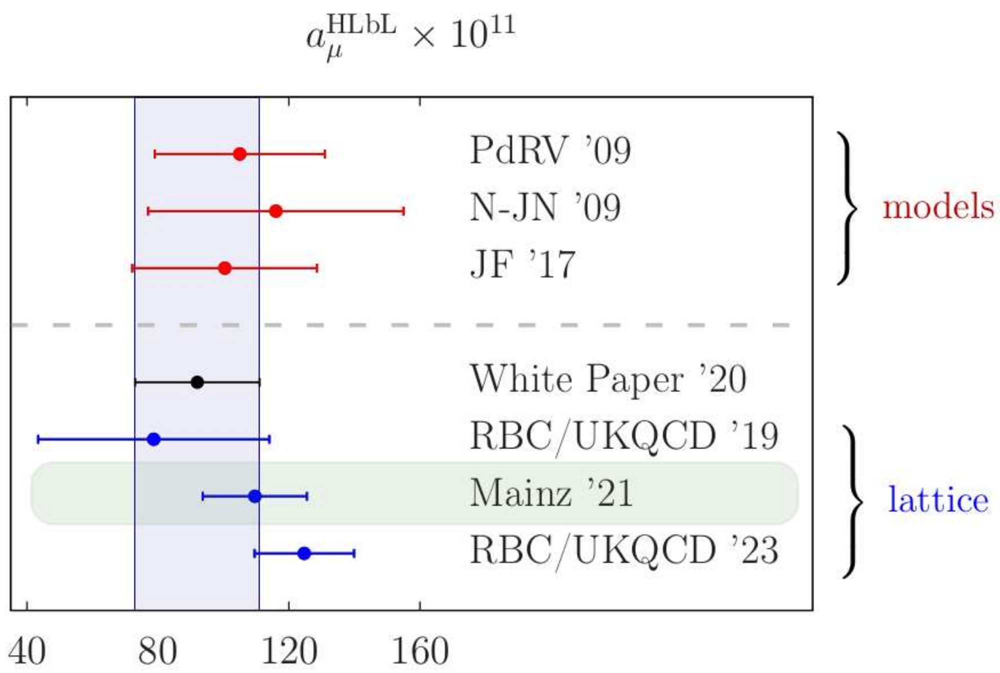
$$a_\mu^{\mathrm{HVP;LO}} = 107.4(11.3)(9.2) \cdot 10^{-11}$$

E.-H. Chao et al., Eur. Phys. J. C 81, 651 (2021)

$$a_\mu^{\mathrm{HVP;LO}} = 124.7(11.5)(9.9) \cdot 10^{-11}$$

T. Blum et al. [RBC/UKQCD], arXiv:2304.04423 (2023)

∼10% accuracy goal seems within reach

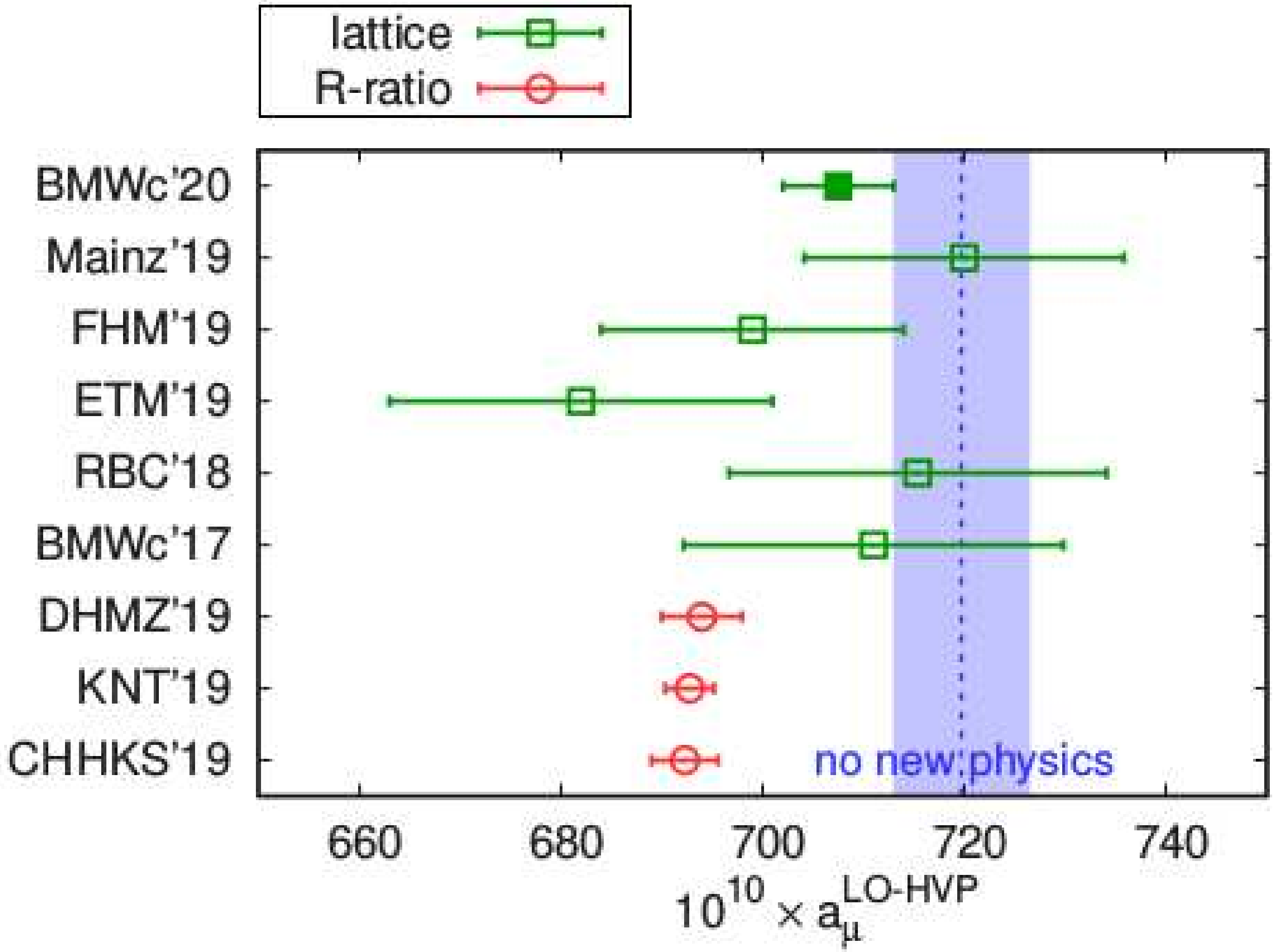


Post-WP results

- New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021)



Post-WP results

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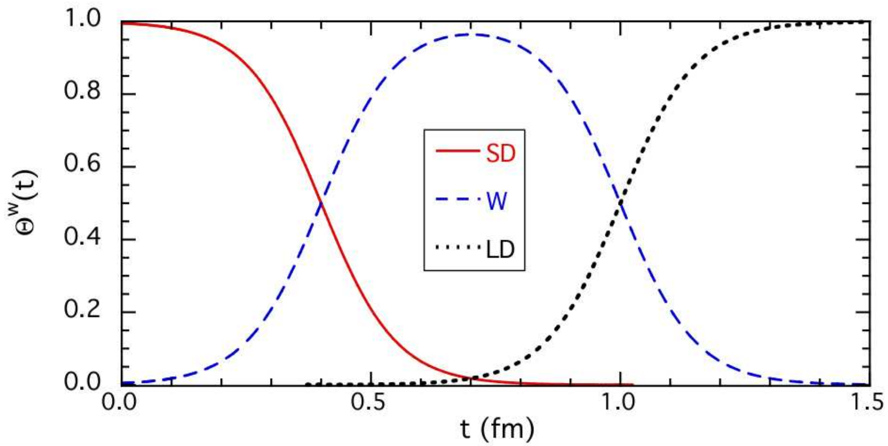
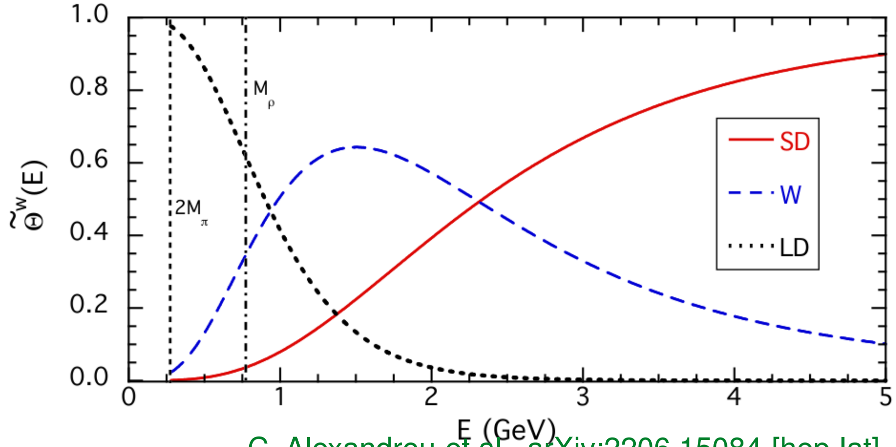
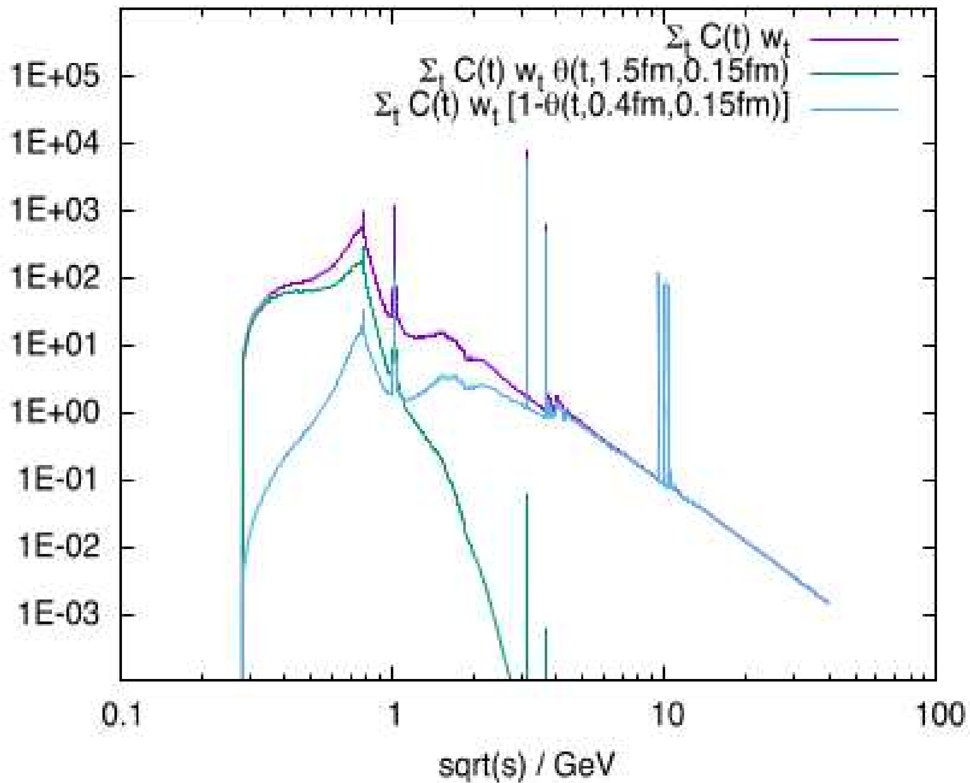




C. Alexandrou et al., arXiv:2206.15084 [hep-lat]



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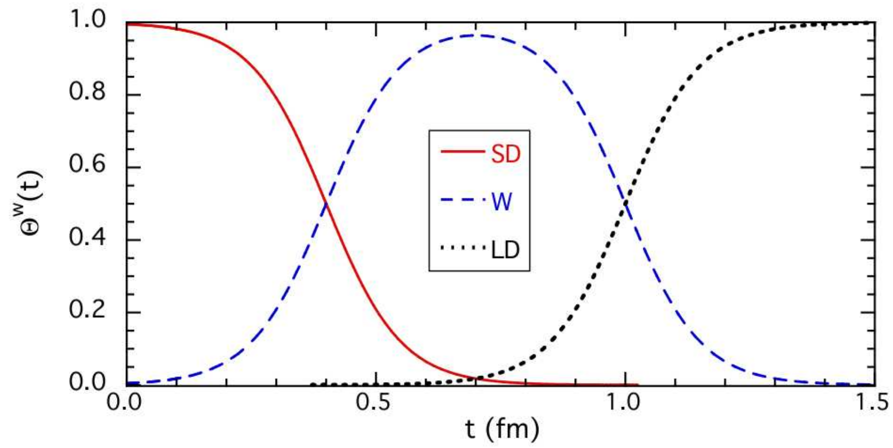

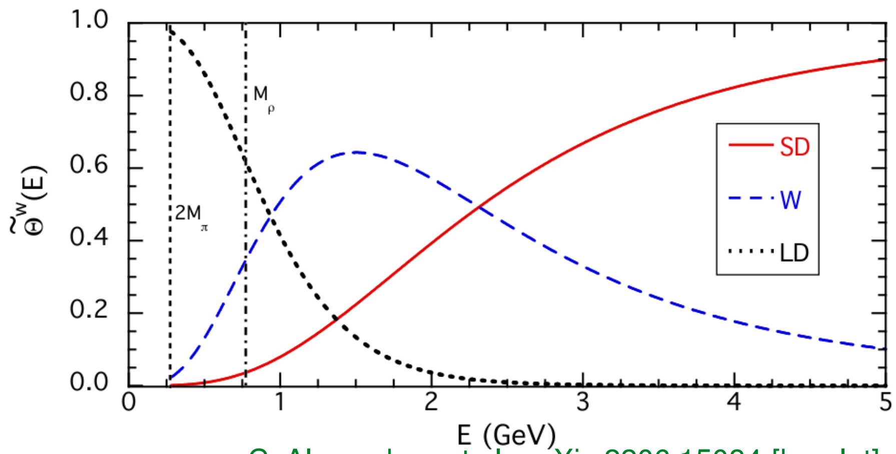

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

$$a_\mu^{\rm SDW} \sim 10\% \text{ of } a_\mu^{\rm HVP;LO}$$

$$a_\mu^{\rm IW} \sim 30\% \text{ of } a_\mu^{\rm HVP;LO}$$

$$a_\mu^{\rm LDW} \sim 60\% \text{ of } a_\mu^{\rm HVP;LO}$$

Post-WP results

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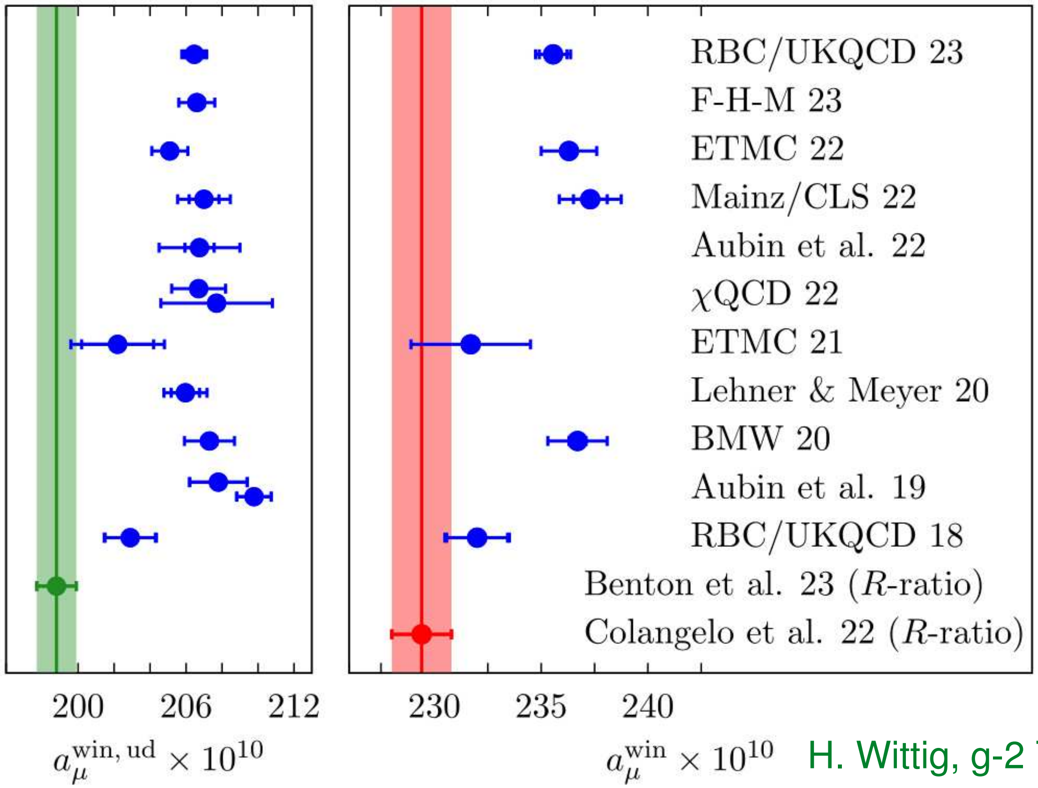
S. Borsanyi et al., Nature 593, 7857 (2021)

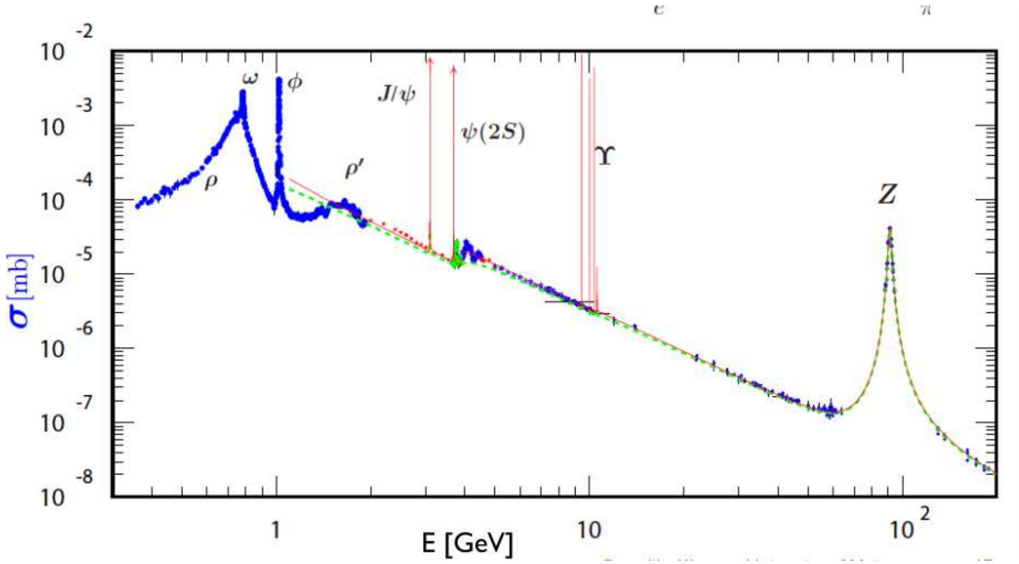


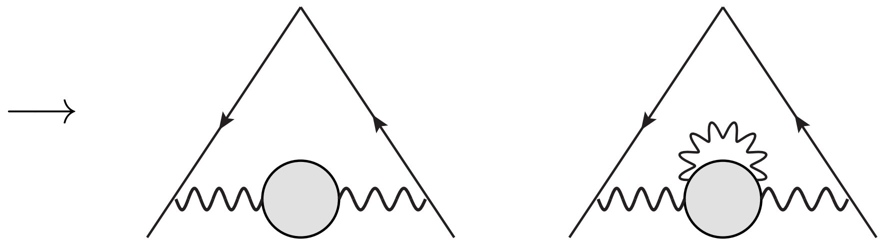
H. Wittig, g-2 TI Worshop, Bern, 4-8 Sept. 2023

To date, no other complete lattice evaluation of the HVP contribution at the same level of precision as BMWc...

Hadronic vacuum polarization: post-WP experimental data from CMD3

- Can be evaluated using available experimental data





- Some tension, for instance, in the $\pi\pi$ channel (region of the ρ resonance)

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi} \cdot 10^{10}$	
CMD2	366.5(3.4)	
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BaBar	370.1(2.7)	BaBar vs. CMD3 2.5σ
BESIII	361.8(3.6)	
SND2k	366.7(3.2)	
CMD3	379.3 (3.0)	F. V. Ignatov *et al.* [CMD-3], arXiv:2302.08834 [hep-ex]

Hadronic vacuum polarization: post-WP experimental data from CMD3

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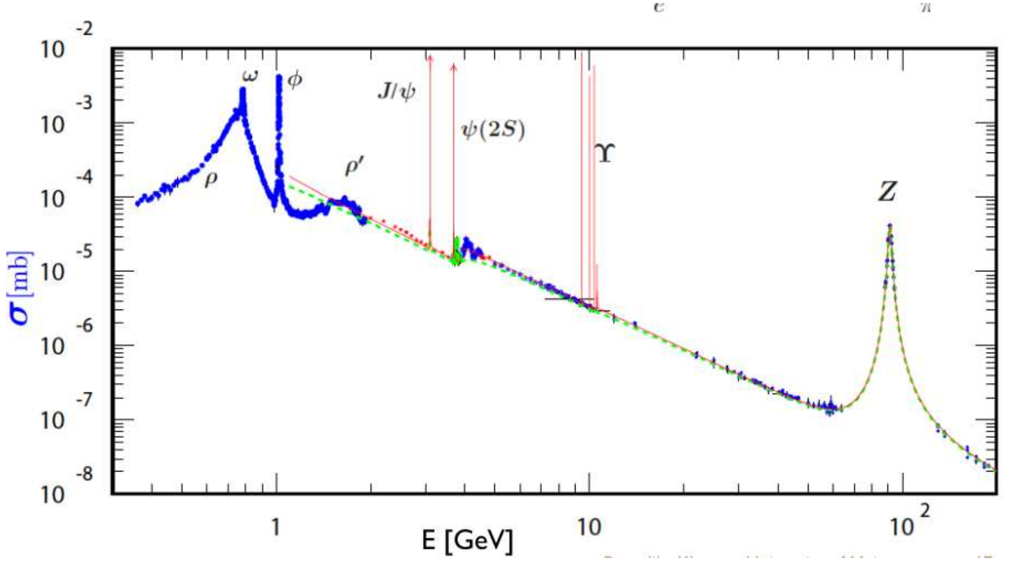
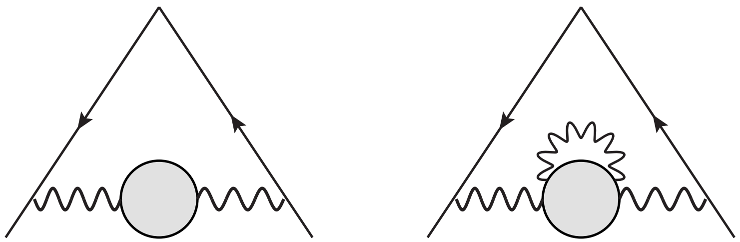


$\longrightarrow$



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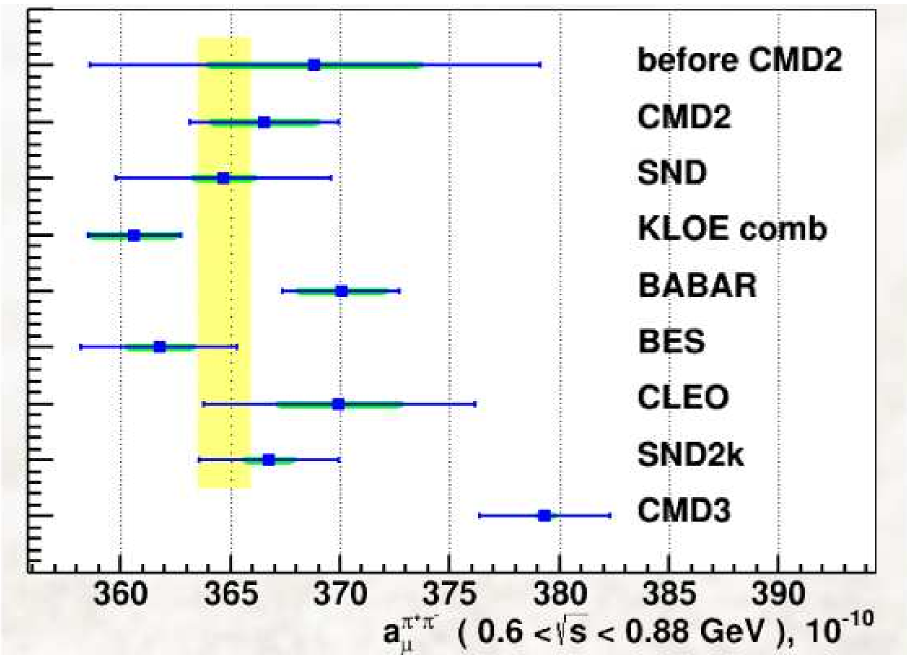


Summary

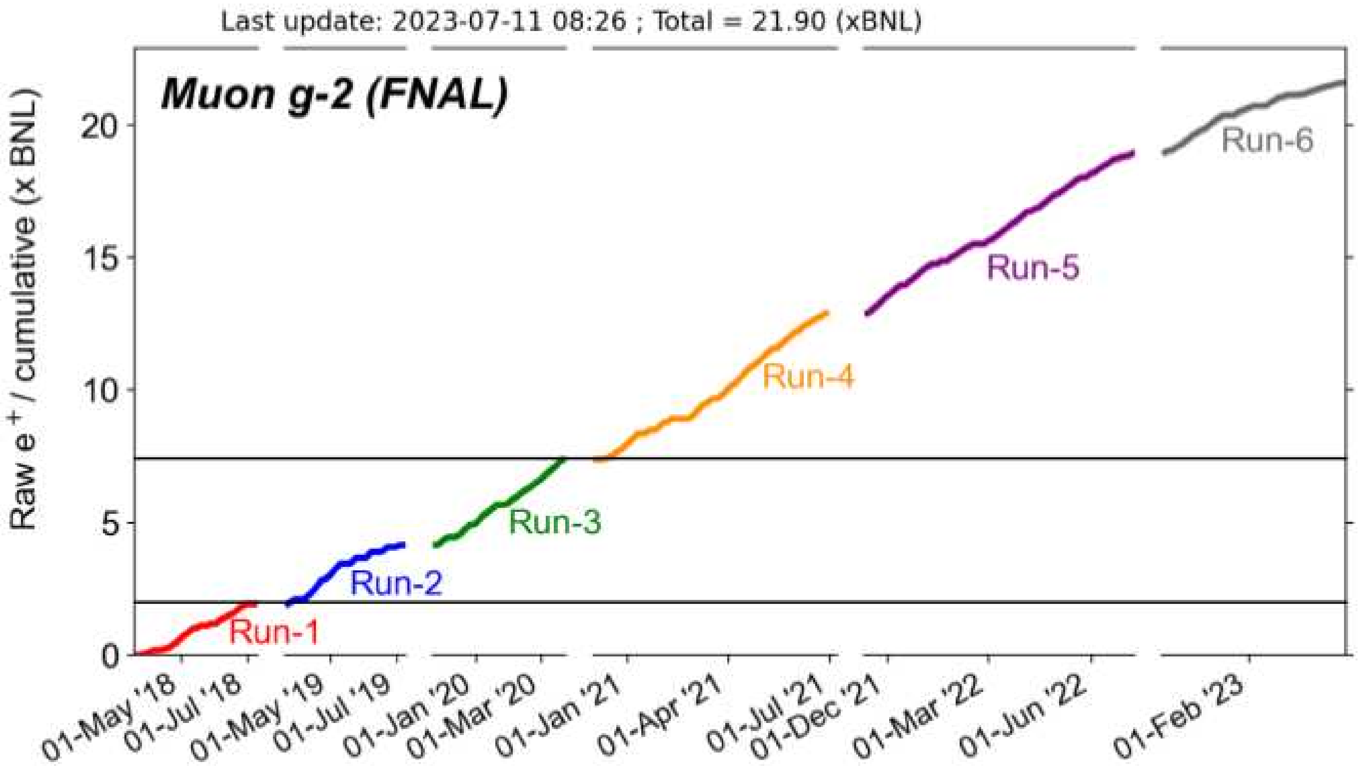
• FNAL-E989 on the right track to achieve a measurement at 0.14ppm

FNAL-E989 Run-2 and Run-3 data

$$a_{\mu^+}^{\rm E989} \;=\; 116\,592\,057(25)\cdot 10^{-11} \;\; [0.21\,\rm ppm]$$

D. P. Aguillard et al. [Muon g-2 Coll.], Phys. Rev. Lett. 131, 161802 (2023)

$\longrightarrow$ talk by A. Driutti



Waiting eagerly for the analysis of the data from the remaining three runs

- Unfortunately, the theory situation is not quite in such a good shape

QED and EW contribution under control at the required level of precision (perturbation theory at high orders)

results on HLxL at $\lesssim 10\%$ within reach (lattice QCD, dispersive approach with exp$^{\mathrm{al}}$ input, e.g. form factors,...)

- Theory situation (as to June 2020) described in detail in the WP outdated as far as HVP is concerned

important tensions in evaluations of HVP

— between KLOE and BABAR

— between CMD3 and earlier experiments

— between BMWc and data-based evaluations (except CMD3)

for more, see https://indico.cern.ch/event/1258310

- More data are being analyzed (BaBar, KLOE)

or will become available in the future (BESIII, BelleII,...)

- Possibilities for inclusive measurements of HVP very interesting
  - in the space-like region (MUonE) $\longrightarrow$ talk by C. M. Carloni Calame
  - or even directly in the time-like region $\longrightarrow$ talk by C. F. Redmer

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  - or even directly in the time-like region $\longrightarrow$ talk by C. F. Redmer

More data does not necessarily mean more clarity

- More data are being analyzed (BaBar, KLOE $\longrightarrow$ talk by Paolo Gauzzi)

or will become available in the future (BESIII, BelleII,...)

- Possibilities for inclusive measurements of HVP very interesting
  - in the space-like region (MUonE) $\longrightarrow$ talk by C. M. Carloni Calame
  - or even directly in the time-like region $\longrightarrow$ talk by C. F. Redmer

More data does not necessarily mean more clarity

- Would like to see independent confirmation of the BMWc result by at least two other lattice collaborations

Thanks for your attention!