On exotic hadron spectroscopy and unitarized EFTs

R. Molina, T. Branz, F. Gil-Dominguez, L.R. Dai and E. Oset

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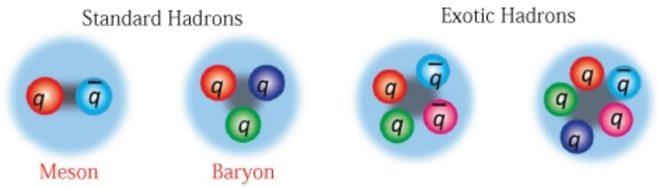
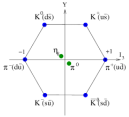
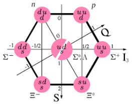
Intro

Hadrons



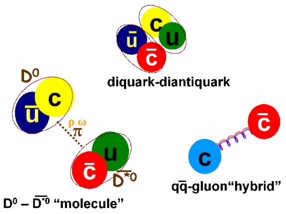
- 'Regular' hadrons: $q\bar{q}$, qqq



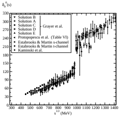


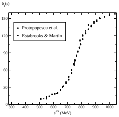
- Exotics: $q\bar{q}q\bar{q}$, $qqqq\bar{q}$, qqg,...

  Not $q\bar{q}$: $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$



Light hadrons





Close to ...

$\longrightarrow$ PP th. $\pi\pi$, $K\bar{K}$... PB $K\Lambda$,$\pi\Sigma$, $\bar{K}N$...

$\longrightarrow$ PV th. $K\bar{K}^*$, $\pi\rho$, $\pi\omega$, $\eta\omega$... SB σN...

$\longrightarrow$ VV th. $\rho\rho$, $K^*\bar{K}^*$... VB $\rho(\omega)N$, K^*N...

$\longrightarrow$ Hybrid, Glueball candidate

Particle	J^P	overall
N	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***

		Overall	Status as seen in —		
Particle	J^P	status	$N\overline{K}$	$\Sigma\pi$	Other channels
$\Lambda(1116)$	$1/2^+$	****			$N\pi$ (weak decay)
$\Lambda(1380)$	$1/2^-$	**	**	**	
$\Lambda(1405)$	$1/2^-$	****	****	****	
$\Lambda(1520)$	$3/2^-$	****	****	****	$\Lambda\pi\pi$, $\Lambda\gamma$
$\Lambda(1600)$	$1/2^+$	****	***	****	$\Lambda\pi\pi$, $\Sigma(1385)\pi$
$\Lambda(1670)$	$1/2^-$	****	****	****	$\Lambda\eta$
$\Lambda(1690)$	$3/2^-$	****	****	***	$\Lambda\pi\pi$, $\Sigma(1385)\pi$
$\Lambda(1710)$	$1/2^+$	*	*	*	
$\Lambda(1800)$	$1/2^-$	***	***	**	$\Lambda\pi\pi$, $\Sigma(1385)\pi$, $N\overline{K}^*$
$\Lambda(1810)$	$1/2^+$	***	**	**	$N\overline{K}_2^*$
$\Lambda(1820)$	$5/2^+$	****	****	****	$\Sigma(1385)\pi$
$\Lambda(1830)$	$5/2^-$	****	****	****	$\Sigma(1385)\pi$
$\Lambda(1890)$	$3/2^+$	****	****	**	$\Sigma(1385)\pi$, $N\overline{K}^*$
$\Lambda(2000)$	$1/2^-$	*	*	*	

• $h_1(1415)$ was $h_1(1380)$	$0^-(1^{+-})$
$a_1(1420)$	$1^-(1^{++})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$

LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

For $I = 1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

See related reviews:

- Form Factors for Radiative Pion and Kaon Decays
- Scalar Mesons below 2 GeV
- $\rho(770)$
- Pseudoscalar and Pseudovector Mesons in the 1400
- $\rho(1450)$ and $\rho(1700)$

• $\pi^\pm$	$1^-(0^-)$	$f_1(1510)$
• π^0	$1^-(0^{-+})$	• $f_2'(1525)$
• η	$0^+(0^{-+})$	$f_2(1565)$
• $f_0(500)$ aka σ; was $f_0(600)$	$0^+(0^{++})$	$\rho(1570)$
		$h_1(1595)$
• $\rho(770)$	$1^+(1^{--})$	• $\pi_1(1600)$
• $\omega(782)$	$0^-(1^{--})$	• $a_1(1640)$
• $\eta'(958)$	$0^+(0^{-+})$	$f_2(1640)$
• $f_0(980)$	$0^+(0^{++})$	• $\eta_2(1645)$
• $a_0(980)$	$1^-(0^{++})$	• $\omega(1650)$
• $\phi(1020)$	$0^-(1^{--})$	• $\omega_3(1670)$
• $h_1(1170)$	$0^-(1^{+-})$	• $\pi_2(1670)$
• $b_1(1235)$	$1^+(1^{+-})$	• $\phi(1680)$
• $a_1(1260)$	$1^-(1^{++})$	• $\rho_3(1690)$
• $f_2(1270)$	$0^+(2^{++})$	• $\rho(1700)$
• $f_1(1285)$	$0^+(1^{++})$	• $a_2(1700)$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_0(1710)$
• $\pi(1300)$	$1^-(0^{-+})$	$\eta(1760)$
• $a_2(1320)$	$1^-(2^{++})$	• $\pi(1800)$
• $f_0(1370)$	$0^+(0^{++})$	$f_2(1810)$
• $\pi_1(1400)$	$1^-(1^{-+})$	$X(1835)$
• $\eta(1405)$	$0^+(0^{-+})$	• $\phi_3(1850)$

Methods: EFT's, Chiral symmetry, Unitarity

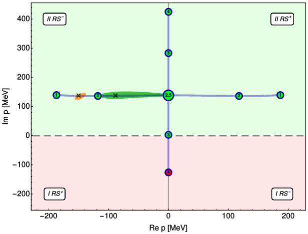
General properties of the scattering amplitudes: Analyticity, Unitarity, Crossing symmetry, applied often in combination with EFT (chiral symmetry)

- Unitarized Chiral Perturbation Theory. Oller, Oset, Pelaez (1997)
- Inverse Amplitude Method. Truon, Herrero, Dobado, Pelaez (1988)
- Roy-Steiner equations based on dispersion relations. Roy, Steiner, Hite (1971)
- N/D method, Oller (1998)
- Bethe-Salpeter ...

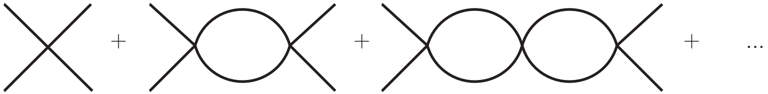
and LQCD!

Example of Application in combination with LQCD data

σ meson. Guo, Alexandru, Molina, Mai, Döring, (2018)



Dynamically generated resonances



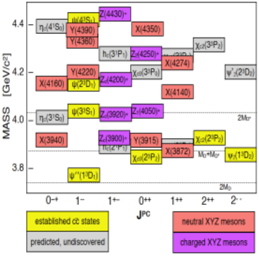
Many suggestions of dynamically generated resonances...

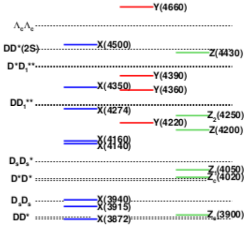
- Oller, Oset, Pelaez, Ramos, Hanhart, Krewald, Speth, Nieves, Inoue, Ruiz-Arriola, Meissner, Zou, Guo, Vicente-Vacas, Garcia-Recio, Molina, Roca, Geng, Alvarez-Ruso, Alarcon, Albaladejo, Nícola ...
  ... and a very long list of authors!...´

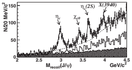
However, no clear statement for them in the light sector since ...

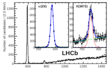
- Most of these exotic candidates can overlap with $q\bar{q}/qqq$ except for those with non-$q\bar{q}$ quantum numbers like $\pi_1(1400)$, $\pi_1(1600)$ ...

Heavy hadrons

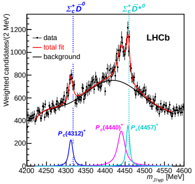








State	M_0 [MeV]	Γ [MeV]
$P_{cs}(4459)^0$	$4458.8 \pm 2.9\,^{+4.7}_{-1.1}$	$17.3 \pm 6.5\,^{+8.0}_{-5.7}$



Clear evidence of exotic states!

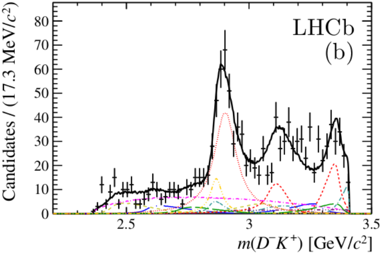
- Hidden-charm charged tetraquarks $Z_c^+ \sim c\bar{d}u\bar{c}$ $(D^{(*)}\bar{D}^{(*)})$.
  Hidden-strange candidate? $a_0(980)$? ... more?
- Hidden-charm (strange) pentaquarks $P^+_{c(s)} \sim c\bar{c}uud(s)$, $(\bar{D}^{(*)}\Sigma_c^{(*)}(\Xi_c^{(*)}))$.
  Hidden-strange candidate? $N^*(1535)$, (strange) $\Lambda(1405)$, ...more?

Wu, Molina, Zou, Oset, PRL (2010). New P_{cs} state discovered by LHCb
(See also Marsé-Valera et al., PRL (2022) Pentaquarks with $S = -2$)

LHCb (2020)

Two states $J^P = 0^+, 1^-$ decaying to $\bar{D}K$. First clear example of an heavy-flavor exotic tetraquark, $\sim \bar{c}\bar{s}ud$.

$$X_0(2866) : M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9\,\mathrm{MeV},$$
$$X_1(2900) : M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5\,\mathrm{MeV}.$$

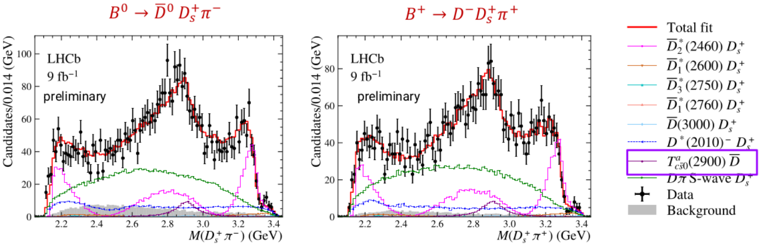


R. Aaij et al. (LHCb Collaboration), PRL125(2020), PRD102(2020)

New exotic tetraquark seen in $D_s^+\pi^+$

LHCb (2022)

One state decaying $T_{c\bar{s}}(2900)$ decaying to $D_s^+\pi^-$ and $D_s^+\pi^+$ has been observed $\sim c\bar{s}u\bar{d}$.



- The analysis favors $J^P = 0^+$
- Mass, $m = 2908 \pm 11 \pm 20$ MeV
- Width, $\Gamma = 136 \pm 23 \pm 11$ MeV

PRD108 (2023)

$D^* K^*$ th.: 2903 MeV

$D_s^* \rho$ th.: 2890 MeV

Local-Hidden-Gauge Formalism

The hidden gauge formalism

Bando, Kugo, Yamawaki, PRL54

Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \tag{1}$$

$$\begin{aligned}
\mathcal{L}^{(2)} &= \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \qquad (2) \\
\mathcal{L}_{III} &= -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu} \rangle + \frac{1}{2} M_V^2 \langle [V_\mu - \frac{i}{g}\Gamma_\mu]^2 \rangle \\
D_\mu U &= \partial_\mu U - ieQA_\mu U + ieUQA_\mu, \qquad U = e^{i\sqrt{2}P/f}
\end{aligned}$$

Upon expansion of $[V_\mu - \frac{i}{g}\Gamma_\mu]^2$, $\mathcal{L}'s$

$$\mathcal{L}_{V\gamma} = -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q \rangle, \mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P] \rangle, \mathcal{L}_{\gamma PP} = ieA_\mu \langle Q[P, \partial_\mu P] \rangle, \ldots$$

$$\frac{F_V}{M_V} = \frac{1}{\sqrt{2}g}, \qquad \frac{G_V}{M_V} = \frac{1}{2\sqrt{2}g}, \qquad F_V = \sqrt{2}f, \qquad G_V = \frac{f}{\sqrt{2}}, \qquad g = \frac{M_V}{2f}$$

Vector-vector scattering Bando,Kugo,Yamawaki

$$\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu} V^{\mu\nu}\rangle$$

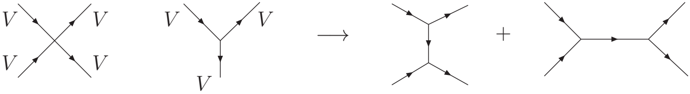
$$\mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu\rangle$$

$$\mathcal{L}_{III}^{(c)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu\rangle$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]$$

$$g = \frac{M_V}{2f}$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$



a) b) c) d)

Flavour exotic states

Flavour exotic states

- 2010. Prediction of several flavour exotic states

PHYSICAL REVIEW D **82,** 014010 (2010)

New interpretation for the $D^*_{s2}(2573)$ and the prediction of novel exotic charmed mesons

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In this manuscript we study the vector-vector interaction within the hidden-gauge formalism in a coupled channel unitary approach. In the sector $C = 1, S = 1, J = 2$ we get a pole in the T matrix around 2572 MeV that we identify with the $D^*_{s2}(2573)$, coupling strongly to the $D^*K^*(D^*_s\phi(\omega))$ channels. In addition we obtain resonances in other exotic sectors which have not been studied before such as $C = 1$, $S = -1$, $C = 2$, $S = 0$ and $C = 2$, $S = 1$. These "flavor-exotic" states are interpreted as $D^*\bar{K}^*$, D^*D^*, and $D^*_sD^*$ molecular states but have not been observed yet. In total we obtain nine states with different spin, isospin, charm, and strangeness of non-$C = 0$, $S = 0$ and $C = 1$, $S = 0$ character, which have been reported before.

DOI: 10.1103/PhysRevD.82.014010 PACS numbers: 14.40.Rt, 12.40.Vv, 13.75.Lb, 14.40.Lb

- Free parameter fixed with $D_{s2}(2573)$; couples to D^*K^*, $c\bar{q}q\bar{s}$
- Flavour exotic states with $I = 0$, $J^P = \{0, 1, 2\}^+$ coupling to $D^*\bar{K}^*$ are predicted, $c\bar{q}s\bar{q}$
- Doubly charm states, $I = 0$; $J^P = 1^+$, close to D^*D^* are predicted, $c\bar{q}c\bar{q}$, and $I = 1/2$; $J^P = 1^+$, close to $D^*D^*_s$ $c\bar{q}c\bar{s}$

Flavour exotic states

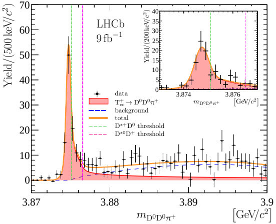
Molina,Branz,Oset, PRD82(2010)

C, S	Channels	$I[J^P]$	$\sqrt{s}$	$\Gamma_A (\Lambda = 1400)$	$\Gamma_B (\Lambda = 1200)$	State	$\sqrt{s}_{\rm exp}$	$\Gamma_{\rm exp}$
$1, -1$	$D^* \bar{K}^*$	$0[0^+]$	2848	23	59	$X_0(2866)$ or $T_{cs}(2900)$	2866	57
		$0[1^+]$	2839	3	3			
		$0[2^+]$	2733	11	36			
$1, 1$	$D^* K^*$, $D_s^* \omega$	$0[0^+]$	2683	20	71			
	$D_s^* \phi$	$0[1^+]$	2707	4×10^{-3}	4×10^{-3}			
		$0[2^+]$	2572	7	23	$D_{s2}(2573)$	2572	20
$1, 1$	$D^* K^*$, $D_s^* \rho$	$1[0^+]$	Cusp structure around $D_s^* \rho$, $D^* K^*$			new $T_{c\bar{s}}(2900)$	2908	136
$1, 1$		$1[1^+]$	Cusp structure around $D_s^* \rho$, $D^* K^*$					
$1, 1$		$1[2^+]$	2786	8	11			
$2, 0$	$D^* D^*$	$0[1^+]$	3969	0	0			
$2, 1$	$D^* D_s^*$	$1/2[1^+]$	4101	0	0			

Table 1: All the quantities here are in MeV. Repulsion in $C = 0, S = 1, I = 1/2$; $C = 1, S = -1, I = 1$; $C = 1, S = 2, I = 1/2$; $C = 2, S = 0, I = 1$ and $C = 2, S = 2, I = 0$ is found.

Form factors in the $D^* D\pi$ vertex; Model A: $F_1(q^2) = \frac{\Lambda_b^2 - m_\pi^2}{\Lambda_b^2 - q^2}$, Titov, Kampfer EPJA7, PRC65 with $\Lambda_b = 1.4, 1.5$ GeV and $g = M_\rho / 2 f_\pi$. Model B: $F_2(q^2) = e^{q^2/\Lambda^2}$ Navarra, Nielsen, Bracco PRD65 (2002), $\Lambda = 1, 1.2$ GeV and $g_D = g^{\rm exp}_{D^* D\pi} = 8.95$ (experimental value). Subtraction constant $\alpha = -1.6$.

$T_{cc}^{+}(3875)$ signal in $D^0 D^0 \pi^+$ ($\sim DD^*$)



BEFORE resolution:

$$m_{\rm exp} = 3875.09 \text{ MeV} + \delta m_{\rm exp}$$

$$\delta m_{\rm exp} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV};$$

$$\Gamma = 410 \pm 165 \pm 43^{+18}_{-38} \; keV$$

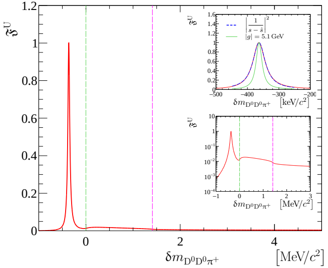
Feijoo, Liang, Oset, PRD104 (2021)

Local Hidden-Gauge Approach

$$\frac{d\Gamma}{dM_{12}^2 \, dM_{23}^2} = \frac{1}{2} \frac{1}{(2\pi)^3} \frac{1}{s^{3/2}} |t|^2,$$

$\Longrightarrow \Gamma = 43$ KeV

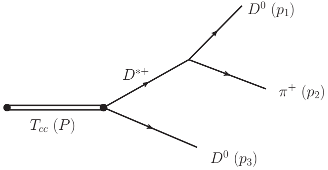
LHCb, Nature (2022):



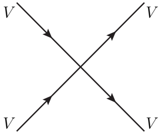
AFTER resolution

$$\delta m_{\rm exp} = -360 \pm 40^{+4}_{-0} \text{ keV};$$

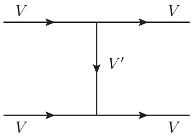
$$\Gamma = 48 \pm 2^{+0}_{-14} keV$$



$C=2; S=0,1$: T_{cc} states



(a)



(b)

$$\mathcal{L}^{(c)}_{III} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle$$

$$\mathcal{L}^{(3V)}_{III} = ig\langle [V_\mu, \partial_\nu V_\mu] V^\nu \rangle$$

Repulsion for $I=1$

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*D^* \to D^*D^*$	0	0	0
1	$D^*D^* \to D^*D^*$	0	$\frac{g^2}{4}(\frac{2}{m^2_{J/\psi}} + \frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})\{(p_1+p_3).(p_2+p_4) + (p_1+p_4).(p_2+p_3)\}$	$-25.4g^2$
2	$D^*D^* \to D^*D^*$	0	0	0

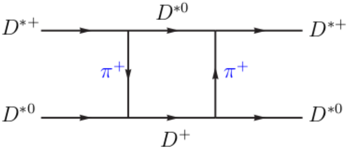
Table 2: Tree level amplitudes for D^*D^* in $I=0$, $C=2$, $S=0$.

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*_s D^* \to D^*_s D^*$	$-4g^2$	$\frac{g^2(p_1+p_4)(p_2+p_3)}{m^2_{K^*}} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m^2_{J/\psi}}$	$19g^2$
1	$D^*_s D^* \to D^*_s D^*$	0	$-\frac{g^2(p_1+p_4)(p_2+p_3)}{m^2_{K^*}} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m^2_{J/\psi}}$	$-19.5g^2$
2	$D^*_s D^* \to D^*_s D^*$	$2g^2$	$\frac{g^2(p_1+p_4)(p_2+p_3)}{m^2_{K^*}} + \frac{g^2(p_1+p_3)(p_2+p_4)}{m^2_{J/\psi}}$	$25.0g^2$

Table 3: Tree level amplitudes for $D^*D^*_s$ in $I=1/2$, $C=2$, $S=1$.

$C = 2; S = 0, 1$: T_{cc} states

Dai, Molina,Oset, PRD105(2022)



$$\mathcal{L}_{VPP} = -ig \, \langle [P, \partial_\mu P] V^\mu \rangle$$

$$\mathcal{L}_{VVP} = \frac{G'}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle$$

$$G' = \frac{3\,g'}{4\pi^2 f}; \quad g' = -\frac{G_V m_\rho}{\sqrt{2} f^2}$$

	$q_{\rm max} = 450$ MeV	$q_{\rm max} = 420$ MeV
$M_{D^* D^*}$	4014.08 MeV	4015.54 MeV
$B_{D^* D^*}$	3.23 MeV	1.56 MeV
$\Gamma_{D^* D^*}$	2.3 MeV	1.5 MeV
$M_{D_s^* D^*}$	4122.46 MeV (cusp)	4122.46 MeV (cusp)
$\Gamma_{D_s^* D^*}$	70 – 100 KeV	70 – 100 KeV

$C = 1, S = 1, I = 1$: The $T_{cs}(2900)$

Molina, Oset PLB811 2020, $\alpha = -1.474$, $\Lambda = 1300$.

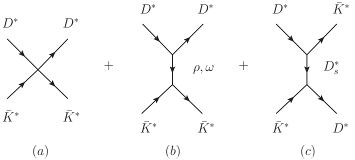
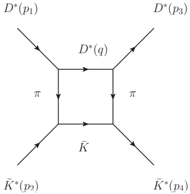


Figure 1: $D^* \bar{K}^* \to D^* \bar{K}^*$ interaction



$$T = [I - VG]^{-1} V$$

$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^* \bar{K}^*$	?
$0(1^+)$	2861	20	$D^* \bar{K}^*$	?
$0(0^+)$	2866	57	$D^* \bar{K}^*$	$T_{cs}(2900)$

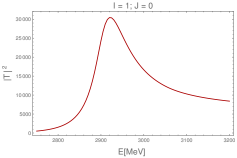
Table 4: Results including the width of the $D^* K$ channel.

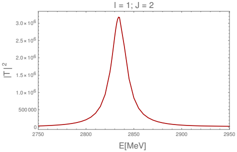
$C = 1, S = 1, I = 1$: The $T_{c\bar{s}}(2900)$

New results, $\alpha = -1.474$ to obtain the $T_{cs}(2900)$ state in $D^* \bar{K}^*$.

Convolution due to the vector meson mass distribution ρ, K^*

$$\tilde{G}(s) \quad = \quad \frac{1}{N} \int_{(M_1-4\Gamma_1)^2}^{(M_1+4\Gamma_1)^2} d\tilde{m}_1^2 \left(-\frac{1}{\pi}\right) \mathcal{I}m \frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1} G(s, \tilde{m}_1^2, M_2^2) \; ,$$



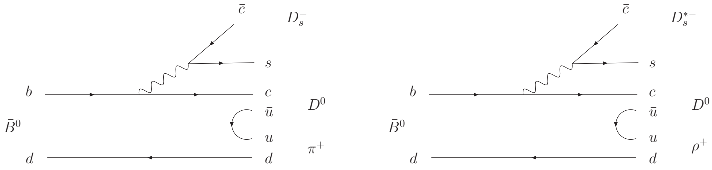


$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$1(0^+)$	2920	130	$D^* K^*, D_s \rho$	$T_{c\bar{s}}(2900)$
$1(1^+)$	2922	145		?
$1(2^+)$	2835	20		?

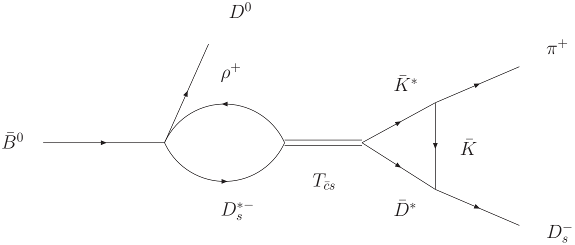
Table 5: PRD107(2023), Exp. $(m, \Gamma) = (2908 \pm 11 \pm 20, 136 \pm 23 \pm 11)$ MeV

Production of the $T_{\bar{c}s}(2900)$

$\bar{B}^0 \to D_s^- D^0 \pi^+$ in B decays



The $T_{\bar{c}s}(2900)$ can be produced by means of external emission

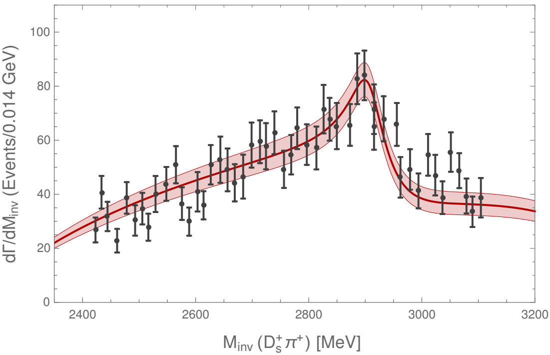


Production of the $T_{\bar{c}s}(2900)$ in B decays

$$T(E) = aG(E)_{D_s^*\rho}\, t_{D_s^*\rho\to\bar{D}^*\bar{K}^*}(E)\, t_L(E) + b \qquad (3)$$

$E = M_{inv}(\pi^+ D_s^-)$; a, b parameters; t_L amplitude for the triangle loop.

$$\boxed{\frac{d\Gamma}{dM_{Inv}} = \frac{1}{(2\pi)^3}\frac{1}{4M_B^2}\, p_D\, \tilde{p}_\pi\, |T|^2}$$



Quark mass dependence of exotic states

Quark mass dependence of the $D(D^*)$ mesons

Heavy Hadron Chiral Perturbation Theory (HHχPT)

E. Jenkins, NPB412 (1994); Gil-Domínguez, Molina PLB (2023)

$$\frac{1}{4}(D+3D^*) = m_H + \alpha_a - \sum_{X=\pi,K,\eta} \beta_a^{(X)} \frac{M_X^3}{16\pi f^2} + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)}\alpha_a\right) \frac{M_X^2}{16\pi^2 f^2} \log\left(M_X^2/\mu^2\right) + c_a$$

$$(D^* - D) = \Delta + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)}\Delta\right) \frac{M_X^2}{16\pi^2 f^2} \log\left(M_X^2/\mu^2\right) + \delta c_a$$

$\mu = 770$ MeV; $g^2 = 0.55$ MeV (Decay of the D^* meson)

$$\left.\begin{aligned} \frac{1}{4}(D+3D^*) &= m_H + f(\sigma,a,b,c,d) \\ (D^* - D) &= \Delta + g(\Delta^{(\sigma)}, \Delta^{(a)}) \end{aligned}\right\}$$

9 parameters, but different collaborations/scale settings, $7 + 2 \times 7 = 21$ parameters, ~ 80 data points

ETMC, PACS, HSC, CLS, RQCD, S.Prelovsek, MILC

$D(D^*)$ quark mass dependence

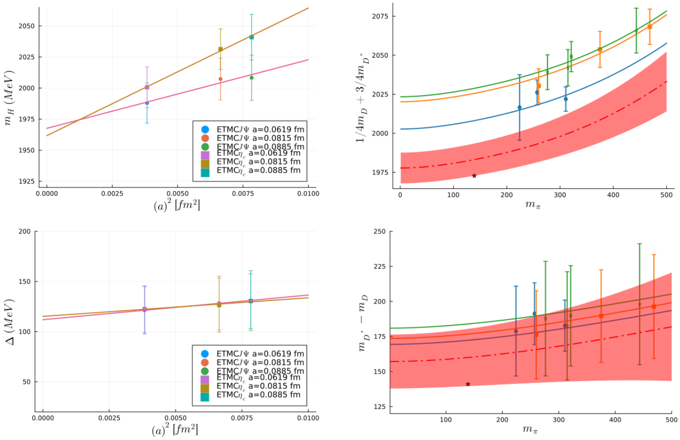


Figure 2: Extrapolation to the physical point of the ETMC data.

Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

Predictions: Cleven, Guo, Hanhart, Meissner (2011); Liu, Orginos, Guo, Meissner, (2013); Previous analyses of LQCD data: Martínez Torres, Oset, Prelovsek, A. Ramos (2015), Yao, Du, Guo, and Meißner, (2015); Albaladejo, Fernandez-Soler, Nieves, Ortega, (2018). LQCD data:

Col.	a(fm)	L(fm)	m_π(MeV)	m_{D_s}(MeV)
	0.12			
HSC	$a_t^{-1} = 6079$ MeV	3.8	239	1967
(2021)	$a_t^{-1} = 5667$ MeV	$1.9 - 2.9$	391	1951
RQCD (2017)	0.071	4.5	150	1977
PACS-CS (2014)	0.0907	2.9	156	1809
Prelovsek et al.	0.1239	2.0	266	1657

Table 6: Charm quark mass settings, m_{D_s}. $m_{D_s}^{phys} = 1968.35 \pm 0.07$ MeV.

Formalism in the finite volume

Infinite volume (Two-meson loop)

$$G = G^{co}(E) = \int_{q<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1+\omega_2}{2\omega_1\omega_2} \frac{2M_i}{E^2-(\omega_1+\omega_2)^2+i\epsilon}$$

where $\omega_i = \sqrt{m_i^2 + |\vec{q}_i\ |^2}$.

Finite volume (Doring, Meißner, Oset, Rusetsky (2011))

$$\vec{q}_i = \frac{2\pi}{L}\vec{n}_i; \qquad T \longrightarrow \tilde{T}; \qquad G(E) \longrightarrow \tilde{G}(E)\ ,$$

$$\tilde{G}(E) = \frac{1}{L^3}\sum_{\vec{q}_i} I(E,\vec{q}_i); \qquad I(E,\vec{q}_i) = \frac{\omega_1(\vec{q}_i)+\omega_2(\vec{q}_i)}{2\omega_1(\vec{q}_i)\omega_2(\vec{q}_i)} \frac{1}{(E)^2-(\omega_1(\vec{q}_i)+\omega_2(\vec{q}_i))^2}$$

$$\tilde{G} = G^{DR} + \lim_{q_{max}\to\infty}\left(\frac{1}{L^3}\sum_{q<q_{max}} I(E,\vec{q}) - \int_{q<q_{max}} \frac{d^3q}{(2\pi^3)} I(E,\vec{q})\right)$$

$$\equiv G^{DR} + \lim_{q_{max}\to\infty} \Delta G\ ,$$

Martinez Torres, Dai, Koren, Jido and Oset (2012)

Formalism in the finite volume

- **Bethe-Salpeter** equation in finite volume, **One-channel case**

$$\tilde{T}^{-1} = V^{-1} - \tilde{G}$$

- Energy levels in the box in the presence of interaction V correspond to the condition

$$\boxed{\det(I - V\tilde{G}) = 0}$$

- One-channel-amplitude in infinite volume T

$$T = (\tilde{G}(E_i) - G(E_i))^{-1}.$$

- Phase shift:

$$\boxed{\tan\delta = -k/(8\pi E \Delta G)}$$

(Similar to the Luscher condition (1986) but relativistic.)

Boosts

Doering, Meißner, Oset, Rusetsky (2012)

$\vec{q}_1, \vec{q}_2 = \vec{P} - \vec{q}_1$, $s \equiv W^2 = (P^0)^2 - \vec{P}^2$, and $\vec{q}^*$ the momenta in the CM frame

$$\int \frac{d^3\vec{q}^{\,*}}{(2\pi)^3} I(|\vec{q}^{\,*}|) \to \tilde{G}(P) = \frac{1}{L^3}\frac{\sqrt{s}}{P^0}\sum_{\vec{n}} I(|\vec{q}^{\,*}(\vec{q})|).$$

$\vec{q}^{\,*}_{1,2} = \vec{q}_{1,2} + \left[\left(\frac{\sqrt{s}}{P^0} - 1\right)\frac{\vec{q}_{1,2}\cdot\vec{P}}{|\vec{P}|^2} - \frac{q^{*0}_{1,2}}{P^0}\right]\vec{P}$; with $\vec{q} = \frac{2\pi}{L}(n_x, n_y, n_z)$, $\vec{P} = \frac{2\pi}{L}(N_x, N_y, N_z)$.

$$\tilde{T}_{lm,l'm'} = V_l\delta_{ll'}\delta_{mm'} + \sum_{l''m''} V_l\tilde{G}_{lm,l''m''}\tilde{T}_{l''m'',lm}$$

$$\boxed{\det(\delta_{ll'}\delta_{mm'} - V_l\tilde{G}_{lm,l'm'}) = 0}$$

Irreducible representations for asymmetric boxes and boost $\vec{P} = \frac{2\pi}{L}(0,0,1)$,

$$\mathrm{I = L = 0 \longrightarrow A^+ : -1 + V_0G_{00,00} = 0}$$

$$\mathrm{I = L = 1 \longrightarrow A_2^- : -1 + V_1G_{10,10} = 0; E^- : -1 + V_1G_{11,11} = 0}$$

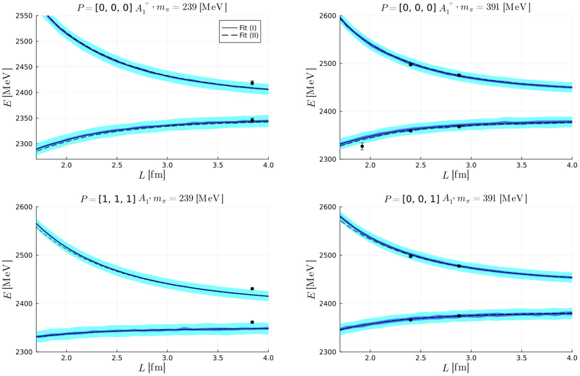
Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

Potential $V(s)$ (consistent with HQSS)

$$V(s) = V_{DK}(s) + V_{\rm ex}(s); \qquad {\rm Fit\,I}: V_{\rm ex} = 0; \qquad \chi^2 = \Delta E^T C^{-1} \Delta E$$

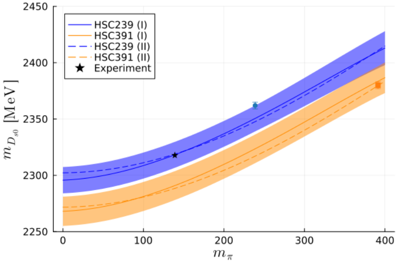
$$V_{DK} = -\frac{s-u}{2f^2}; \quad V_{\rm ex} = \frac{V_{c\bar{s}}^2}{s - m_{c\bar{s}}^2}; V_{c\bar{s}}(s) = -\frac{c}{f}\sqrt{M_D m_{c\bar{s}}}\frac{s + m_K^2 - M_D^2}{\sqrt{s}}$$

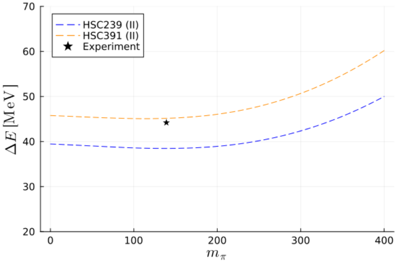
Fitting parameters: $a = a_1 + a_2 m_\pi$ (subtraction constant), m_{cs}, c.



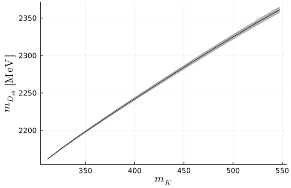
Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

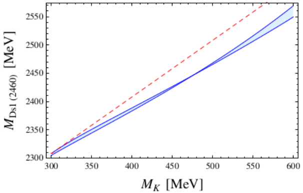
Pion and kaon mass dependence



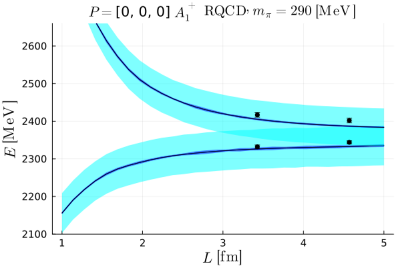


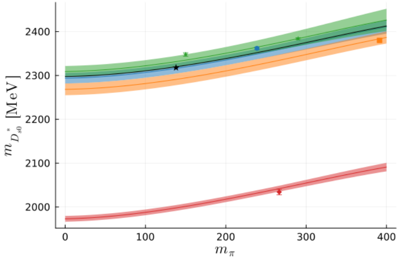
$m_\pi = 236$ MeV; $a_t^{-1} = 5.667$ GeV; $a_t M_{\eta_c} = 0.2412$, $M_{\eta_c} = 2986$ MeV; $m_\pi = 391$ MeV; $a_t^{-1} = 6.079$ GeV; $a_t M_{\eta_c} = 0.2735$; $M_{\eta_c} = 2963$ MeV; Similar trends than in Cleven, Guo, Hanhart, Meißner (2010)

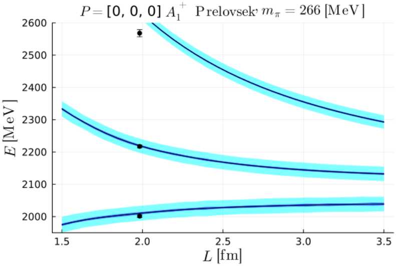


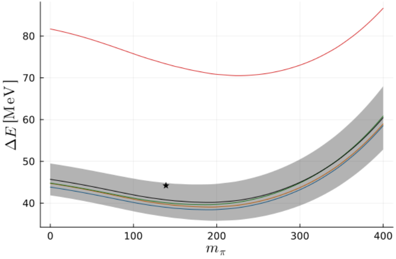


Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

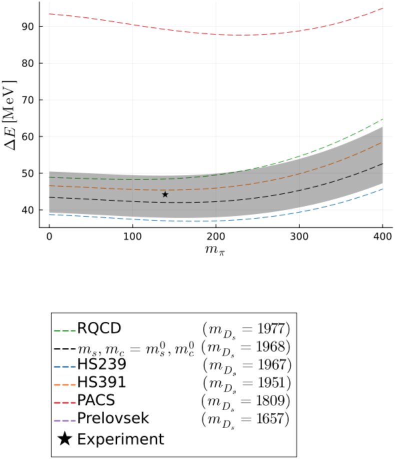


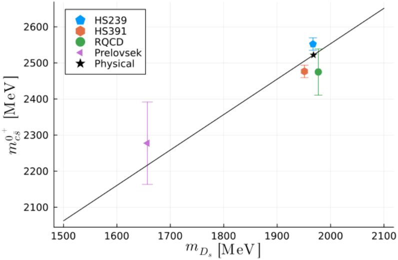






Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$





	c	$a_1^{0^+}$	$a_1^{1^+}$	$a_2[(m_\pi^0)^{-2}]$
Fit I	-	$-1.87(1)$	$-2.05(2)$	$-0.040(2)$
Fit II	1.05(1)	$-1.34(1)$	$-1.44(1)$	$-0.002(1)$

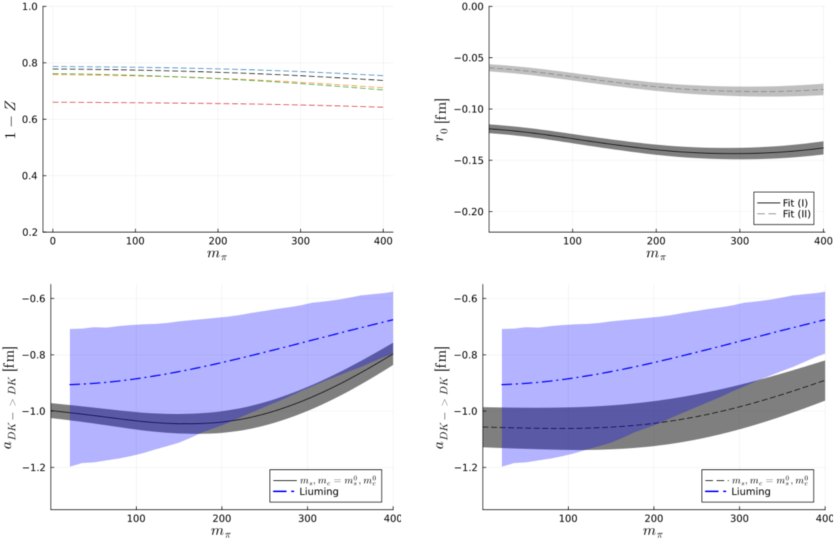
	$m_{c\bar{s}}^{0^+}$	$m_{c\bar{s}}^{1^+}$
HSC239	2552(26)	-
HSC391	2476(25)	-
RQCD	2475(25)	2553(25)
Prelovsek	2277(23)	2356(23)

$m_{D_s} - m_{D_s^0} \simeq m_{c\bar{s}} - m_{c\bar{s}^0}$. Compatible with the assumption taken in PRD, Albaladejo, Fernandez, Nieves (2018)

Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$

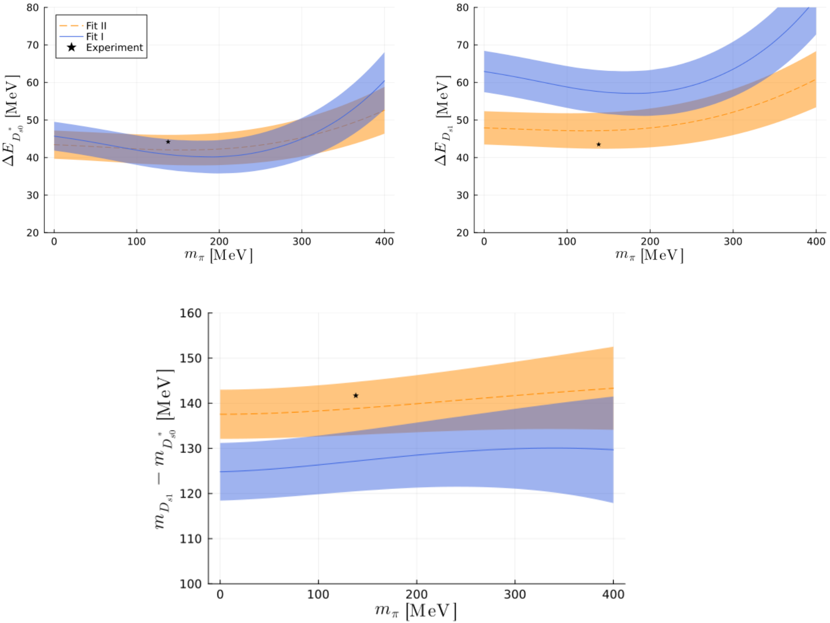
Compositeness and Scattering parameters

See also Dai,Song,Oset,PLB (2023)



Comparison with Liu, Orginos, Guo, Meissner (2013)

Quark mass dependence of $D_{s0}(2317)$ and $D_{s1}(2460)$



Conclusions

Conclusions

- The HGF has predicted many exotic states. Some of them discovered. A new table of exotic particles is coming ...
- The $X_0(2866)$ or $T_{cs}(2900)$ is compatible with a $D^*\bar{K}^*$ resonance decaying to $D\bar{K}$. Proposed reactions to observe the 1^+ state:
  $\bar{B}^0 \to D^{*+}\bar{D}^{*0}K^-$, Dai, Molina, Oset (2022),
  $\bar{B}^0 \to D^{*+}K^-\bar{K}^{*0}$; and the 2^+ state:
  $B^+ \to D^+D^-K^+$, Bayar and Oset (2022).
- The $T_{c\bar{s}}(2900)$ is more likely to be a failed bound state, or cusp structure around the D^*K^*, $D_s^*\rho$ thresholds.
- The combination of LQCD with EFT's is a usefool tool to extract the properties of resonances with high accuracy

Back up slide