MENU 2023

The 16th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon

EXPLORING THE HADRON STRUCTURE WITH GPDS AND TMDS

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DI PAVIA

STRONG 2020

INFN
Istituto Nazionale
di Fisica Nucleare

Emergent phenomena in QCD

"the whole is more than the sum of its parts"

``What proton is depends on how you look at it, or rather on how hard you hit it''

A. Cooper-Sarkar, CERN Courier, June, 2019

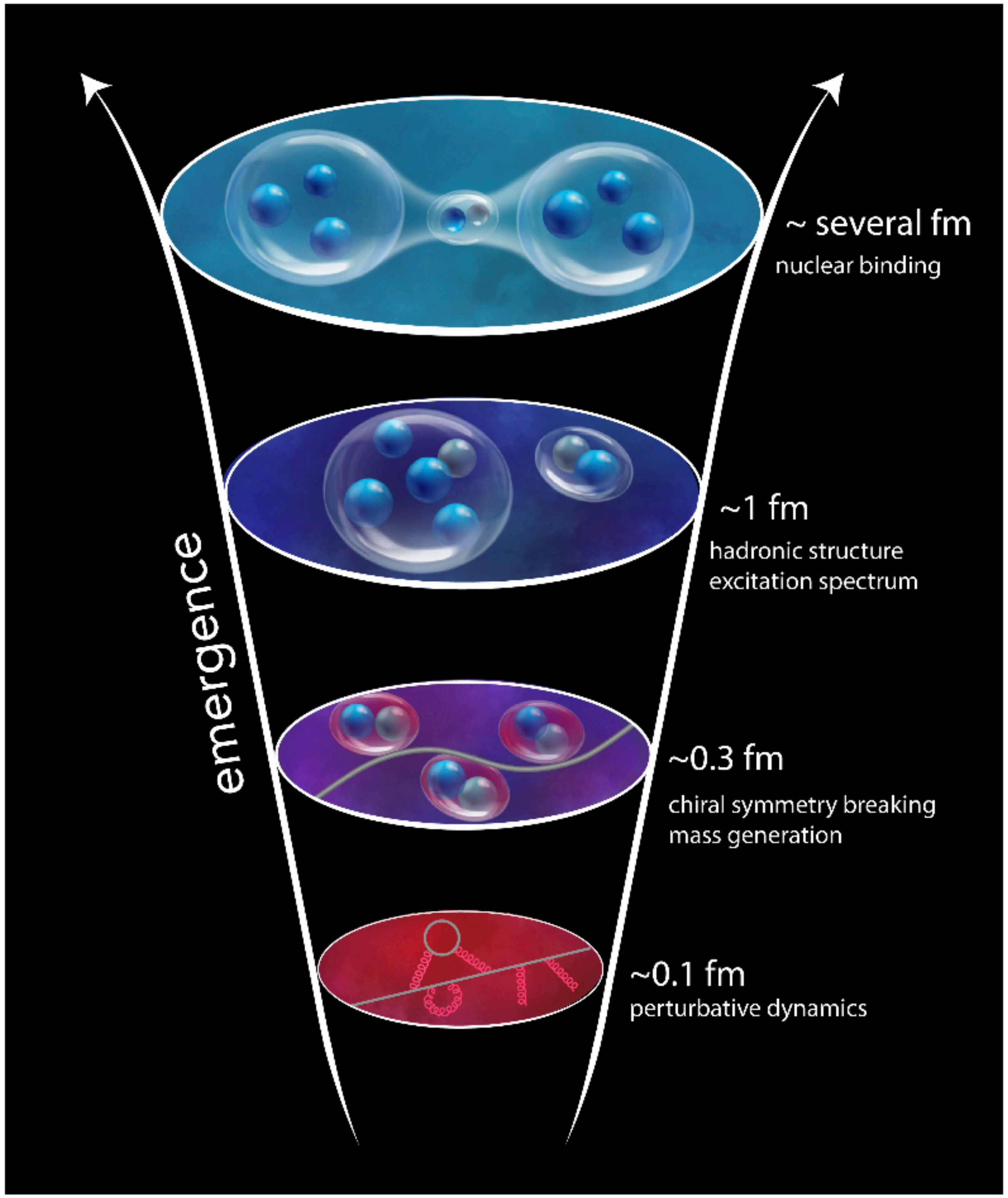
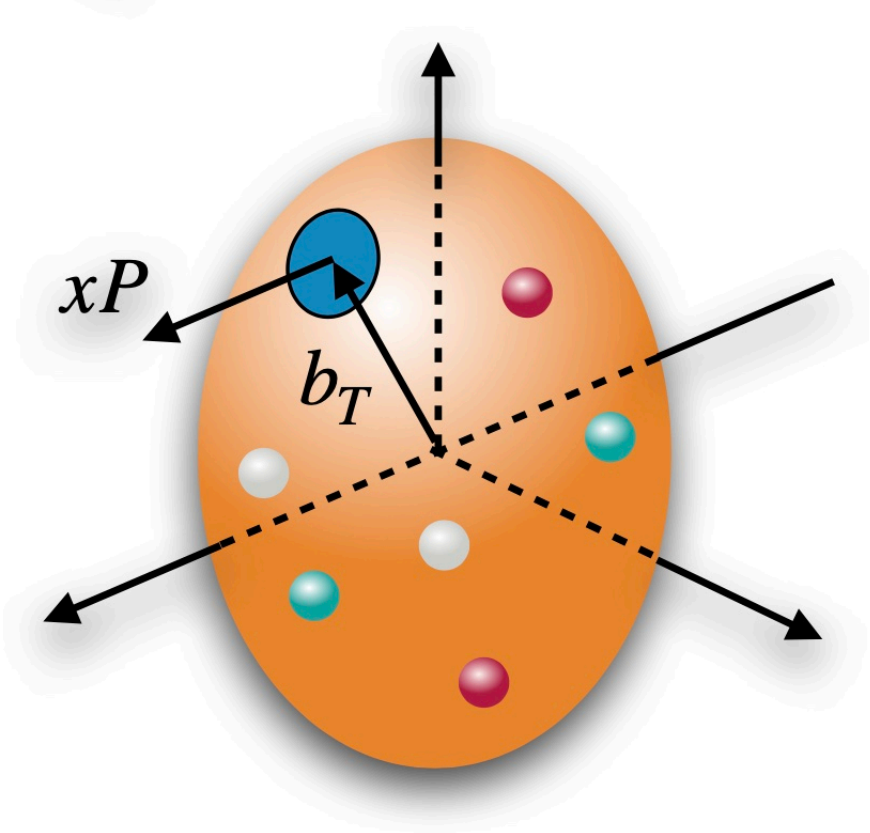


Fig. from arXiv: 2306.09360

Two-scale processes:
length resolution scale
soft momentum scale to probe the emergent regimes at different scales

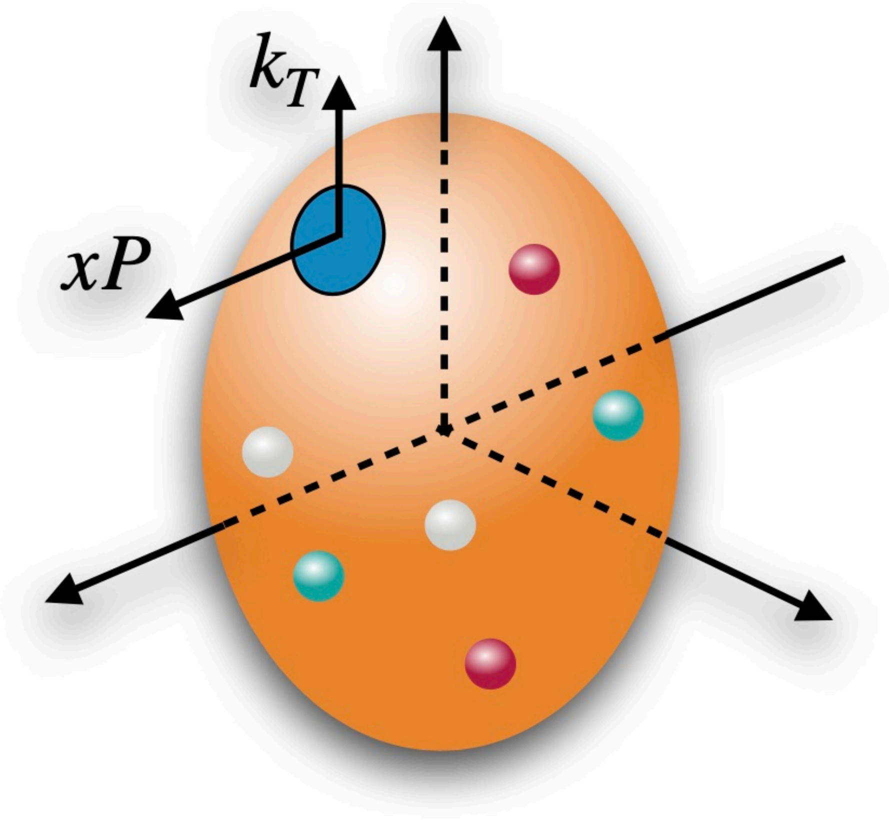
GPDs

Generalized Parton Distributions



TMDs

Transverse Momentum Dependent Distributions

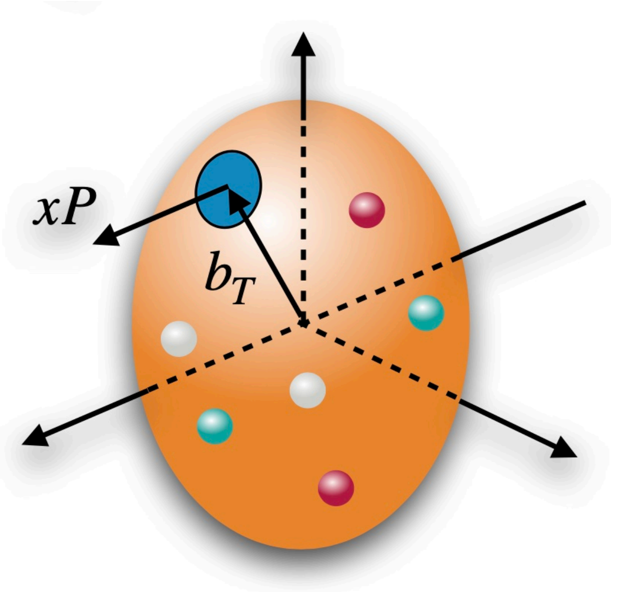


Resolution scale $1/Q^2 <<$ ⟶ Parton degrees of freedom

Emergence from QCD ⟶ 3D structure of the nucleon

in space and in momentum

Key information from GPDs



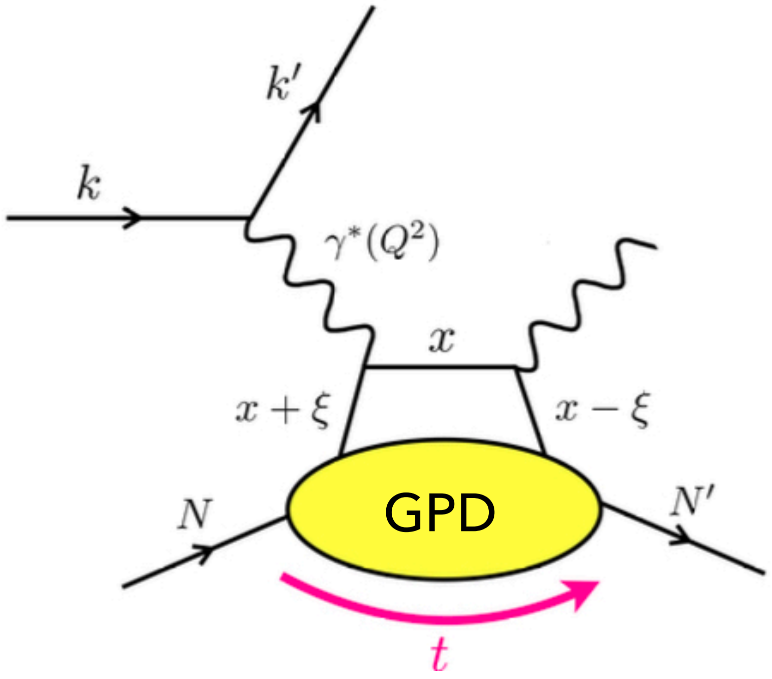
- Multidimensional picture of the proton in the 1+2D
- Access to Form Factors of Energy Momentum Tensor

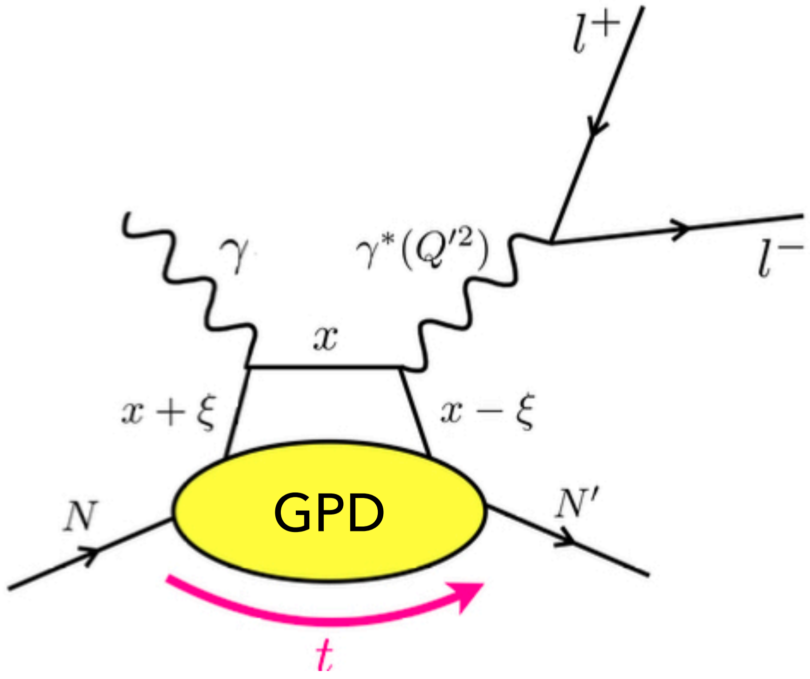
⟶ “mechanical” properties of the nucleon

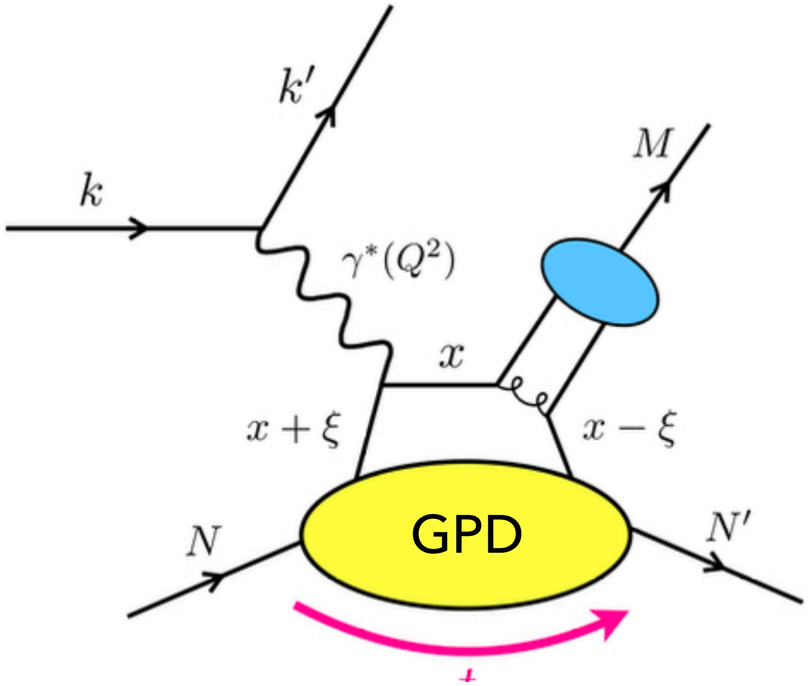
⟶ quark and gluon contribution to mass of the nucleon

- Sum rule for Angular Momentum

How to measure GPDs







- ‣accessible in exclusive reactions: universality of GPDs
- ‣factorization for large Q^2, $|t| << Q^2$, W^2
- ‣depend on 3 variables: x, ξ, $t = \Delta^2$
- ‣Compton form factors $\mathrm{Im}\mathcal{H} \overset{\mathrm{LO}}{=} H(\xi,\xi,t)$ $\mathrm{Re}\mathcal{H} \overset{\mathrm{LO}}{=} \mathcal{P}\int_{-1}^{1} \mathrm{d}x \frac{H(x,\xi,t)}{x-\xi}$

GPD table: leading twist

quark polarization

nucleon polarization

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T, \ \tilde{H}_T$

*similar classification for gluon GPDs

GPDs in **black** survive in the collinear limit and reduce to the PDFs

GPDs in **red** vanish if there is no quark orbital angular momentum

$$(\text{at } \xi = 0) \quad \vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$$

Impact Parameter Distributions

Burkardt., IJMA 18 (2003) 173

x-dependent transverse squared charge radius

$$H(x,0,\vec{b}_\perp) = \int_{-\infty}^{+\infty} \mathrm{d}^2\vec{\Delta}_\perp\, H(x,0,t)\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \quad\Longrightarrow\quad \langle \vec{b}_\perp^{\,2}(x)\rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp\, \vec{b}_\perp^{\,2}\, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp\, H(x,0,b_\perp)}$$

$(t = -\vec{\Delta}_\perp^2)$ $\quad \xi = 0$ extrapolation from data

x-dependent transverse squared radius

x-dependent transverse squared charge radius

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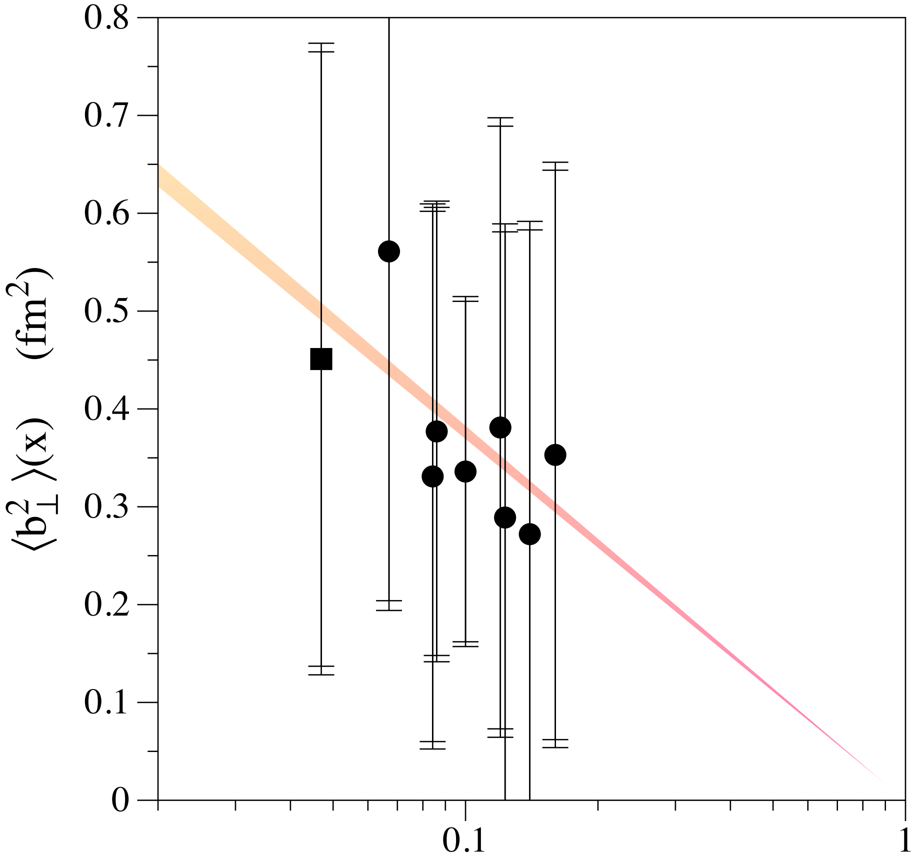
x-dependent transverse squared radius

CLAS and Hall A data



x-range of data: $0.05 \leq x \leq 0.2$

The errors are large,
but slowly we are getting some 3D information

x-dependent transverse squared charge radius

$$H(x,0,\vec{b}_\perp) = \int_{-\infty}^{+\infty} \mathrm{d}^2\vec{\Delta}_\perp \, \underset{\downarrow}{H(x,0,t)} \, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \longrightarrow \langle \vec{b}_\perp^{\,2}(x) \rangle = \frac{\int \mathrm{d}^2\vec{b}_\perp \, \vec{b}_\perp^{\,2} \, H(x,0,b_\perp)}{\int \mathrm{d}^2\vec{b}_\perp H(x,0,b_\perp)}$$

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x-dependent transverse squared radius

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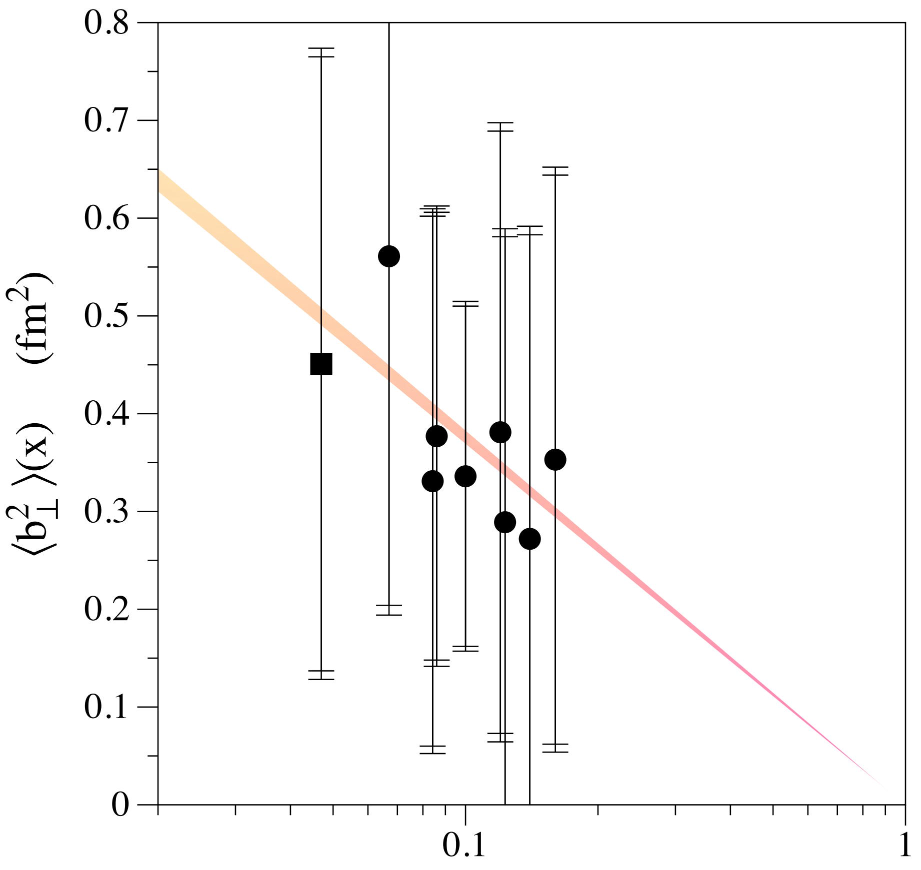
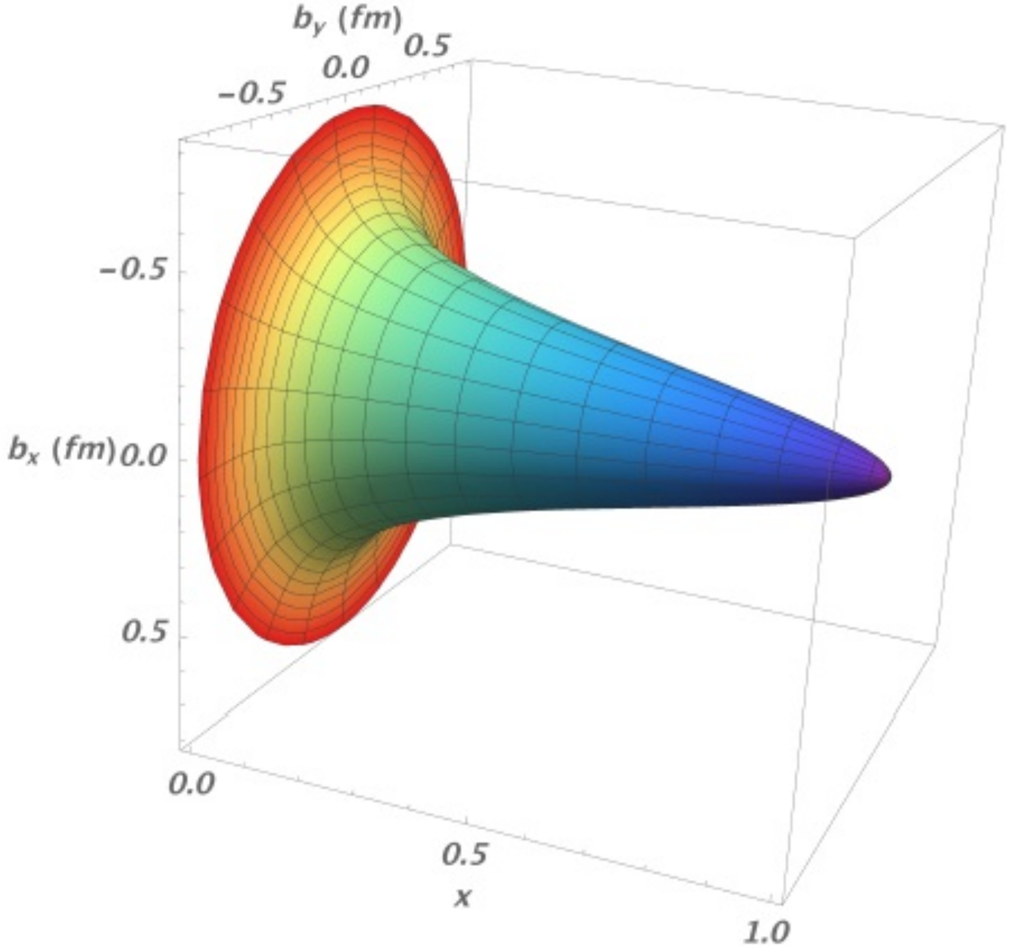


x-range of data: $0.05 \leq x \leq 0.2$

extrapolating in the unmeasured x-range $\longrightarrow$

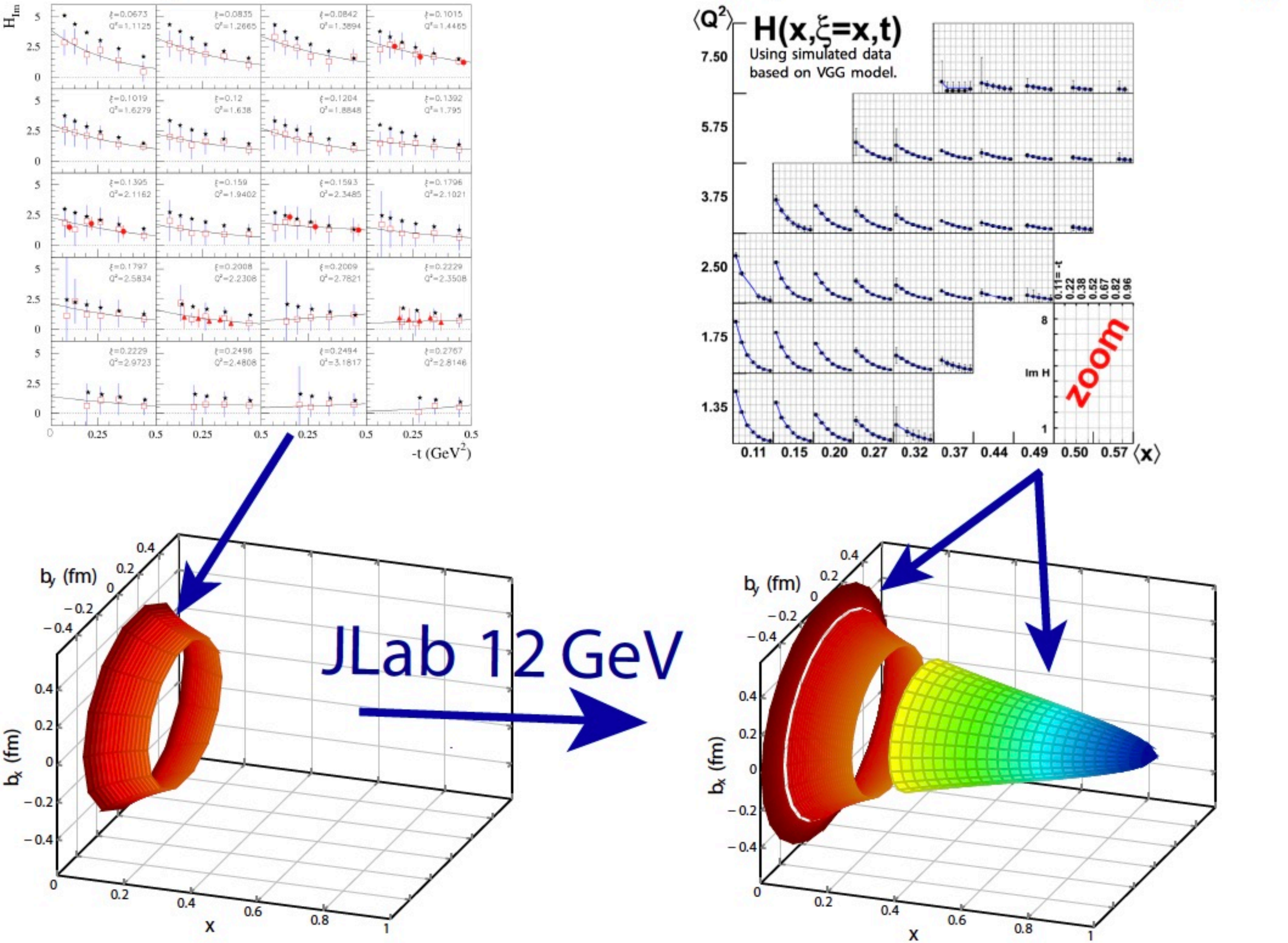


As x $\longrightarrow$ 1, the active parton carries all the momentum and represents the centre of momentum

Dupré et al., PRD95(2017)011501

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)

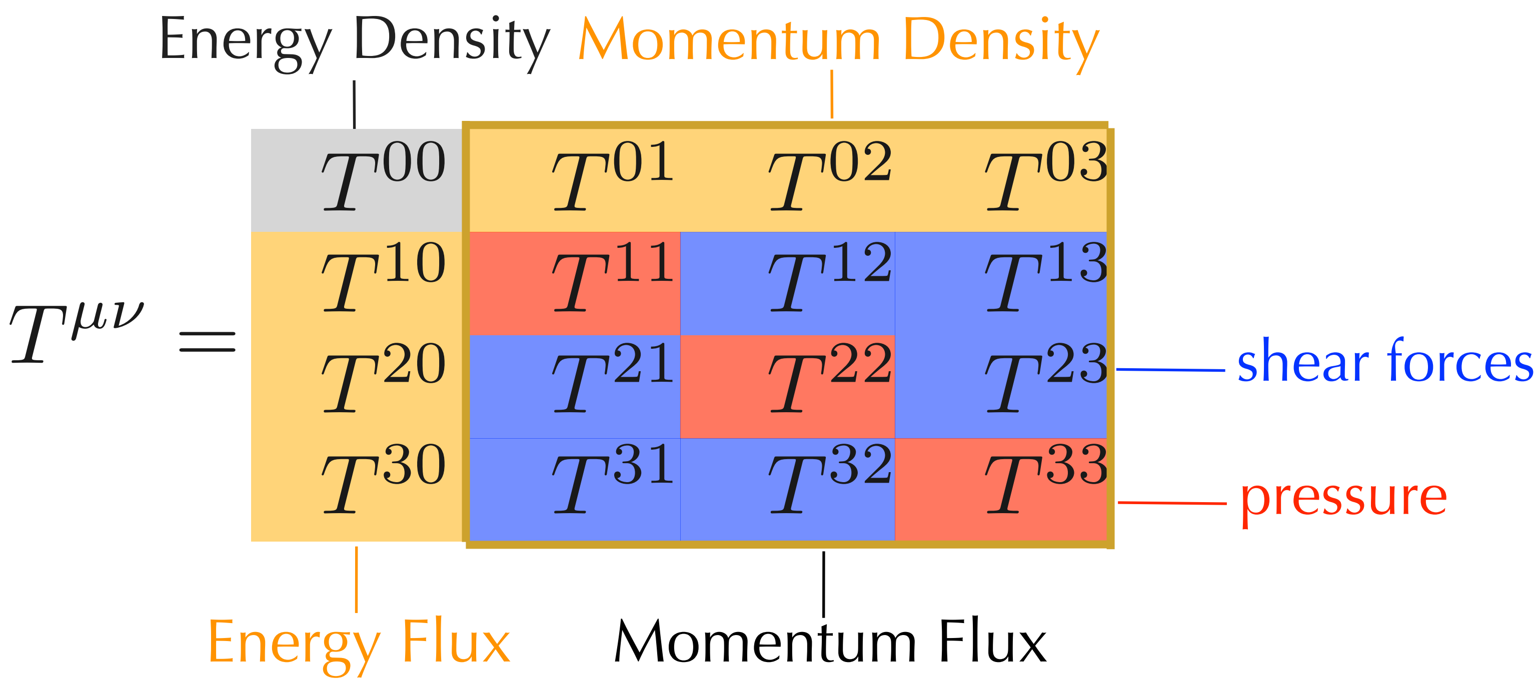
CLAS12 projections E12-06-119 with DVCS A_{UL} and A_{LU}



JLab 12 GeV

Courtesy of Z. Meziani

Form Factors of Energy Momentum Tensor



Energy Density

Momentum Density

$$T^{\mu\nu} = \begin{matrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{matrix}$$

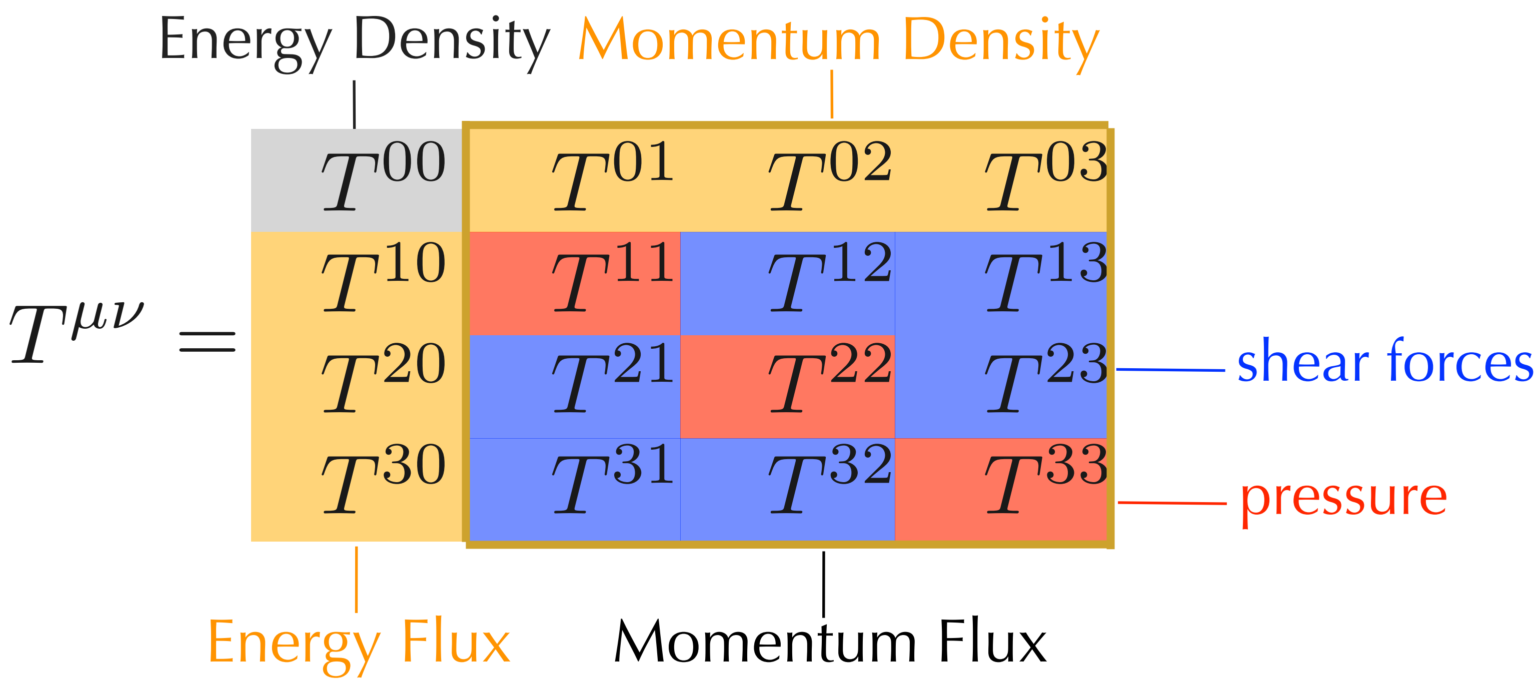
shear forces

pressure

Energy Flux

Momentum Flux

Form Factors of Energy Momentum Tensor



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shear forces

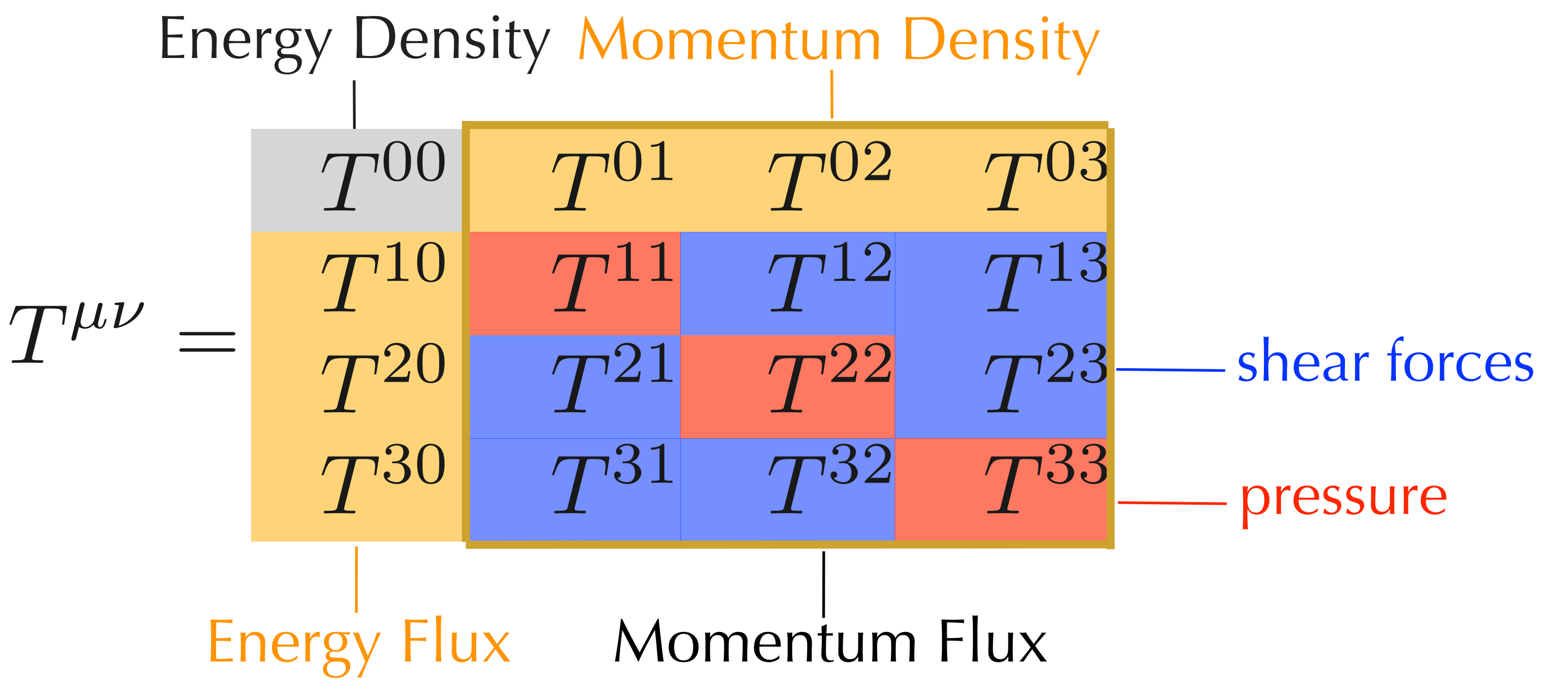
pressure

Energy Flux

Momentum Flux

$$\langle p | \, T^{Q,G}_{\mu\nu} \, | p' \rangle = \bar{u}(p') \left[M_2^{Q,G}(t) \frac{P_\mu P_\nu}{M_N} + J^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \pm \bar{c}(t) g_{\mu\nu} \right] u(p)$$

Form Factors of Energy Momentum Tensor



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Relation with second-moments of GPDs:

$$\sum_q \int \mathrm{d}x\, x\, H^q(x,\xi,t) = M_2^Q(t) + \frac{4}{5}\, d_1^Q(t)\xi^2$$

$$\sum_q \int \mathrm{d}x\, x\, E^q(x,\xi,t) = 2J^Q(t) - M_2^Q(t) - \frac{4}{5}\, d_1^Q(t)\xi^2$$

“Charges” of the EMT Form Factors at t=0

$M_2(0)$	nucleon momentum carried by parton
$J(0)$	angular momentum of partons
$d_1(0)$	D-term (“stability” of the nucleon)

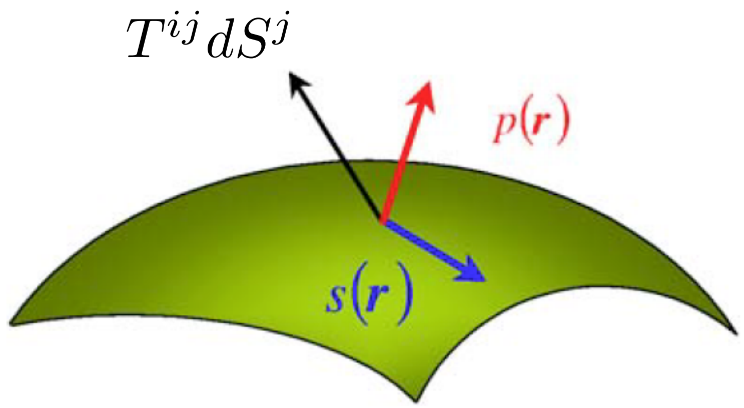
D(t) form factor from data

Fourier transform in coordinate space

$$T^{Q}_{ij}(\vec{r}) = \underset{\substack{\downarrow \\ \text{shear forces}}}{s(\vec{r})} \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij} \right) + \underset{\substack{\downarrow \\ \text{pressure}}}{p(\vec{r})}\, \delta_{ij}$$

- talk of J. Panteleeva

“mechanical properties” of nucleon



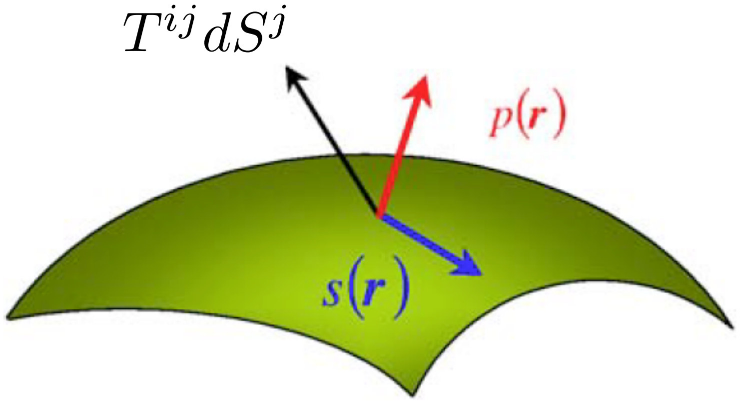
D(t) form factor from data

➡ Fourier transform in coordinate space

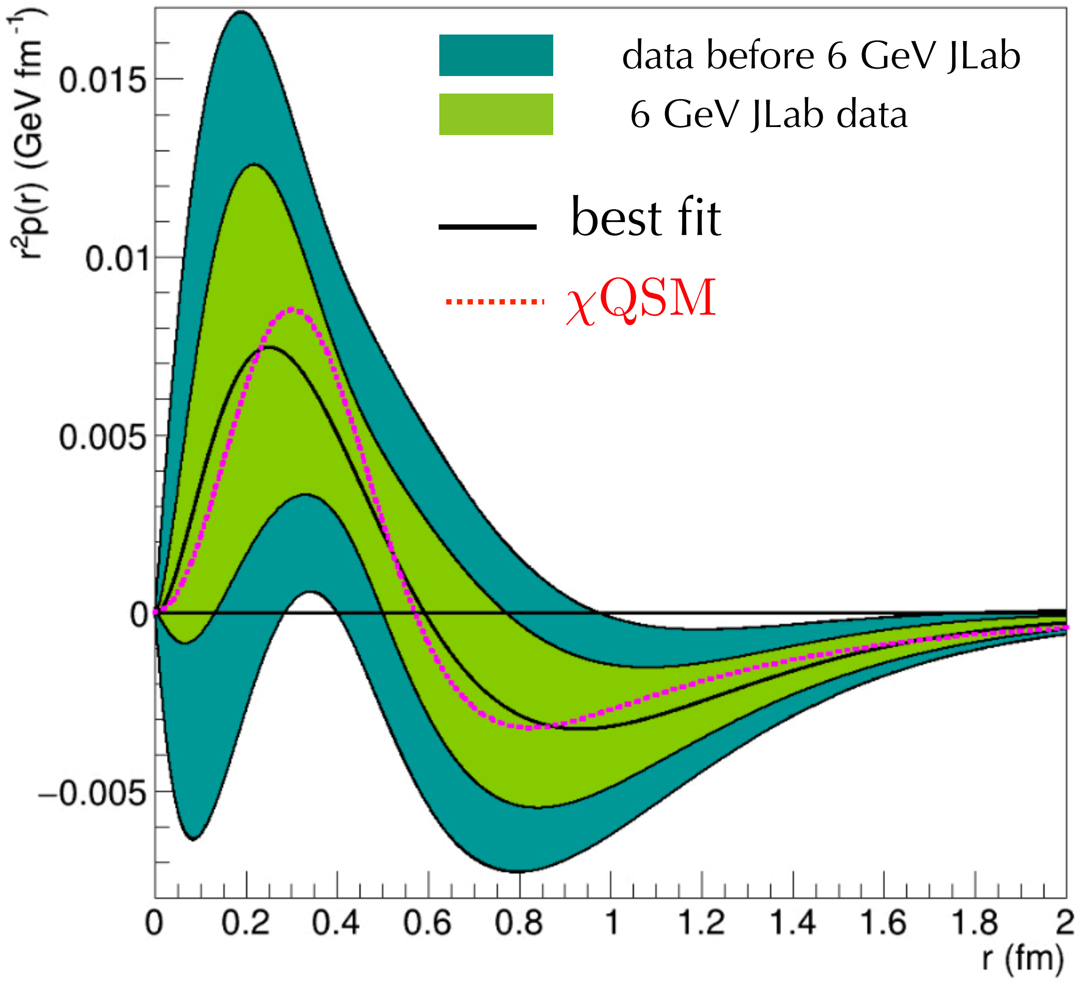
“mechanical properties” of nucleon

$$T^Q_{ij}(\vec{r}) = \underset{\text{shear forces}}{\underset{\downarrow}{s(\vec{r})}} \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij} \right) + \underset{\text{pressure}}{\underset{\downarrow}{p(\vec{r})}}\, \delta_{ij}$$

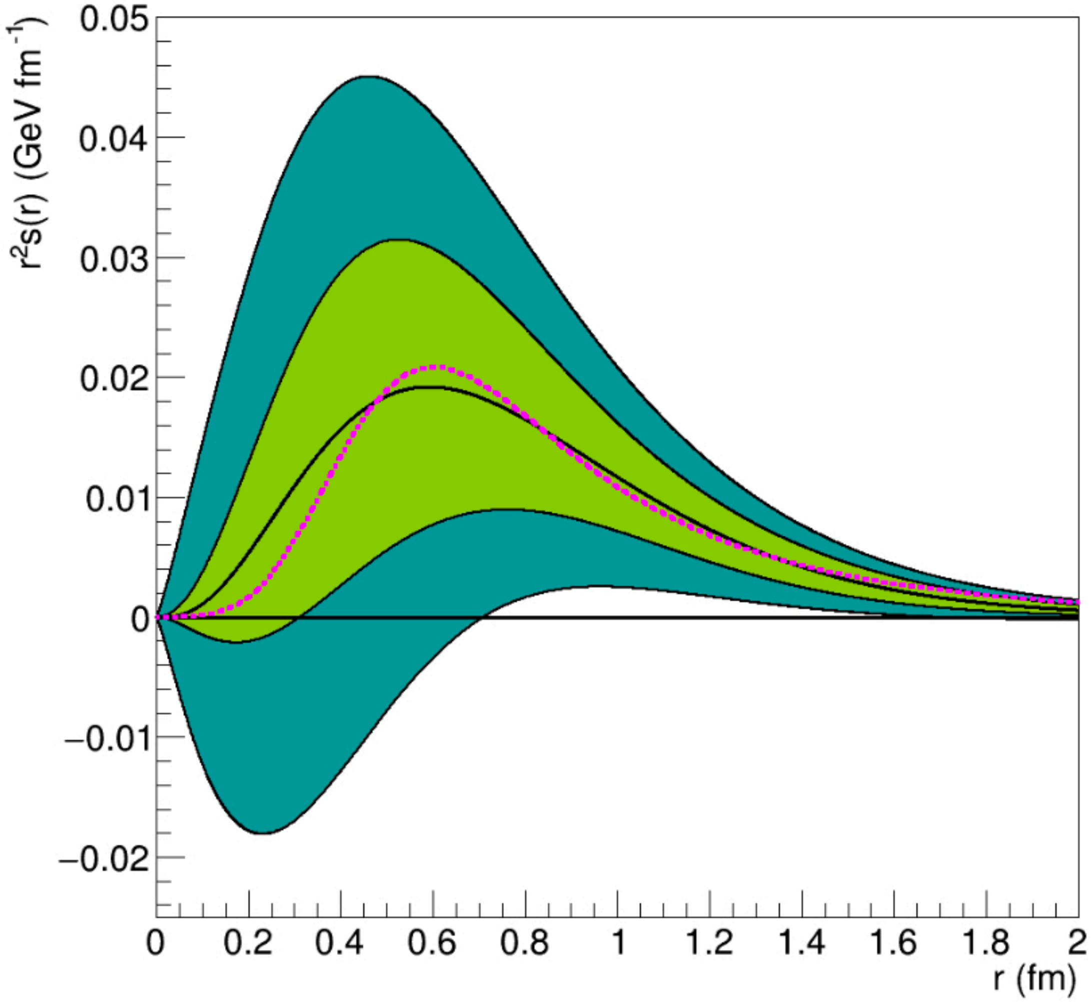
- talk of J. Panteleeva



$$p(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}D(r)$$



$$s(r) = -\frac{1}{4M_N}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}D(r)$$



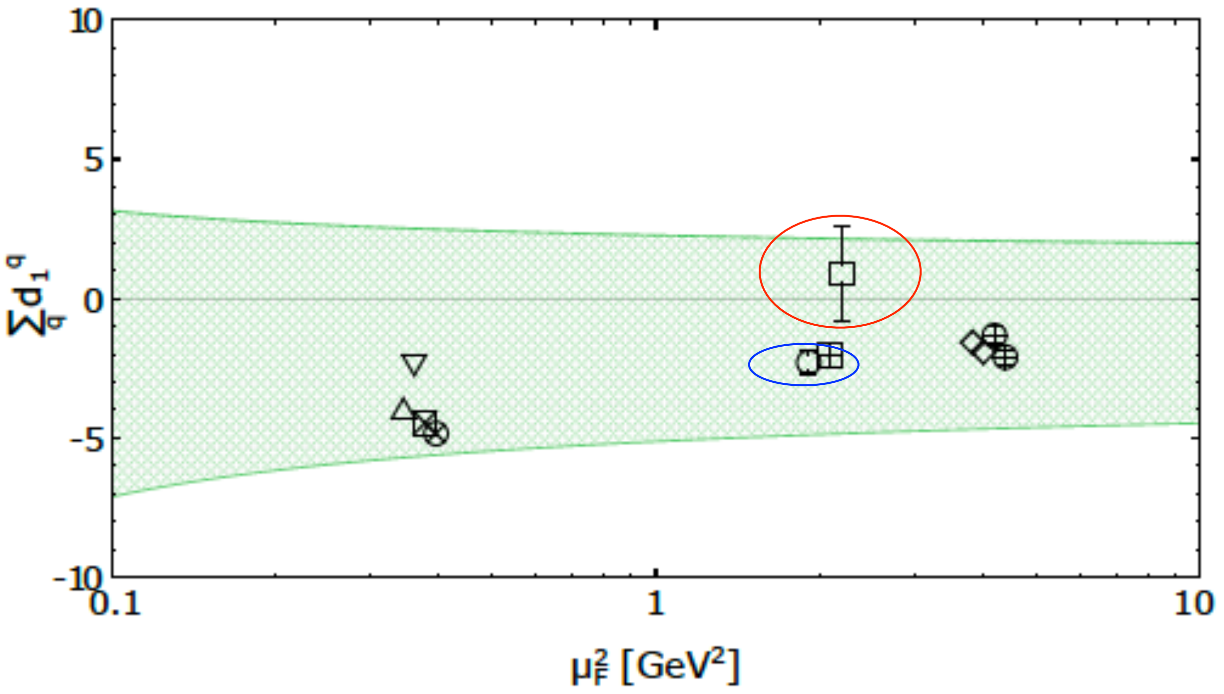
Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705; Polyakov, Schweitzer, IJMA 33 (2018) 1830025

Necessary to verify model assumptions in the exp extraction with more data coming from JLab, COMPASS and the future EIC, EIcC

Kumericki, Nature 570 (2019) 7759; Dutrieux et al, Eur. Phys. J. C81 (2021) 4

🟩

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param., Girod et al.

CLAS data, with neural networks Kumericki

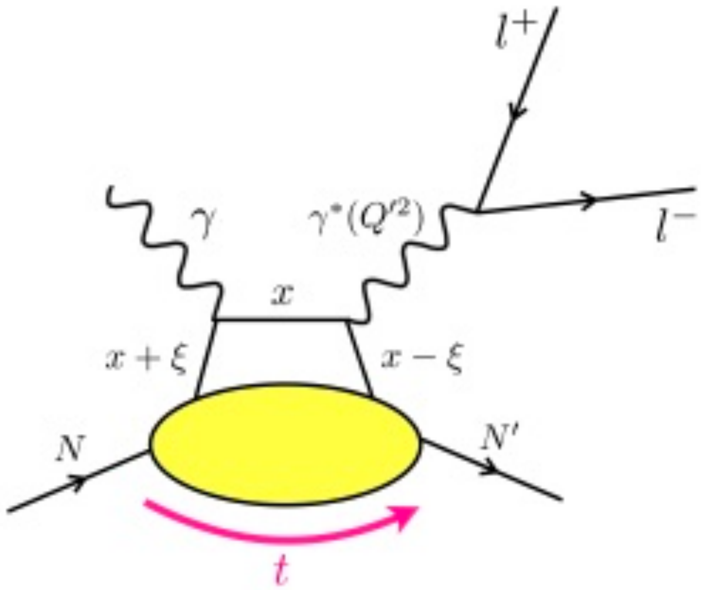
Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
□	0.88 ± 1.69	2.2	2	from experimental data
◇	-1.59	4	2	t-channel saturated model
	-1.92	4	2	t-channel saturated model
△	-4	0.36	3	χQSM
▽	-2.35	0.36	2	χQSM
⊠	-4.48	0.36	2	Skyrme model
⊞	-2.02	2	3	LFWF model
⊗	-4.85	0.36	2	χQSM
⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)

$$\sum_q d_1^q < 0$$

in all model calculations
for a stable proton

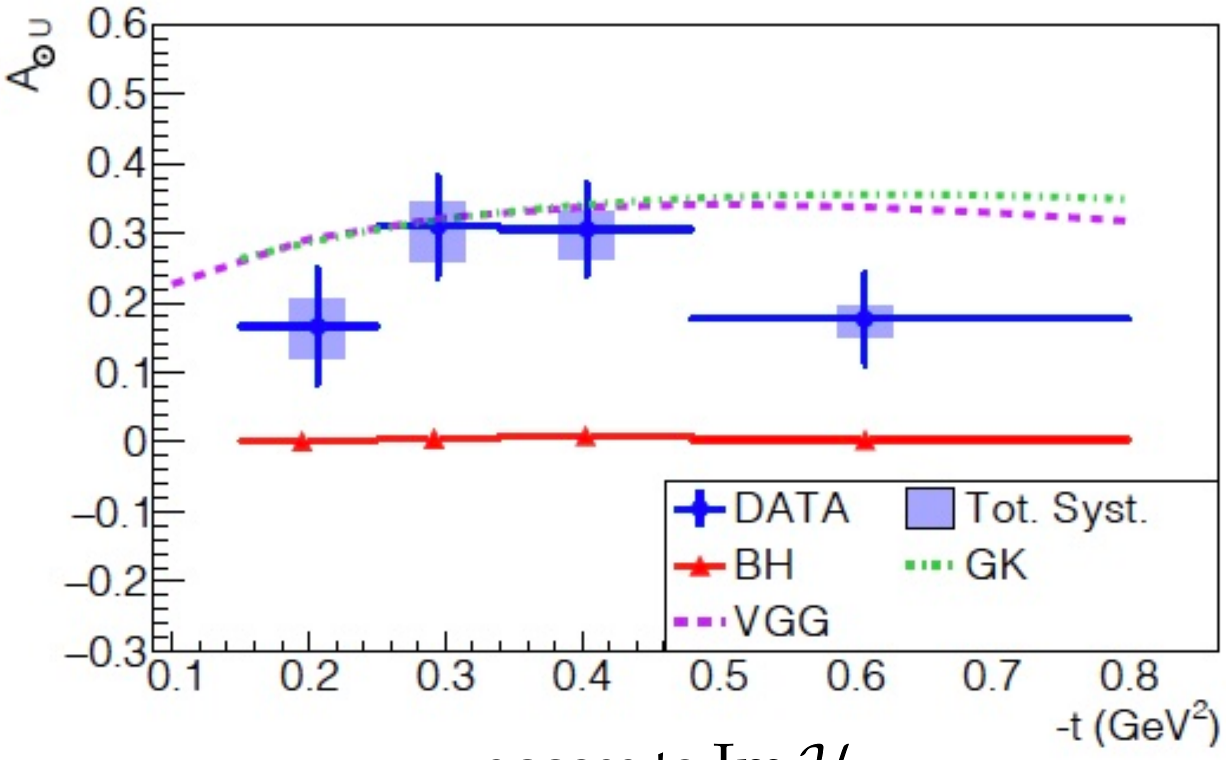
Timelike Compton scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)



photon polarization asymmetry

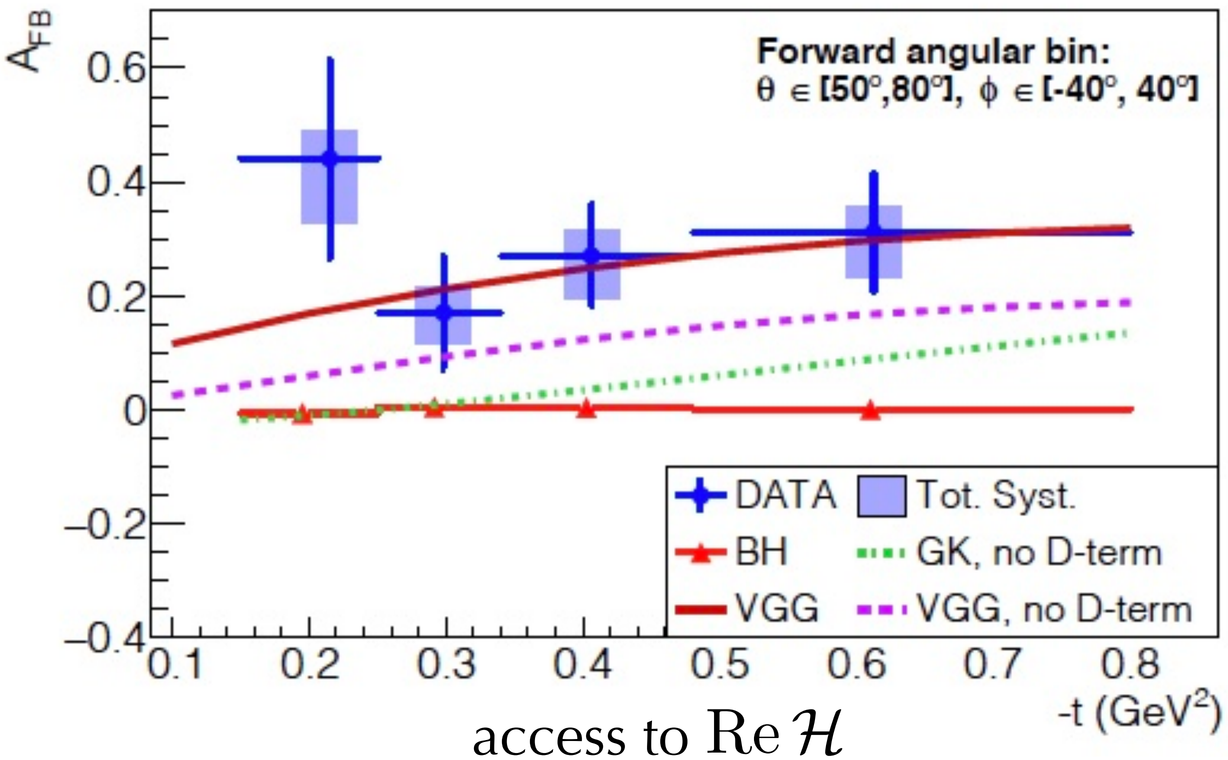
$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$



access to $\mathrm{Im}\,\mathcal{H}$

forward-backward asymmetry

$$A_{FB} = \frac{d\sigma(\theta,\phi) - d\sigma(180^\circ - \theta, 180^\circ + \phi)}{d\sigma(\theta,\phi) + d\sigma(180^\circ - \theta, 180^\circ + \phi)}$$



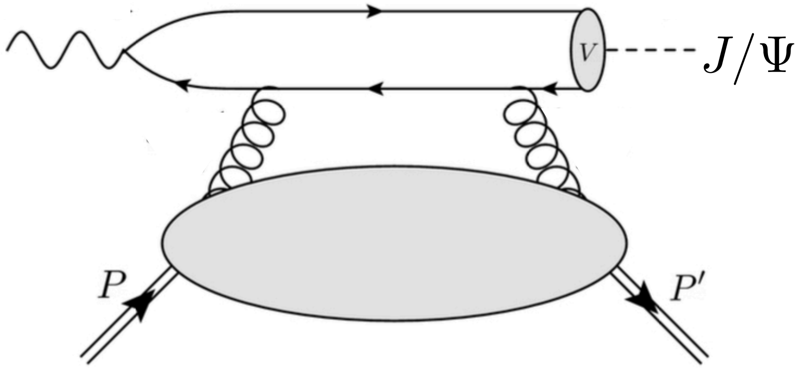
access to $\mathrm{Re}\,\mathcal{H}$

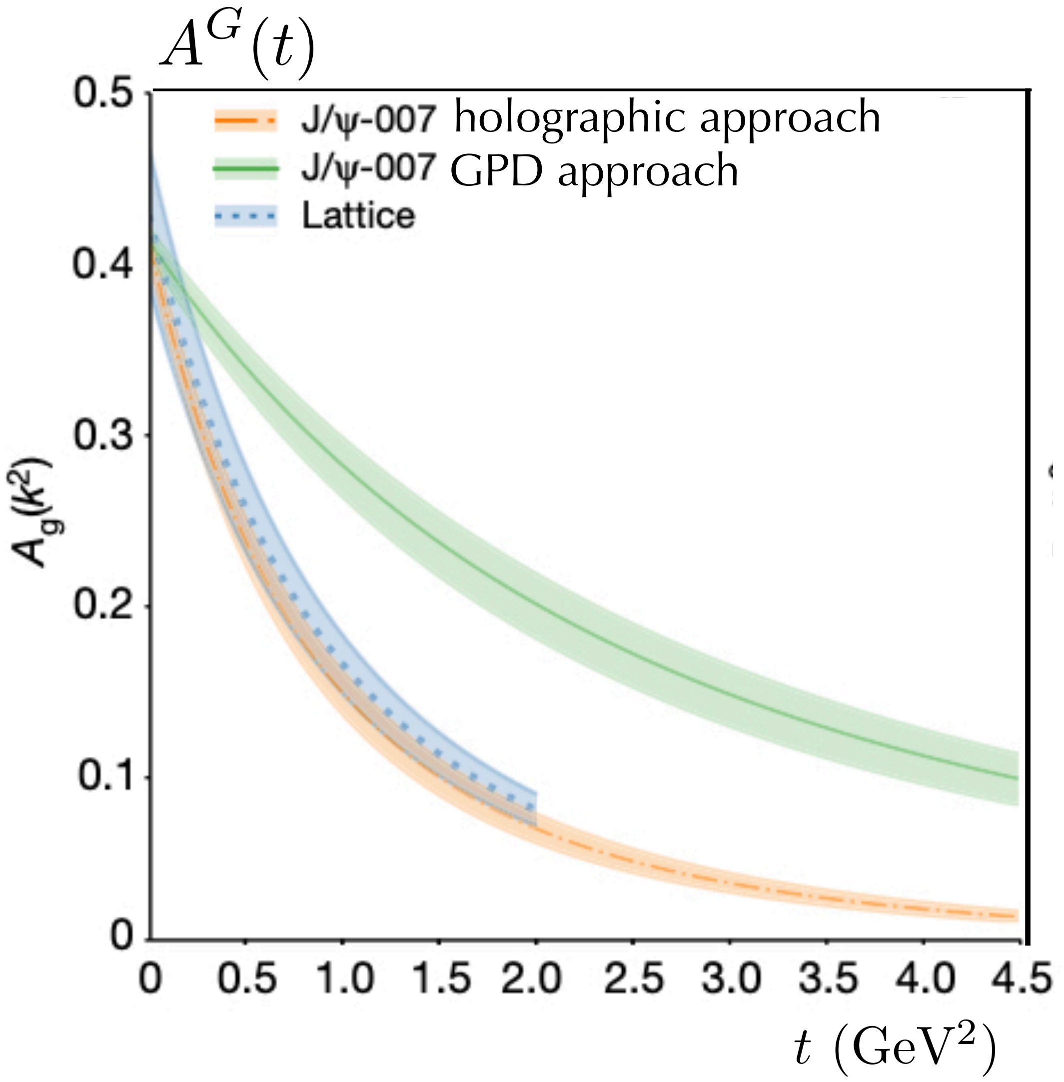
- ✓ Test of the universality of GPDs
- ✓ Further data from JLab12 and future EIC
- ✓ New promising path towards the extraction of $\mathrm{Re}\,\mathcal{H}$ and then the D-term (also with positron beam)

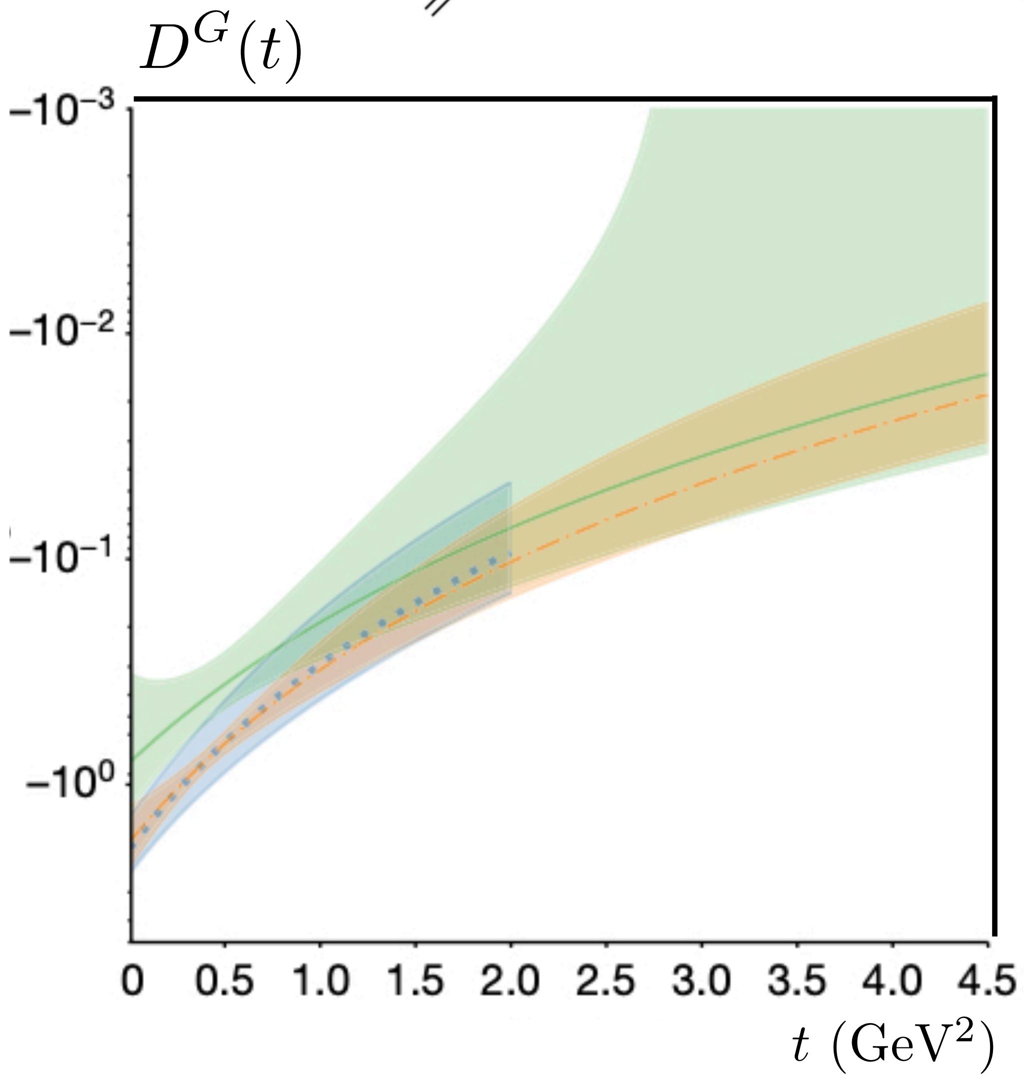
talk of E. Voutier

Gluonic EMT Form Factors

Duran et al., Nature 615 (2023) 7954



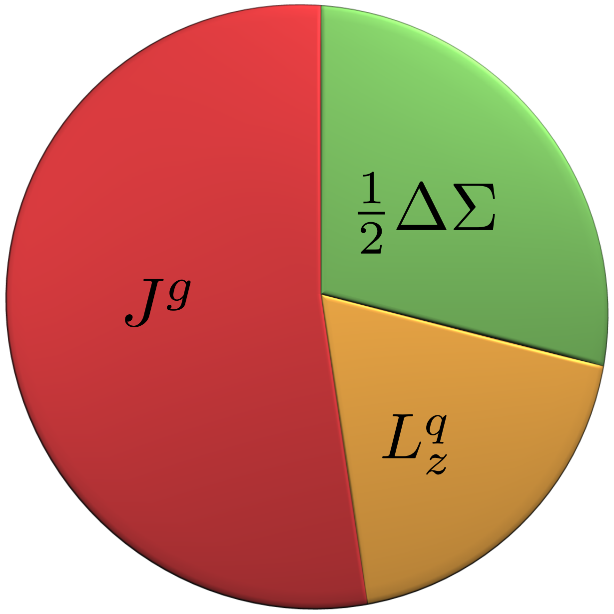




- proof of concept of feasibility to extract gluonic structure
- further measurements planned with SOLID at JLab
- JLab22 crucial for these measurements: high luminosity and leverage in t
- EIC: complementary measurements for Υ photo- and electro-production, but require $L=100$ fb^{-1}
  - 📌 talk of S. Fazio

Angular Momentum Relation

X. Ji, PRL 78 (1997) 610



$$\frac{1}{2} = J^q + J^g \qquad\qquad L_z^q = J^q - \frac{1}{2}\Delta\Sigma$$

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x\, \left(\underset{\substack{\downarrow\\ \text{at } \xi=0 \text{ unpolarized PDF}}}{H^{q,g}(x,\xi,0)} + \underset{\substack{\downarrow\\ \text{not directly accessible}}}{E^{q,g}(x,\xi,0)}\right)$$

- Requires extrapolation to t=0
- Requires spanning x at fixed values of ξ ($\xi = 0$ is the most convenient)
- $J^{q,g}(x) \neq \frac{1}{2}[xH^{q,g}(x,0,0) + E^{q,g}(x,0,0)] \longrightarrow$ not angular momentum density

Angular momentum of the proton from GPD measurements

$$J^q = \frac{1}{2}\int_{-1}^{1} dx\, x\, (H^q(x,\xi,0) + E^q(x,\xi,0)) \qquad L^q = J^q - \frac{1}{2}\Delta\Sigma$$

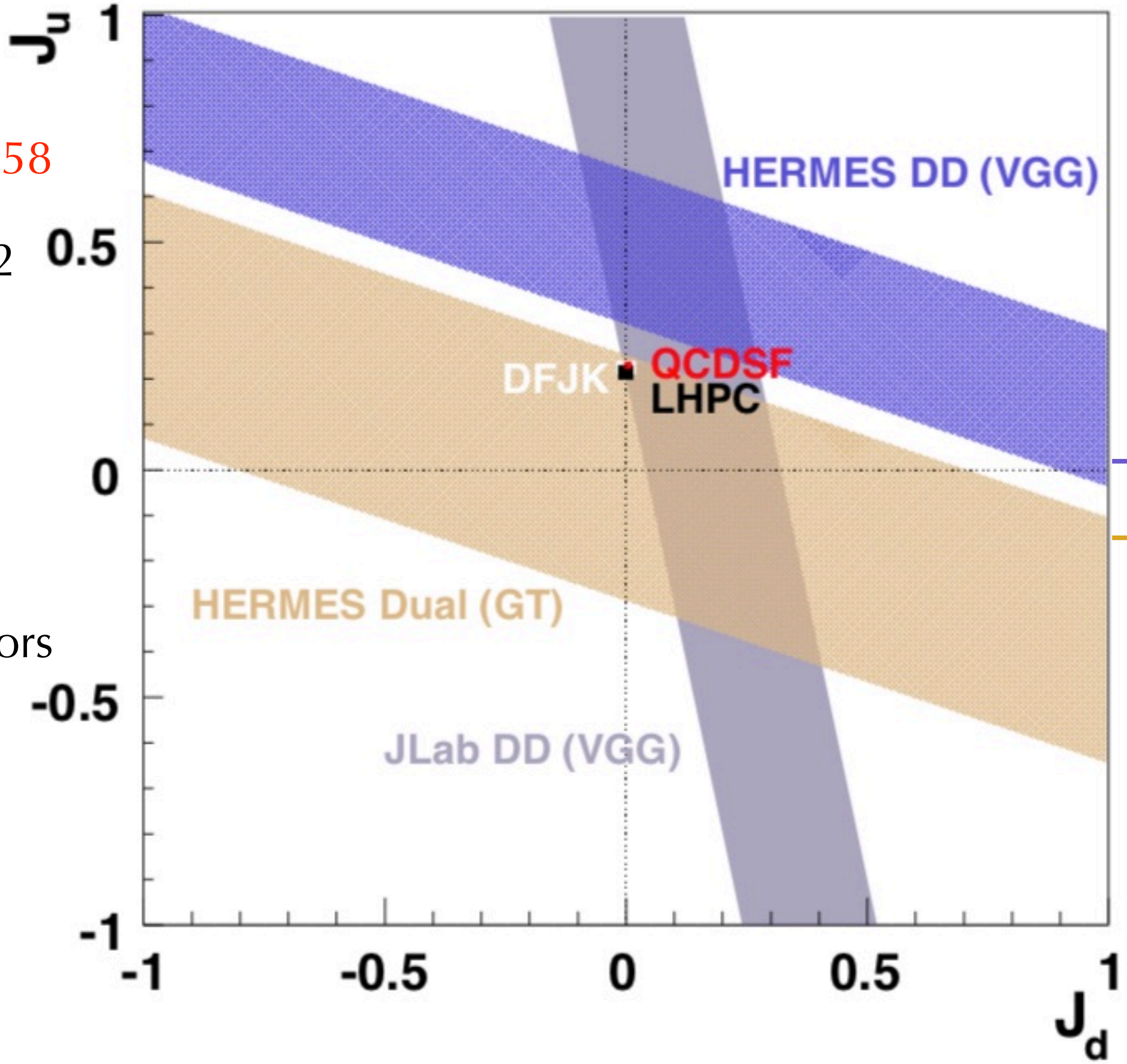
Lattice results

QCDSF: PoS (Lattice 2007) 158

LHPC: PRD77 (2008) 094502

GPDs extracted from form factors

DFJK, EPJC39 (2005) 1



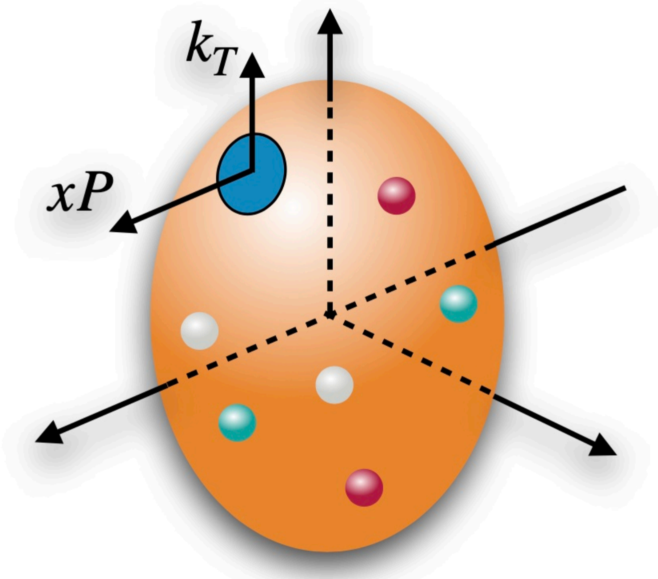
extractions from HERMES data using two different models

JLab Hall A, Phys. Rev. Lett. 99 (2007) 242501

Hermes Coll., JHEP 06 (2008) 066

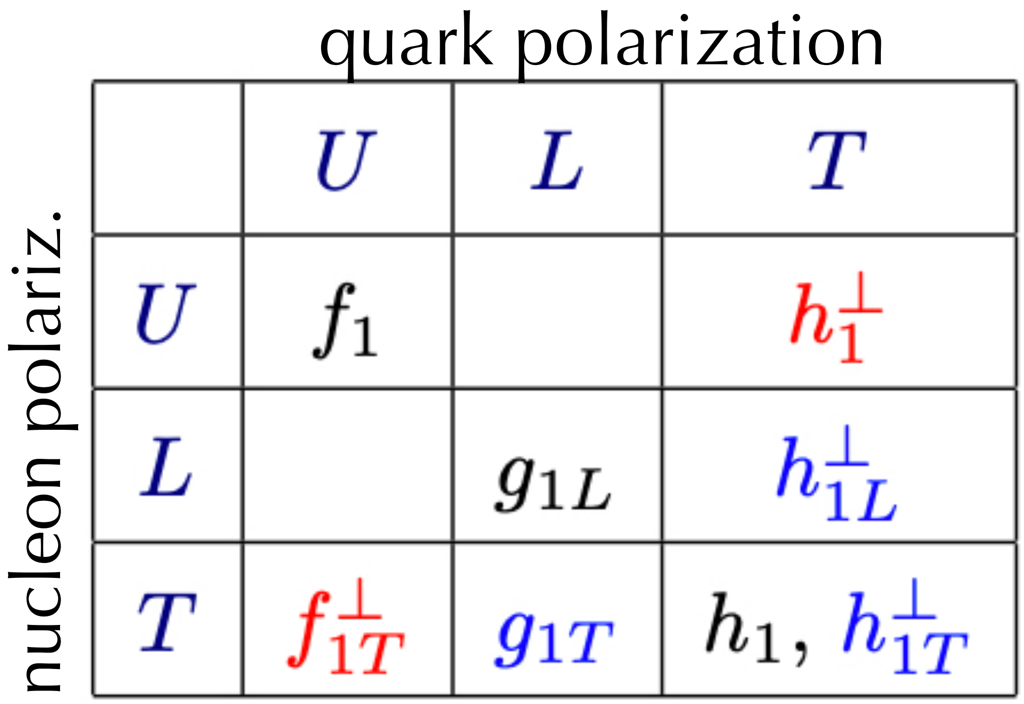
Improved accuracy with JLab12 and future EIC measurements!

Key information from TMDs



- Complete momentum spectrum of single particle
- Transverse momentum size as function of x (3D map) at different Q^2
- Spin-Spin and Spin-Orbit Correlations of partons
- Information on parton orbital angular momentum
  (no direct model-independent relation)

TMD table: leading twist



quark polarization

nucleon polariz.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

*similar classification for gluon TMDs

TMDs in **black** survive integration over transverse momentum and reduce to the PDFs

TMDs in **blue** and **red** vanish if there is no quark orbital angular momentum

TMDs in **red** are time-reversal odd

talk of L. Rossi and A. Vossen

TMD table: leading twist

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- Very good knowledge of x dependence of f_1 and g_{1L}

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- Good knowledge of the k_T dependence of f_1 (also for the pion)

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- Fair knowledge of the Sivers and transversity (mainly x dependence)

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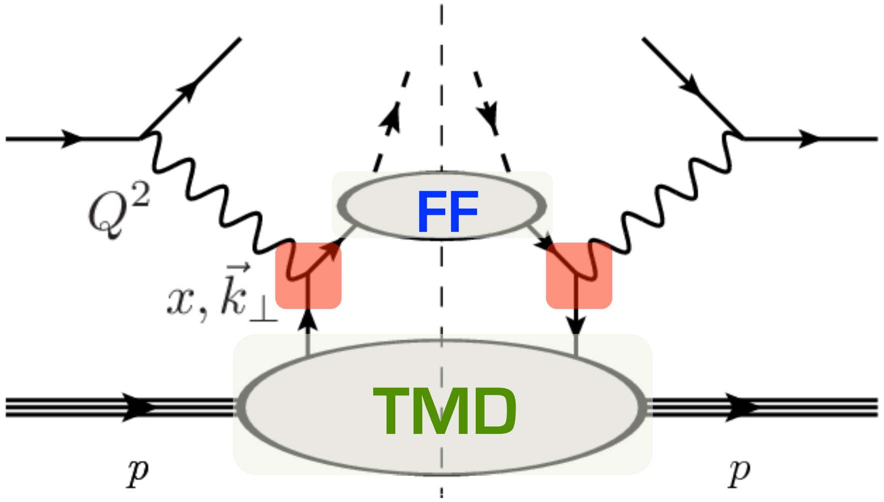
- Very good knowledge of x dependence of f_1 and g_{1L}
- Good knowledge of the k_T dependence of f_1 (also for the pion)
- Fair knowledge of the Sivers and transversity (mainly x dependence)
- Some hints about all other

talk of L. Rossi and A. Vossen

How to measure TMDs

SIDIS

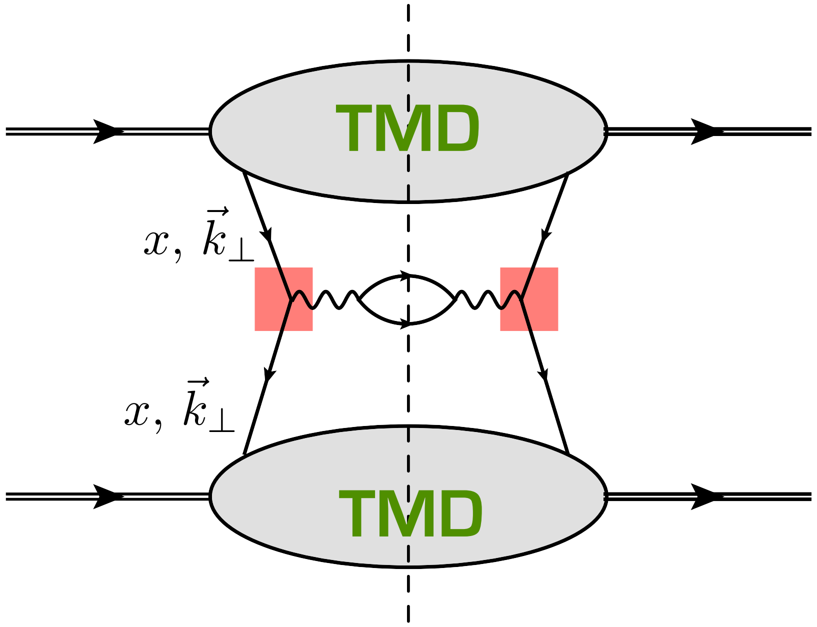
$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X$$



$$\mathrm{d}\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \mathrm{d}\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Drell-Yan

$$h(P_1) + h(P_2) \rightarrow \ell^+(l) + \ell^-(l')$$



$$\mathrm{d}\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \overline{\mathrm{TMD}}(x, \vec{k}_\perp) \otimes \mathrm{d}\hat{\sigma}_{hard}$$

✓Factorization

✓Universality

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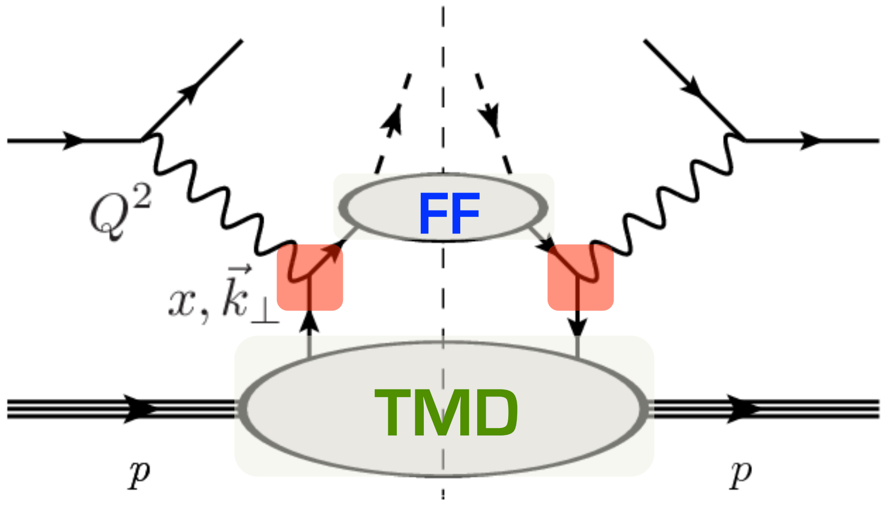

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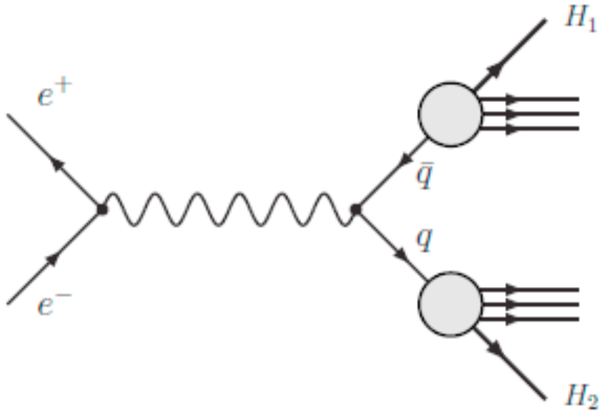
↓

Fragmentation Functions

$e^+e^- \to hh'X$



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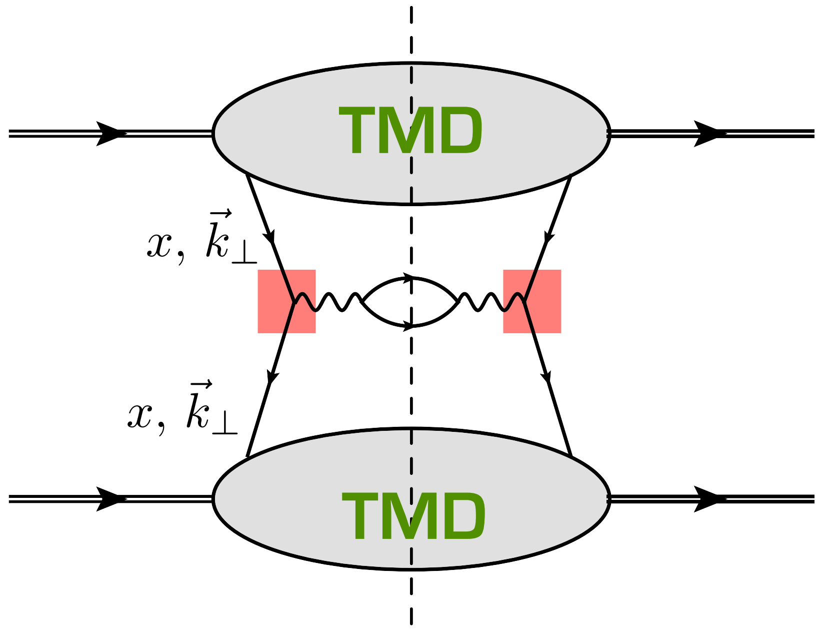

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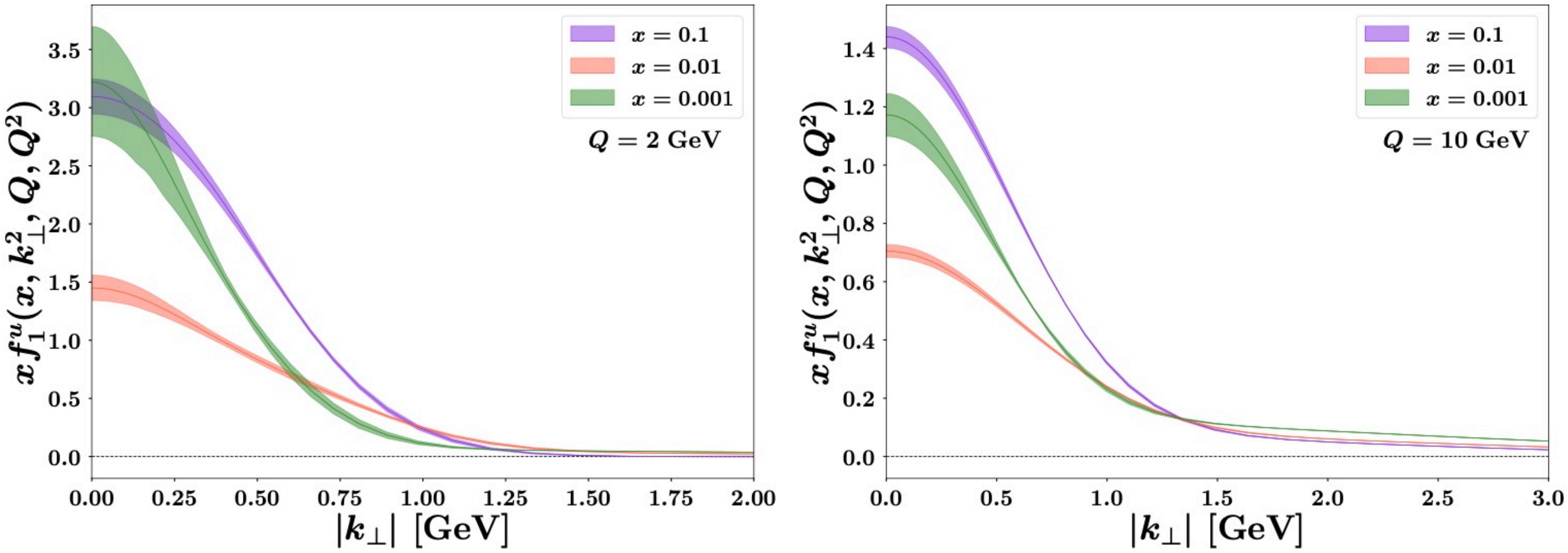
Quark unpolarized TMD extractions

	Framework	HERMES	COMPASS	DY	Z Production	N of points
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059
SV 2017 arXiv:1706.01473	NNLL	✗	✗	✓	✓	309
BSV 2019 arXiv:1902.08474	NNLL	✗	✗	✓	✓	457
Pavia 2019 arXiv:1912.07550	NNNLL	✗	✗	✓	✓	353
SV 2019 arXiv:1912.06532	NNNLL	✓	✓	✓	✓	1039
MAP 2022 arXiv:2206.07598	NNNLL	✓	✓	✓	✓	2031

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$



Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, (MAP 2022), JHEP 10 (2022) 127

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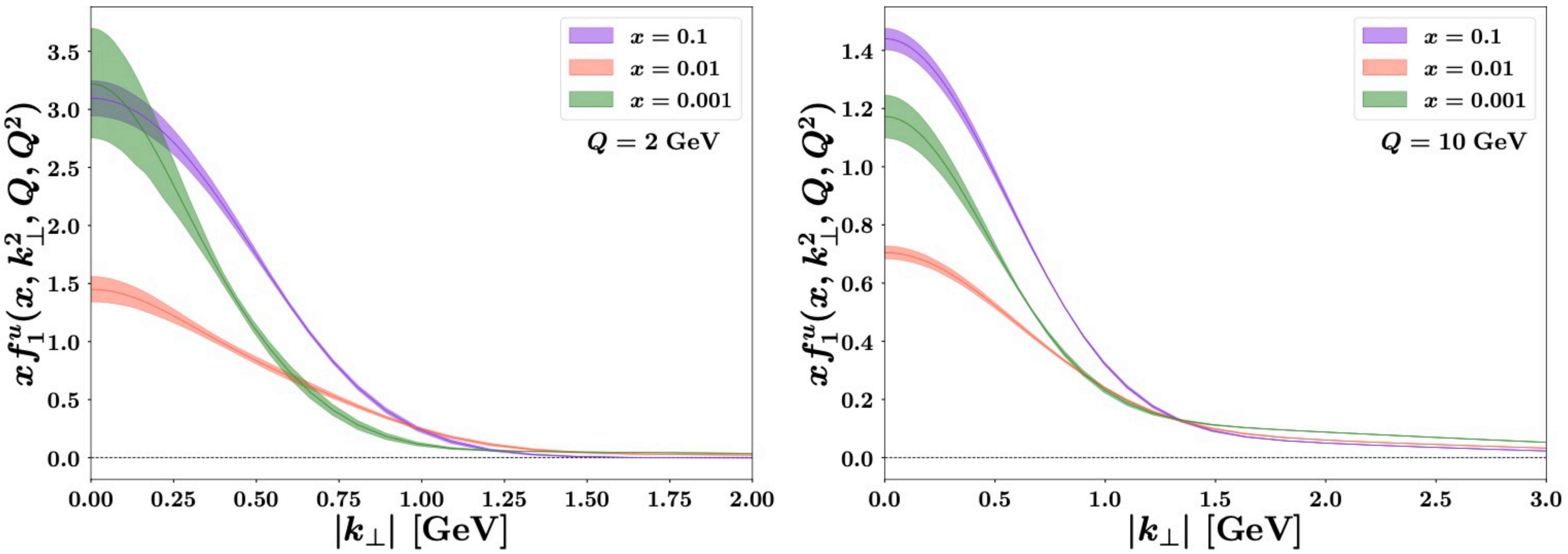
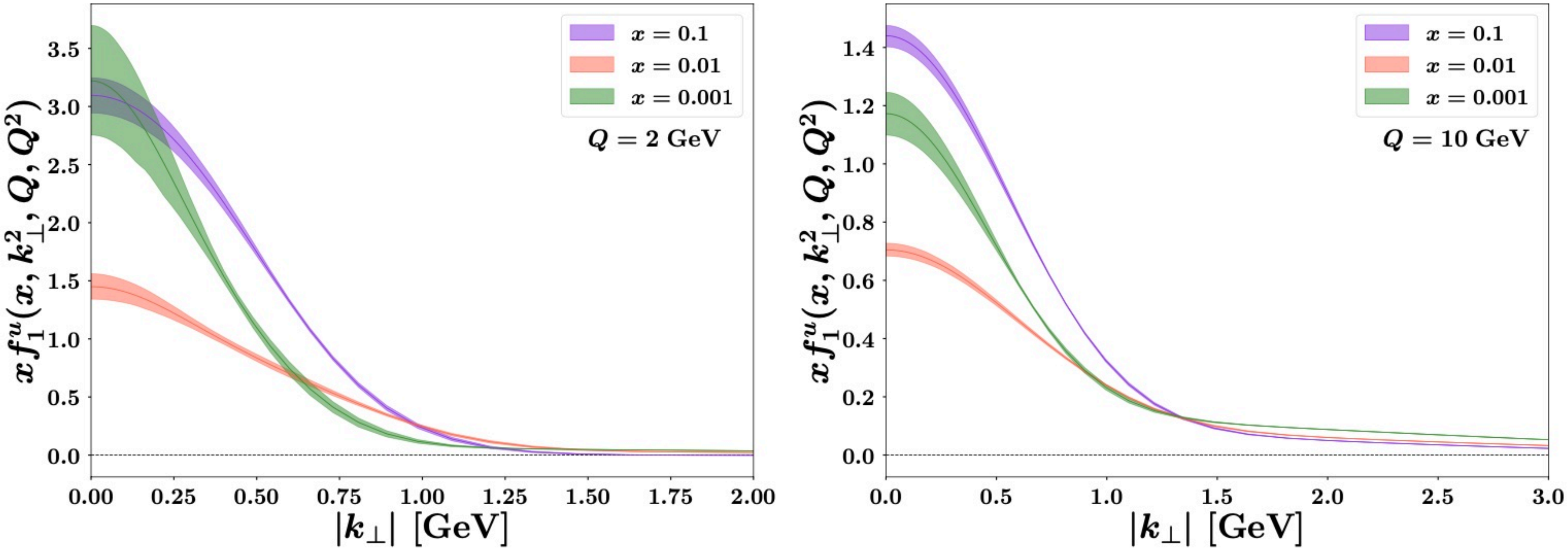


Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, (MAP 2022), JHEP 10 (2022) 127

Open issues:

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$



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Open issues:

- Flavor dependence

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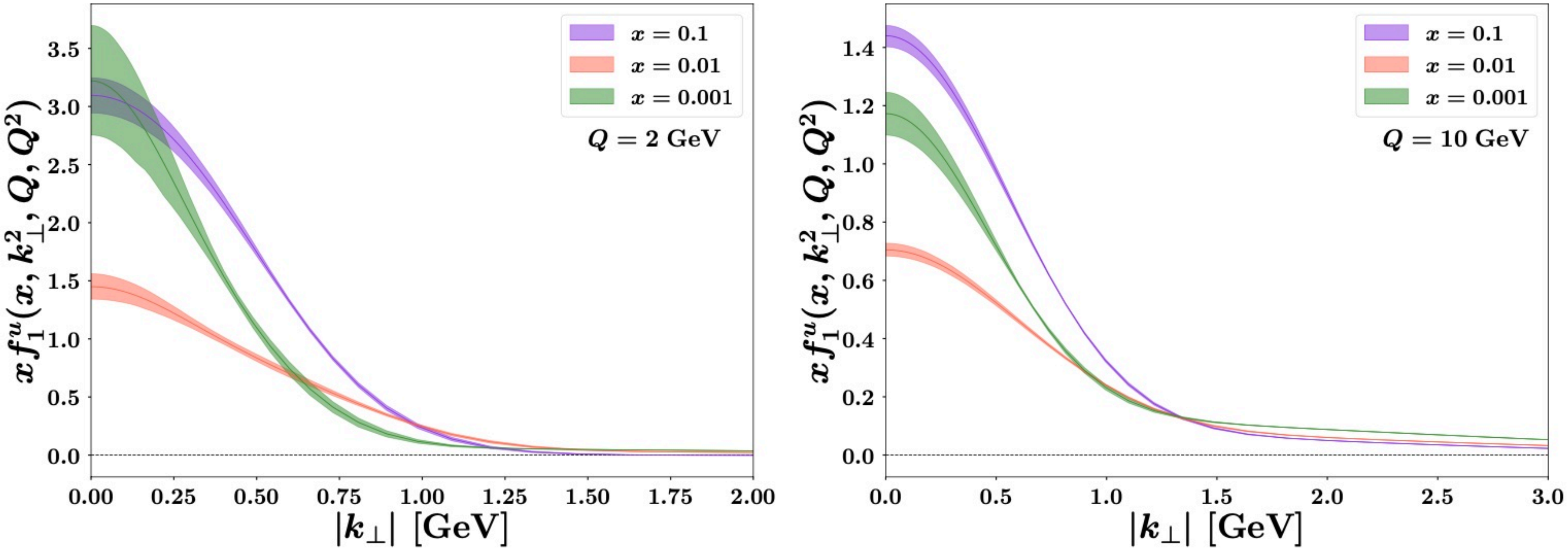

Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Piacenza, Radici, Signori, (MAP 2022), JHEP 10 (2022) 127

Open issues:

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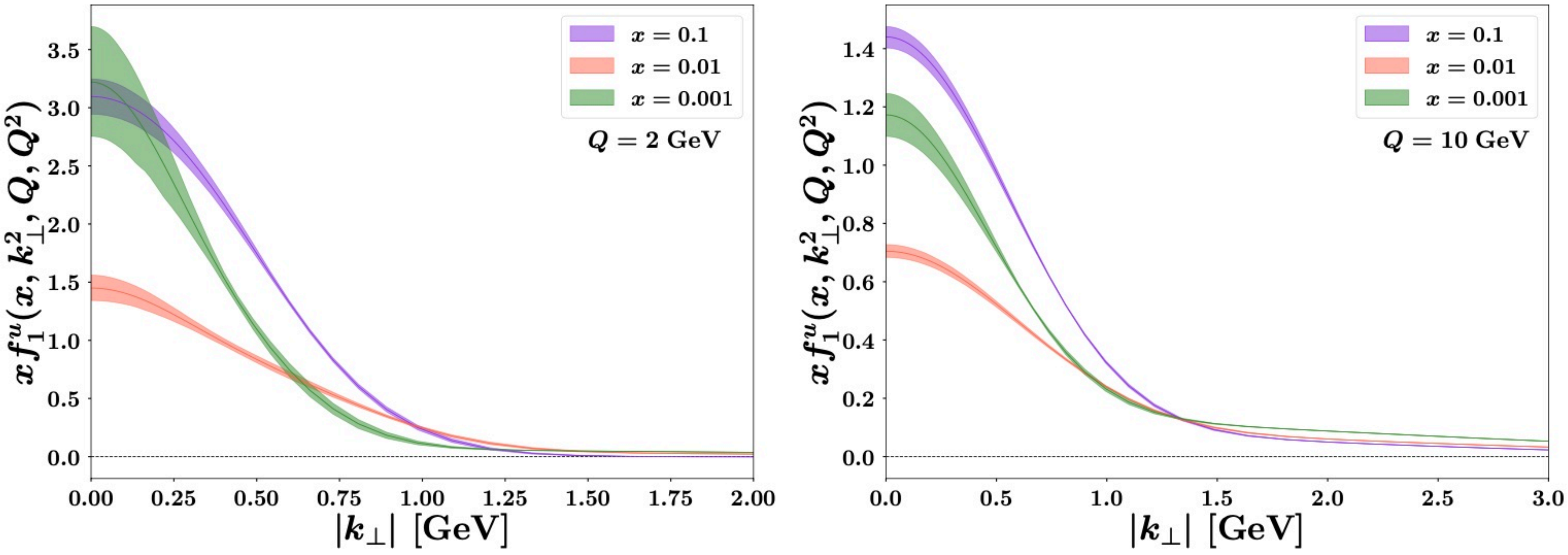
- Improvements on the knowledge of the fragmentation function

Quark unpolarized TMD extractions $f_1(x, \vec{k}_\perp)$



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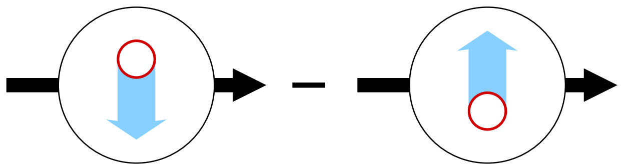
Open issues:

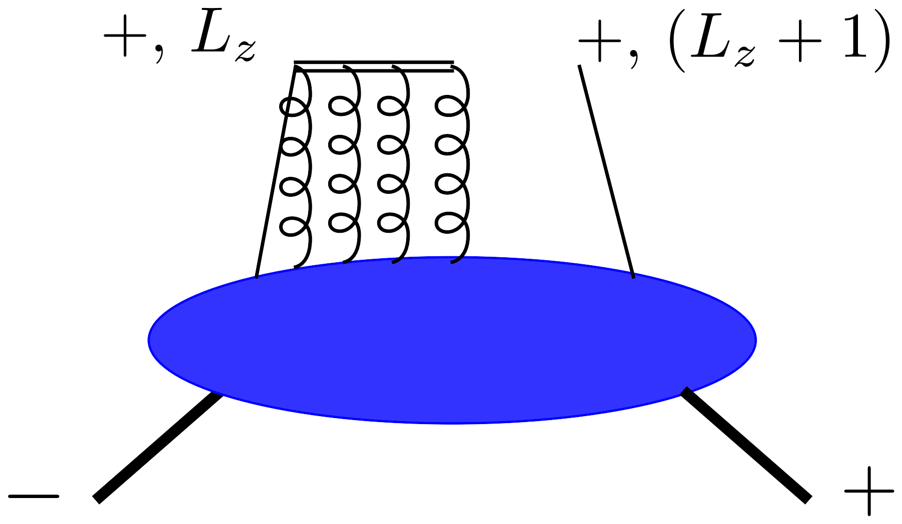
- Flavor dependence
- Improvements on the knowledge of the fragmentation function
- More data needed to the test the formalism and functional form of parametrization

Sivers function

$f_{1T}^{\perp} =$



unpolarized quarks in ⊥ pol. nucleon



the helicity mismatch requires orbital angular momentum (OAM)

non trivial correlation between quark OAM and nucleon transverse spin

no counterpart in IPD and PDF case

non-zero ONLY with final(initial)-state interaction

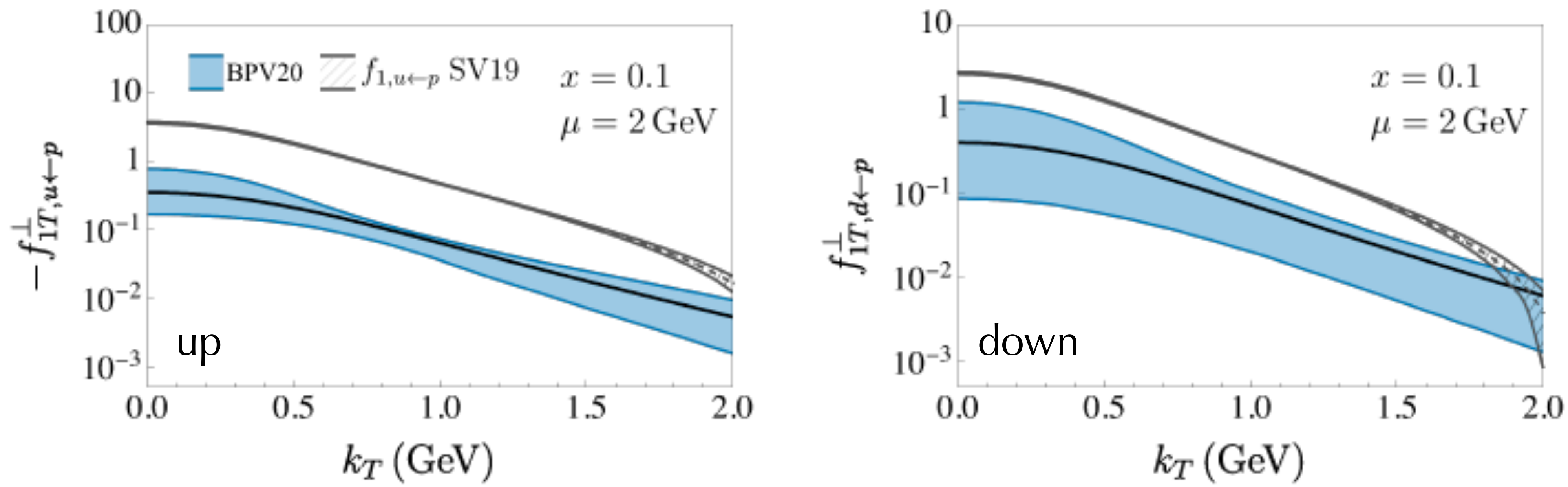
$$f_{1T}^{\mathrm{SIDIS}}(x, k_{\perp}) = -f_{1T}^{\mathrm{DY}}(x, k_{\perp})$$

first hints of sign change from STAR and COMPASS data

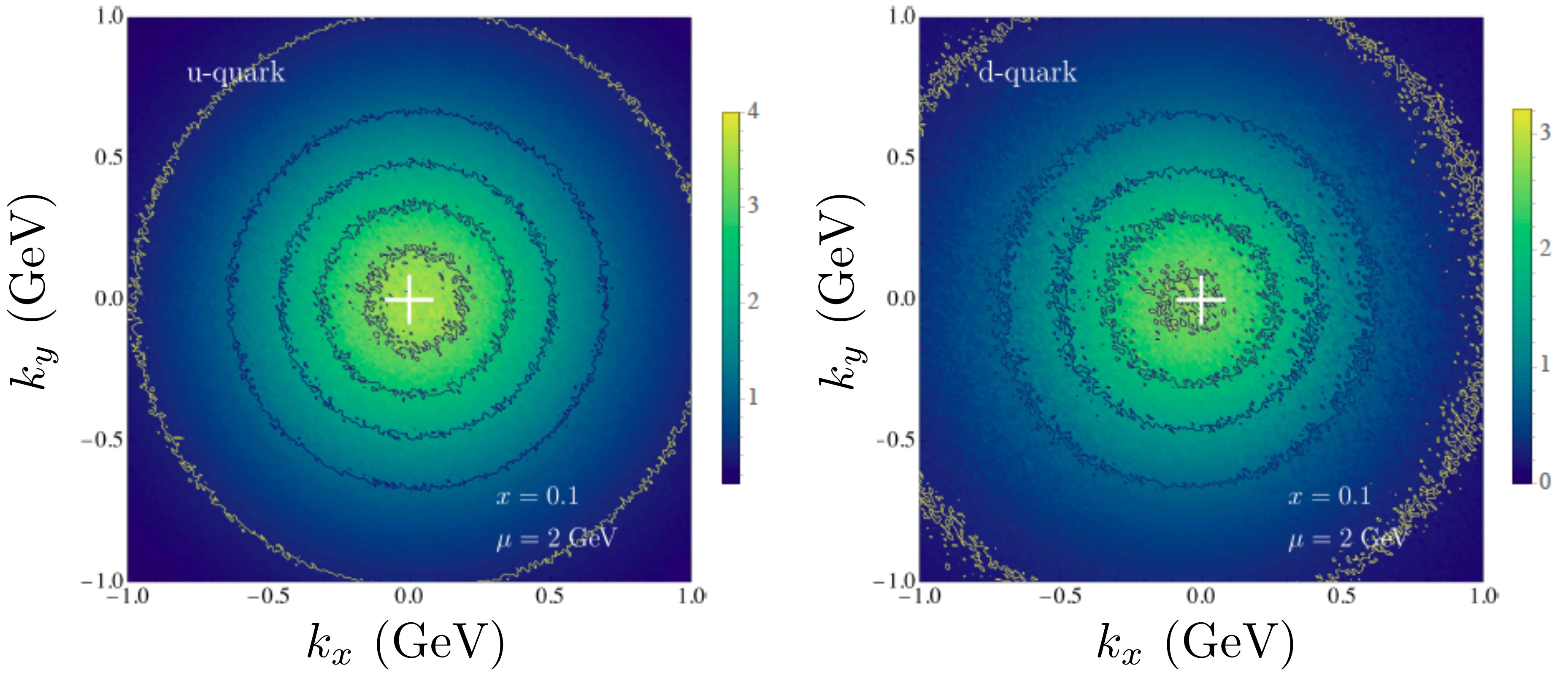
Global fit to SIDIS, DY, W$^{\pm}$/Z boson production

f_1

$f_{1T}^{\perp}$



$$\rho_{UT_y}(x,\vec{k}_\perp,S_y) = f_1(x,k_\perp) - \frac{k_x}{M} f_{1T}^{\perp}(x,k_\perp)$$



M. Bury, A. Prokudin, A. Vladimirov, JHEP 05 (2021) 151

See also extraction also from MAP Coll., JAM20 Coll., Echevarria et al.

Library and Plotting tools for collinear parton distributions

LHAPDF

lhapdf.hepforge.org

xFitter

www.xfitter.org

APFEL++

github.com/vbertone/apfelxx

apfel.mi.infn.it

Dedicated Softwares to study GPDs

PARTONS

partons.cea.fr

PARtonic

Tomography

Of

Nucleon

Software

GeParD

Dedicated software to study and fit TMDs

arTeMiDe

teorica.fis.ucm.es/artemide

TMD lib and TMD Plotter

tmdlib.hepforge.org

NangaParbat

MapCollaboration/NangaParbat

Next Efforts: combine different inputs to understand PDFs, TMDs and GPDs in an unified framework

Backup Slides

Future from JLab22 upgrade and EIC

EIC and JLab22 complementary to:

- Cover larger energy domain to ensure convergence in dispersion analysis of GPDs
- Span a larger range of t for a meaningful FT
- JLab22 bridges between EIC (gluon components) and JLab12 (valence region)

High luminosity at JLab22 gives unique possibility to measure new processes so-far unexplored

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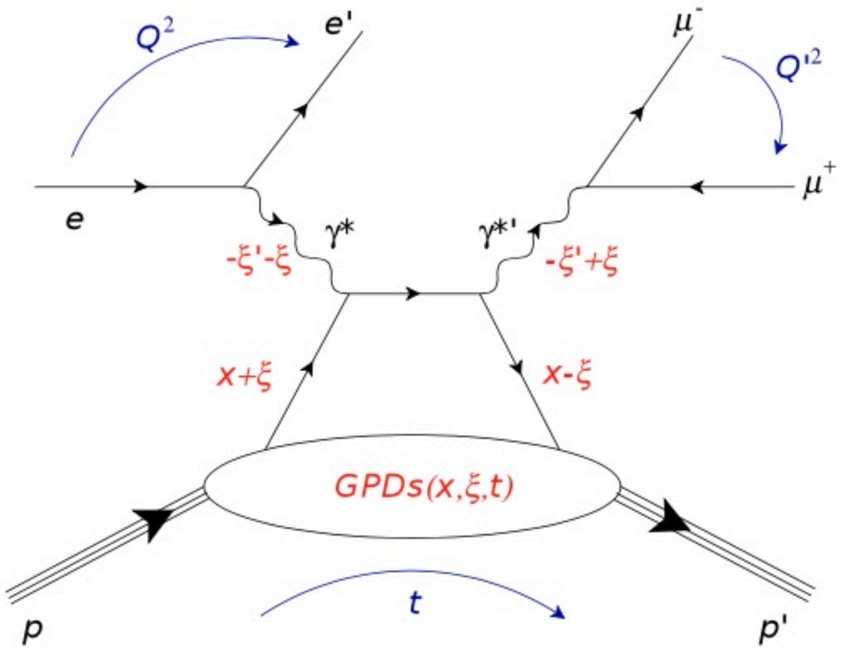
$$e + p \to e' + (l^+ l^-) + p$$



- dilepton electroproduction suppressed by a factor $\alpha_{QED} \sim 10^{-2}$ compared to DVCS
- disentangle the longitudinal momentum variables by varying the dilepton mass

EMT and the proton mass

- Forward matrix element of total EMT

$$\langle T^{\mu\nu}\rangle \equiv \langle p|T^{\mu\nu}|p\rangle = 2p^\mu p^\nu$$

Proton mass

$$n\,\langle T^\mu{}_\mu\rangle = n\,\langle T^{00}\rangle\Big|_{\vec{p}=0} = \frac{\langle H_{\rm QCD}\rangle}{\langle p|p\rangle}\Big|_{\vec{p}=0} = M$$

$(n = \frac{1}{2M}$ depends on normalization of state)

$$H_{\rm QCD} = \int d^3x \mathcal{H}_{QCD} = \int d^3x\, T^{00}$$

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$$H_{\rm QCD} = \int d^3x \mathcal{H}_{QCD} = \int d^3x \, T^{00}$$

- Forward matrix element quark and gluon contributions

$$\langle T^{\mu\nu}_{i,R} \rangle = 2 p^\mu p^\nu \, A_i(0) + 2 M^2 g^{\mu\nu} \bar{C}_i(0)$$

Conservation of full EMT:

$$A_q(0) + A_g(0) = 1 \qquad\qquad \bar{C}_q(0) + \bar{C}_g(0) = 0$$

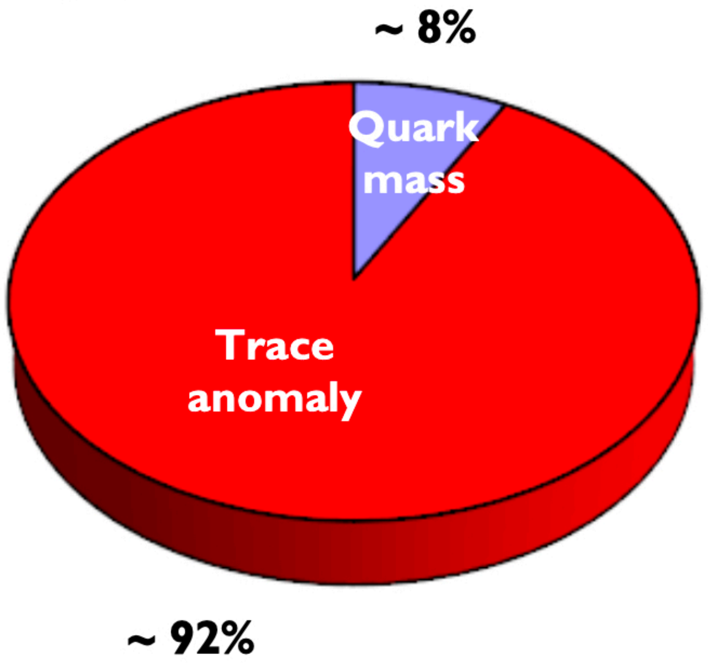
in forward limit, matrix elements of EMT fully determined by two form factors

any mass sum rule for the proton related to at most two independent numbers

Mass decompositions in D2 scheme

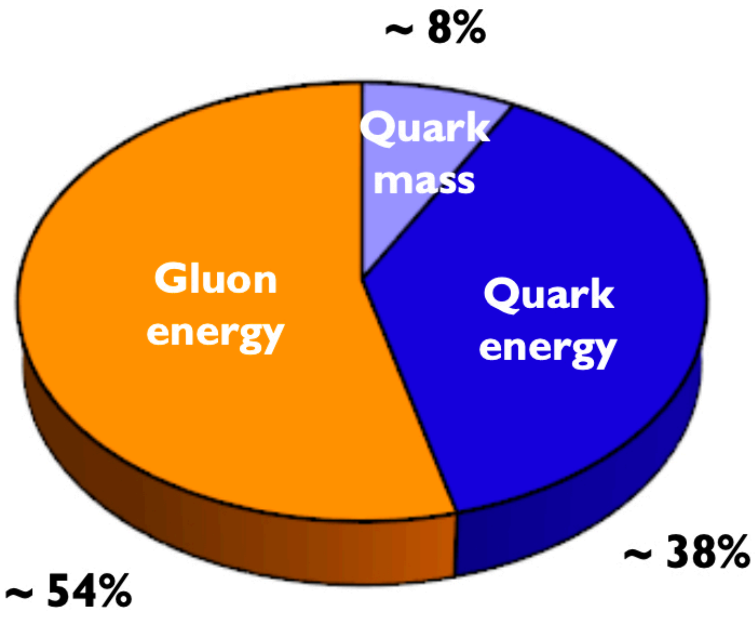
Trace decomposition

$\mu = 2\,\mathrm{GeV}$



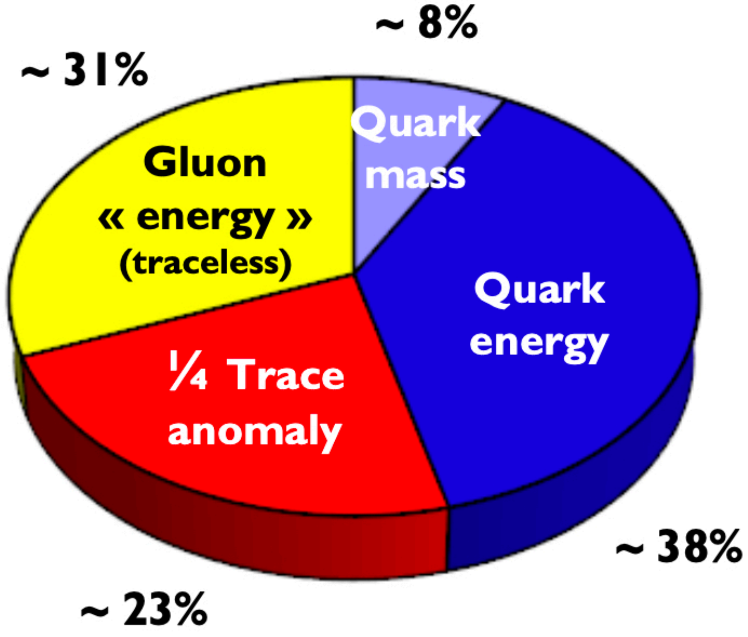
$$\overline{M}_q^{\mathrm{D2}} = M_m$$

3-term
energy decomposition



$$M_q^{\mathrm{D2}} = M_{q[\mathrm{Ji}]}$$

4-term
sum rule (Ji)



$$M_g^{\mathrm{D2}} = M_{g[\mathrm{Ji}]} + M_a$$