Gravitational form factors of the delta resonance in chiral EFT

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Menu 2023, 16-20th Oct. 2023

Outline:

1. Introduce GFFs and D-term
2. Calculations on spin-3/2 GFFs
3. ChPT results for GFFs & Local spatial densities
4. Summary

$$2P = (p'+p) = (2E, \vec{0})$$
$$\Delta = (p'-p) = (0, \vec{\Delta})$$
$$t = \Delta^2$$

Proton electromagnetic Form Factors:

$$\langle p', s'|\hat{j}^\mu(x)|p, s\rangle = \bar{u}'\left[\gamma^\mu F_1(t) + \frac{1}{2m} i\sigma^{\mu\nu}\Delta_\nu F_2(t)\right] u\, e^{i(p'-p)x}$$

Dirac FF Pauli FF

Sachs FFs

$$G_E(t) = F_1(t) + \frac{t}{4M^2}F_2(t) \xrightarrow{t\to 0} \text{charge}$$
$$G_M(t) = F_1(t) + F_2(t) \longrightarrow \text{magnetic moment}$$

Charge density in Breit frame:

$$\rho(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3} G_E\left(-\vec{\Delta}^2\right) e^{-i\vec{\Delta}\cdot\vec{r}}$$

Charge radius:

$$\langle r^2\rangle = \frac{\int d^3\vec{r} r^2 \rho(\vec{r})}{\int d^3\vec{r}\rho(\vec{r})}$$

Proton EMT FFs (ie: gravitational form factors GFFs):

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p, s\rangle = \bar{u}'\Big[A^a(t)\frac{P_\mu P_\nu}{m} \xrightarrow{t\to 0} \text{mass}$$
$$(a = q, g) \qquad + J^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m} \longrightarrow \text{spin}$$
$$+ D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} \longrightarrow \text{D-term}$$
$$+ m\,\bar{c}^a(t) g_{\mu\nu}\Big] u\, e^{i(p'-p)x}$$

mass, spin: external properties

D-term: "internal" property

3+1

Ji 1995 & 1997;
Polyakov, 1999, 2003

"Druck"= pressure

✤ Free fermion: $D_{\text{fermion}} = 0 \rightarrow \neq 0$: interaction! Hudson & Schweitzer, 2018

Interpretation: Static EMT

Epelbaum, Gegelia, Lange, Meißner, Polyakov, PRL 2022
Panteleeva, Epelbaum, Gegelia, Meißner, 2023
Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2023

- Definition in Breit frame (Polyakov, 2003)

$$T^{\mu\nu}(\boldsymbol{r},\sigma',\sigma) = \sum_a T_a^{\mu\nu}(\boldsymbol{r},\sigma',\sigma)$$
$$= \sum_a \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \langle p',\sigma'|\hat{T}_a^{\mu\nu}(0)|p,\sigma\rangle$$

- Energy (mass) densities

$$T^{00}(\boldsymbol{r},\sigma',\sigma) = \varepsilon_0(r)\delta_{\sigma'\sigma} + \varepsilon_2(r)\hat{Q}^{ij}_{\sigma'\sigma}Y_2^{ij}(\Omega_r)$$

- Spin density

$$J^i(\boldsymbol{r},\sigma',\sigma) = \sum_a J_a^i(\boldsymbol{r},\sigma',\sigma) = \epsilon^{ijk} r^j \sum_a T_a^{0k}(\boldsymbol{r},\sigma',\sigma)$$

$$\rho_J(r) = -r\frac{d}{dr}\int \frac{d^3\Delta}{(2\pi)^3} e^{-\boldsymbol{\Delta}\cdot\boldsymbol{r}} \mathcal{J}_1(t) \quad \text{(averaged)}$$

(Kim, BDS, 2020)

- Pressure and shear forces: ("mechanical properties")

(Polyakov, BDS, 2019, Panteleeva, Polyakov, 2020)

$$T^{ij}(\boldsymbol{r},\sigma',\sigma) = p_0(r)\delta^{ij}\delta_{\sigma'\sigma} + s_0(r)Y_2^{ij}\delta_{\sigma'\sigma} + \left(p_2(r) + \frac{1}{3}p_3(r) - \frac{1}{9}s_3(r)\right)\hat{Q}^{ij}_{\sigma'\sigma}$$
$$+ \left(s_2(r) - \frac{1}{2}p_3(r) + \frac{1}{6}s_3(r)\right) 2\left[\hat{Q}^{ip}_{\sigma'\sigma}Y_2^{pj} + \hat{Q}^{jp}_{\sigma'\sigma}Y_2^{pi} - \delta^{ij}\hat{Q}^{pq}_{\sigma'\sigma}Y_2^{pq}\right]$$
$$+ \hat{Q}^{pq}_{\sigma'\sigma}Y_2^{pq}\left[\left(\frac{2}{3}p_3(r) + \frac{1}{9}s_3(r)\right)\delta^{ij} + \left(\frac{1}{2}p_3(r) + \frac{5}{6}s_3(r)\right)Y_2^{ij}\right]$$

- Radii: (energy, spin, mechanical)

$$\langle r_E^2\rangle = \frac{1}{m}\int d^3r\, r^2 \varepsilon_0(r)$$

$$\langle r_J^2\rangle = \frac{\int d^3r\, r^2 \rho_J(r)}{\int d^3r\, \rho_J(r)}$$

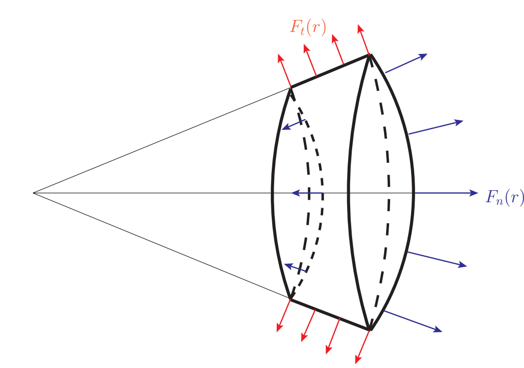
$$\langle r_n^2\rangle_{\text{mech}} = \frac{\int d^3r\, r^2\left[p_n(r) + \frac{2}{3}s_n(r)\right]}{\int d^3r\left[p_n(r) + \frac{2}{3}s_n(r)\right]}$$

- Local stability condition & D-terms:

(Polyakov & Schweitzer, 2018)

$$\left.\frac{dF_r}{dS_r}\right|_{\text{unp}} = p_0(r) + \frac{2}{3}s_0(r) \geq 0$$

$$\mathcal{D}_0 = m\int d^3r\, r^2 p_0(r) = -\frac{4}{15}m\int d^3r\, r^2 s_0(r) \leq 0$$



See Panteleeva's talk for novel definitions of local densities

spin-0 GFFs and its D-term

Kobzarev & Okun 1962; Pagels 1966;

Definition: $\langle p' | \hat{T}^a_{\mu\nu}(x) | p\rangle = \left[2P_\mu P_\nu\, A^a(t) + \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\, D^a(t) + 2\ m^2\ \bar{c}^a(t)\, g_{\mu\nu}\right] e^{i(p'-p)x}$ 2+1

Free Klein-Gordon field (no interaction):

Callan, Coleman, Jackiw 1970
Collins, 1976,
Hudson & Schweitzer, 2017

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\Phi)(\partial^\mu\Phi) - V_0(\Phi)\,, \quad V_0(\Phi) = \frac{1}{2}m^2\Phi^2$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \ \Rightarrow\ D \equiv \lim_{t\to 0} D(t) = -1$$

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Action in cured spacetime with conformal symmetry requires a non-minimal coupling term:

$$S_{\rm grav} = \int \mathrm{d}^n x \, \sqrt{-g} \left(\frac{1}{2} \, g^{\mu\nu} (\partial_\mu \Phi)(\partial_\nu \Phi) - V(\Phi) - \frac{1}{2} \, h \, R \, \Phi^2 \right) \, , \qquad h = \frac{1}{4} \left(\frac{n-2}{n-1} \right)$$

Generate one "improvement term" in EMT (not vanish in flat limit)

$$\theta^{\mu\nu}_{\rm improve} = -h(\partial^\mu \partial^\nu - g^{\mu\nu} \Box) \, \Phi(x)^2$$

$$(\text{with } \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu)$$

$$T^{\mu\nu} \;\Rightarrow\; T^{\mu\nu} + \theta^{\mu\nu}_{\rm improve} \;\Rightarrow\; D = -\frac{1}{3}$$

spin-0 GFFs and its D-term

Kobzarev & Okun 1962; Pagels 1966;

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(with $\Box = g^{\mu\nu}\nabla_\mu \nabla_\nu$)

$$T^{\mu\nu} \;\Rightarrow\; T^{\mu\nu} + \theta^{\mu\nu}_{\mathrm{improve}} \;\Rightarrow\; D = -\frac{1}{3}$$

- Even infinitesimally small interactions can drastically impact D-term
- Cannot arbitrarily add "total derivatives" to the EMT
- h removes UV divergences up to three loops in dimensional regularization

spin-1 GFFs

Definition：（Holstein, 2006; Cosyn et al, 2019; Polyakov, BDS, 2019）

$$\begin{aligned}
\langle p',\sigma'|\hat{T}^a_{\mu\nu}(x)|p,\sigma\rangle = \Bigg[& 2P_\mu P_\nu\left(-\epsilon'^*\cdot\epsilon\, A^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}\, A^a_1(t)\right) \\
& + 2\left[P_\mu(\epsilon'^*_\nu\,\epsilon\cdot P + \epsilon_\nu\,\epsilon'^*\cdot P) + P_\nu(\epsilon'^*_\mu\,\epsilon\cdot P + \epsilon_\mu\,\epsilon'^*\cdot P)\right]\; J^a(t) \\
& + \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\left(\epsilon'^*\cdot\epsilon\, D^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}\, D^a_1(t)\right) \\
& + \Bigg[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu)\Delta^2 - (\epsilon'^*_\mu\Delta_\nu + \epsilon'^*_\nu\Delta_\mu)\,\epsilon\cdot P \\
& + (\epsilon_\mu\Delta_\nu + \epsilon_\nu\Delta_\mu)\,\epsilon'^*\cdot P - 4g_{\mu\nu}\,\epsilon'^*\cdot P\,\epsilon\cdot P\Bigg]\, E^a(t) \\
& + \left(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu - \frac{\epsilon'^*\cdot\epsilon}{2}\, g_{\mu\nu}\right) m^2\, \bar{f}^a(t) \\
& + g_{\mu\nu}\left(\epsilon'^*\cdot\epsilon\, m^2\, \bar{c}^a_0(t) \,+\, \epsilon'^*\cdot P\,\epsilon\cdot P\, \bar{c}^a_1(t)\right)\Bigg]\, e^{i(p'-p)x}
\end{aligned}$$

6 conserving

3 non-conserving

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$$
\begin{aligned}
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& + 2\left[P_\mu(\epsilon'^*_\nu\,\epsilon\cdot P + \epsilon_\nu\,\epsilon'^*\cdot P) + P_\nu(\epsilon'^*_\mu\,\epsilon\cdot P + \epsilon_\mu\,\epsilon'^*\cdot P)\right]\; J^a(t) \\
& + \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\left(\epsilon'^*\cdot\epsilon\, D_0^a(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}\, D_1^a(t)\right) \\
& + \Big[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu)\Delta^2 - (\epsilon'^*_\mu\Delta_\nu + \epsilon'^*_\nu\Delta_\mu)\,\epsilon\cdot P \\
& + (\epsilon_\mu\Delta_\nu + \epsilon_\nu\Delta_\mu)\,\epsilon'^*\cdot P - 4g_{\mu\nu}\,\epsilon'^*\cdot P\,\epsilon\cdot P\Big]\; E^a(t) \\
& + \left(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu - \frac{\epsilon'^*\cdot\epsilon}{2}\, g_{\mu\nu}\right) m^2\, \bar{f}^a(t) \\
& + g_{\mu\nu}\left(\epsilon'^*\cdot\epsilon\, m^2\, \bar{c}_0^a(t) + \epsilon'^*\cdot P\,\epsilon\cdot P\,\bar{c}_1^a(t)\right)\Bigg]\, e^{i(p'-p)x}
\end{aligned}
$$

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Spin operators: Polyakov & Schweitzer, 2018

$$
\hat{S}^{\lambda}_{\sigma'\sigma} = \sqrt{S(S+1)}\; C^{S\sigma'}_{S\sigma 1\lambda}
$$

$$
\hat{Q}^{ij} = \frac{1}{2}\left[\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij}\right]
$$

$$
\epsilon^\mu(p,\sigma) = \left(\frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m}, \hat{\epsilon}_\sigma + \frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m(m+E)}\vec{p}\right) \quad (\text{for } S=1)
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& + \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\Big(\epsilon'^*\cdot\epsilon\, D_0^a(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2} D_1^a(t)\Big) \\
& + \Big[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu)\Delta^2 - (\epsilon'^*_\mu\Delta_\nu + \epsilon'^*_\nu\Delta_\mu)\,\epsilon\cdot P \\
& + (\epsilon_\mu\Delta_\nu + \epsilon_\nu\Delta_\mu)\,\epsilon'^*\cdot P - 4g_{\mu\nu}\,\epsilon'^*\cdot P\,\epsilon\cdot P\Big] E^a(t) \\
& + \Big(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu - \frac{\epsilon'^*\cdot\epsilon}{2} g_{\mu\nu}\Big) m^2\, \bar{f}^a(t) \\
& + g_{\mu\nu}\Big(\epsilon'^*\cdot\epsilon\, m^2\, \bar{c}_0^a(t) + \epsilon'^*\cdot P\,\epsilon\cdot P\,\bar{c}_1^a(t)\Big)\Big] e^{i(p'-p)x}
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\hat{Q}^{ij} = \frac{1}{2}\left[\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij}\right]
$$

$$
\epsilon^\mu(p,\sigma) = \left(\frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m}, \hat{\epsilon}_\sigma + \frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m(m+E)}\vec{p}\right) \quad (\text{for } S=1)
$$

Multipole expansion: (Polyakov, BDS, 2019)

$$
\begin{aligned}
\langle \hat{T}^{00}_a(0)\rangle &= 2m^2\mathcal{E}_0^a(t)\,\delta_{\sigma'\sigma} + \hat{Q}^{kl}\,\Delta^k\Delta^l\,\mathcal{E}_2^a(t)\,, \\
\langle \hat{T}^{0j}_a(0)\rangle &= i\epsilon^{jkl}\hat{S}^k_{\sigma'\sigma}\Delta^l\, m\,\mathcal{J}^a(t)\,, \\
\langle \hat{T}^{ij}_a(0)\rangle &= \frac{1}{2}(\Delta^i\Delta^j - \delta^{ij}\vec{\Delta}^2)\mathcal{D}_0^a(t)\,\delta_{\sigma'\sigma} \\
&\quad + \Big(\Delta^j\Delta^k\hat{Q}^{ik} + \Delta^i\Delta^k\hat{Q}^{jk} - \vec{\Delta}^2\hat{Q}^{ij} - \delta^{ij}\Delta^k\Delta^l\hat{Q}^{kl}\Big)\,\mathcal{D}_2^a(t) \\
&\quad + \frac{1}{2m^2}(\Delta^i\Delta^j - \delta^{ij}\vec{\Delta}^2)\Delta^k\Delta^l\hat{Q}^{kl}\,\mathcal{D}_3^a(t) \\
&\quad + \text{non-conserving terms}
\end{aligned}
$$

spin-1 GFFs

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$$\langle p',\sigma'|\hat{T}^a_{\mu\nu}(x)|p,\sigma\rangle = \Big[2P_\mu P_\nu\Big(-\epsilon'^*\cdot\epsilon\, A^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}A^a_1(t)\Big)$$
$$+ 2\left[P_\mu(\epsilon'^*_\nu\,\epsilon\cdot P + \epsilon_\nu\,\epsilon'^*\cdot P) + P_\nu(\epsilon'^*_\mu\,\epsilon\cdot P + \epsilon_\mu\,\epsilon'^*\cdot P)\right]\,J^a(t)$$
$$+ \frac{1}{2}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)\Big(\epsilon'^*\cdot\epsilon\,D^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}D^a_1(t)\Big)$$
$$+ \Big[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu)\Delta^2 - (\epsilon'^*_\mu\Delta_\nu + \epsilon'^*_\nu\Delta_\mu)\,\epsilon\cdot P$$
$$+ (\epsilon_\mu\Delta_\nu + \epsilon_\nu\Delta_\mu)\,\epsilon'^*\cdot P - 4g_{\mu\nu}\,\epsilon'^*\cdot P\,\epsilon\cdot P\Big]\,E^a(t)$$
$$+ \Big(\epsilon_\mu\epsilon'^*_\nu + \epsilon'^*_\mu\epsilon_\nu - \frac{\epsilon'^*\cdot\epsilon}{2}g_{\mu\nu}\Big)\,m^2\,\bar{f}^a(t)$$
$$+ g_{\mu\nu}\Big(\epsilon'^*\cdot\epsilon\,m^2\,\bar{c}^a_0(t) + \epsilon'^*\cdot P\,\epsilon\cdot P\,\bar{c}^a_1(t)\Big)\Big]\,e^{i(p'-p)x}$$

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Spin operators: Polyakov & Schweitzer, 2018

$$\hat{S}^\lambda_{\sigma'\sigma} = \sqrt{S(S+1)}\,C^{S\sigma'}_{S\sigma 1\lambda}$$
$$\hat{Q}^{ij} = \frac{1}{2}\Big[\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij}\Big]$$
$$\epsilon^\mu(p,\sigma) = \Big(\frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m},\,\hat{\epsilon}_\sigma + \frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m(m+E)}\vec{p}\Big)\quad (\text{for } S=1)$$

Multipole expansion: (Polyakov, BDS, 2019)

$$\langle\hat{T}^{00}_a(0)\rangle = 2m^2\mathcal{E}^a_0(t)\,\delta_{\sigma'\sigma} + \hat{Q}^{kl}\,\Delta^k\Delta^l\,\mathcal{E}^a_2(t)\,,$$
$$\langle\hat{T}^{0j}_a(0)\rangle = i\epsilon^{jkl}\hat{S}^k_{\sigma'\sigma}\Delta^l\,m\,\mathcal{J}^a(t)\,,$$
$$\langle\hat{T}^{ij}_a(0)\rangle = \frac{1}{2}(\Delta^i\Delta^j - \delta^{ij}\vec{\Delta}^2)\mathcal{D}^a_0(t)\,\delta_{\sigma'\sigma}$$
$$+ \Big(\Delta^j\Delta^k\hat{Q}^{ik} + \Delta^i\Delta^k\hat{Q}^{jk} - \vec{\Delta}^2\hat{Q}^{ij} - \delta^{ij}\Delta^k\Delta^l\hat{Q}^{kl}\Big)\,\mathcal{D}^a_2(t)$$
$$+ \frac{1}{2m^2}(\Delta^i\Delta^j - \delta^{ij}\vec{\Delta}^2)\Delta^k\Delta^l\hat{Q}^{kl}\,\mathcal{D}^a_3(t)$$
$$+ \text{non-conserving terms}$$

Gravitational multipole form factors

$$\mathcal{E}^a_0(t) = A^a_0(t) - \frac{t}{m^2}\frac{5}{12}A^a_0(t) + \cdots$$
$$\mathcal{E}^a_2(t) = -A^a_0(t) + 2J^a(t) - E^a(t) + \cdots$$
$$\mathcal{J}^a(t) = J^a(t) - \frac{t}{4m^2}\Big[J^a(t) - E^a(t)\Big] + \cdots$$
$$\mathcal{D}^a_0(t) = -D^a_0(t) + \frac{4}{3}E^a(t) + \cdots$$
$$\mathcal{D}^a_2(t) = -E^a(t)$$
$$\mathcal{D}^a_3(t) = \frac{1}{4}\Big[2D^a_0(t) - 2E^a(t) + D^a_1(t)\Big] + \cdots$$

spin-3/2 GFFs

Rarita-Schwinger spinor:
$u^\mu = \sum C^{\frac{3}{2}\sigma}_{1\lambda\frac{1}{2}s} u_s(p)\epsilon^\mu_\lambda$

Definition: (Cosyn et al, 2019)

$$
\begin{aligned}
\langle \hat{T}^{\mu\nu}_a(0)\rangle = -\bar{u}^{\alpha'}(p')\Bigg[&\frac{P^\mu P^\nu}{m}\left(g_{\alpha'\alpha}F^a_{1,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{1,1}(t)\right)\\
&+\frac{(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)}{4m}\left(g_{\alpha'\alpha}F^a_{2,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{2,1}(t)\right)\\
&+mg^{\mu\nu}\left(g_{\alpha'\alpha}F^a_{3,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{3,1}(t)\right)\\
&+\frac{i}{2}\frac{(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})\Delta_\rho}{m}\left(g_{\alpha'\alpha}\underbrace{F^a_{4,0}(t)}_{\text{(spin)}} - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{4,1}(t)\right)\\
&-\frac{1}{m}(\Delta^\mu g^\nu_{\alpha'}\Delta_\alpha + \Delta^\nu g^\mu_{\alpha'}\Delta_\alpha + \Delta^\mu g^\nu_\alpha\Delta_{\alpha'} + \Delta^\nu g^\mu_\alpha\Delta_{\alpha'}\\
&-2g^{\mu\nu}\Delta_{\alpha'}\Delta_\alpha - g^\mu_{\alpha'}g^\nu_\alpha\Delta^2 - g^\nu_{\alpha'}g^\mu_\alpha\Delta^2)F^a_{5,0}(t)\\
&+m(g^\mu_{\alpha'}g^\nu_\alpha + g^\nu_{\alpha'}g^\mu_\alpha)F^a_{6,0}(t)\Bigg]u^\alpha(p,\sigma)
\end{aligned}
$$

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&+ \frac{(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)}{4m}\left(g_{\alpha'\alpha}F^a_{2,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{2,1}(t)\right) \\
&+ mg^{\mu\nu}\left(g_{\alpha'\alpha}F^a_{3,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{3,1}(t)\right) \\
&+ \frac{i}{2}\frac{(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})\Delta_\rho}{m}\left(g_{\alpha'\alpha}\underbrace{F^a_{4,0}(t)}_{\text{(spin)}} - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{4,1}(t)\right) \\
&- \frac{1}{m}(\Delta^\mu g^\nu_{\alpha'}\Delta_\alpha + \Delta^\nu g^\mu_{\alpha'}\Delta_\alpha + \Delta^\mu g^\nu_\alpha\Delta_{\alpha'} + \Delta^\nu g^\mu_\alpha\Delta_{\alpha'} \\
&- 2g^{\mu\nu}\Delta_{\alpha'}\Delta_\alpha - g^\mu_{\alpha'}g^\nu_\alpha\Delta^2 - g^\nu_{\alpha'}g^\mu_\alpha\Delta^2)F^a_{5,0}(t) \\
&+ m(g^\mu_{\alpha'}g^\nu_\alpha + g^\nu_{\alpha'}g^\mu_\alpha)F^a_{6,0}(t)\Bigg]u^\alpha(p,\sigma)
\end{aligned}$$

7 conserving

3 non-conserving

Octupole operator:

$$\begin{aligned}
\hat{O}^{ijk} = \frac{1}{6}\Big[&\hat{S}^i\hat{S}^j\hat{S}^k + \hat{S}^j\hat{S}^i\hat{S}^k + \hat{S}^k\hat{S}^j\hat{S}^i \\
&+ \hat{S}^j\hat{S}^k\hat{S}^i + \hat{S}^i\hat{S}^k\hat{S}^j + \hat{S}^k\hat{S}^i\hat{S}^j \\
&- \frac{6S(S+1)-2}{5}(\delta^{ij}\hat{S}^k + \delta^{ik}\hat{S}^j + \delta^{kj}\hat{S}^i)\Big]
\end{aligned}$$

n-rank irreducible tensors:

$$Y^{i_1 i_2 \ldots i_n}_n(\Omega_p) = \frac{(-1)^n}{(2n-1)!!}p^{n+1}\partial^{i_1}\partial^{i_2}\ldots\partial^{i_n}\frac{1}{p}$$

spin-3/2 GFFs

Rarita-Schwinger spinor:
$u^\mu = \sum C^{\frac{3}{2}\sigma}_{1\lambda\frac{1}{2}s} u_s(p)\epsilon^\mu_\lambda$

Definition: (Cosyn et al, 2019)

$$
\begin{aligned}
\langle \hat{T}^{\mu\nu}_a(0)\rangle = -\bar{u}^{\alpha'}(p')\Bigg[&\frac{P^\mu P^\nu}{m}\left(g_{\alpha'\alpha}F^a_{1,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{1,1}(t)\right)\\
&+\frac{(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)}{4m}\left(g_{\alpha'\alpha}F^a_{2,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{2,1}(t)\right)\\
&+mg^{\mu\nu}\left(g_{\alpha'\alpha}F^a_{3,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{3,1}(t)\right)\\
&+\frac{i}{2}\frac{(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})\Delta_\rho}{m}\left(g_{\alpha'\alpha}\underbrace{F^a_{4,0}(t)}_{\text{(spin)}} - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{4,1}(t)\right)\\
&-\frac{1}{m}(\Delta^\mu g^\nu_{\alpha'}\Delta_\alpha + \Delta^\nu g^\mu_{\alpha'}\Delta_\alpha + \Delta^\mu g^\nu_\alpha\Delta_{\alpha'} + \Delta^\nu g^\mu_\alpha\Delta_{\alpha'}\\
&-2g^{\mu\nu}\Delta_{\alpha'}\Delta_\alpha - g^\mu_{\alpha'}g^\nu_\alpha\Delta^2 - g^\nu_{\alpha'}g^\mu_\alpha\Delta^2)F^a_{5,0}(t)\\
&+m(g^\mu_{\alpha'}g^\nu_\alpha + g^\nu_{\alpha'}g^\mu_\alpha)F^a_{6,0}(t)\Bigg]u^\alpha(p,\sigma)
\end{aligned}
$$

7 conserving

3 non-conserving

Octupole operator:

$$
\begin{aligned}
\hat{O}^{ijk} = \frac{1}{6}\Bigg[&\hat{S}^i\hat{S}^j\hat{S}^k + \hat{S}^j\hat{S}^i\hat{S}^k + \hat{S}^k\hat{S}^j\hat{S}^i\\
&+\hat{S}^j\hat{S}^k\hat{S}^i + \hat{S}^i\hat{S}^k\hat{S}^j + \hat{S}^k\hat{S}^i\hat{S}^j\\
&-\frac{6S(S+1)-2}{5}(\delta^{ij}\hat{S}^k + \delta^{ik}\hat{S}^j + \delta^{kj}\hat{S}^i)\Bigg]
\end{aligned}
$$

n-rank irreducible tensors:

$$
Y^{i_1 i_2 \ldots i_n}_n(\Omega_p) = \frac{(-1)^n}{(2n-1)!!}p^{n+1}\partial^{i_1}\partial^{i_2}\ldots\partial^{i_n}\frac{1}{p}
$$

Multipole expansion: (Kim, BDS, 2020)

$$
\langle \hat{T}^{00}_a(0)\rangle = 2mE\left[\mathcal{E}^a_0(t)\delta_{\sigma'\sigma} + \left(\frac{\sqrt{-t}}{m}\right)^2\hat{Q}^{kl}_{\sigma'\sigma}Y^{kl}_2\mathcal{E}^a_2(t)\right]
$$

$$
\langle \hat{T}^{0i}_a(0)\rangle = 2mE\left[\frac{\sqrt{-t}}{m}i\epsilon^{ikl}Y^l_1\hat{S}^k_{\sigma'\sigma}\mathcal{J}^a_1(t) + \left(\frac{\sqrt{-t}}{m}\right)^3 i\epsilon^{ikl}Y^{lmn}_3\hat{O}^{kmn}_{\sigma'\sigma}\mathcal{J}^a_3(t)\right]
$$

$$
\begin{aligned}
\langle \hat{T}^{ij}_a(0)\rangle = 2mE\Bigg[&\frac{1}{4m^2}(\Delta^i\Delta^j + \delta^{ij}\Delta^2)D^a_0(t)\delta_{\sigma'\sigma}\\
&+\frac{1}{4m^4}\hat{Q}^{kl}_{\sigma'\sigma}(\Delta^i\Delta^j + \delta^{ij}\Delta^2)\Delta^k\Delta^l D^a_3(t)\\
&+\frac{1}{2m^2}\left(\hat{Q}^{ik}_{\sigma'\sigma}\Delta^j\Delta^k + \hat{Q}^{jk}_{\sigma'\sigma}\Delta^i\Delta^k + \hat{Q}^{ij}_{\sigma'\sigma}\Delta^2 - \delta^{ij}\hat{Q}^{kl}_{\sigma'\sigma}\Delta^k\Delta^l\right)D^a_2(t)\\
&+\text{non-conserving terms}\Bigg]
\end{aligned}
$$

spin-3/2 GFFs

Rarita-Schwinger spinor:
$$u^\mu = \sum C^{\frac{3}{2}\sigma}_{1\lambda\frac{1}{2}s} u_s(p)\epsilon^\mu_\lambda$$

Definition: (Cosyn et al, 2019)

$$\begin{aligned}\langle \hat{T}^{\mu\nu}_a(0)\rangle = -\bar{u}^{\alpha'}(p')\Bigg[&\frac{P^\mu P^\nu}{m}\left(g_{\alpha'\alpha}F^a_{1,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{1,1}(t)\right)\\ &+\frac{(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)}{4m}\left(g_{\alpha'\alpha}F^a_{2,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{2,1}(t)\right)\\ &+mg^{\mu\nu}\left(g_{\alpha'\alpha}F^a_{3,0}(t) - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{3,1}(t)\right)\\ &+\frac{i}{2}\frac{(P^\mu\sigma^{\nu\rho} + P^\nu\sigma^{\mu\rho})\Delta_\rho}{m}\left(g_{\alpha'\alpha}\underset{\text{(spin)}}{F^a_{4,0}(t)} - \frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{4,1}(t)\right)\\ &-\frac{1}{m}(\Delta^\mu g^\nu_{\alpha'}\Delta_\alpha + \Delta^\nu g^\mu_{\alpha'}\Delta_\alpha + \Delta^\mu g^\nu_\alpha\Delta_{\alpha'} + \Delta^\nu g^\mu_\alpha\Delta_{\alpha'}\\ &-2g^{\mu\nu}\Delta_{\alpha'}\Delta_\alpha - g^\mu_{\alpha'}g^\nu_\alpha\Delta^2 - g^\nu_{\alpha'}g^\mu_\alpha\Delta^2)F^a_{5,0}(t)\\ &+m(g^\mu_{\alpha'}g^\nu_\alpha + g^\nu_{\alpha'}g^\mu_\alpha)F^a_{6,0}(t)\Bigg]u^\alpha(p,\sigma)\end{aligned}$$

7 conserving

3 non-conserving

Octupole operator:

$$\begin{aligned}\hat{O}^{ijk} =& \frac{1}{6}\Big[\hat{S}^i\hat{S}^j\hat{S}^k + \hat{S}^j\hat{S}^i\hat{S}^k + \hat{S}^k\hat{S}^j\hat{S}^i\\ &+\hat{S}^j\hat{S}^k\hat{S}^i + \hat{S}^i\hat{S}^k\hat{S}^j + \hat{S}^k\hat{S}^i\hat{S}^j\\ &-\frac{6S(S+1)-2}{5}(\delta^{ij}\hat{S}^k + \delta^{ik}\hat{S}^j + \delta^{kj}\hat{S}^i)\Big]\end{aligned}$$

n-rank irreducible tensors:

$$Y^{i_1 i_2 \dots i_n}_n(\Omega_p) = \frac{(-1)^n}{(2n-1)!!}p^{n+1}\partial^{i_1}\partial^{i_2}\dots\partial^{i_n}\frac{1}{p}$$

Multipole expansion: (Kim, BDS, 2020)

$$\langle \hat{T}^{00}_a(0)\rangle = 2mE\left[\mathcal{E}^a_0(t)\delta_{\sigma'\sigma} + \left(\frac{\sqrt{-t}}{m}\right)^2\hat{Q}^{kl}_{\sigma'\sigma}Y^{kl}_2\mathcal{E}^a_2(t)\right]$$

$$\langle \hat{T}^{0i}_a(0)\rangle = 2mE\left[\frac{\sqrt{-t}}{m}i\epsilon^{ikl}Y^l_1\hat{S}^k_{\sigma'\sigma}\mathcal{J}^a_1(t) + \left(\frac{\sqrt{-t}}{m}\right)^3 i\epsilon^{ikl}Y^{lmn}_3\hat{O}^{kmn}_{\sigma'\sigma}\mathcal{J}^a_3(t)\right]$$

$$\begin{aligned}\langle \hat{T}^{ij}_a(0)\rangle = 2mE\Bigg[&\frac{1}{4m^2}(\Delta^i\Delta^j + \delta^{ij}\Delta^2)D^a_0(t)\delta_{\sigma'\sigma}\\ &+\frac{1}{4m^4}\hat{Q}^{kl}_{\sigma'\sigma}(\Delta^i\Delta^j + \delta^{ij}\Delta^2)\Delta^k\Delta^l D^a_3(t)\\ &+\frac{1}{2m^2}\left(\hat{Q}^{ik}_{\sigma'\sigma}\Delta^j\Delta^k + \hat{Q}^{jk}_{\sigma'\sigma}\Delta^i\Delta^k + \hat{Q}^{ij}_{\sigma'\sigma}\Delta^2 - \delta^{ij}\hat{Q}^{kl}_{\sigma'\sigma}\Delta^k\Delta^l\right)D^a_2(t)\\ &+\text{non-conserving terms}\Bigg]\end{aligned}$$

Gravitational multipole form factors

$$\mathcal{E}^a_0(t) = F^a_{1,0}(t) + F^a_{3,0}(t) - \frac{t}{m^2}\frac{5}{12}F^a_{1,0}(t) + \cdots$$

$$\mathcal{E}^a_2(t) = -\frac{1}{6}F^a_{1,0}(t) - \frac{1}{6}F^a_{1,1}(t) + \cdots$$

$$\mathcal{J}^a_1(t) = \frac{1}{3}F^a_{4,0}(t) - \frac{1}{3}F^a_{6,0}(t) + \cdots$$

$$\mathcal{J}^a_3(t) = -\frac{1}{6}\left[F^a_{4,0}(t) + F^a_{4,1}(t)\right] + \frac{t}{24m^2}F^a_{4,1}(t)$$

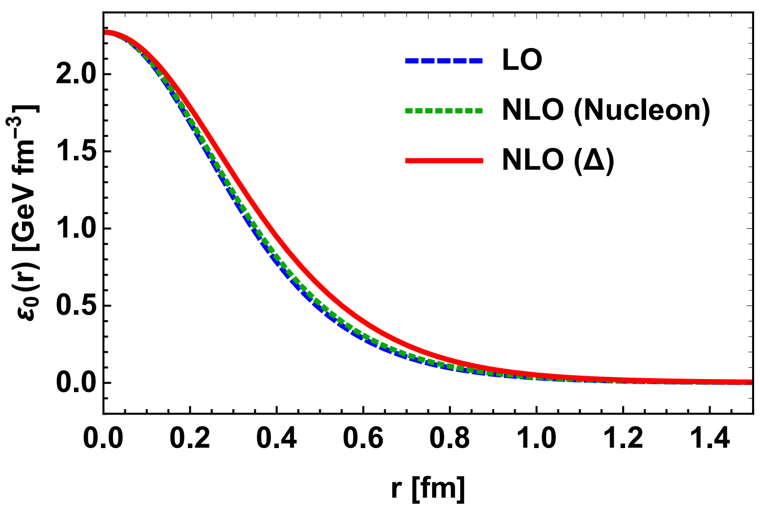
$$\rightarrow D^a_0(t) = F^a_{2,0}(t) - \frac{16}{3}F^a_{5,0}(t) + \cdots$$

$$D^a_2(t) = \frac{4}{3}F^a_{5,0}(t)$$

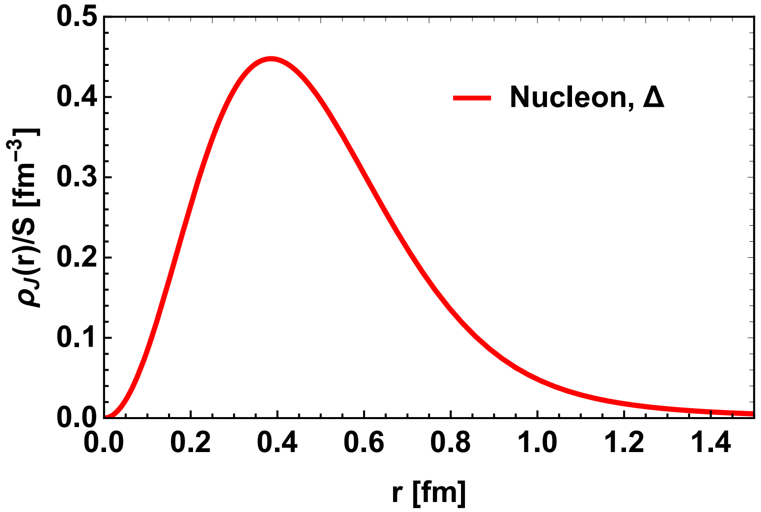
$$D^a_3(t) = -\frac{1}{6}F^a_{2,0}(t) - \frac{1}{6}F^a_{2,1}(t) + \cdots$$

Δ densities by SU(2) Skyrme model (Kim, BDS, 2020)

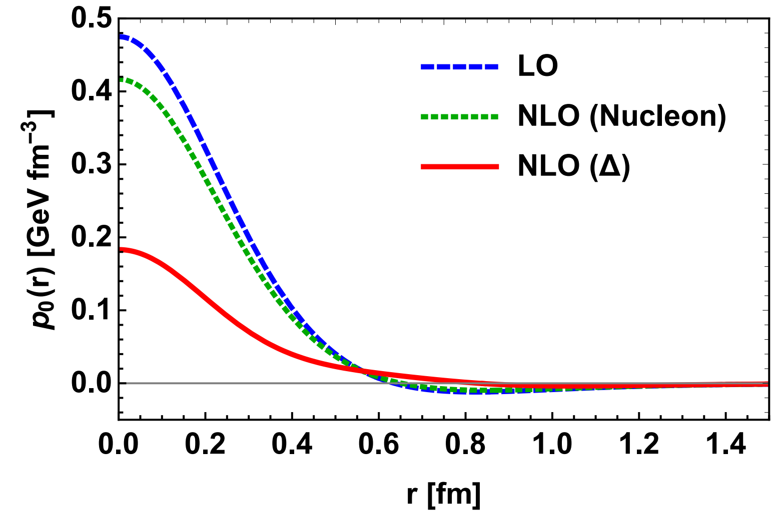
(energy/mass)

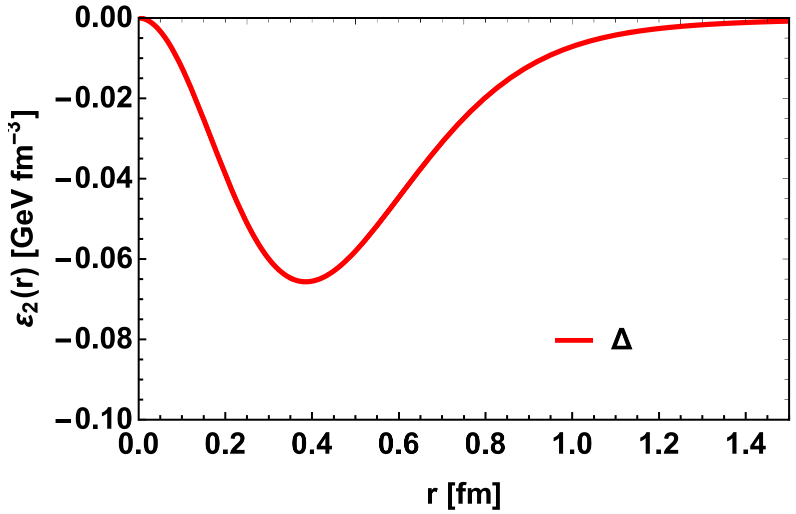


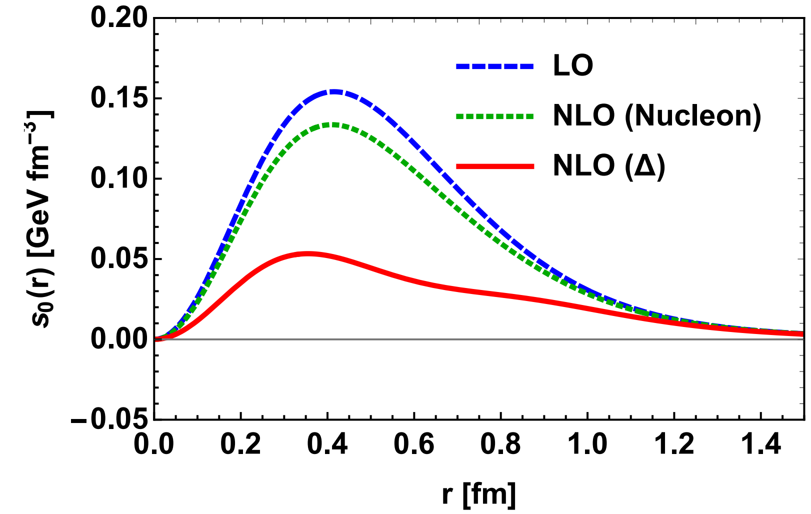
(spin)



(pressure & shear forces: "mechanical")





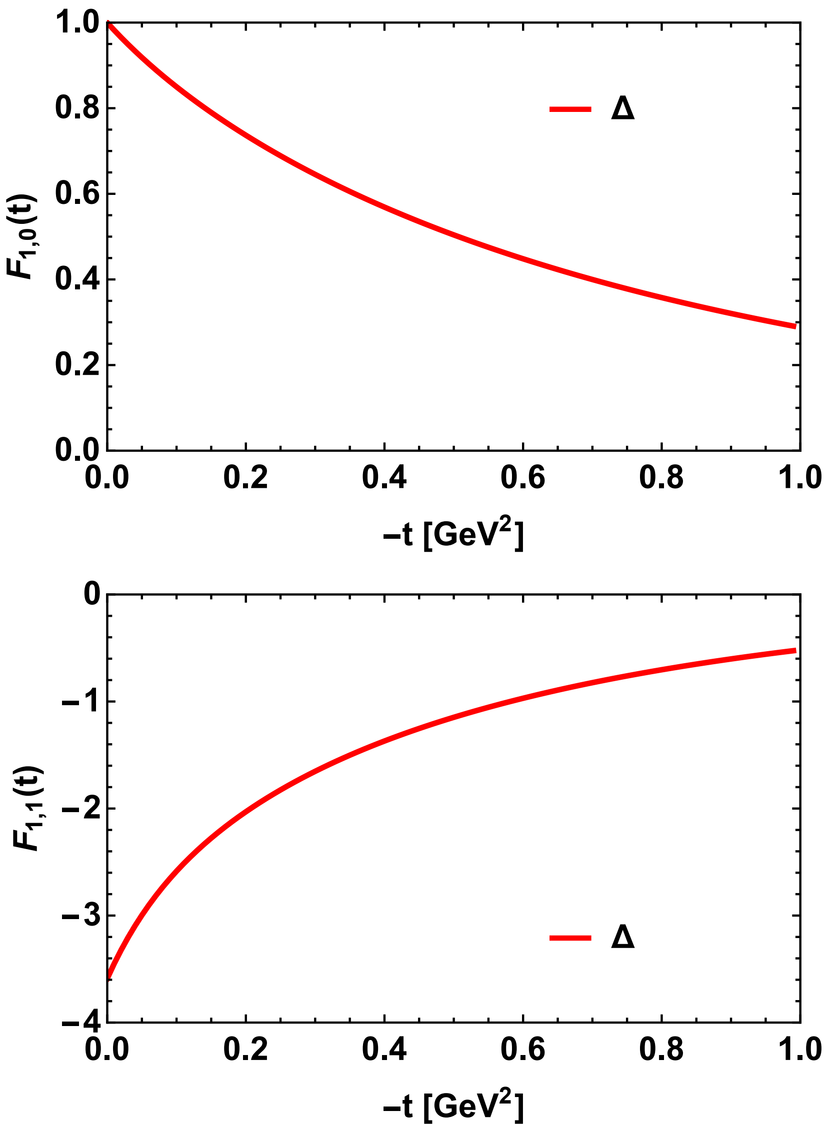


$$\langle r_E^2\rangle = 0.54\,\mathrm{fm}^2\ (\mathrm{LO})$$
$$\langle r_E^2\rangle = 0.57\,\mathrm{fm}^2\ (\mathrm{NLO},\ \mathrm{Nucleon})$$
$$\langle r_E^2\rangle = 0.64\,\mathrm{fm}^2\ (\mathrm{NLO},\ \Delta)$$
$$\mathcal{Q}^{ij}_{\sigma'\sigma} = -0.0181\ Q^{ij}_{\sigma'\sigma}\mathrm{GeV}\cdot\mathrm{fm}^2$$

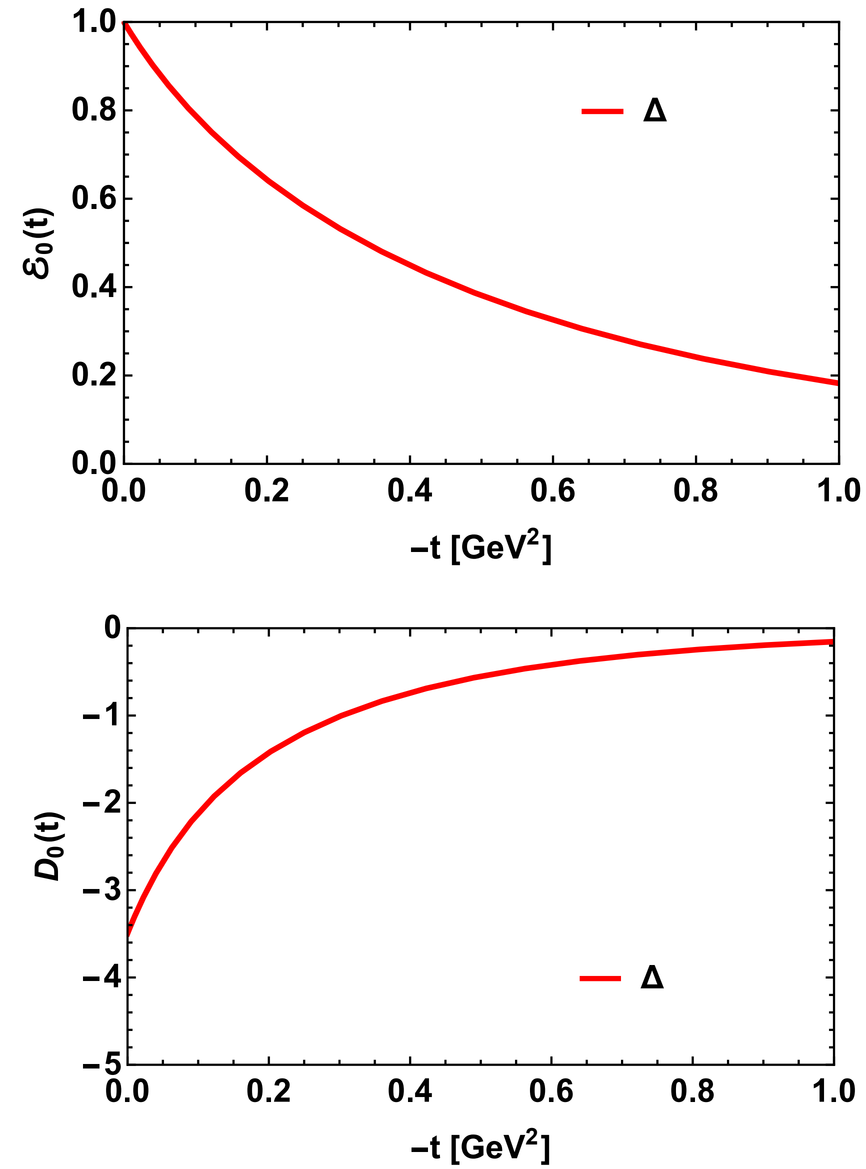
$$\langle r_J^2\rangle_{N,\Delta} = 0.92\,\mathrm{fm}^2$$

$$\langle r_0^2\rangle_{\mathrm{mech}} = 0.61\,\mathrm{fm}^2\ (\mathrm{LO})$$
$$\langle r_0^2\rangle_{\mathrm{mech}} = 0.63\,\mathrm{fm}^2\ (\mathrm{NLO},\ \mathrm{Nucleon})$$
$$\langle r_0^2\rangle_{\mathrm{mech}} = 0.85\,\mathrm{fm}^2\ (\mathrm{NLO},\ \Delta)$$
$$\langle r_3^2\rangle_{\mathrm{mech}} = 0.33\,\mathrm{fm}^2$$

Δ GFFs/GMFFs by SU(2) Skyrme model (Kim, BDS, 2020)



+ …



+ …

large-N_c behaviors: (GMFFs)

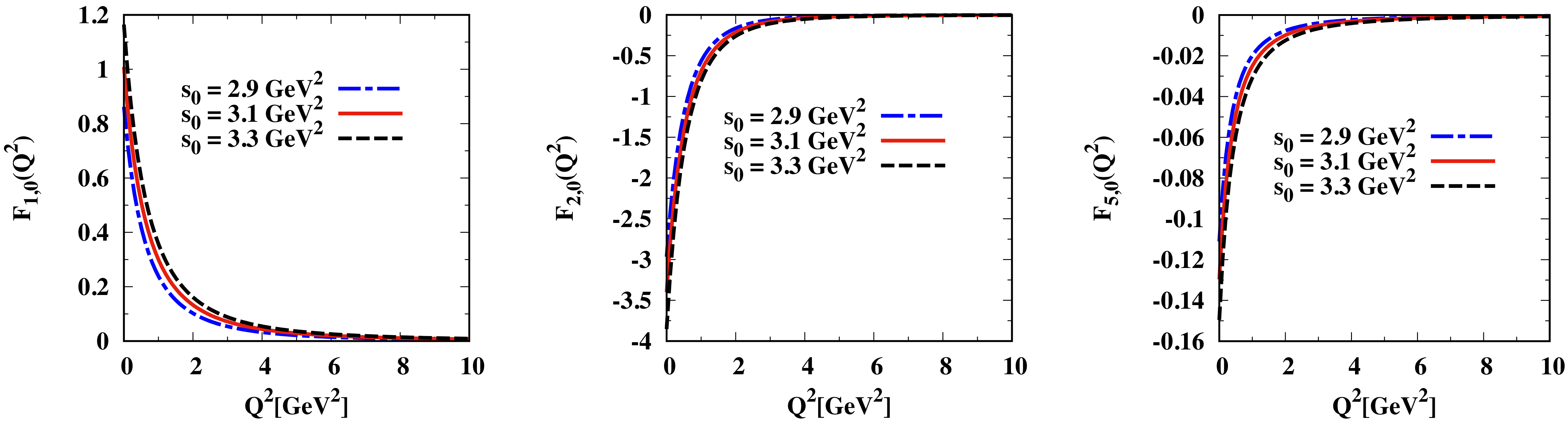
$$\mathcal{E}_0(t) \sim \mathcal{O}(N_c^0), \quad \mathcal{E}_2(t) \sim \mathcal{O}(N_c^0), \quad \mathcal{J}_0(t) \sim \mathcal{O}(N_c^0), \quad \mathcal{J}_3(t) \sim \mathcal{O}(N_c^0),$$
$$D_0(t) \sim \mathcal{O}(N_c^2), \quad D_2(t) \sim \mathcal{O}(N_c^0), \quad D_3(t) \sim \mathcal{O}(N_c^2)$$

✔

(Generalized) D-terms

$$\mathcal{D}_0^{\Delta} = -3.53 < 0 \text{ (stable!)}$$ ✔
$$\mathcal{D}_0^{N} = -3.63$$
$$\mathcal{D}_2 = 0$$
$$\mathcal{D}_3 = -0.50$$

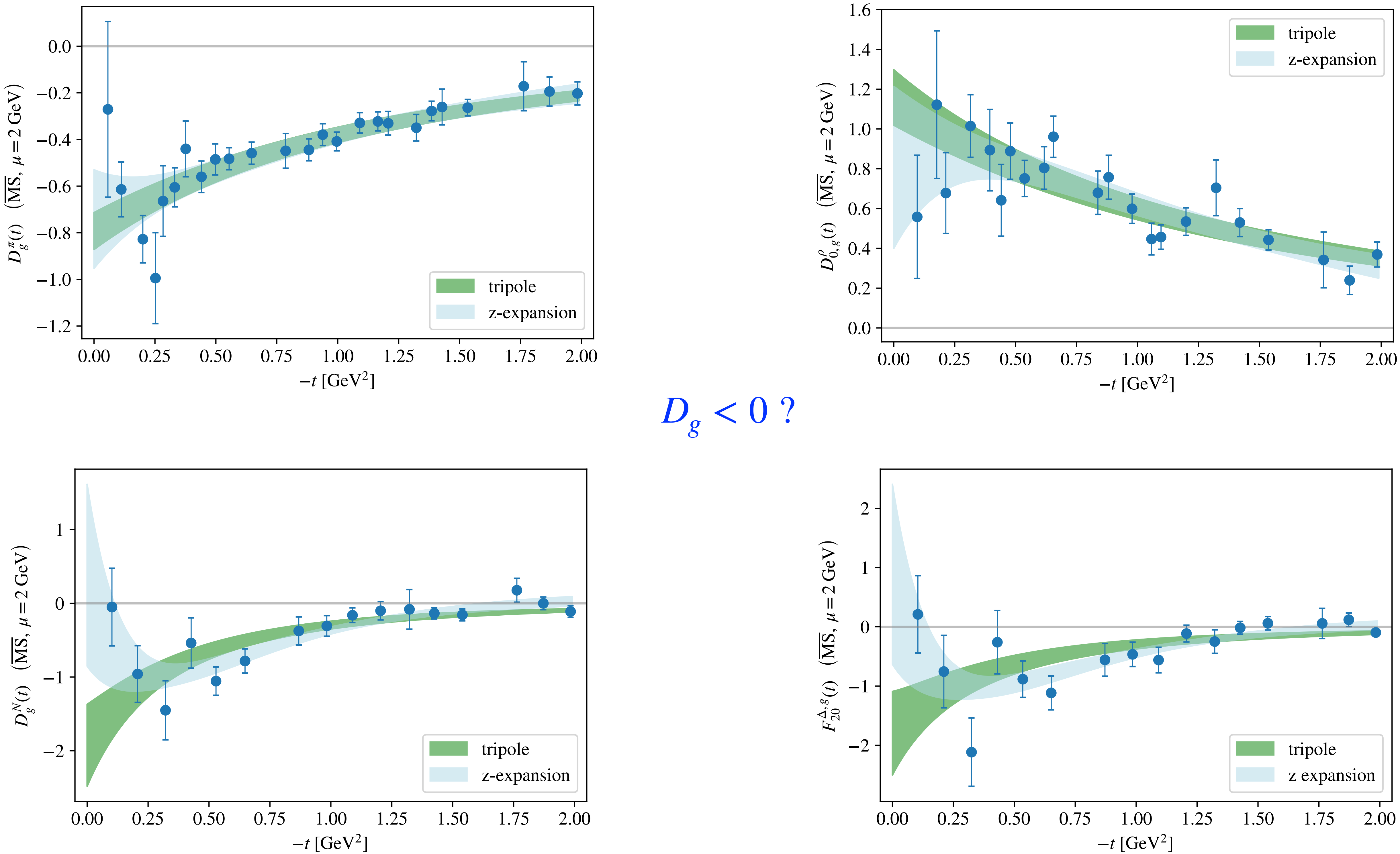
QCD-Sum-Rule approach for spin-3/2 GFFs Dehghan, Azizi, Özdem, 2023



Model	$\mathcal{D}_0^\Delta$	$\mathcal{D}_2^\Delta$	$\mathcal{D}_3^\Delta$	$\mathcal{D}_0^N$	$\langle r_E^2 \rangle$ (fm^2)
This Work	-2.71 ± 0.34	0.000 ± 0.002	-0.43 ± 0.06	-3.57 ± 0.46	0.67 ± 0.04
Skyrme model [52, 53]	-2.65	0	-0.38	-3.40	0.64
Skyrme model [54]	-3.53	0	-0.50	-3.63	0.64

TABLE III. A comparison of D-terms and mass radius obtained in the present study with those from other models.

Lattice QCD calculation for gluonic GFFs Pefkou, Hackett, Shanahan, 2022



$D_g < 0$?

Δ in **quark-diquark model** Fu, BDS, Dong, 2022

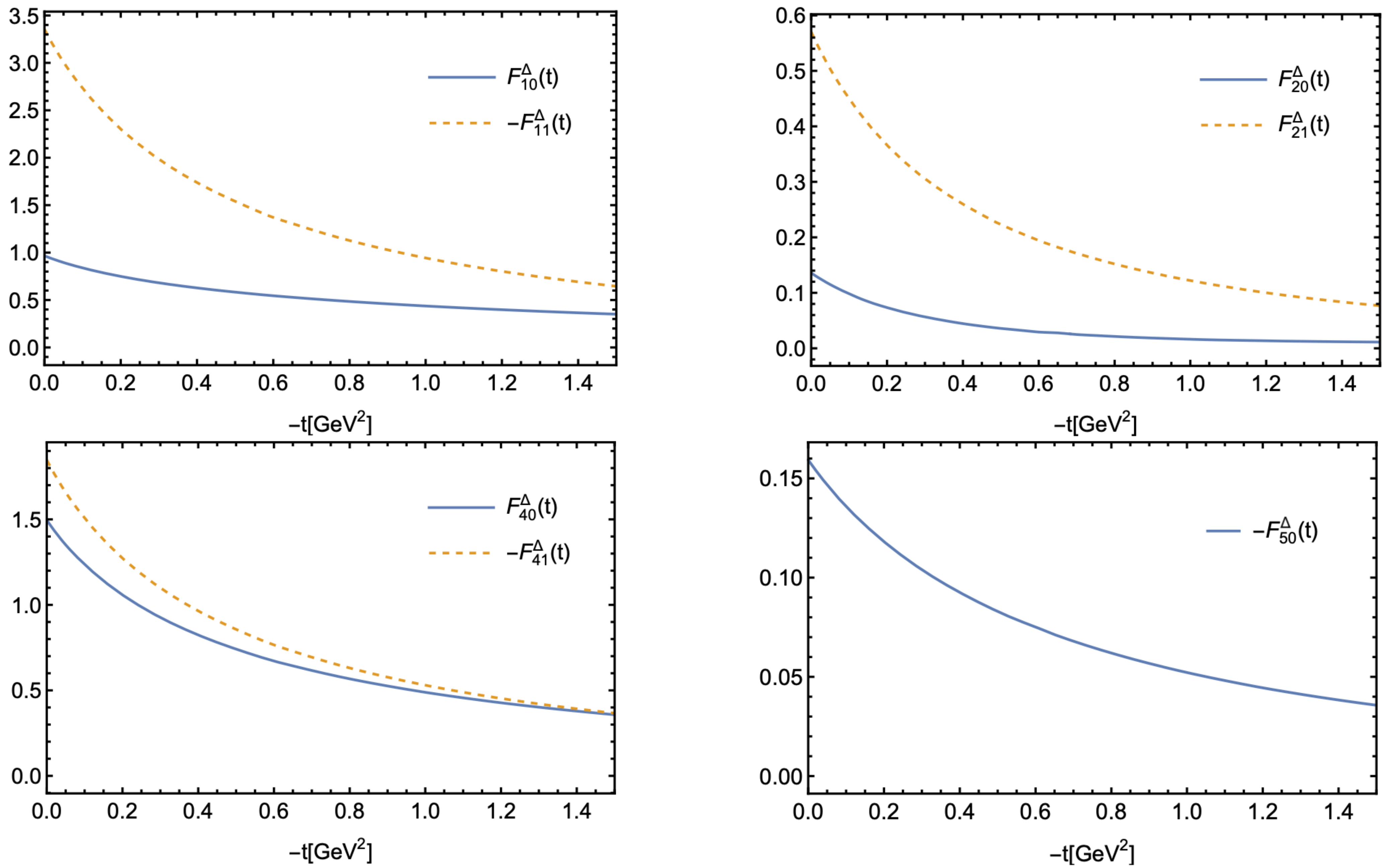


Figure 6: Calculated GFFs of $F^T_{10,11,20,21,40,41,50}$ as functions of $-t$ for Δ.

With D-term $D = 0.986 > 0$?

Quark model is too rough?

- $D_{\text{hydrogen}} > 0$ Ji, Liu 2021, 2022
  Not applicable to low-density objects?

ChPT actions for Δ in curved space-time Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

Rarita-Schwinger fields: $\Psi_\mu(x) = \sum_{s_\Delta} \int \frac{d^3p}{(2\pi)^3} \frac{M_\Delta}{E} \left[b(\vec{p}, s_\Delta) u_\mu(\vec{p}, s_\Delta) e^{-ip\cdot x} + d^\dagger(\vec{p}, s_\Delta) v_\mu(\vec{p}, s_\Delta) e^{ip\cdot x} \right]$

Derivatives on fields:

$\pi \qquad D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$

$N \qquad \overset{\rightarrow}{\nabla}_\mu \Psi = \partial_\mu \Psi + \frac{i}{2} \omega_\mu^{ab} \sigma_{ab} \Psi + \left(\Gamma_\mu - i v_\mu^{(s)} \right) \Psi$

$\Delta \qquad \nabla_\mu^{ij} \Psi_\nu^j = \left[\delta^{ij} \partial_\mu + \delta^{ij} \Gamma_\mu - i \delta^{ij} v_\mu^{(s)} - i \epsilon^{ijk} \mathrm{Tr}\left(\tau^k \Gamma_\mu \right) + \frac{i}{2} \delta^{ij} \omega_\mu^{ab} \sigma_{ab} \right] \Psi_\nu^j - \Gamma_{\mu\nu}^\alpha \Psi_\alpha^i$

Spin connection: $\omega_\mu^{ab} = -\frac{1}{2} g^{\nu\lambda} e_\lambda^a \left(\partial_\mu e_\nu^b - e_\sigma^b \Gamma_{\mu\nu}^\sigma \right)$

Christoffel symbol: $\Gamma_{\alpha\beta}^\lambda = \frac{1}{2} g^{\lambda\sigma} \left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta} \right)$

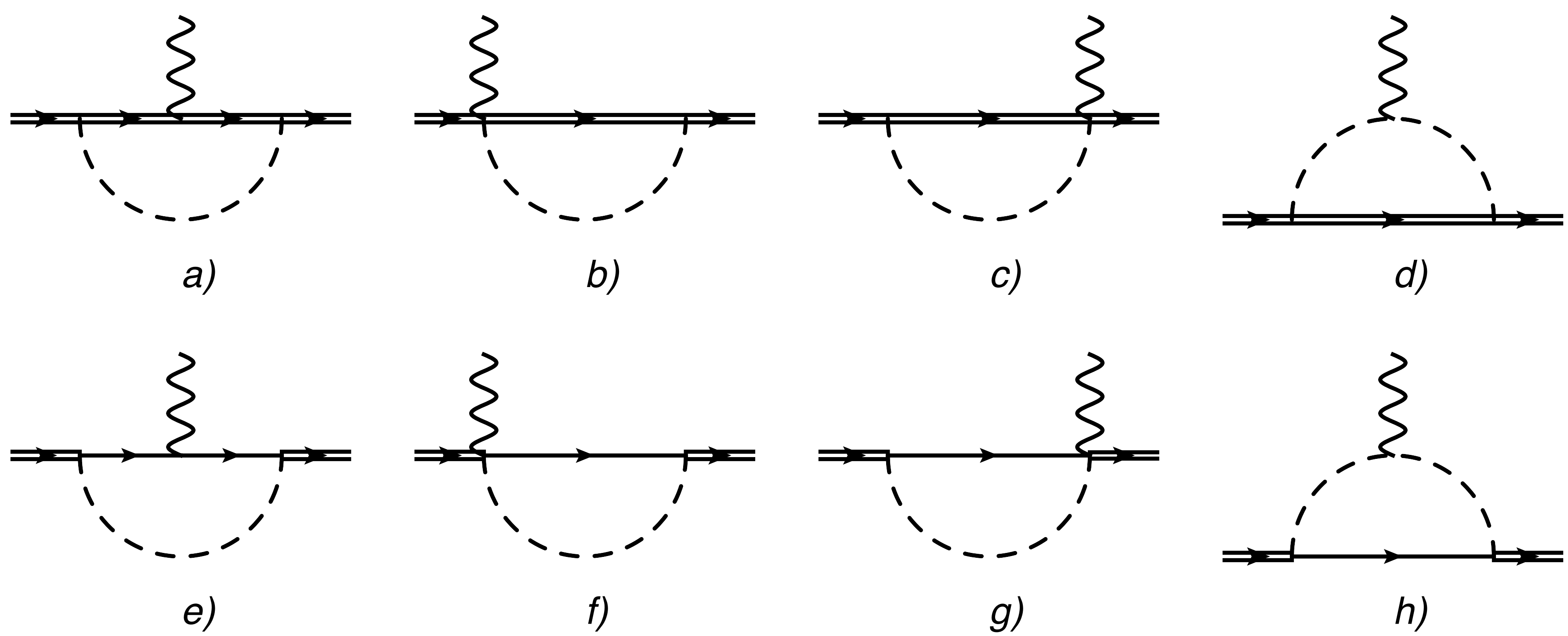
Vielbein fields e_μ^a: $e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu}$ connects Lorentz indices μ and Dirac indices a: $\gamma_\mu \equiv e_\mu^a \gamma_a$

$\epsilon-$counting scheme (small scale expansion) Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

To calc delta matrix elements of order 3:

Pion mass M:	1	Loop momenta:	1
Derivatives on N or Δ:	0	Pion lines:	-2
Masses m_Δ, m_N:	0	Nucleon lines:	-1
$\delta = m_\Delta - m_N$:	1	Delta lines:	-1
Momentum transfer:	1	$L^{(N)}$ vertices :	N

Use EOMS (extended on-mass-shell) scheme to remove divergent parts and power counting violating pieces. Renormalization scale chosen as $\mu = m_N$.



Actions Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

$$S_\pi^{(2)} = \int d^4x\sqrt{-g}\,\left\{\frac{F^2}{4}\,g^{\mu\nu}\,\mathrm{Tr}(D_\mu U(D_\nu U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\}$$

$$S_{\pi\mathrm{N}}^{(1)} = \int d^4x\sqrt{-g}\left\{\bar\Psi\, i\gamma^\mu \overleftrightarrow{\nabla}_\mu\Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi\gamma^\mu\gamma_5 u_\mu\Psi\right\}$$

$$\begin{aligned} S_{\pi\Delta}^{(1)} = -\int d^4x\sqrt{-g}\Big[& g^{\mu\nu}\,\bar\Psi^i_\mu\, i\gamma^\alpha\overleftrightarrow{\nabla}_\alpha\Psi^i_\nu - m_\Delta\, g^{\mu\nu}\bar\Psi^i_\mu\Psi^i_\nu - g^{\lambda\sigma}\left(\bar\Psi^i_\mu i\gamma^\mu\overleftrightarrow{\nabla}_\lambda\Psi^i_\sigma + \bar\Psi^i_\lambda i\gamma^\mu\overleftrightarrow{\nabla}_\sigma\Psi^i_\mu\right) \\ & + i\bar\Psi^i_\mu\gamma^\mu\gamma^\alpha\gamma^\nu\overleftrightarrow{\nabla}_\alpha\Psi^i_\nu + m_\Delta\bar\Psi^i_\mu\gamma^\mu\gamma^\nu\Psi^i_\nu + \frac{g_1}{2}\,g^{\mu\nu}\bar\Psi^i_\mu u_\alpha\gamma^\alpha\gamma_5\Psi^i_\nu + \frac{g_2}{2}\bar\Psi^i_\mu\left(u^\mu\gamma^\nu + u^\nu\gamma^\mu\right)\gamma_5\Psi^i_\nu \\ & + \frac{g_3}{2}\bar\Psi^i_\mu u_\alpha\gamma^\mu\gamma^\alpha\gamma_5\gamma^\nu\Psi^i_\nu\Big] \end{aligned}$$

- Off-shell parameter $A = -1$
- LECs $g_2 = g_3 = -g_1$

$$S_{\pi N\Delta}^{(1)} = -\int d^4x\sqrt{-g}\; g_{\pi N\Delta}\;\bar\Psi_{\mu,i}\left(g^{\mu\nu} - \gamma^\mu\gamma^\nu\right)u_{\nu,i}\Psi + \mathrm{H.c.}$$

$$S_{\pi\Delta,a}^{(2)} = \int d^4x\sqrt{-g}\,a_1\,\bar\Psi^i_\mu\Theta^{\mu\alpha}(z)\,\langle\chi_+\rangle\,g_{\alpha\beta}\Theta^{\beta\nu}(z')\Psi^i_\nu$$

Actions Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

$$\left.\begin{array}{ll} \text{Riemann tensor:} & R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma} \\ \text{Ricci tensor:} & R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} \\ \text{Ricci scalar:} & R = g^{\mu\nu} R^{\lambda}{}_{\mu\lambda\nu} \end{array}\right\} \text{Chiral order } = 2$$

$$\begin{aligned} S^{(2)}_{\pi\Delta,b} = \int d^4x \sqrt{-g} \Big[& h_1 R\, g^{\alpha\beta} \bar{\Psi}^i_\alpha \Psi^i_\beta + h_2 R\, \bar{\Psi}^i_\alpha \gamma^\alpha \gamma^\beta \Psi^i_\beta + i h_3 R \left(g^{\alpha\lambda} \bar{\Psi}^i_\alpha \gamma^\beta \overrightarrow{\nabla}_\lambda \Psi^i_\beta - g^{\beta\lambda} \bar{\Psi}^i_\alpha \gamma^\alpha \overleftarrow{\nabla}_\lambda \Psi^i_\beta \right) \\ & + h_4 R^{\mu\nu}\, \bar{\Psi}^i_\mu \Psi^i_\nu + 2 i h_5 R^{\mu\nu}\, g^{\alpha\beta} \bar{\Psi}^i_\alpha \gamma_\mu \overleftrightarrow{\nabla}_\nu \Psi^i_\beta + i h_6 R^{\mu\nu} g^{\alpha\beta} \left(\bar{\Psi}^i_\alpha \gamma_\mu \overrightarrow{\nabla}_\beta \Psi^i_\nu - \bar{\Psi}^i_\nu \gamma_\mu \overleftarrow{\nabla}_\beta \Psi^i_\alpha \right) \\ & + i h_7 R^{\mu\nu} \left(\bar{\Psi}^i_\alpha \gamma^\alpha \overrightarrow{\nabla}_\mu \Psi^i_\nu - \bar{\Psi}^i_\nu \gamma^\alpha \overleftarrow{\nabla}_\mu \Psi^i_\alpha \right) + h_8 R^{\mu\nu} \left(\bar{\Psi}^i_\alpha \gamma^\alpha \gamma_\mu \Psi^i_\nu + \bar{\Psi}^i_\nu \gamma_\mu \gamma^\alpha \Psi^i_\alpha \right) \\ & + i h_9 R^{\mu\nu} \left(\bar{\Psi}^i_\kappa \gamma^\kappa \gamma^\alpha \gamma_\mu \overrightarrow{\nabla}_\nu \Psi^i_\alpha - \bar{\Psi}^i_\alpha \gamma_\mu \gamma^\alpha \gamma^\kappa \overleftarrow{\nabla}_\nu \Psi^i_\kappa \right) + i h_{10} R^{\mu\nu\alpha\beta} \bar{\Psi}^i_\alpha \sigma_{\mu\nu} \Psi^i_\beta \\ & + i \left[h_{11}\, R^{\mu\nu\alpha\beta} + h_{12}\, R^{\mu\alpha\nu\beta} \right] \left(\bar{\Psi}^i_\alpha \gamma_\mu \overrightarrow{\nabla}_\nu \Psi^i_\beta - \bar{\Psi}^i_\beta \gamma_\mu \overleftarrow{\nabla}_\nu \Psi^i_\alpha \right) + h_{13} R^{\mu\alpha\nu\beta} \bar{\Psi}^i_\alpha \gamma_\mu \gamma_\nu \Psi^i_\beta \\ & + i \left[h_{14}\, R^{\mu\nu\alpha\beta} + h_{15}\, R^{\mu\alpha\nu\beta} \right] \left(\bar{\Psi}^i_\kappa \gamma^\kappa \gamma_\mu \gamma_\nu \overrightarrow{\nabla}_\alpha \Psi^i_\beta - \bar{\Psi}^i_\beta \gamma_\nu \gamma_\mu \gamma^\kappa \overleftarrow{\nabla}_\alpha \Psi^i_\kappa \right) \Big] \end{aligned}$$

1, EMT surface terms DO matters! (Recall the spin-0 case. Hudson,Schweitzer, 2017)

2, Absorb power-counting violating terms

EMT vertices Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

Actions in curved space-time: $S = \int d^4x \sqrt{-g}\,\mathcal{L}$

Calc EMT: $T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g^{\mu\nu}} \quad \Rightarrow \quad T_{\mu\nu} = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\,e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\,e^a_\mu\right]$

EMTs:

$$T^{(2)}_{\pi,\mu\nu} = \frac{F^2}{4}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) - \frac{\eta_{\mu\nu}}{2}\left\{\frac{F^2}{4}\,\mathrm{Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\} + (\mu \leftrightarrow \nu)$$

$$T^{(1)}_{\pi N,\mu\nu} = \frac{i}{2}\bar\Psi\,\gamma_\mu \overleftrightarrow{D}_\nu \Psi + \frac{g_A}{4}\bar\Psi\,\gamma_\mu\gamma_5 u_\nu \Psi - \frac{\eta_{\mu\nu}}{2}\left(\bar\Psi\, i\gamma^\alpha \overleftrightarrow{D}_\alpha \Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi\gamma^\alpha\gamma_5 u_\alpha \Psi\right)$$
$$+ (\mu \leftrightarrow \nu)$$

$$T^{(1)}_{\pi N\Delta,\mu\nu} = \frac{1}{2} g_{\pi N\Delta}\ \eta_{\mu\nu}\left[\bar\Psi^i_\alpha u^\alpha_i \Psi + \bar\Psi u^\alpha_i \Psi^i_\alpha - \bar\Psi^i_\alpha \gamma^\alpha\gamma^\beta u^i_\beta \Psi - \bar\Psi\gamma^\beta\gamma^\alpha u^i_\beta \Psi^i_\alpha\right]$$
$$- g_{\pi N\Delta}\ \left(\bar\Psi^i_\mu u^i_\nu \Psi + \bar\Psi u^i_\nu \Psi^i_\mu\right)$$

GFFs at Tree order Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

$$F_{1,0,\text{tree}}(t) \;=\; 1-\frac{t}{m_\Delta^2}+\frac{t\,(2h_5 m_\Delta+2h_{10}-h_{13})}{m_\Delta}-\frac{(-2h_6+2h_{11}+h_{12})\,t^2}{2m_\Delta^2}\,,$$

$$F_{1,1,\text{tree}}(t) \;=\; -4-4m_\Delta\,(h_{12}m_\Delta-2h_{10}+h_{13})+(4h_6-2\,(2h_{11}+h_{12}))\,t\,,$$

GFFs at One-Loop order ($t=0$)

$$F_{1,0,\text{loop}}(0)=0\,,$$

$$\begin{aligned}F_{1,1,\text{loop}}(0)=&-\frac{5g_1^2 m_N(3\pi M-49\delta)}{648\pi^2F^2}+\frac{g_{\pi N\Delta}^2 m_N}{144\pi^2F^2\,(M^2-\delta^2)}\Bigg(-53\delta^3+24\delta\,(M^2-\delta^2)\ln\frac{M}{m_N}\\&+24i\pi\delta^2\sqrt{\delta^2-M^2}-12i\pi M^2\sqrt{\delta^2-M^2}\\&+12\,(M^2-2\delta^2)\,\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}+53\delta M^2\Bigg)+\mathcal{O}(\epsilon^2)\end{aligned}$$

Slopes of the GFFs: $F_{i,j}(t)=F_{i,j}(0)+s_{F_{i,j}}t+\mathcal{O}(t^2)$

$$\begin{aligned}s_{F_{1,0}}=&\frac{g_1^2(8\delta-255\pi M)}{10368\pi^2F^2m_N}+\frac{g_{\pi N\Delta}^2}{576\pi^2F^2m_N\,(M^2-\delta^2)}\Bigg(25\delta(\delta^2-M^2)+24\delta\,(\delta^2-M^2)\ln\frac{M}{m_N}\\&-12i\pi(2\delta^2-M^2)\sqrt{\delta^2-M^2}-12\,(M^2-2\delta^2)\,\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\Bigg)+\mathcal{O}(\epsilon^2)\end{aligned}$$

Long-range behavior of local spatial densities Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2023

Energy densities:

$$\rho_0^E(r) = \frac{25 g_1^2}{1536 F^2 m_\Delta} \frac{1}{r^6} - \frac{10 g_1^2}{81 \pi^2 F^2 m_\Delta^2} \frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right)$$

$$\rho_2^E(r) = \frac{35 g_1^2}{6144 F^2 m_\Delta} \frac{1}{r^6} + \frac{35 g_1^2}{162 \pi^2 F^2 m_\Delta^2} \frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right),$$

$$\rho_0^E(r) > 0$$

Spin densities:

$$\rho_1^J(r) = \frac{5 g_1^2}{162 \pi^2 F^2 m_\Delta} \frac{1}{r^5} - \frac{125 g_1^2}{3072 F^2 m_\Delta^2} \frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^7}\right),$$

$$\rho_3^J(r) = -\frac{625 g_1^2}{24576 F^2 m_\Delta^2} \frac{1}{r^6} + \frac{5 g_1^2}{54 \pi^2 F^2 m_\Delta^3} \frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right),$$

Pressure & shear force densities:

$$p_0(r) = -\frac{25 g_1^2}{2304 F^2 m_\Delta} \frac{1}{r^6} - \frac{75 g_1^2}{1024 F^2 m_\Delta^3} \frac{1}{r^8} + \mathcal{O}\left(\frac{1}{r^9}\right),$$

$$s_0(r) = \frac{5 g_1^2}{96 F^2 m_\Delta} \frac{1}{r^6} + \frac{15 g_1^2}{64 F^2 m_\Delta^3} \frac{1}{r^8} + \mathcal{O}\left(\frac{1}{r^8}\right)$$

$$\frac{2}{3} s_0(r) + p_0(r) > 0$$

Note: delta resonances are unstable particles, our expressions satisfy the general stability conditions. It agrees with the observation by other approaches.

Summary and Outlook

1. Parameterization of the matrix elements of EMT defines GFFs, which relate to the fundamental properties, mass, spin, D-term.
2. D-terms for particles with different spins are discussed.
3. Spin-$3/2$ GFFs are calculated in different approaches, Skyme model, Lattice QCD (gluon part), QCD-sum-rules.
4. But their predictions are quit different and it motivate us to carry out the ChPT calculation. Still one need more input data to fix the LECs to obtain the GFFs and its D-term. Lattice QCD calculation for spin $3/2$ quark GFFs are excepted.
5. ChPT calculation for $N - \Delta$ gravitational transition FFs is on-going.

Thanks for your attention!

Backup

Generalized parton distributions（GPDs）of spin-3/2 Fu, BDS, Dong 2022, 2023

Vector matrix elements：

$$
\begin{aligned}
V_{\lambda'\lambda} &= \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ix(P\cdot z)} \left\langle p',\lambda' \left| \bar{\psi}(-z/2)\, \not{n} \psi(z/2) \right| p,\lambda \right\rangle \Big|_{z^+=0,\mathbf{z}=\mathbf{0}} \\
&= -\bar{u}_{\alpha'}(p',\lambda')\, \mathcal{H}^{\alpha'\alpha}(x,\xi,t)\, u_\alpha(p,\lambda)
\end{aligned}
$$

8 independent unpolarized GPDs:

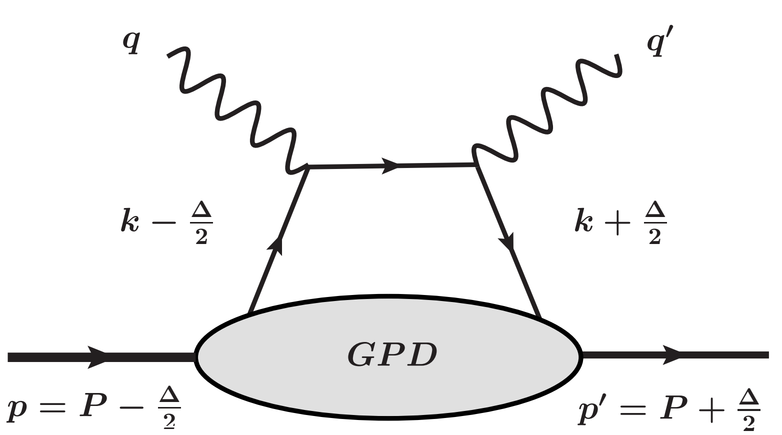
$$
\begin{aligned}
\mathcal{H}^{\alpha'\alpha} &= H_1 g^{\alpha'\alpha} + H_2 \frac{P^{\alpha'}P^{\alpha}}{M^2} + H_3 \frac{n^{[\alpha'}P^{\alpha]}}{P\cdot n} + H_4 \frac{M^2 n^{\alpha'}n^{\alpha}}{(P\cdot n)^2} + H_5 \frac{M g^{\alpha'\alpha}\not{n}}{P\cdot n} \\
&+ H_6 \frac{P^{\alpha'}P^{\alpha}\not{n}}{MP\cdot n} + H_7 \frac{M n^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2} + H_8 \frac{M^3 n^{\alpha'}n^{\alpha}\not{n}}{(P\cdot n)^3}
\end{aligned}
$$

quark $\rightarrow$ gluon

$$\bar{\psi}\gamma^\mu\psi \;\rightarrow\; F^{\beta'\mu}F_\mu{}^\beta$$

$$\bar{\psi}\gamma^\mu\gamma^5\psi \;\rightarrow\; F^{\beta'\mu}\tilde{F}_\mu{}^\beta$$

$$\tilde{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$$



Sum rules for spin-3/2 GPDs

Mellin moments for deriving sum-rules:

$$(P^+)^{n+1} \int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left[\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\right]_{z^+=0,\, \boldsymbol{z}=0}$$

$$= \left(i\frac{\mathrm{d}}{\mathrm{d}z^-}\right)^n \left[\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\right]\Big|_{z=0} = \bar{q}(0)\,\gamma^+ (i\overleftrightarrow{\partial}^+)^n\, q(0)$$

$n \to 0$ $n \to 1$

- Δ GPDs $\leftrightarrow$ EM FFs

$$M\int_{-1}^{1} \mathrm{d}x\, H_i(x,\xi,t) = G_i(t) \quad \text{with} \quad i = 1,2,5,6,$$

$$M\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_i(x,\xi,t) = \xi\tilde{G}_i(t) \quad \text{with} \quad i = 1,2,$$

$$M\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_i(x,\xi,t) = \tilde{G}_i(t) \quad \text{with} \quad i = 5,6,$$

$$M\int_{-1}^{1} \mathrm{d}x\, H_j(x,\xi,t) = M\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_j(x,\xi,t) = 0 \quad \text{with} \quad j = 3,4,7,8.$$

- Δ GPDs $\leftrightarrow$ GFFs

$$M\int_{-1}^{1} \mathrm{d}x\, xH_1(x,\xi,t) = F^T_{1,0}(t) + \xi^2 F^T_{2,0}(t) - 2F^T_{4,0}(t),$$

$$M\int_{-1}^{1} \mathrm{d}x\, xH_2(x,\xi,t) = 2F^T_{1,1}(t) + 2\xi^2 F^T_{2,1}(t) - 4F^T_{4,1}(t),$$

$$M\int_{-1}^{1} \mathrm{d}x\, xH_3(x,\xi,t) = 8\xi F^T_{5,0}(t),$$

$$M\int_{-1}^{1} \mathrm{d}x\, xH_4(x,\xi,t) = \frac{2t}{M^2} F^T_{5,0}(t) + 2F^T_{6,0}(t),$$

$$M\int_{-1}^{1} \mathrm{d}x\, xH_5(x,\xi,t) = 2F^T_{4,0}(t),$$

$$M\int_{-1}^{1} \mathrm{d}x\, xH_6(x,\xi,t) = 4F^T_{4,1}(t),$$

$$M\int_{-1}^{1} \mathrm{d}x\, xH_i(x,\xi,t) = 0, \quad \text{with} \quad i = 7,8.$$

Free massive vector particle

- Proca Lagrangian + a non-minimal term (?):

$$S_{\rm grav} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + \frac{1}{2}hRA_\mu A^\mu\right) \longrightarrow$$

Holstein, 2006; Polyakov, BDS, 2019

Table II: The free theory values of the total EMT FFs.

EMT FFs	$\mathcal{E}_0(t)$	$\mathcal{E}_2(t)$	$\mathcal{J}(t)$	$\mathcal{D}_0(t)$	$\mathcal{D}_2(t)$	$\mathcal{D}_3(t)$
free theory	1	0	1	$\frac{1}{3} - 4h$	-1	0

- all GFFs are t-independent: free of interaction
- $D_\rho \leq 0 \overset{?}{\leftrightarrow} h \geq \dfrac{1}{12}$: seems NOT allowed ...

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- all GFFs are t-independent: free of interaction
- $D_\rho \leq 0 \overset{?}{\leftrightarrow} h \geq \frac{1}{12}$: seems NOT allowed ...

- conformal transformation: (Dabrowski, 2009)

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x), \quad \widetilde{m} = \Omega^{-1} m,$$

$$\widetilde{A}_\mu = A_\mu, \quad \widetilde{A}^\mu = \tilde{g}^{\mu\nu}\widetilde{A}_\nu = \Omega^{-2}A^\mu,$$

$$\widetilde{U}_{\mu\nu} = U_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- choices of S: conformal invariance (CI) (or not)

$$S^0_{\rm grav} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu\right), \quad \text{(CI)}$$

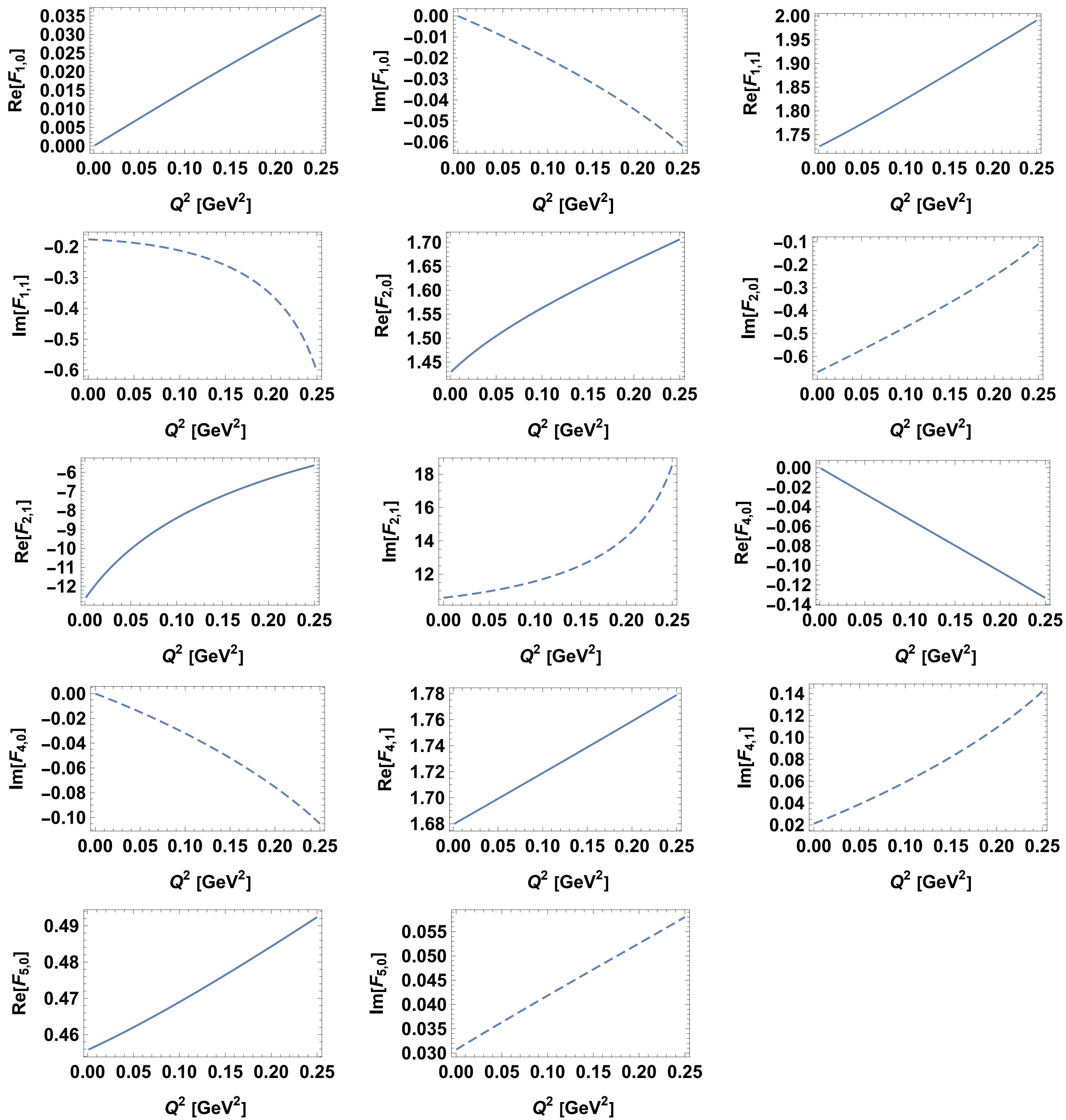
$$S_{\rm grav} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + \frac{1}{2}hRA^2_\mu\right), \quad (\text{not CI for } h \neq 0) \longrightarrow$$ Ricci scalar term breaks CI !

$$S^2_{\rm grav} = \int d^4x\sqrt{-g}\left(\frac{1}{2}A_\mu \Box A_\mu - \frac{1}{2}A_\mu\nabla^\mu\nabla^\nu A_\nu + \frac{1}{2}m^2 A^2_\mu\right), \quad \text{(not CI)}$$

$$S^3_{\rm grav} = \int d^4x\sqrt{-g}\left(\frac{1}{2}A_\mu \Box A_\mu - \frac{1}{2}A_\mu\nabla^\mu\nabla^\nu A_\nu + \frac{1}{2}m^2 A^2_\mu - \frac{1}{2}R_{\mu\nu}A^\mu A^\nu\right), \quad (\text{CI and give same } D_0 \text{ as } S^0_{\rm grav}!)$$

- Riemann tensor $R_{\mu\nu\rho\sigma}$, Weyl tensor $C_{\mu\nu\rho\sigma}$, etc., but NO suitable mass-dim-4 terms!

One-Loop contributions to GFFs



Δ（Rarita–Schwinger）fields

Isospin doublet spin 3/2 field:

$$\Psi_\mu(x) = \sum_{s_\Delta} \int \frac{d^3p}{(2\pi)^3} \frac{M_\Delta}{E} \left[b\left(\vec{p}, s_\Delta\right) u_\mu\left(\vec{p}, s_\Delta\right) e^{-ip\cdot x} + d^\dagger\left(\vec{p}, s_\Delta\right) v_\mu\left(\vec{p}, s_\Delta\right) e^{ip\cdot x} \right]$$

Attaching an additional isovector index i = 1,2,3 to it and use the subsidiary condition:

$$\tau^i \Psi^i_\mu(x) = 0$$

to eliminate two degrees of freedom.

For the three isospin doublets we use the representation

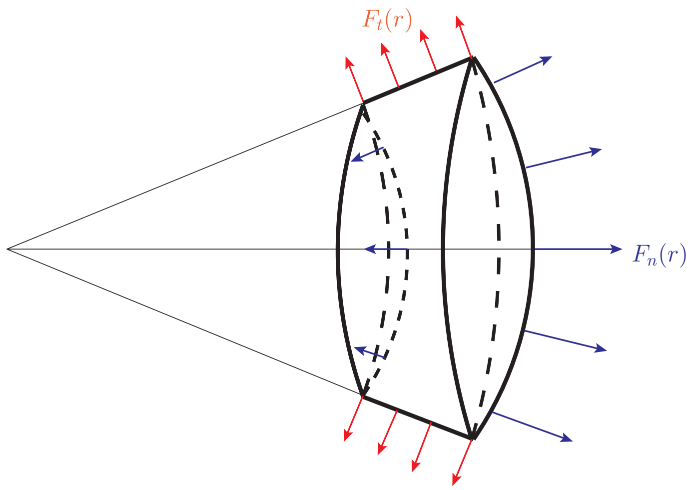
$$\begin{aligned}
\Psi^1_\mu &= \frac{1}{\sqrt{2}} \left[\Delta^{++} - \frac{1}{\sqrt{3}} \Delta^0, \frac{1}{\sqrt{3}} \Delta^+ - \Delta^- \right]^T_\mu, \\
\Psi^2_\mu &= -\frac{i}{\sqrt{2}} \left[\Delta^{++} + \frac{1}{\sqrt{3}} \Delta^0, \frac{1}{\sqrt{3}} \Delta^+ + \Delta^- \right]^T_\mu, \\
\Psi^3_\mu &= \sqrt{\frac{2}{3}} \left[\Delta^+, \Delta^0 \right]^T_\mu .
\end{aligned}$$

$p(r)$ and $s(r)$, normal/tangential force, stability conditions

- Force acting on the area element $d\mathbf{S} = \mathbf{dS_r}\hat{\mathbf{e}}_\mathbf{r} + \mathbf{dS}_\theta\hat{\mathbf{e}}_\theta + \mathbf{dS}_\phi\hat{\mathbf{e}}_\phi$ • (Panteleeva, Polyakov 2020)

$$\frac{dF_r}{dS_r} = \delta_{\sigma'\sigma}\left(p_0(r) + \frac{2}{3}s_0(r)\right) + \hat{Q}^{rr}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r) + p_3(r) + \frac{2}{3}s_3(r)\right), \longrightarrow \text{Normal force}$$

$$\frac{dF_\theta}{dS_r} = \hat{Q}^{\theta r}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r)\right), \qquad \frac{dF_\phi}{dS_r} = \hat{Q}^{\phi r}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r)\right), \longrightarrow \text{Tangential forces}$$



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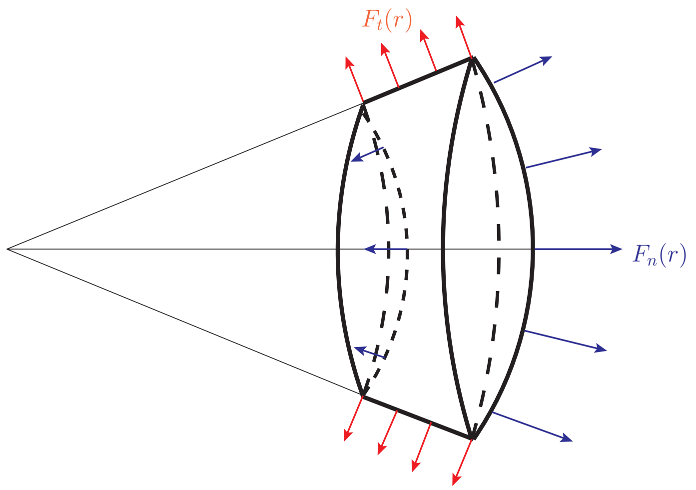

- Stability condition (von Laue 1911): $\int d^3r\, p_n(r) = 0$

- Local stability condition : $\left.\frac{dF_r}{dS_r}\right|_{\text{unp}} = p_0(r) + \frac{2}{3}s_0(r) \geq 0$

(unpolarized / spherically symmetric hadron)

(Polyakov & Schweitzer, 2018)

- D-term(unp): $\mathcal{D}_0 = m\int d^3r\, r^2 p_0(r) = -\frac{4}{15}m\int d^3r\, r^2 s_0(r) \leq 0$

$p(r)$ and $s(r)$, normal/tangential force, stability conditions

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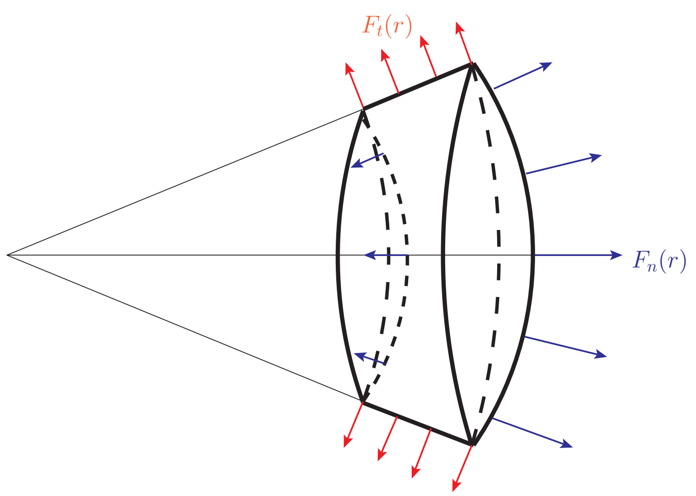

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Equilibrium relation ($\partial_\mu \hat{T}^{\mu\nu} = 0$): $\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$

(Goeke, et al, 2007)

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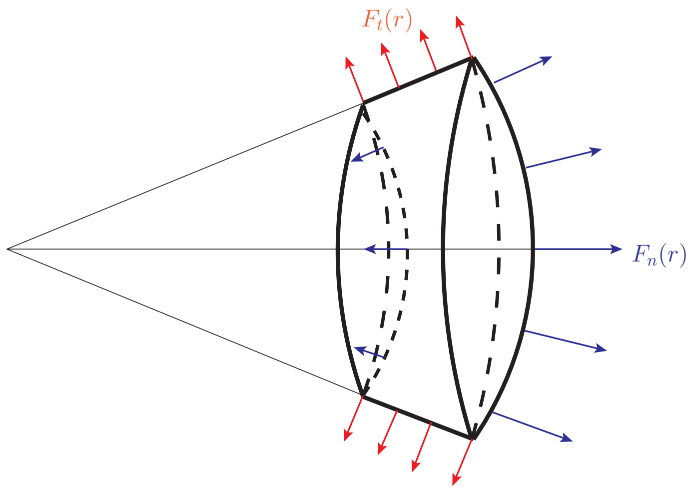
- Force acting on the area element $d\mathbf{S} = \mathbf{dS_r \hat{e}_r} + \mathbf{dS_\theta \hat{e}_\theta} + \mathbf{dS_\phi \hat{e}_\phi}$ • (Panteleeva, Polyakov 2020)

$$\frac{dF_r}{dS_r} = \delta_{\sigma'\sigma}\left(p_0(r) + \frac{2}{3}s_0(r)\right) + \hat{Q}^{rr}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r) + p_3(r) + \frac{2}{3}s_3(r)\right), \longrightarrow \text{Normal force}$$

$$\frac{dF_\theta}{dS_r} = \hat{Q}^{\theta r}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r)\right), \qquad \frac{dF_\phi}{dS_r} = \hat{Q}^{\phi r}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r)\right), \longrightarrow \text{Tangential forces}$$



- Stability condition (von Laue 1911): $\int d^3r\, p_n(r) = 0$

- Local stability condition : $\left.\frac{dF_r}{dS_r}\right|_{\text{unp}} = p_0(r) + \frac{2}{3}s_0(r) \geq 0$

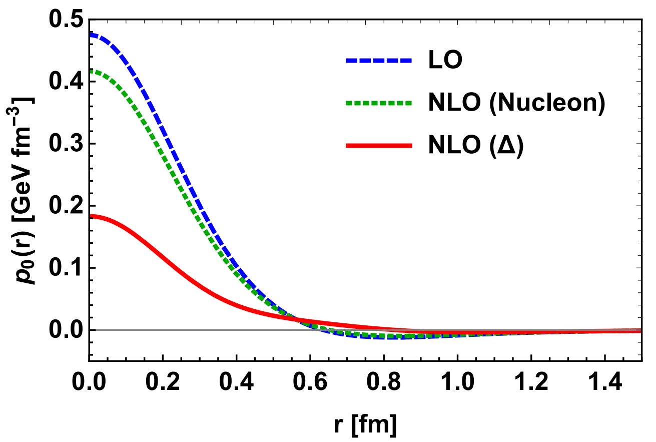
(unpolarized / spherically symmetric hadron)

(Polyakov & Schweitzer, 2018)

- D-term(unp): $\mathcal{D}_0 = m\int d^3r\, r^2 p_0(r) = -\frac{4}{15}m\int d^3r\, r^2 s_0(r) \leq 0$

Equilibrium relation ($\partial_\mu \hat{T}^{\mu\nu} = 0$): $\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$

(Goeke, et al, 2007)



$p_0(r)$ in Skyrme model

(Kim, BDS, 2021)

GFFs at One-Loop order $(t=0)$

$$
\begin{aligned}
F_{1,0,\text{loop}}(0) &= 0\,,\\
F_{1,1,\text{loop}}(0) &= -\frac{5g_1^2 m_N(3\pi M-49\delta)}{648\pi^2F^2}\\
&+ \frac{g_{\pi N\Delta}^2 m_N}{144\pi^2F^2\left(M^2-\delta^2\right)}\left(-53\delta^3+24\delta\left(M^2-\delta^2\right)\ln\frac{M}{m_N}+24i\pi\delta^2\sqrt{\delta^2-M^2}-12i\pi M^2\sqrt{\delta^2-M^2}\right.\\
&+ \left.12\left(M^2-2\delta^2\right)\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}+53\delta M^2\right)+\mathcal{O}(\epsilon^2)\,,\\
F_{2,0,\text{loop}}(0) &= -\frac{g_1^2 m_N(25\pi M-1068\delta)}{2160\pi^2F^2}\\
&+ \frac{g_{\pi N\Delta}^2 m_N\left(29\delta+48\delta\ln\frac{M}{m_N}-48i\pi\sqrt{\delta^2-M^2}+48\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\right)}{288\pi^2F^2}+\mathcal{O}(\epsilon^2)\,,\\
F_{2,1,\text{loop}}(0) &= -\frac{g_1^2m_N^3}{54\pi F^2M}+\frac{g_{\pi N\Delta}^2 M m_N^3\sqrt{\frac{\delta^2}{M^2}-1}\left(\ln\left(\sqrt{\frac{\delta^2}{M^2}-1}+\frac{\delta}{M}\right)-i\pi\right)}{15\pi^2F^2\left(M^2-\delta^2\right)}+\mathcal{O}(\epsilon^0)\,,\\
F_{4,0,\text{loop}}(0) &= 0\,,\\
F_{4,1,\text{loop}}(0) &= \frac{5g_{\pi N\Delta}^2m_N^2}{576\pi^2F^2}+\frac{235g_1^2m_N^2}{2592\pi^2F^2}+\mathcal{O}(\epsilon)\,,\\
F_{5,0,\text{loop}}(0) &= -\frac{g_1^2 m_N(150\pi M-3323\delta)}{25920\pi^2F^2}\\
&+ \frac{g_{\pi N\Delta}^2 m_N\left(5\delta+2\delta\ln\frac{M}{m_N}-2i\pi\sqrt{\delta^2-M^2}+2\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\right)}{96\pi^2F^2}+\mathcal{O}(\epsilon^2)\,.
\end{aligned}
$$

Slopes of the GFFs

$$F_{i,j}(t) \;=\; F_{i,j}(0) + s_{F_{i,j}} t + \mathcal{O}(t^2)$$

$$\begin{aligned}
s_{F_{1,0}} &= \frac{g_1^2(8\delta - 255\pi M)}{10368\pi^2 F^2 m_N} \\
&+ \frac{g_{\pi N\Delta}^2}{576\pi^2 F^2 m_N\,(M^2-\delta^2)}\Bigg(25\delta(\delta^2-M^2) + 24\delta\left(\delta^2-M^2\right)\ln\frac{M}{m_N} - 12i\pi(2\delta^2-M^2)\sqrt{\delta^2-M^2} \\
&- 12\left(M^2-2\delta^2\right)\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\Bigg) + \mathcal{O}(\epsilon^2)\,, \\
s_{F_{1,1}} &= \frac{g_1^2 m_N}{432\pi F^2 M} + \frac{g_{\pi N\Delta}^2 m_N\left(\delta^3 + M^2\left(-\delta + i\pi\sqrt{\delta^2-M^2}\right) - M^2\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\right)}{120\pi^2F^2\left(M^2-\delta^2\right)^2} + \mathcal{O}(\epsilon^0)\,, \\
s_{F_{2,0}} &= -\frac{g_1^2 m_N}{108\pi F^2 M} + \frac{g_{\pi N\Delta}^2 m_N\left(\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M} - i\pi\right)}{60\pi^2F^2\sqrt{\delta^2-M^2}} + \mathcal{O}(\epsilon^0)\,, \\
s_{F_{2,1}} &= \frac{g_{\pi N\Delta}^2 m_N^3\left(-\delta^3 + M^2\left(\delta - i\pi\sqrt{\delta^2-M^2}\right) + M^2\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M}\right)}{140\pi^2F^2M^2\left(M^2-\delta^2\right)^2} - \frac{g_1^2 m_N^3}{504\pi F^2 M^3} + \mathcal{O}(\epsilon^{-2})\,, \\
s_{F_{4,0}} &= \frac{g_{\pi N\Delta}^2\left(163\delta^2 - 96\left(M^2-\delta^2\right)\ln\frac{M}{m_N} - 96i\pi\delta\sqrt{\delta^2-M^2} + 96\delta\sqrt{\delta^2-M^2}\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M} - 163M^2\right)}{4608\pi^2F^2\left(M^2-\delta^2\right)} \\
&+ \frac{g_1^2\left(877 - 150\ln\frac{M}{m_N}\right)}{25920\pi^2F^2} + \mathcal{O}(\epsilon)\,, \\
s_{F_{4,1}} &= 0 + \mathcal{O}(\epsilon^{-1})\,, \\
s_{F_{5,0}} &= \frac{g_1^2 m_N}{3456\pi F^2 M} + \frac{g_{\pi N\Delta}^2 m_N\left(\ln\frac{\delta+\sqrt{\delta^2-M^2}}{M} - i\pi\right)}{960\pi^2F^2\sqrt{\delta^2-M^2}} + \mathcal{O}(\epsilon^0)\,.
\end{aligned}$$

Localized Wave Packet

Epelbaum, Gegelia, Lange, Meißner, Polyakov, PRL 2022
Panteleeva, Epelbaum, Gegelia, Meißner, 2022
Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

Heisenberg-picture:

$$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\, \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}} |p, s\rangle$$

Localized Wave Packet

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Normalization in ZAMF:
(Zero Average Momentum Frame)

$$\int d^3p\, |\phi(s, \mathbf{p})|^2 = 1$$

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Spherically Sym & Dimensionless:

$$\phi(\mathbf{p}) = R^{3/2}\, \tilde{\phi}(R\mathbf{p})$$

Localized Wave Packet

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EM parameterization:
(spin-3/2)

$$\begin{aligned} \langle p_f, s'|J_\mu|p_i, s\rangle \;=\; & -\bar{u}^\beta(p_f, s')\Bigg[\frac{P_\mu}{m}\left(g_{\alpha\beta}F^V_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}\, F^V_{1,1}(q^2)\right) \\ & + \frac{i}{2m}\sigma_{\mu\rho}q^\rho\left(g_{\alpha\beta}F^V_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}\, F^V_{2,1}(q^2)\right)\Bigg] u^\alpha(p_i, s) \end{aligned}$$

Localized Wave Packet

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Panteleeva, Epelbaum, Gegelia, Meißner, 2022
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Localize:
$$\begin{aligned} j^\mu_\phi(s', s, \mathbf{r}) \;&\equiv\; \langle \Phi, \mathbf{X}, s'|\hat{J}^\mu(\mathbf{x}, 0)|\Phi, \mathbf{X}, s\rangle \\ &= -\int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, \bar{u}^\beta\left(P + \frac{q}{2}, \sigma'\right)\Bigg[\frac{P_\mu}{m}\left(g_{\alpha\beta}F^V_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}\, F^V_{1,1}(q^2)\right) \\ &+ \frac{i}{2m}\sigma_{\mu\rho}q^\rho\left(g_{\alpha\beta}F^V_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2}\, F^V_{2,1}(q^2)\right)\Bigg] u^\alpha\left(P - \frac{q}{2}, \sigma\right)\phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right)e^{-i\mathbf{q}\cdot\mathbf{r}} \end{aligned}$$

$\mathbf{P} \equiv \mathbf{Q}/R, \quad R \to 0$ Only large $\mathbf{P}$ region contributes!

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Localized Wave Packet

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$\mathbf{P} \equiv \mathbf{Q}/R, \quad R \to 0$ Only large $\mathbf{P}$ region contributes!

“Naive” Breit Frame is problematic:
first $m \to \infty$ then $R \to 0$

Pressure and Shear Forces

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

$$\tilde{t}^{ij}_{\phi,2}(s', s, \mathbf{r}) \longrightarrow \begin{aligned} p_0(r) &= \tilde{v}_0(r) - \frac{1}{6m^2}\partial^2 w_0(r)\,, \quad & s_0(r) &= -\frac{1}{2m^2} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr} w_0(r)\,, \\ p_2(r) &= 0, & s_2(r) &= 0, \\ p_3(r) &= m^2 \tilde{v}_1(r) - \frac{1}{6}\partial^2 w_1(r)\,, & s_3(r) &= -\frac{1}{2} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr} w_1(r)\,, \end{aligned}$$

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Conservation of EMT:

$$\partial_\mu t^{\mu\nu}_\phi(s', s, \mathbf{r}, t)|_{t=0} = \partial_0 t^{0\nu}_\phi(s', s, \mathbf{r}, t)|_{t=0} + \partial_i t^{i\nu}_\phi(s', s, \mathbf{r}, t)|_{t=0} = 0.$$

Breit Frame only has 2nd term

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Breit Frame only has 2nd term

Differential eqs:

$$p_n'(r) + \frac{2}{3} s_n'(r) + \frac{2}{r} s_n(r) = h_n'(r), \quad \text{with } n = 0, 2, 3,$$

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Breit Frame only has 2nd term

Differential eqs:

$$p'_n(r) + \frac{2}{3}s'_n(r) + \frac{2}{r}s_n(r) = h'_n(r)\,, \quad \text{with } n = 0, 2, 3,$$

von Laue stability condition:

$$\int d^3r\, p_n(r) \;=\; 0, \quad \text{with} \;\; n = 0, 2, 3,$$

as long as $\lim_{q_\perp^2\to 0}\left(q_\perp^2\right)^\delta F_{2,0}\left(-q_\perp^2\right) = 0$ and $\lim_{q_\perp^2\to 0}\left(q_\perp^2\right)^\delta F_{2,1}\left(-q_\perp^2\right) = 0$, for $\delta > 0$.

Pressure and Shear Forces

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

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Generalized D-terms: $\displaystyle \mathcal{D}_n = -\frac{4}{15} m^2 \int \mathrm{d}^3r\, r^2 s_n(r) = m^2 \int d^3r\, r^2 \left[p_n(r) - h_n(r)\right], \quad \text{with} \quad n = 0,2,3.$

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Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

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$$\mathcal{D}_n = -\frac{4}{15}m^2\int \mathrm{d}^3r\, r^2 s_n(r) = m^2\int d^3r\, r^2\left[p_n(r) - h_n(r)\right], \quad \text{with} \quad n = 0,2,3.$$

Internal forces:

$$\frac{dF_r}{dS_r} = N_{\phi,R,2}\left[\left(p_0(r) + \frac{2}{3}s_0(r)\right)\delta_{s's} + \left(p_2(r) + \frac{2}{3}s_2(r)\right)\hat{Q}^{rr}_{s's} + \dots\,, \qquad \frac{dF_\theta}{dS_r} = \dots\,, \qquad \frac{dF_\varphi}{dS_r} = \dots$$

EMT densities

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022
Freese, Miller, 2022
Panteleeva, Epelbaum, Gegelia, Meißner, 2022

$$
\begin{aligned}
t_\phi^{\mu\nu}(\mathbf{r}) &\equiv \langle \Phi, \mathbf{X}, s' | \hat{T}^{\mu\nu}(\mathbf{x}, 0) | \Phi, \mathbf{X}, s \rangle \\
&= -\int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\, \bar{u}^\beta\left(P + \frac{q}{2}, \sigma'\right) \left[\frac{P_\mu P_\nu}{m} \left(g_{\alpha\beta} F_{1,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{1,1}(q^2) \right) \right. \\
&+ \frac{q_\mu q_\nu - \eta_{\mu\nu} q^2}{4m} \left(g_{\alpha\beta} F_{2,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{2,1}(q^2) \right) + \frac{i}{2} \frac{(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho})\, q^\rho}{m} \left(g_{\alpha\beta} F_{4,0}(q^2) - \frac{q_\alpha q_\beta}{2m^2} F_{4,1}(q^2) \right) \\
&- \frac{1}{m} \left(g_{\nu\beta} q_\mu q_\alpha + g_{\mu\beta} q_\nu q_\alpha + g_{\nu\alpha} q_\mu q_\beta + g_{\mu\alpha} q_\nu q_\beta - 2 g_{\mu\nu} q_\alpha q_\beta \right. \\
&- \left. \left. g_{\mu\beta} g_{\nu\alpha} q^2 - g_{\nu\beta} g_{\mu\alpha} q^2 \right) F_{5,0}(q^2) \right] u^\alpha \left(P - \frac{q}{2}, \sigma \right) \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2} \right) \phi^\star\left(\mathbf{P} + \frac{\mathbf{q}}{2} \right) e^{-i\mathbf{q}\cdot\mathbf{r}}. \qquad (21)
\end{aligned}
$$

EMT densities

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022
Freese, Miller, 2022
Panteleeva, Epelbaum, Gegelia, Meißner, 2022

$$
\begin{aligned}
t_\phi^{\mu\nu}(\mathbf{r}) \;&\equiv\; \langle \Phi, \mathbf{X}, s'|\hat{T}^{\mu\nu}(\mathbf{x},0)|\Phi,\mathbf{X},s\rangle \\
&= -\int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\,\bar{u}^\beta\left(P+\frac{q}{2},\sigma'\right)\left[\frac{P_\mu P_\nu}{m}\left(g_{\alpha\beta}F_{1,0}(q^2)-\frac{q_\alpha q_\beta}{2m^2}F_{1,1}(q^2)\right)\right. \\
&+ \frac{q_\mu q_\nu-\eta_{\mu\nu}q^2}{4m}\left(g_{\alpha\beta}F_{2,0}(q^2)-\frac{q_\alpha q_\beta}{2m^2}F_{2,1}(q^2)\right)+\frac{i}{2}\frac{(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\,q^\rho}{m}\left(g_{\alpha\beta}F_{4,0}(q^2)-\frac{q_\alpha q_\beta}{2m^2}F_{4,1}(q^2)\right) \\
&- \frac{1}{m}\,(g_{\nu\beta}q_\mu q_\alpha+g_{\mu\beta}q_\nu q_\alpha+g_{\nu\alpha}q_\mu q_\beta+g_{\mu\alpha}q_\nu q_\beta-2g_{\mu\nu}q_\alpha q_\beta \\
&- \left. g_{\mu\beta}g_{\nu\alpha}q^2-g_{\nu\beta}g_{\mu\alpha}q^2)\,F_{5,0}(q^2)\right]u^\alpha\left(P-\frac{q}{2},\sigma\right)\phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right)e^{-i\mathbf{q}\cdot\mathbf{r}}.
\end{aligned}
\tag{21}
$$

Using multipole expansion:

$$
t_\phi^{00}(s',s,\mathbf{r}) \;=\; N_{\phi,R}\int\frac{d^3q}{(2\pi)^3}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\left\{\mathcal{E}_0(q_\perp^2)\,\delta_{s's}+\left[\mathcal{E}_1(q_\perp^2)\,\hat{n}^k\hat{n}^l+\mathcal{E}_2(q_\perp^2)\,\frac{q_\perp^k q_\perp^l}{m^2}\right]\hat{Q}^{kl}_{s's}\right\}, \tag{22a}
$$

$$
\begin{aligned}
t_\phi^{0i}(s',s,\mathbf{r}) \;&=\; i\,N_{\phi,R}\int\frac{d^3q}{(2\pi)^3}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\int d^2\hat{n}\,\left\{\;\left[\mathcal{C}_0(q_\perp^2)\,\epsilon^{kln}\hat{n}^l\hat{n}^i+\mathcal{C}_1(q_\perp^2)\,\epsilon^{iln}\left(\delta^{kl}-\hat{n}^k\hat{n}^l\right)\right]\frac{q_\perp^n}{m}\,\hat{S}^k_{s's}\right. \\
&+ \left[\left(\mathcal{C}_2(q_\perp^2)\hat{n}^t\hat{n}^z+\mathcal{C}_3(q_\perp^2)\frac{q_\perp^t q_\perp^z}{m^2}\right)\epsilon^{kln}\hat{n}^l\hat{n}^i\right. \\
&+ \left.\left.\left(\mathcal{C}_4(q_\perp^2)\hat{n}^t\hat{n}^z+\mathcal{C}_5(q_\perp^2)s\frac{q_\perp^t q_\perp^z}{m^2}\right)\epsilon^{iln}\left(\delta^{kl}-\hat{n}^k\hat{n}^l\right)\right]\frac{q_\perp^n}{m}\,\hat{O}^{ktz}_{s's}\right\},
\end{aligned}
\tag{22b}
$$

$$
t_\phi^{ij}(s',s,\mathbf{r}) \;=\; t_{\phi,0}^{ij}(s',s,\mathbf{r})+t_{\phi,2}^{ij}(s',s,\mathbf{r}), \tag{22c}
$$

$\sim 1/R$
motion of system

$\sim R$
internal pressure & shear forces
(needs higher order contributions)

$$
\begin{aligned}
N_{\phi,R} \;&=\; \frac{1}{R}\int_0^\infty dQ\,Q^3|\tilde{\phi}(|\mathbf{Q}|)|^2\,, \\
N_{\phi,R,2} \;&=\; \frac{m^2R}{2}\int_0^\infty dQ\,Q|\tilde{\phi}(|\mathbf{Q}|)|^2\,.
\end{aligned}
$$

long-range behavior under ChPT

Alharazin, BDS, Epelbaum, Gegelia, Meißner, 2022

$$\rho_0^E(r) = \frac{25g_1^2}{1536F^2m_\Delta}\frac{1}{r^6} - \frac{10g_1^2}{81\pi^2F^2m_\Delta^2}\frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{51}$$

$$\rho_2^E(r) = \frac{35g_1^2}{6144F^2m_\Delta}\frac{1}{r^6} + \frac{35g_1^2}{162\pi^2F^2m_\Delta^2}\frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{52}$$

$$\rho_1^J(r) = \frac{5g_1^2}{162\pi^2F^2m_\Delta}\frac{1}{r^5} - \frac{125g_1^2}{3072F^2m_\Delta^2}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^7}\right), \tag{53}$$

$$\rho_3^J(r) = -\frac{625g_1^2}{24576F^2m_\Delta^2}\frac{1}{r^6} + \frac{5g_1^2}{54\pi^2F^2m_\Delta^3}\frac{1}{r^7} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{54}$$

$$p_0(r) = -\frac{25g_1^2}{2304F^2m_\Delta}\frac{1}{r^6} - \frac{75g_1^2}{1024F^2m_\Delta^3}\frac{1}{r^8} + \mathcal{O}\left(\frac{1}{r^9}\right), \tag{55}$$

$$s_0(r) = \frac{5g_1^2}{96F^2m_\Delta}\frac{1}{r^6} + \frac{15g_1^2}{64F^2m_\Delta^3}\frac{1}{r^8} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{56}$$

$$p_3(r) = \frac{85g_1^2m_\Delta}{221184F^2}\frac{1}{r^4} - \frac{155g_1^2}{196608F^2m_\Delta}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^8}\right), \tag{57}$$

$$s_3(r) = -\frac{25g_1^2m_\Delta}{9216F^2}\frac{1}{r^4} + \frac{15g_1^2}{4096F^2m_\Delta}\frac{1}{r^6} + \mathcal{O}\left(\frac{1}{r^8}\right). \tag{58}$$

general stability conditions: $\rho_0^E(r) > 0$ and $\frac{2}{3}s_0(r) + p_0(r) > 0$

Note: necessary but not sufficient for a system to be stable