Nucleon Structure lattice QCD

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Extended Twisted Mass Collaboration

STIMULATE
European Joint Doctorates

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European Joint Doctorate

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Outline

- Introduction
  - State-of-the-art lattice QCD simulations
- 3D structure of the nucleon
  - First and second Mellin moments
    - ➡ Charges
    - ➡ Axial form factors, arXiv: 2309.05774
    - ➡Spin content of the nucleon
  - Direct computation of parton distributions
- Conclusions

Quantum ChromoDynamics (QCD)

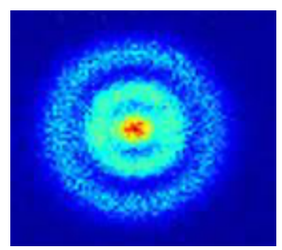
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

Fritzsch, Gell-Mann and Leutwyler, Phys. Lett. 47B (1973) 365

- Unique properties:
  - ★Confinement
  - ★Asymptotic freedom
  - ★Mass generation via interaction

QED

Quantum theory of the electromagnetic force mediated by exchange of photons



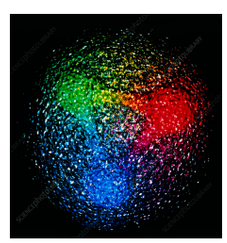
Hydrogen atom

$$m_{\text{Hydrogen}} = \underbrace{0.51\ \text{MeV}}_{m_{e-}} + \underbrace{938.29\ \text{MeV}}_{m_{p+}} - \underbrace{13.6\ \text{eV}}_{\text{E}_{\text{binding}}}$$

A. Stodolna et al., PRL 110 (2013) 213001

QCD

Quantum theory of the strong force mediated by exchange of gluons



Artist impression

Proton

$$m_p = \underbrace{2.3\ \text{MeV}}_{2\times m_u} + \underbrace{4.7\ \text{MeV}}_{m_d} + \underbrace{929\ \text{MeV}}_{\text{E}_{\text{binding}}}$$

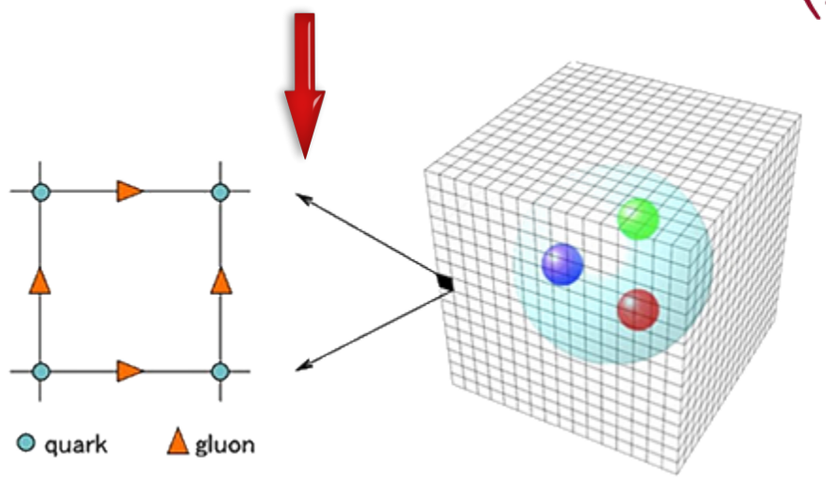
99% of proton mass from interaction!

Lattice QCD provides an *ab initio* method to study a wide class of strong interaction phenomena

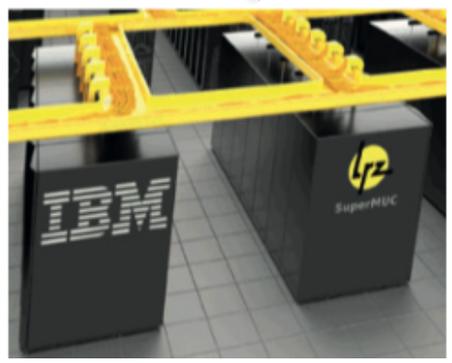
- Lattice QCD uses directly $\mathcal{L}_{QCD}$ or the action $S_{QCD} = \int d^4x\, \mathcal{L}_{QCD}$

Lattice QCD

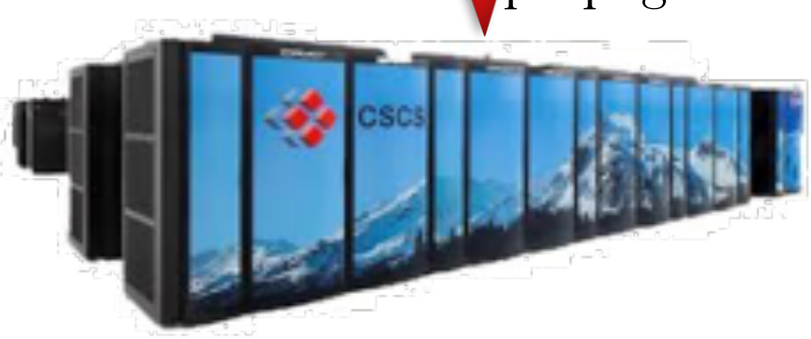
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



Simulation of gauge ensembles {U}



Quark & gluon propagators



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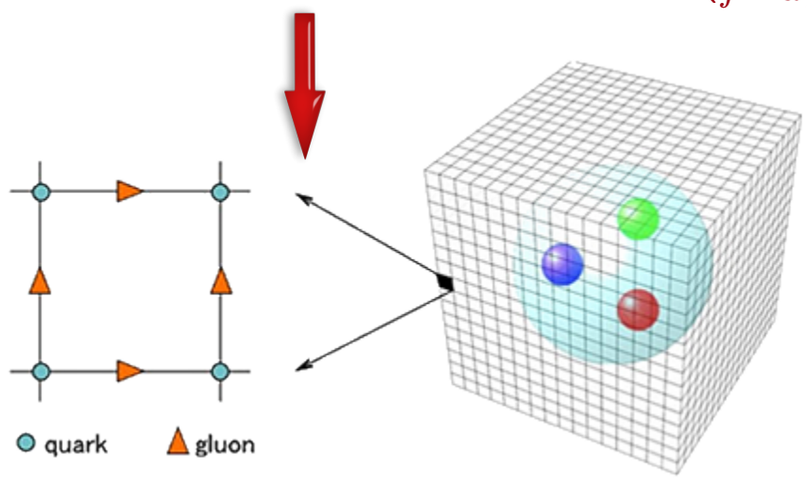
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U]\; O(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



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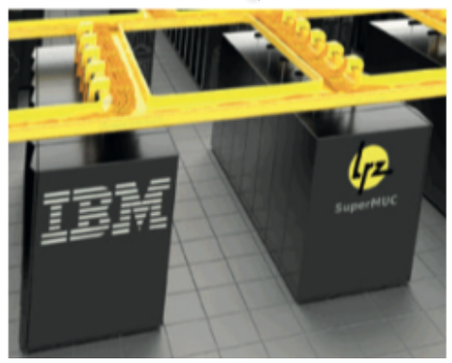
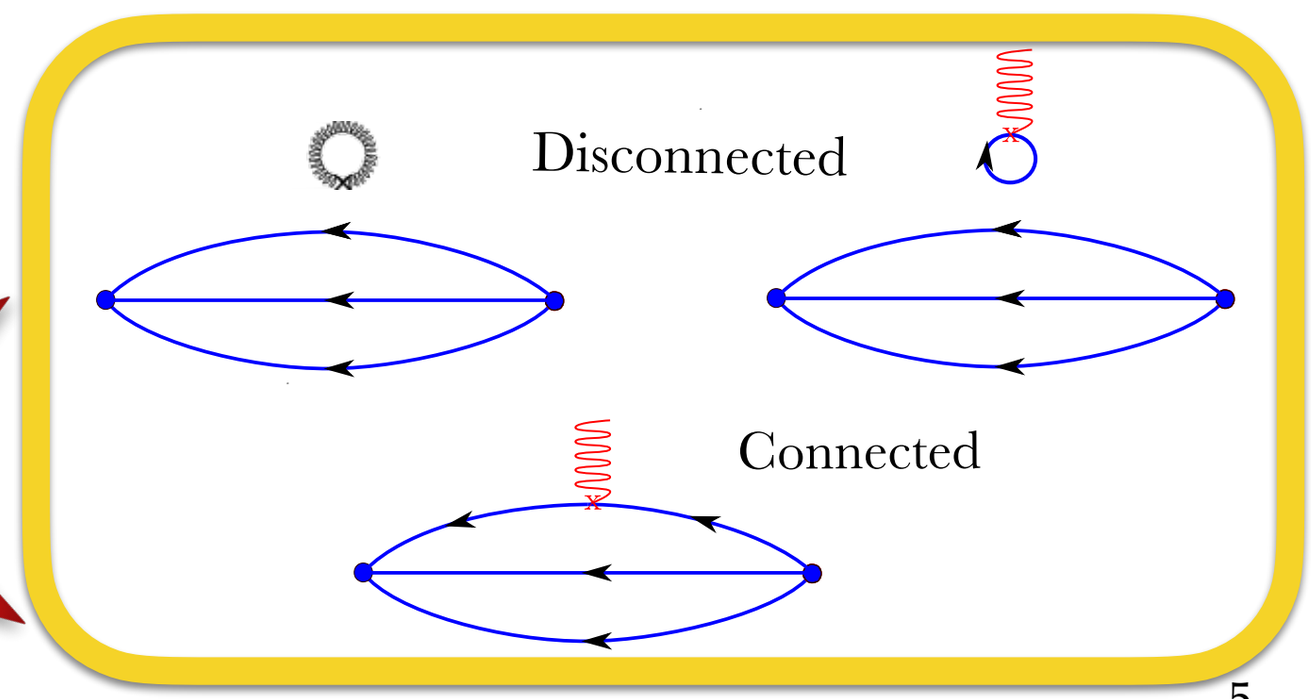


Quark & gluon propagators

contractions



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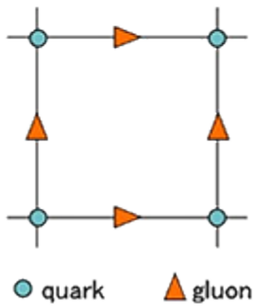

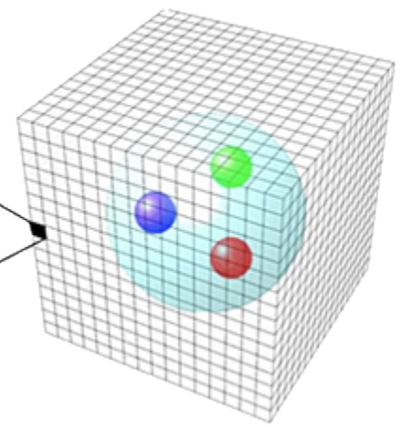

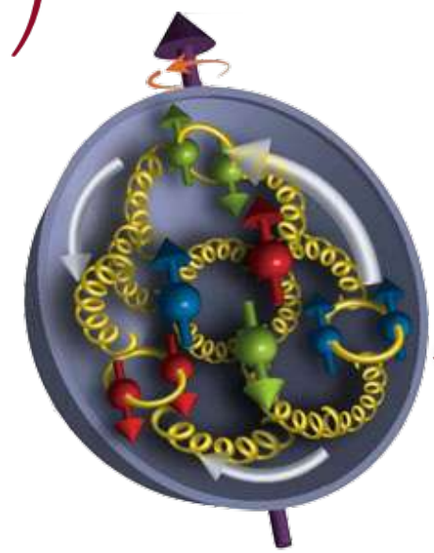

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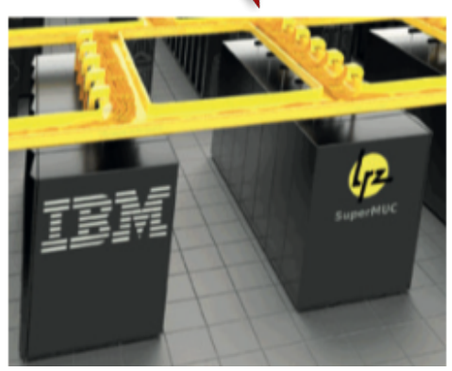
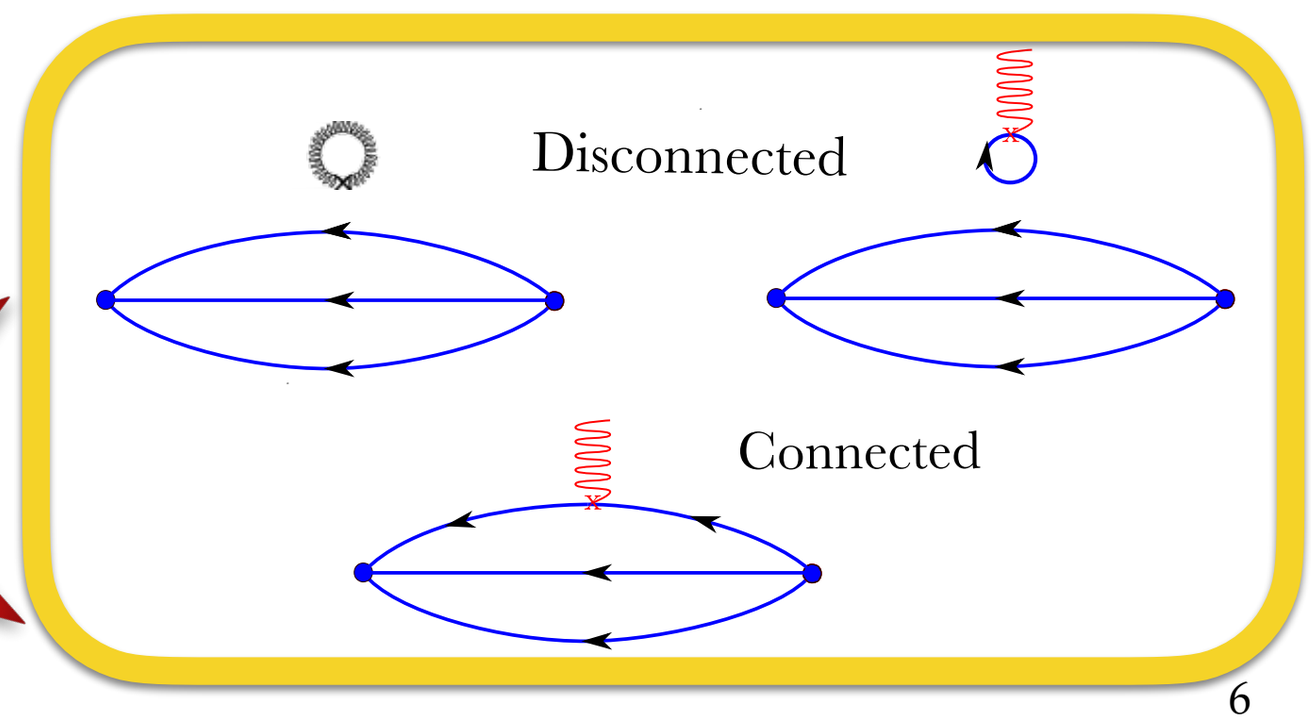


Data Analysis

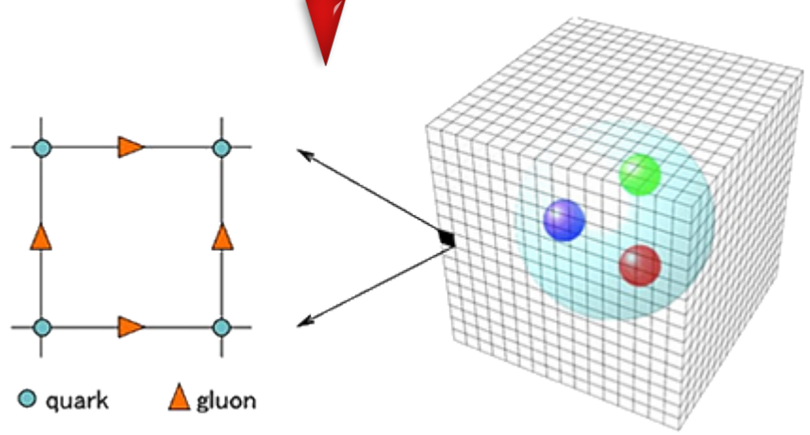
Quark & gluon propagators

contractions



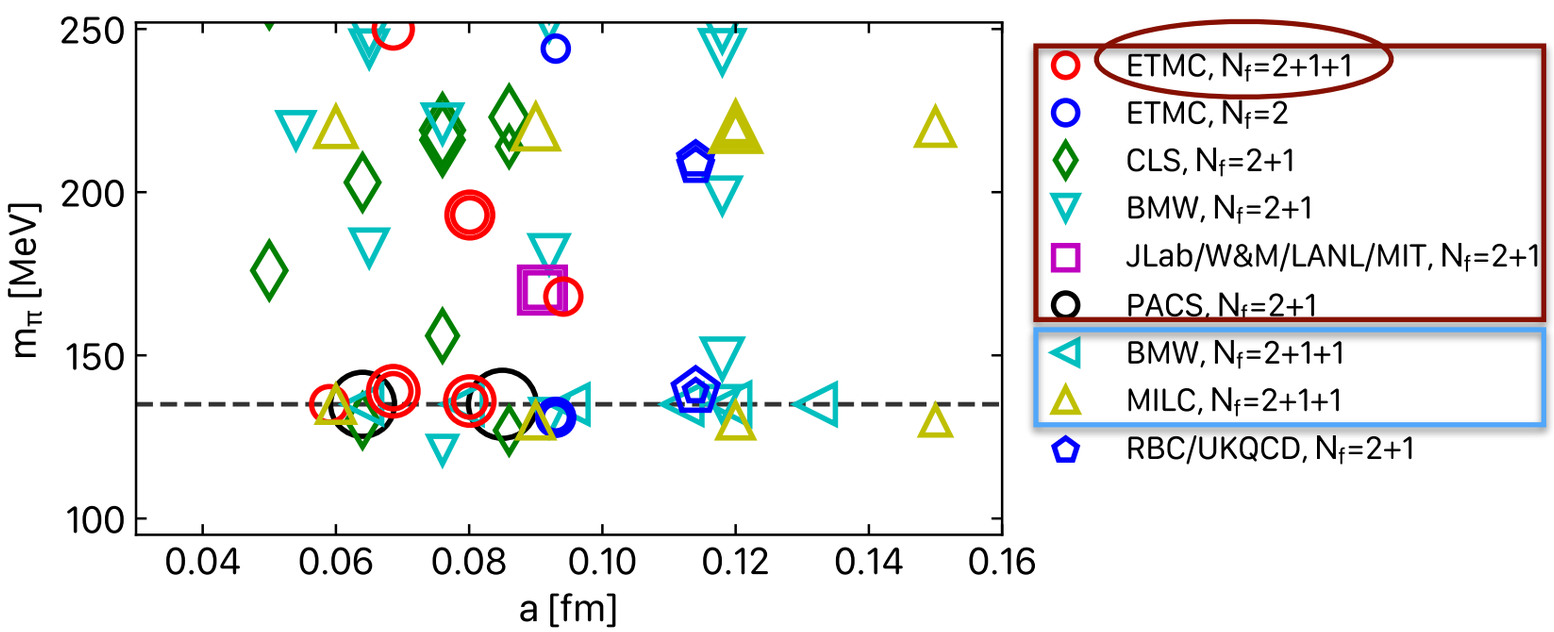
Simulations of lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



Simulation of gauge ensembles {U}:

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



Wilson-type

Staggered

Domain wall

Simulations of lattice QCD

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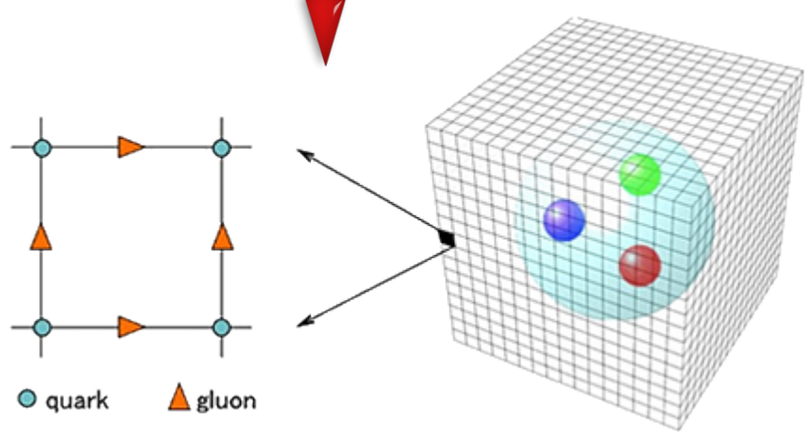

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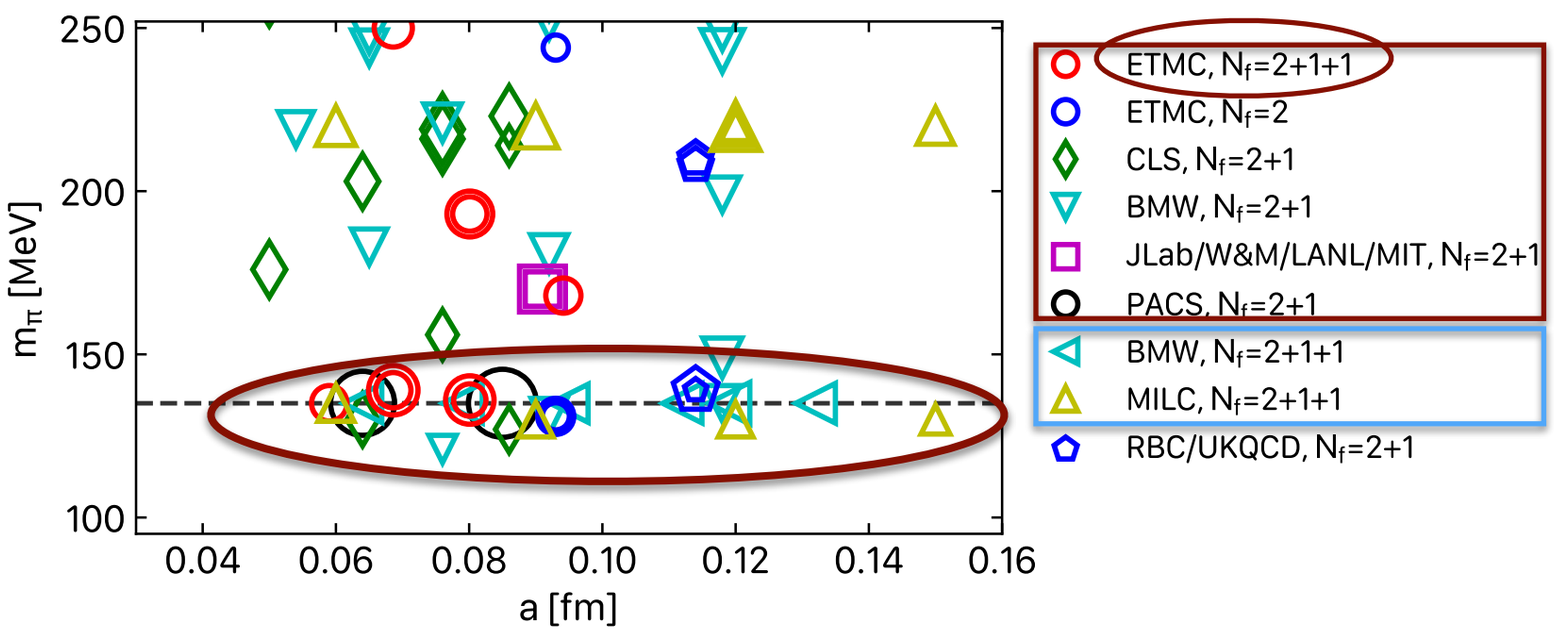

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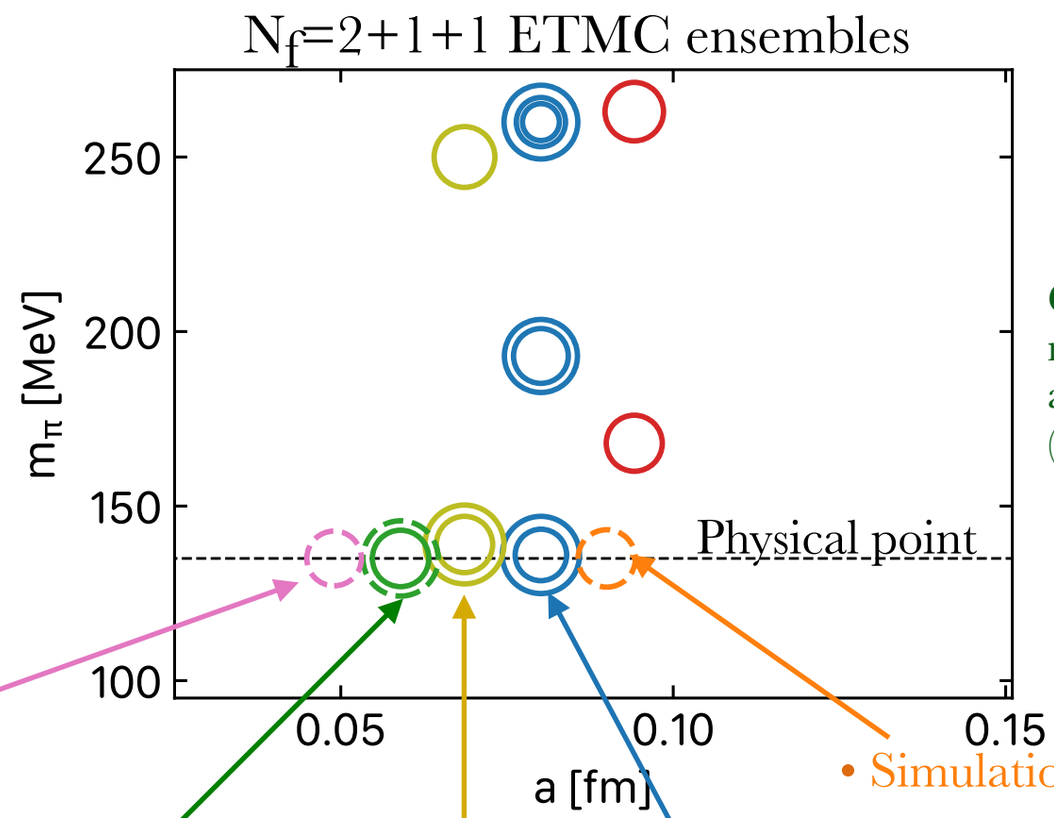
Staggered

Domain wall

Gauge ensembles generated by ETMC

5 ensembles completed and 3 under production at physical pion mass

- 5 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
- Two volumes at a=0.08 fm, 0.07 fm and 0.06 fm of Lm_π~3.6 (5.1 fm) and Lm_π ~5.4 (7.7 fm) completed



C. A. *et al.* (ETMC), "Simulating twisted mass fermions at physical light, strange and charm quark masses" Phys. Rev. D98 (2018) 054518

- Simulation ongoing for 112^3x224, a~0.05 fm
- Simulation ongoing 48^3 x 96 a~0.09 fm
- Analysis completed for 64^3x128 a~0.08 fm
- Analysis ongoing for 96^3x192 a~0.08 fm
- Analysis completed for 80^3x160, a~0.07 fm
- Analysis ongoing for 112^3x224, a~0.07 fm
- Analysis completed for 96^3x192, a~0.06 fm
- Simulation ongoing for 112^3x224, a~0.06 fm

Gauge ensembles generated by ETMC

5 ensembles completed and 3 under production at physical pion mass

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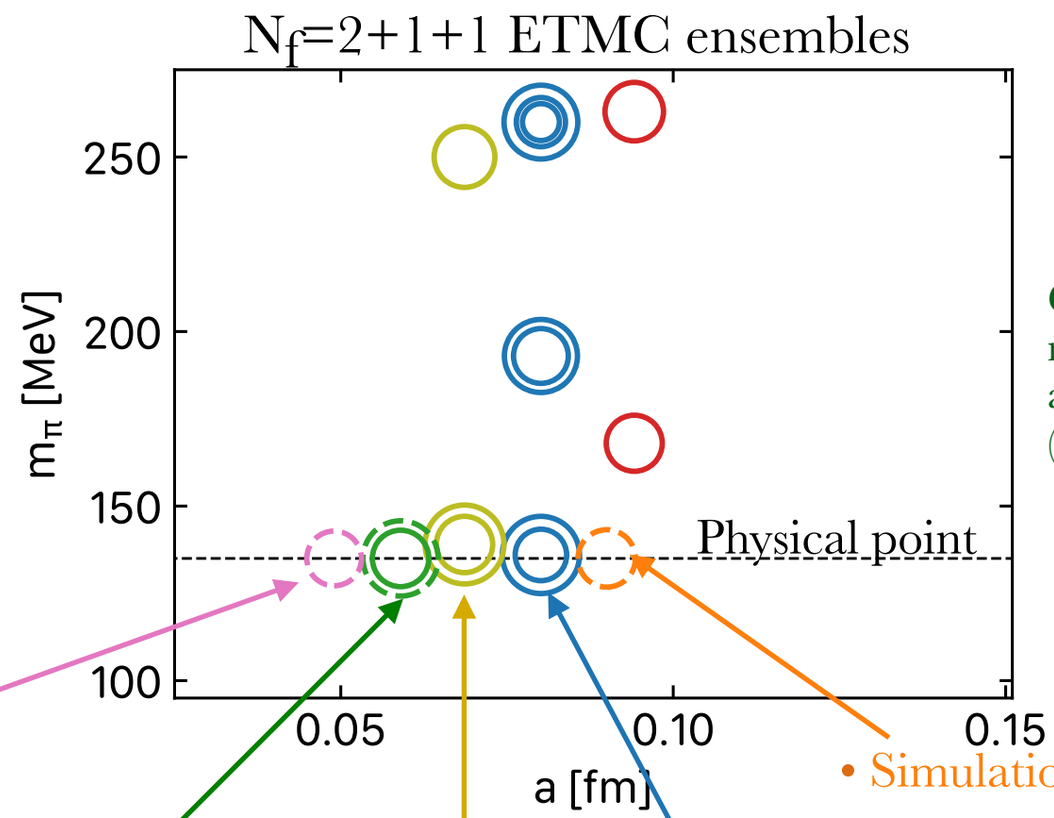

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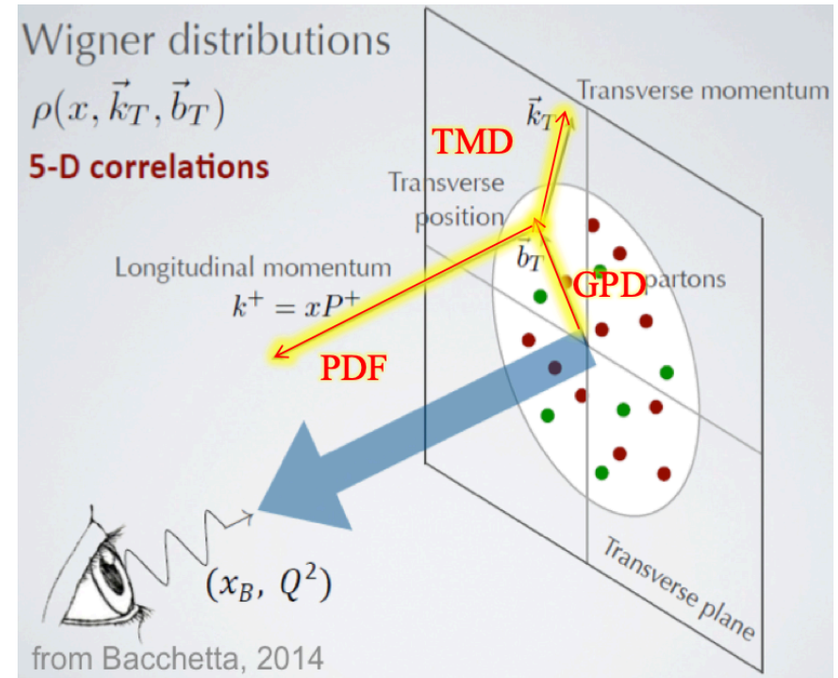
Results in this talk from the analysis of 3 physical point ensembles

- B-ensemble: 64^3 x 128, a~0.08 fm
- C-ensemble: 80^3x160, a~0.07 fm
- D-ensemble:96^3x192, a~0.06 fm

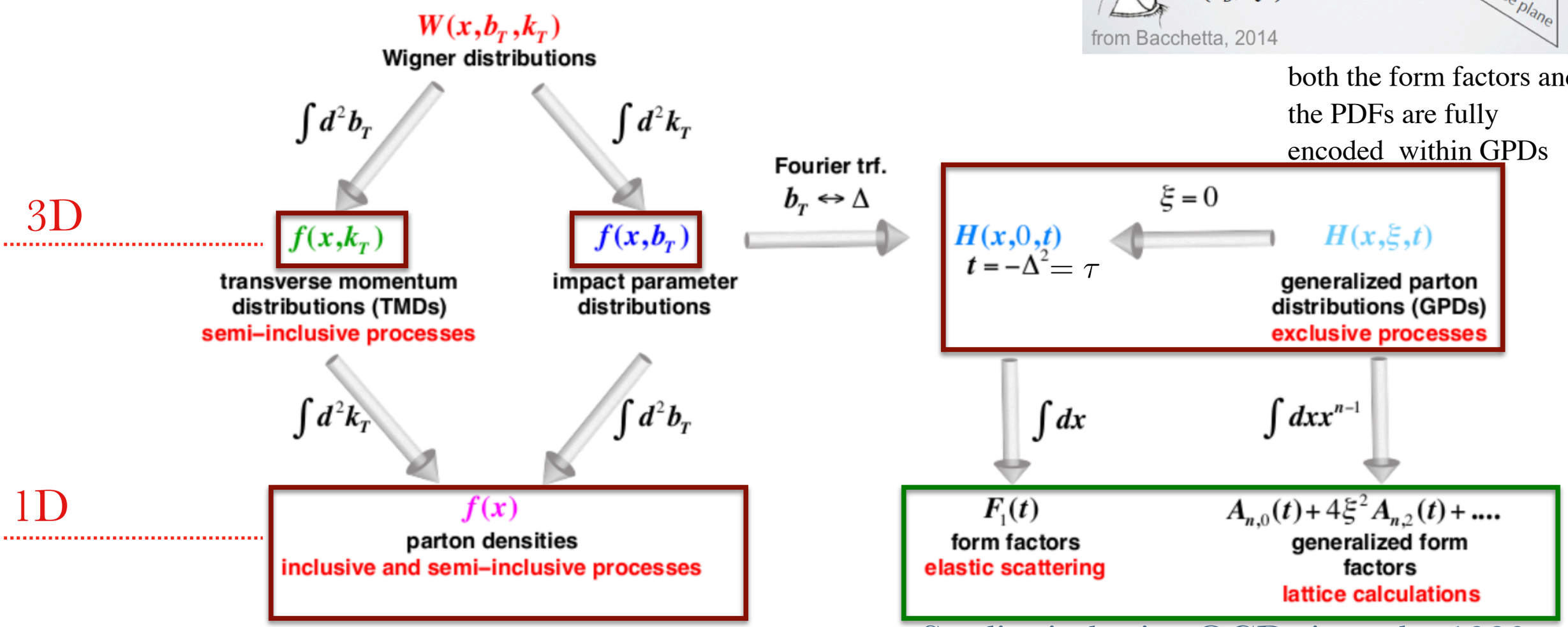
3D structure of the nucleon

- Understanding the 3D-structure of the nucleon from its fundamental constituents, the quarks and the gluons, is major goal of nuclear physics and a key aim of on-going experiments and the future EIC

- Lattice QCD can contribute towards this goal - many recent developments to compute Mellin moments but also directly parton distributions



both the form factors and the PDFs are fully encoded within GPDs

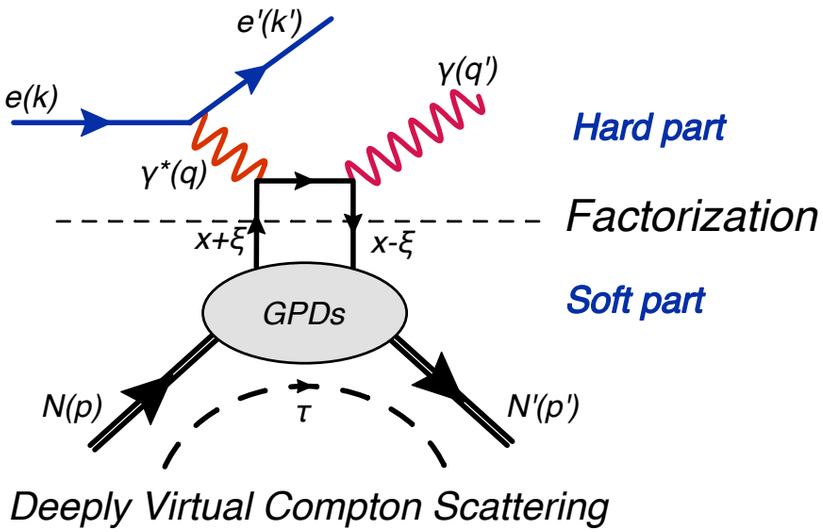


Studies in lattice QCD since the 1980s

EIC white paper, arXiv:1212.1701

Generalised Parton Distributions (GPDs)

- High energy scattering processes: Factorization into a hard partonic subprocess, calculable in perturbation theory, and a universal non-perturbative parton distribution

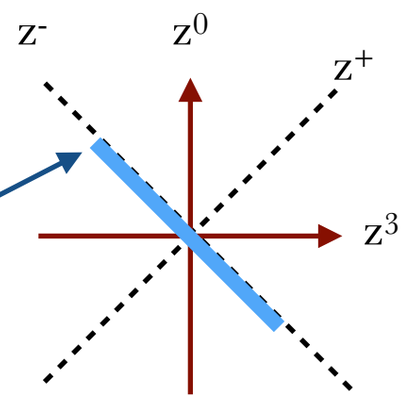


- D. Mueller *et al.*, Fortschr. Phys. 42, 101 (1994)
- A. V. Radyushkin, Phys. Lett. B380, 417 (1996), hep-ph/9604317
- A. V. Radyushkin, Phys. Lett. B385, 333 (1996), hep-ph/9605431
- A. V. Radyushkin, Phys. Rev. D56, 5524 (1997), hep-ph/9704207
- X. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249.
- X. Ji, Phys. Rev. D55, 7114 (1997), hep-ph/9609381
- X. Ji, J. Phys. G24, 1181 (1998), hep-ph/9807358

- GPDs are light cone matrix elements

$$F_\Gamma(x,\xi,\tau) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle N(p')|\bar{\psi}(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$

Correlator along z^-



Γ structure defines 3 different types of GDPs

- $P^+ = \dfrac{p'^+ + p}{2}$
- $\tau = -Q^2 = (p' - p)^2$
- $\xi = \dfrac{p^+ - p'^+}{2P^+}$: skewness

Computation of Mellin moments of GPDs

- Light-cone matrix elements cannot be computed using a Euclidean lattice formulation of QCD
- Expansion of light-cone operator leads to a tower of local twist-2 operators —> connected to moments that can be computed in lattice QCD

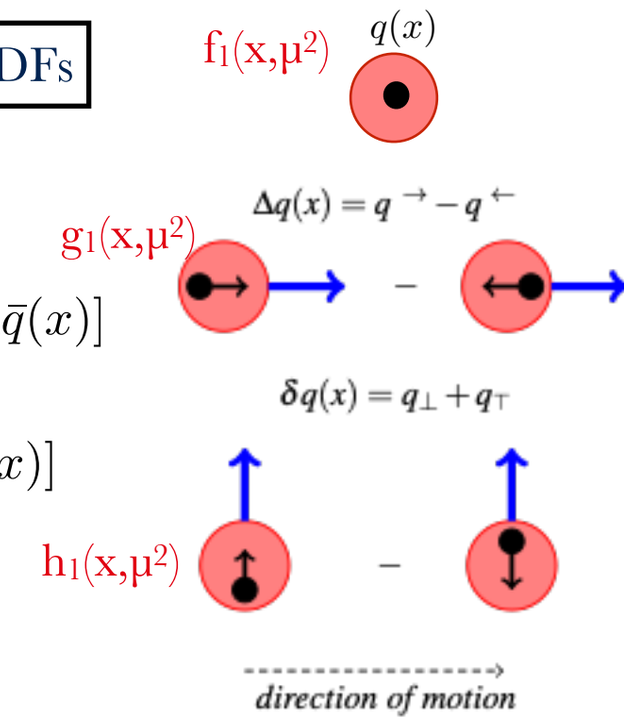
Forward matrix elements give moments of PDFs

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \quad \overset{unpolarized}{\rightarrow} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n\,[q(x) - (-1)^n\bar{q}(x)]$$

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \quad \overset{helicity}{\rightarrow} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n\,[\Delta q(x) + (-1)^n\Delta\bar{q}(x)]$$

$$\mathcal{O}_T^{\rho\mu_1\cdots\mu_n} = \bar{\psi}\sigma^{\rho\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \quad \overset{transversity}{\rightarrow} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n\,[\delta q(x) - (-1)^n\delta\bar{q}(x)]$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$



Twist-2 PDFs

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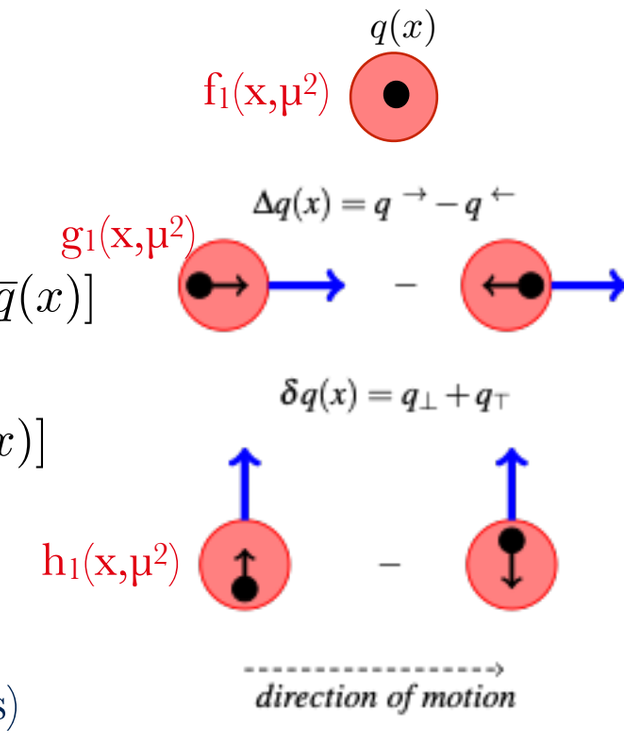

Twist-2 PDFs

- Off-diagonal matrix elements yield moments of GPDs or the generalised form factors (GFFs)

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1} \left[(2\xi)^i A_{ni}(\tau) + \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

$$\int_{-1}^{1} dx\, x^{n-1} E(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1} \left[(2\xi)^i B_{ni}(\tau) - \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

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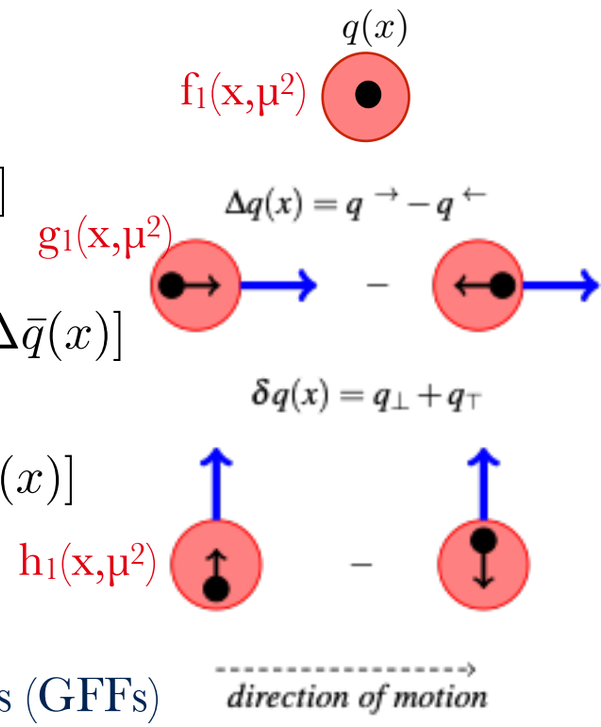

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- For off-diagonal matrix elements we obtain moments of GPDs or the generalised form factors (GFFs)

Twist-2 PDFs

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Ph. Hagler, Phys. Rept. 490 (2010) 49

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Special cases: n=1,2 for the nucleon

- n=1: τ=0 —> charges g_V, g_A, g_T

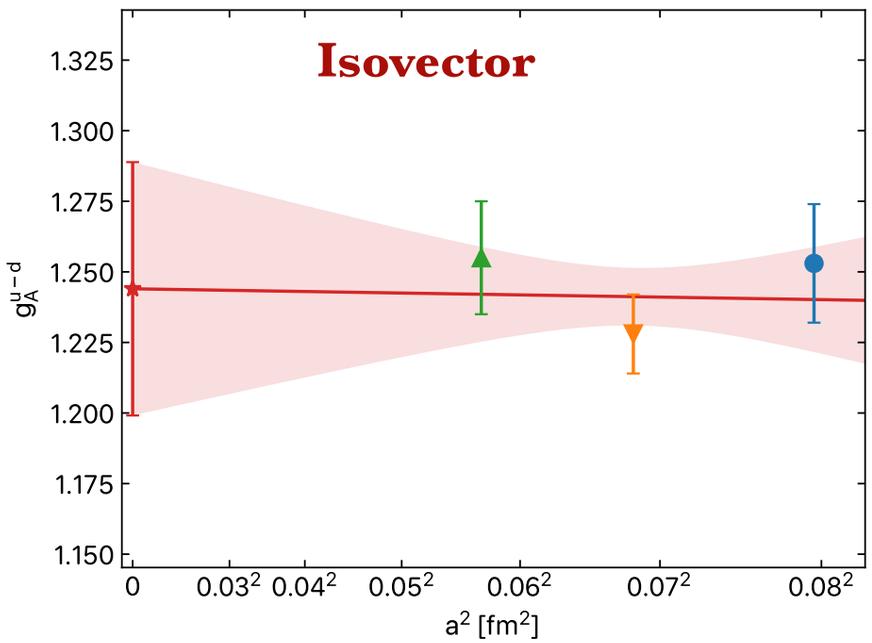
  $\tau \neq 0$ —> form factors: $A_{10}(\tau) = F_1(\tau), \quad B_{10}(\tau) = F_2(\tau), \quad \tilde{A}_{10}(\tau) = G_A(\tau), \quad \tilde{B}_{10}(\tau) = G_p(\tau)$
- n=2: generalised form factors: $A_{20}(\tau), \quad B_{20}(\tau), \quad C_{20}(\tau), \quad \tilde{A}_{20}(\tau), \quad \tilde{B}_{20}(\tau)$

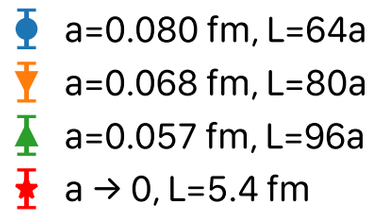
$$\langle x\rangle_q = A_{20}(0), \quad \langle x\rangle_{\Delta q} = \tilde{A}_{20}(0), \quad \langle x\rangle_{\delta q} = A_{20}^T(0) \quad \text{and } J_q = \tfrac{1}{2}[A_{20}(0) + B_{20}(0)] = \tfrac{1}{2}\Delta\Sigma_q + L_q$$

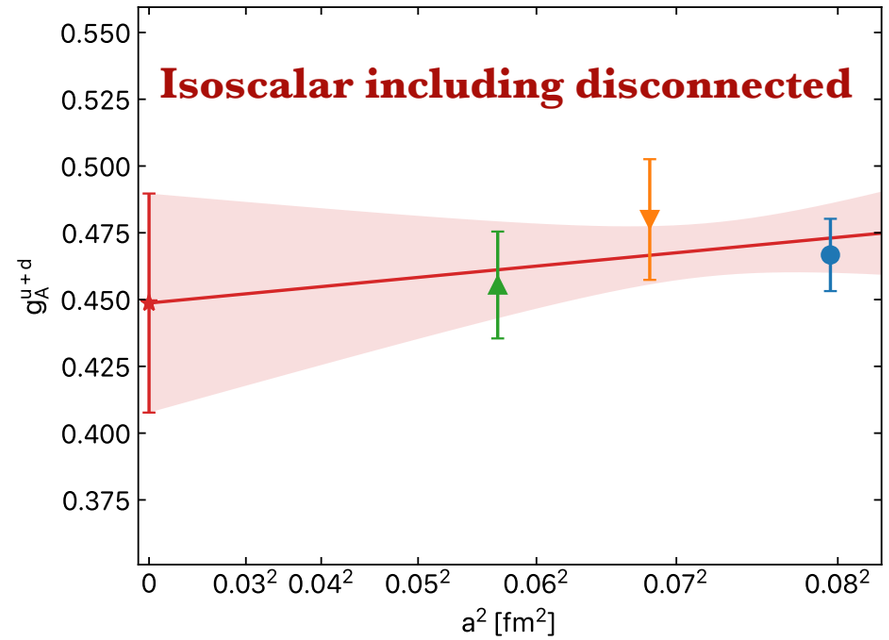
- Spin and momentum sums: $\sum_q \left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}, \quad \sum_q \langle x\rangle_q + \langle x\rangle_g = 1$

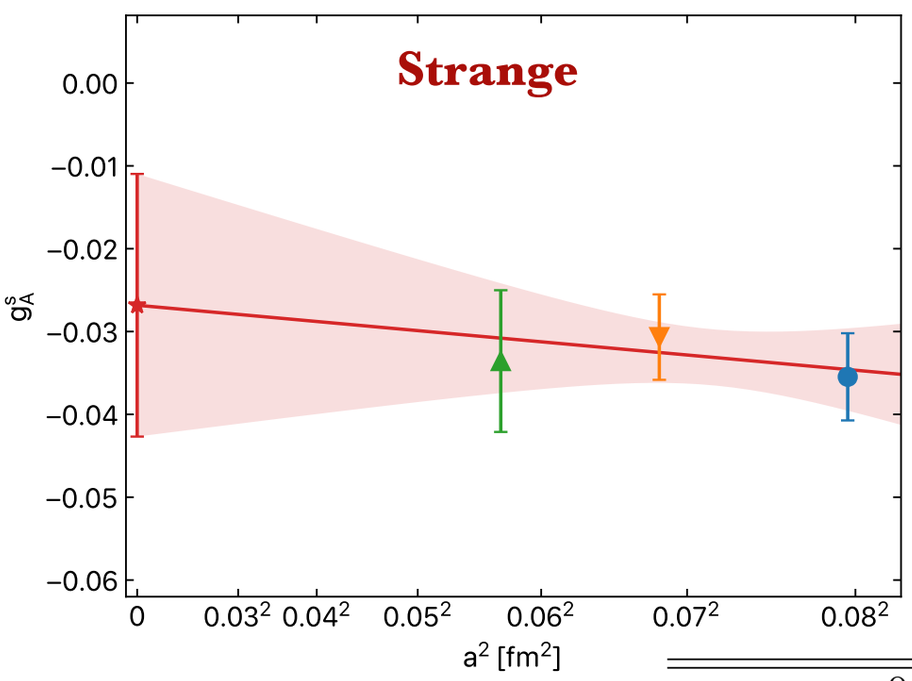
Continuum results

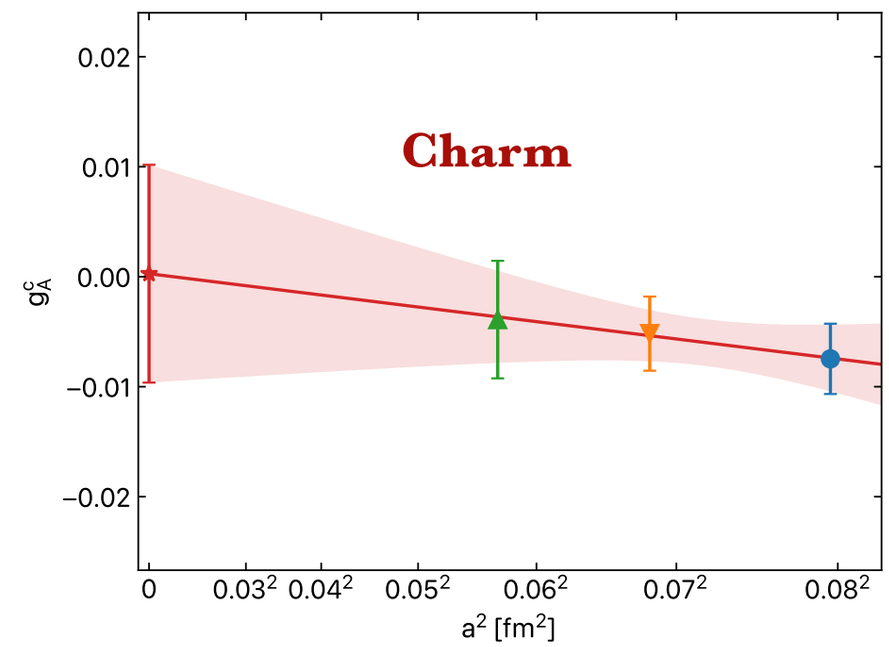
- Axial charges extracted directly from the forward matrix element











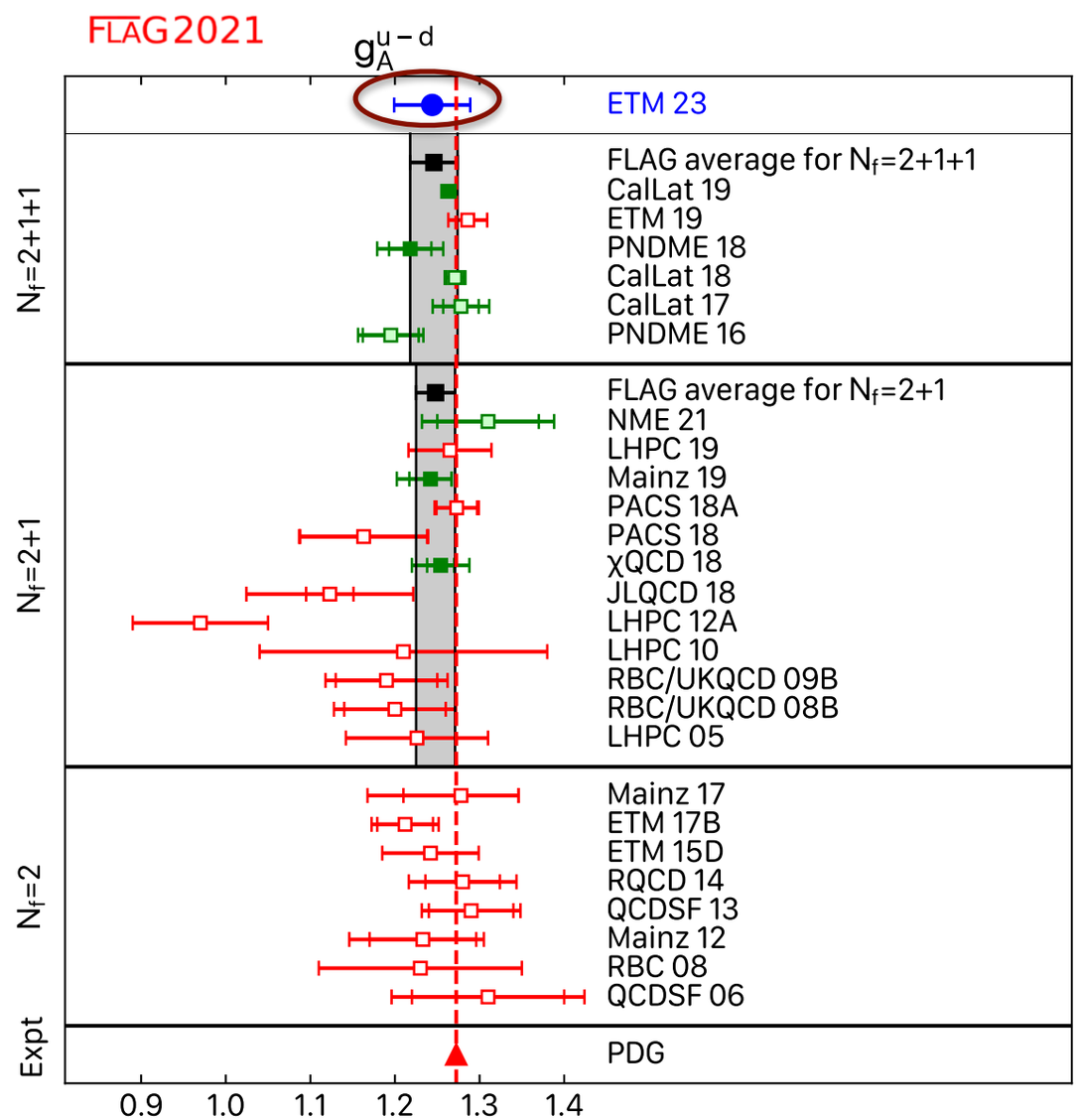
	a=0.080 fm	a=0.068 fm	a=0.057 fm	a→ 0
g_A^{u-d}	1.253(21)	1.228(14)	1.255(20)	1.244(45)
g_A^{u+d}	0.467(14)	0.480(23)	0.455(20)	0.449(41)
g_A^s	-0.0355(53)	-0.0307(52)	-0.0336(86)	-0.027(16)
g_A^c	-0.0075(32)	-0.0052(34)	-0.0039(53)	0.0003(99)

With our two additional lattice spacings we expect more stability in the results and reduced errors at the continuum limit

Nucleon isovector charges

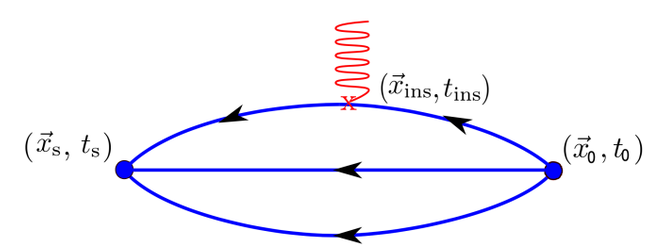
$$g_V = \langle 1 \rangle_{u-d}$$
$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$
$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- $g_V = 1$
- $g_A = 1.2723 \pm 0.0023$ ☜ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001

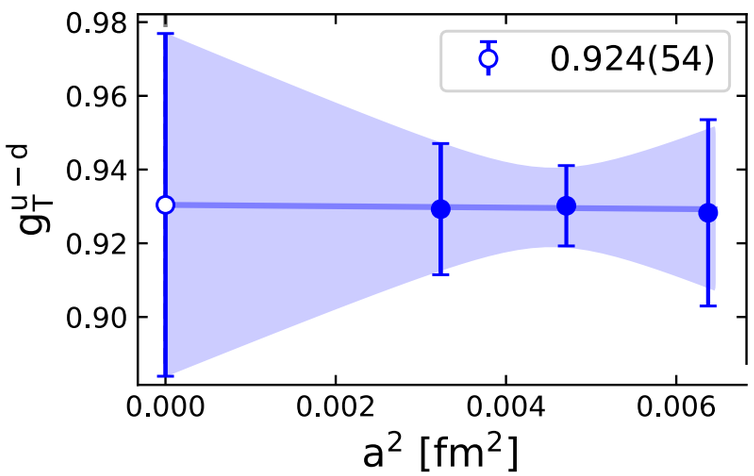


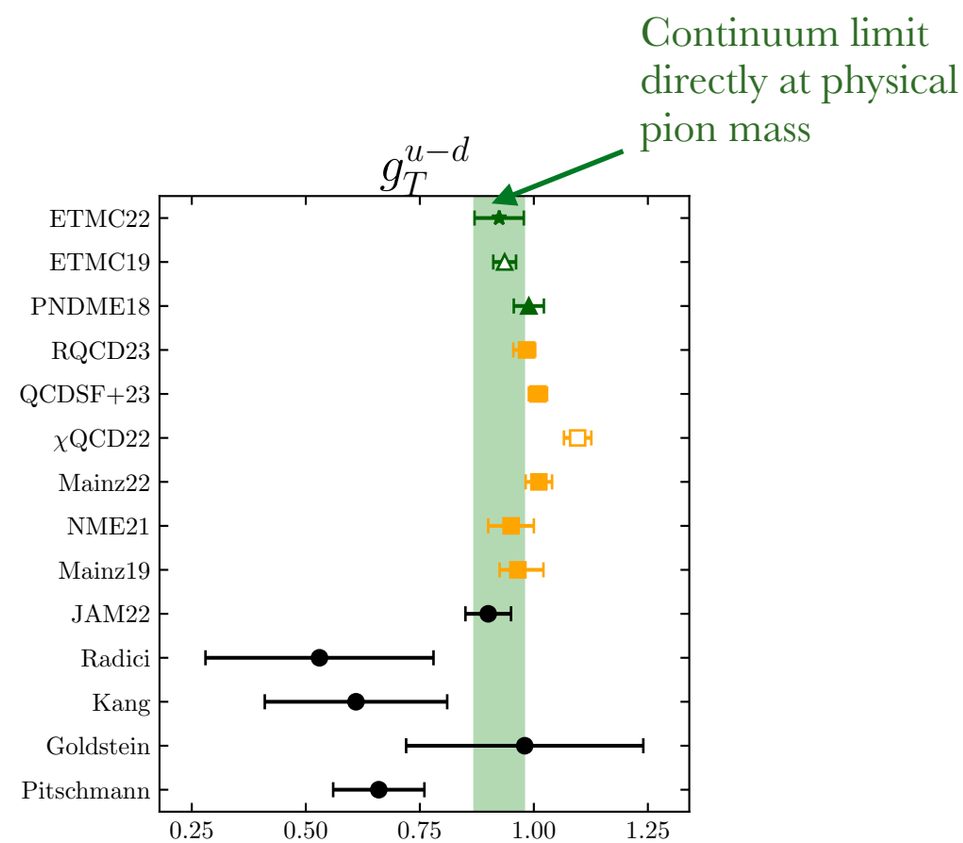
Lattice QCD results on g_A consistent with experimental value

Nucleon isovector (u-d) tensor charge



- Only connected contributions



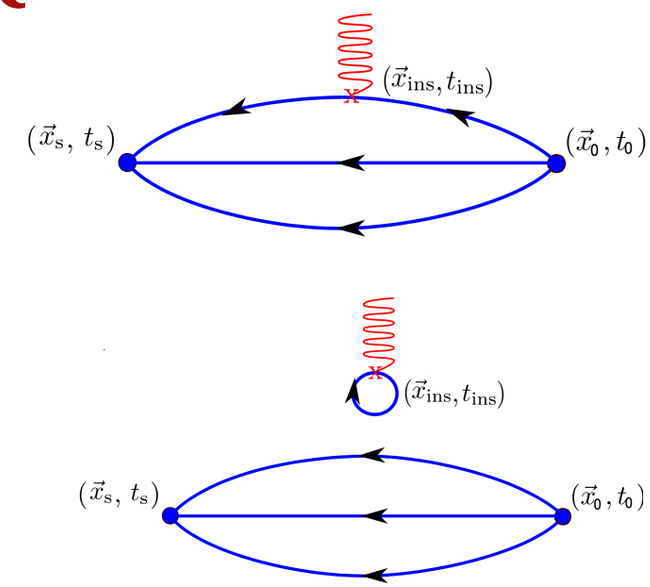


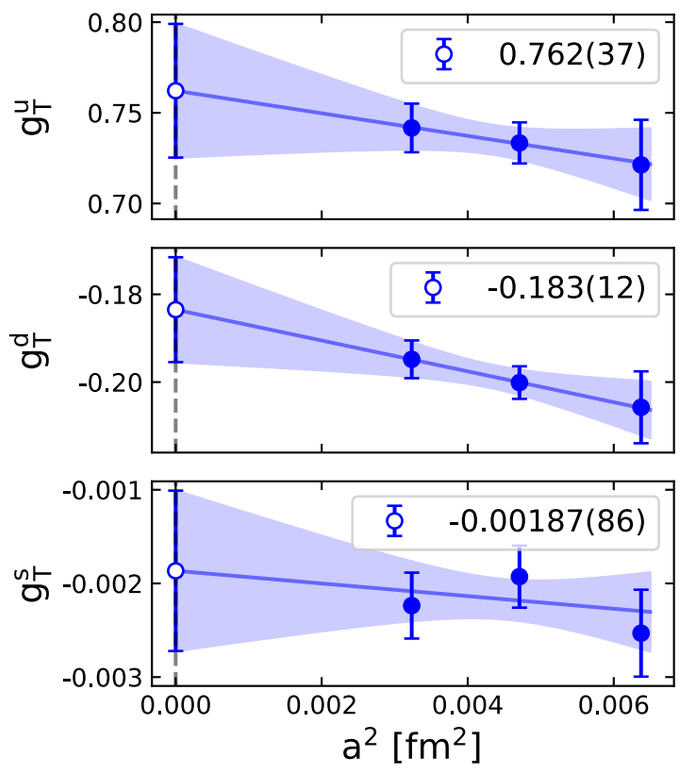
- Precision results on the isovector tensor charge - input for phenomenology e.g. JAM3D-22 analysis

Phys.Rev.D 106 (2022) 3, 034014, arXiv:2205.00999

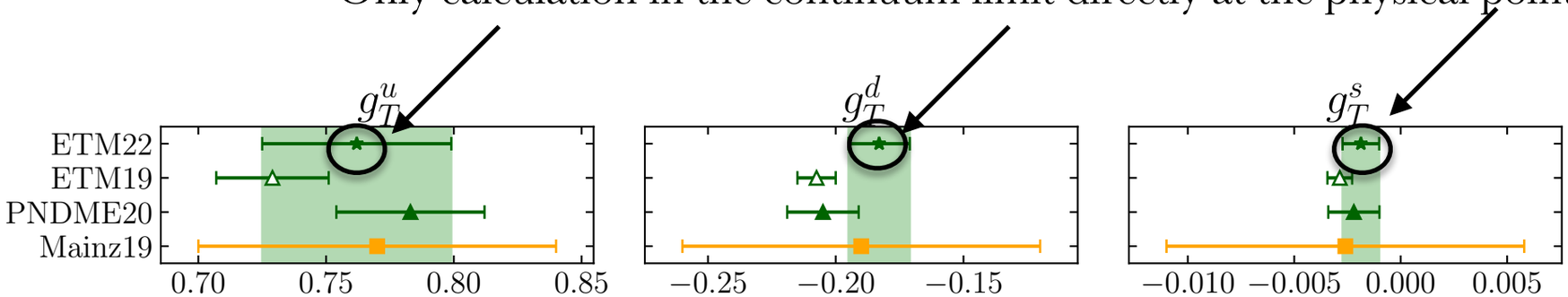
Flavor diagonal tensor charge

- Evaluate both connected and disconnected contributions
- Obtain flavor diagonal tensor charge for the first time in the continuum using only physical point ensembles - input for phenomenology





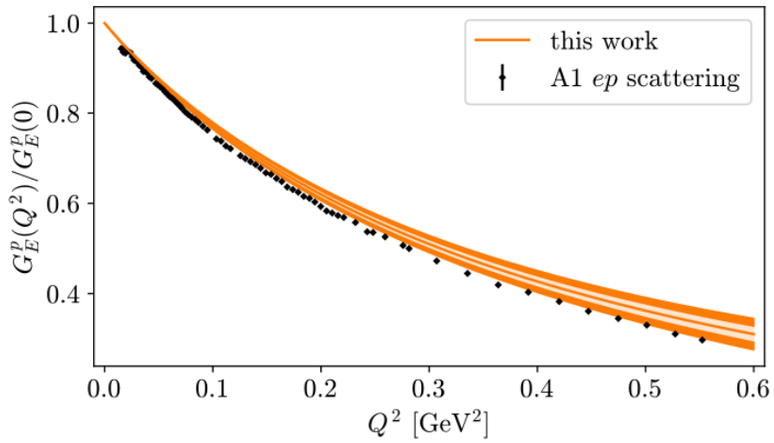
Only calculation in the continuum limit directly at the physical point

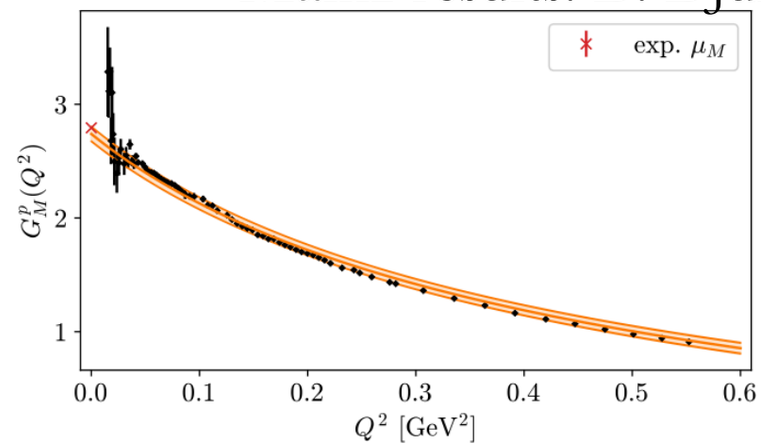


Precision era of lattice QCD for first Mellin moments including flavor diagonal

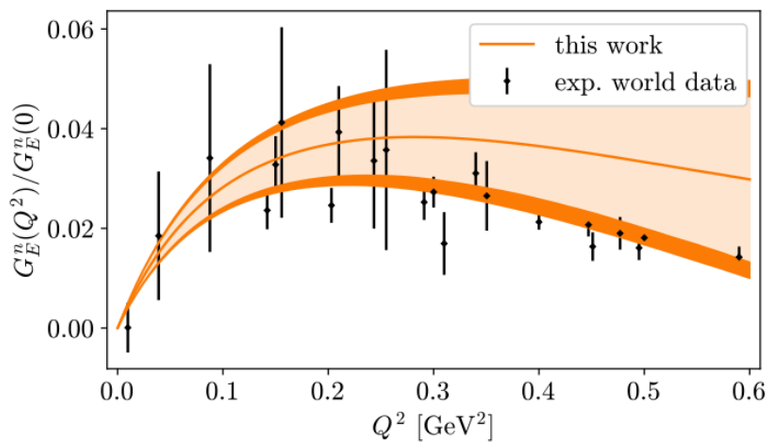
Electromagnetic form factors

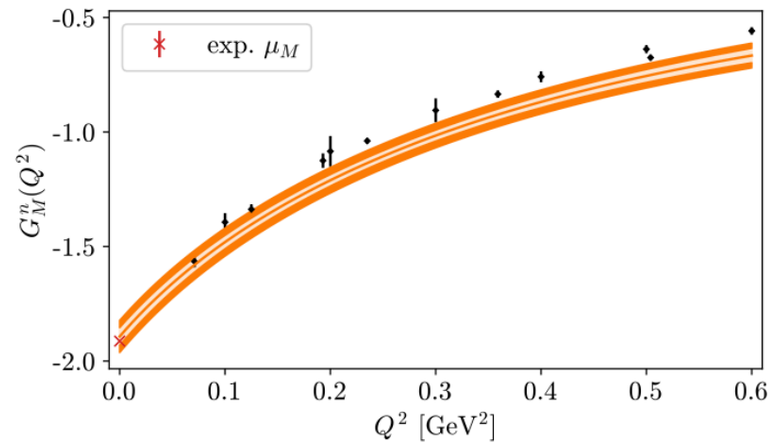
Mainz results: D. Djukanovic *et al.*, arXiv:2309.06590



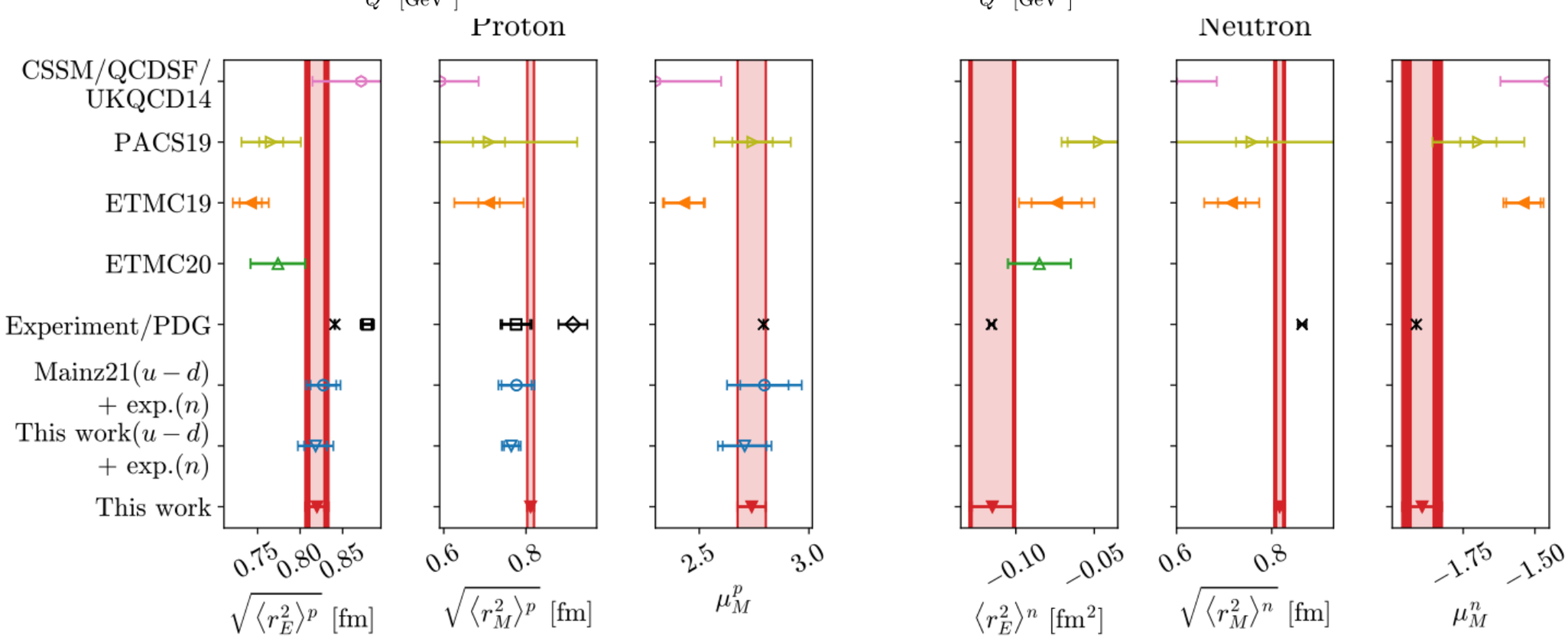


Only one ensemble at physical pion mass —> chiral extrapolation needed





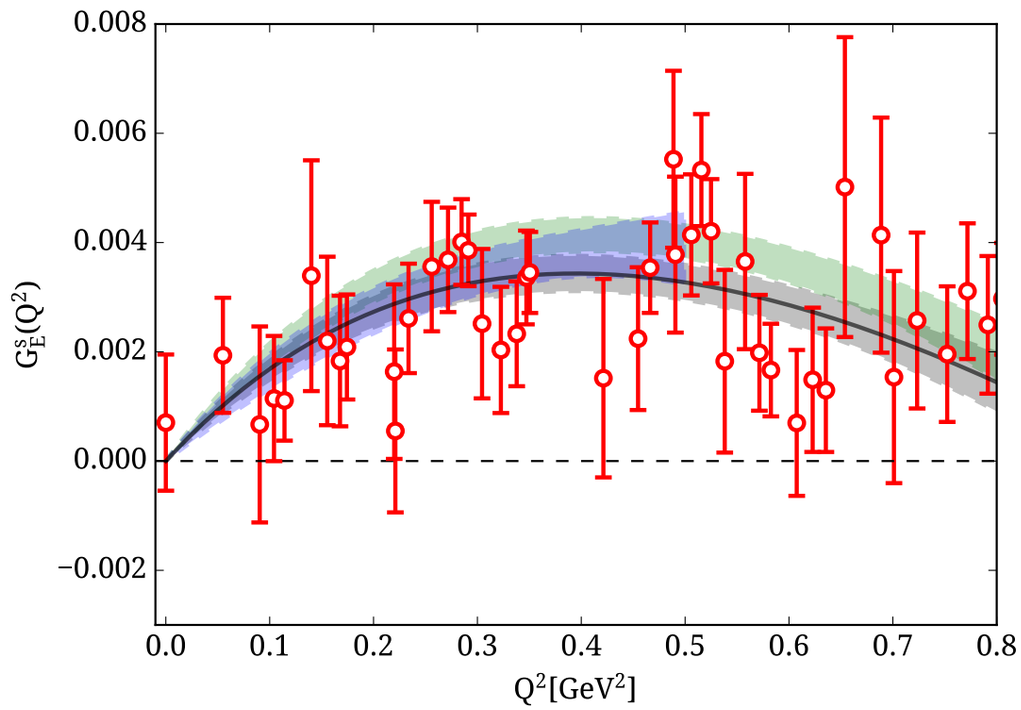
However, impressive accuracy



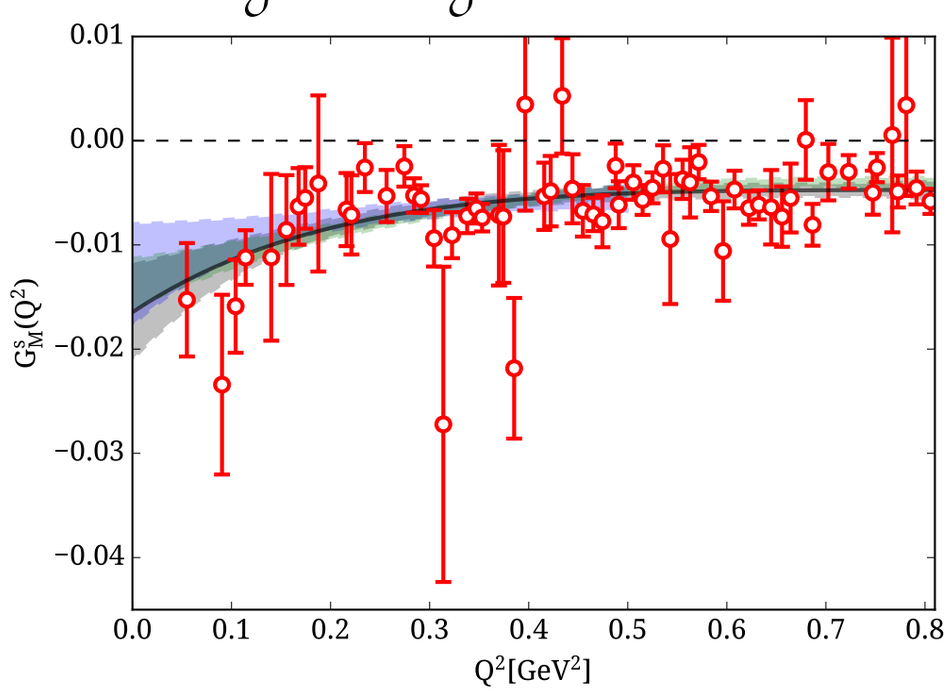
Strangeness of the nucleon

- Sea quark effects can be accurately determined for EM form factors —> provide precise input to experiments

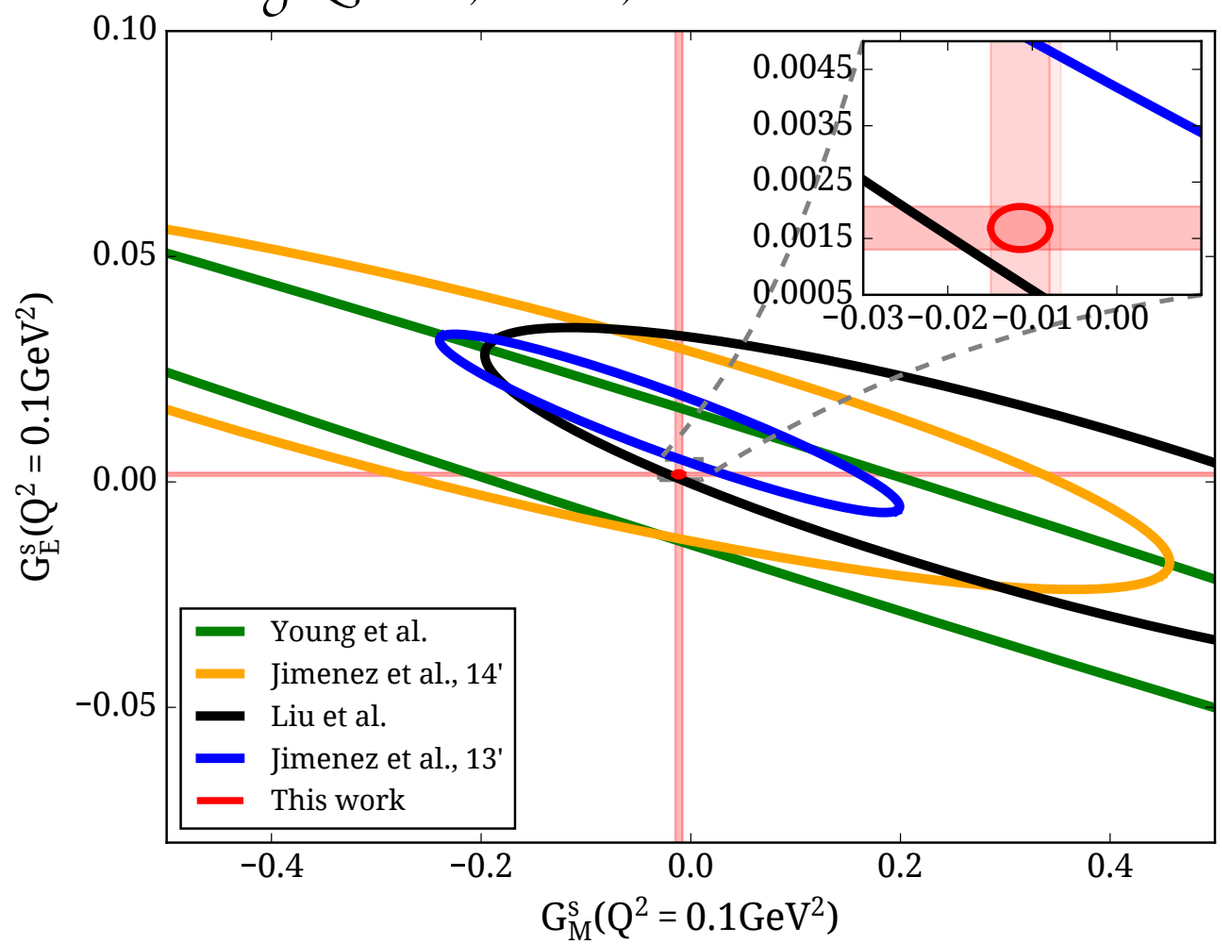
B-ensemble: 64^3 x 128, a~0.08 fm



Negative magnetic moment



Significant input to experimental searches
e.g Q-Weak, SolID, etc



C. A. *et al.*, Phys. Rev. D 101 (2020) 031501, arXiv:1909.10744

Axial and pseudoscalar form factors

Extract from lattice QCD

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N} G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$\langle N(p',s')|P_5|N(p,s)\rangle = G_5(Q^2)\bar{u}_N(p',s')\gamma_5 u_N(p,s) \qquad q^2=-Q^2$$

- Chiral symmetry breaking leads to: $\partial^\mu A_\mu = F_\pi m_\pi \psi_\pi$
- Axial Ward-Takahashi identity leads to PCAC : $\partial^\mu A_\mu = 2m_q P, \quad m_q = m_u = m_d$

$$\psi_\pi = \frac{2m_q P}{F_\pi m_\pi^2}$$

- Take nucleon matrix elements :

$$G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{m_q}{m_N} G_5(Q^2)$$

$$G_5(Q^2) = \frac{F_\pi m_\pi^2}{m_q} \frac{G_{\pi NN}(Q^2)}{m_\pi^2 + Q^2}$$

← Goldberger-Treiman relation

Background

Extract from lattice QCD

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$\langle N(p',s')|P_5|N(p,s)\rangle = G_5(Q^2)\bar{u}_N(p',s')\gamma_5 u_N(p,s) \qquad q^2=-Q^2$$

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- Take nucleon matrix elements :

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

$$G_5(Q^2) = \frac{F_\pi m_\pi^2}{m_q}\frac{G_{\pi NN}(Q^2)}{m_\pi^2 + Q^2}$$ ← Goldberger-Treiman relation

- Pion pole dominance:

$$G_P(Q^2) = \frac{4m_N^2}{Q^2+m_\pi^2}G_A(Q^2)\Big|_{Q^2\to -m_\pi^2}$$

$$G_A(Q^2) = \frac{F_\pi}{m_N}G_{\pi NN}(Q^2)\Big|_{Q^2\to -m_\pi^2}$$

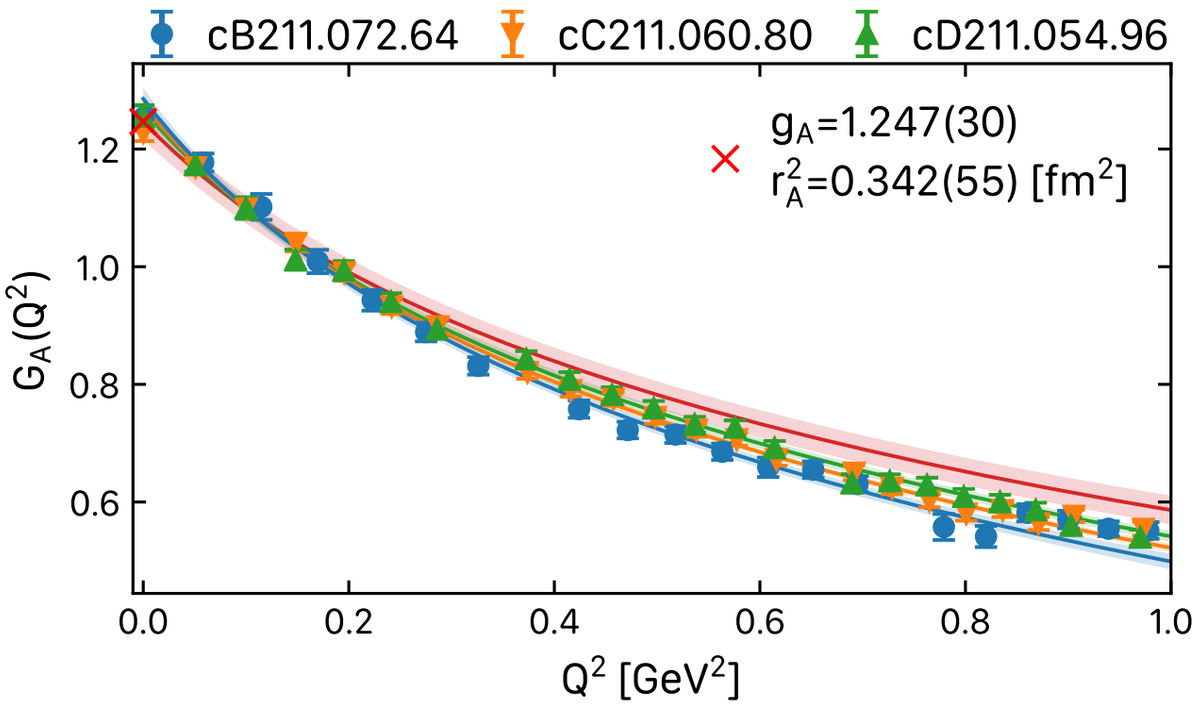
- At the pion pole we get the pion nucleon coupling: $g_{\pi NN} \equiv G_{\pi NN}(Q^2 = -m_\pi^2)$

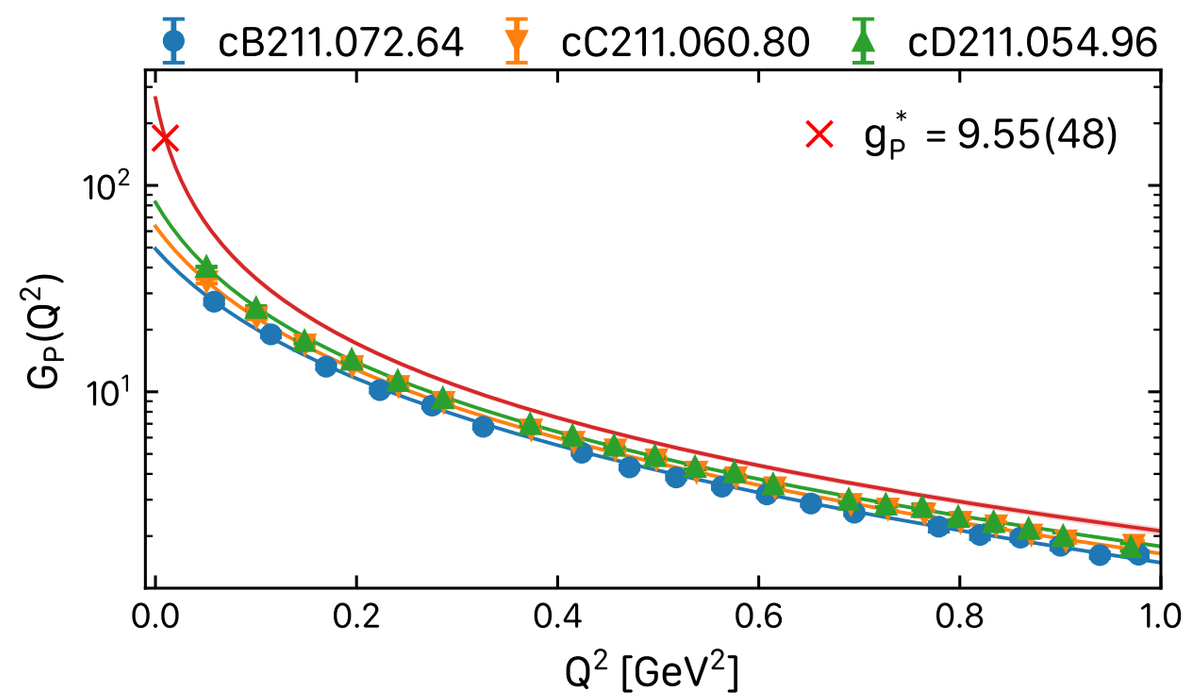
$$\lim_{Q^2\to -m_\pi^2}(Q^2+m_\pi^2)G_P(Q^2) = 4m_N F_\pi g_{\pi NN}$$

$$g_{\pi NN} = m_N G_A(-m_\pi^2)/F_\pi \xrightarrow{m_\pi\to 0} \frac{m_N}{F_\pi}g_A$$ and $\Delta_{GT} = 1 - \frac{g_A m_N}{g_{\pi NN}F_\pi}$ is the GT discrepancy

Results

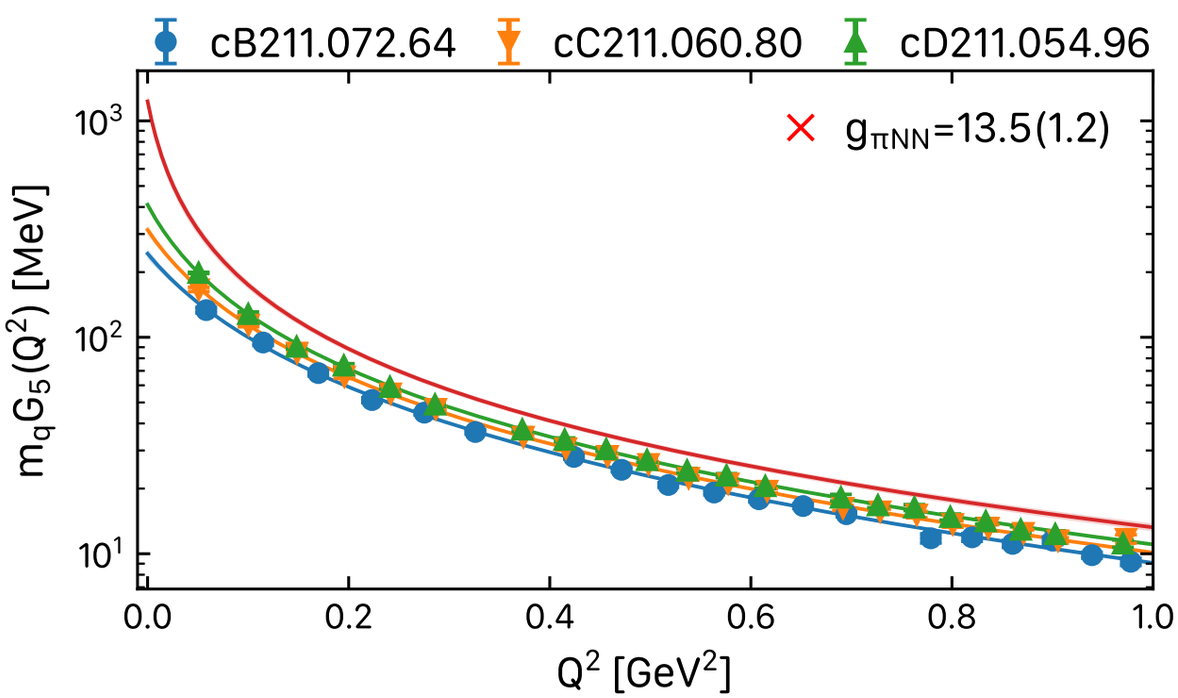
- **Axial and induced pseudoscalar form factors**

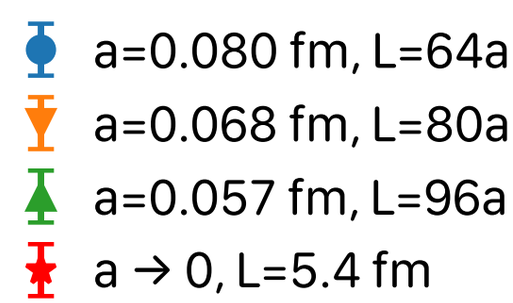




- **Pseudoscalar form factor**

$$g_P^* \equiv \frac{m_\mu}{2m_N} G_P(0.88\, m_\mu^2)$$





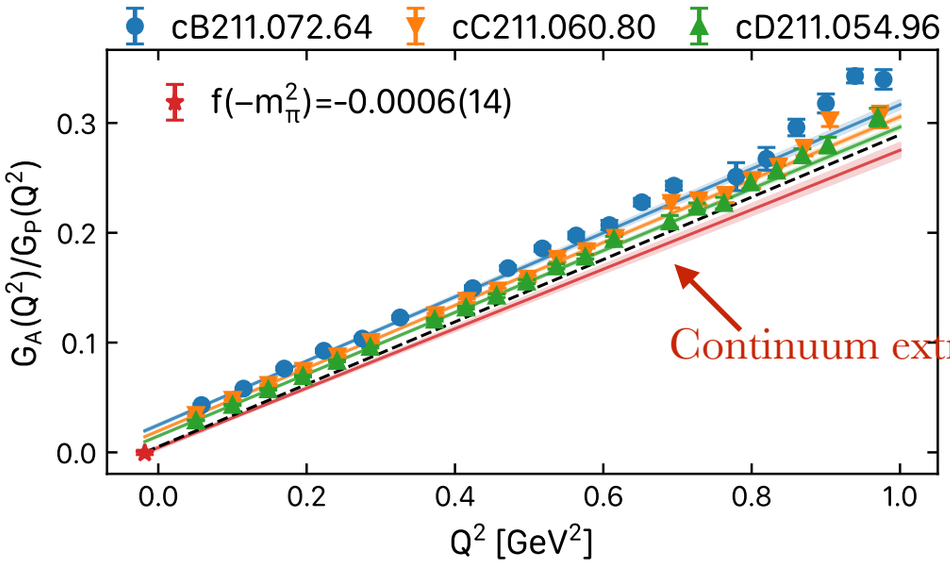
❋ Dipole and z-expansion fits, various ranges ⟶ model average using AIC

PCAC and pion pole dominance (PPD)

- Check PCAC and PPD relations

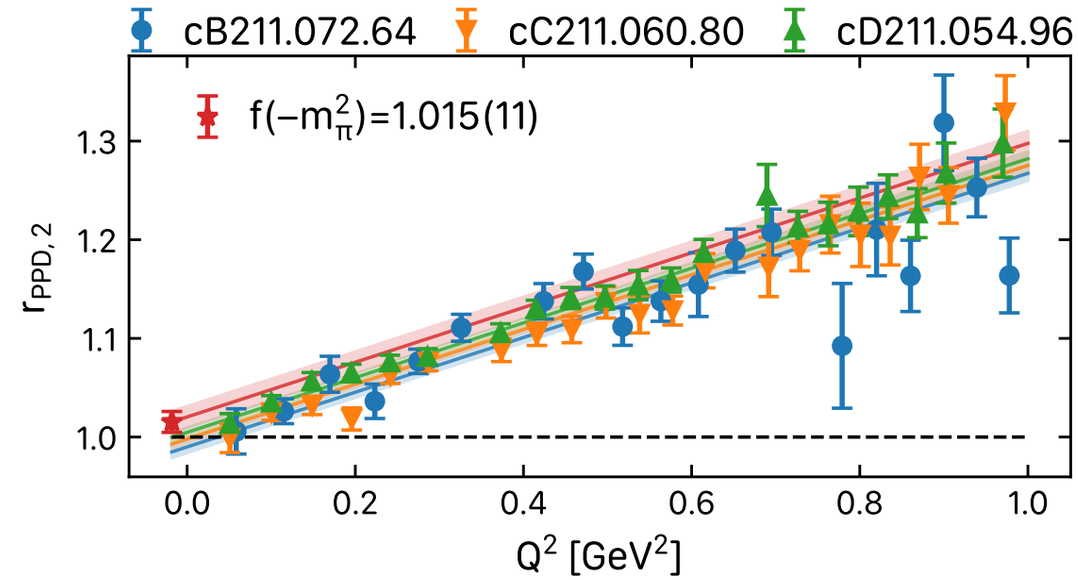
$$\frac{G_A(Q^2)}{G_P(Q^2)} = \frac{Q^2 + m_\pi^2}{4m_N^2}\bigg|_{Q^2 \to -m_\pi^2}$$

$$r_{\mathrm{PPD},2} = \frac{4m_N}{m_\pi^2}\frac{m_q G_5(Q^2)}{G_P(Q^2)}\bigg|_{Q^2 \to -m_\pi^2}$$



$c_1 \sim 1/4m_N^2$

$c_0 \sim m_\pi^2/4\, m_N^2$



Continuum extrapolation using:

$$f(Q^2, a^2) = c_0 + c_1 Q^2 + c_2 a^2 + c_3 a^2 Q^2$$

Q^2 dependent deviation among from pion pole

$$\frac{4m_N}{m_\pi^2}\frac{m_q G_5(Q^2)}{G_P(Q^2)} = 1 + \epsilon\left(1 + \frac{Q^2}{m_\pi^2}\right)$$

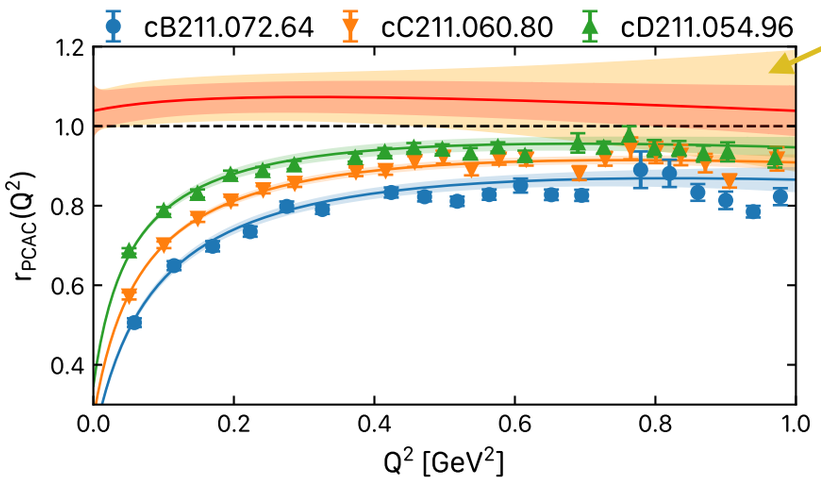
From the slope ε we can compute GT deviation:

$$\Delta_{GT} = -0.0216(43)(10) \longleftarrow \text{2\% in ChPT}$$

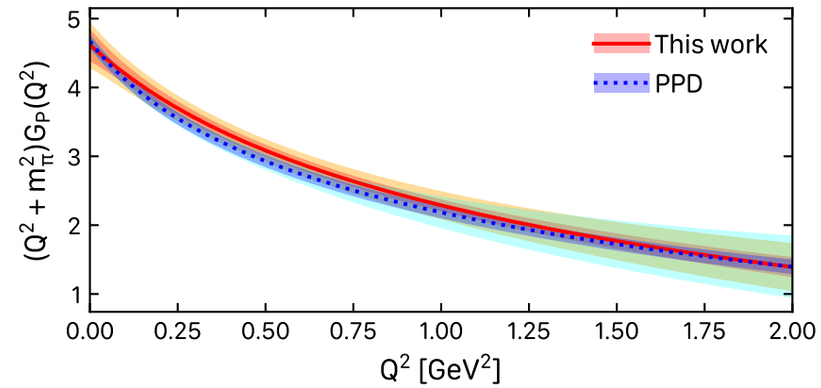
$$\Delta_{\mathrm{GT}} \equiv \frac{2\bar{d}_{18} m_\pi^2}{g_A} \qquad \bar{d}_{18} = -0.0288(60)(18)\ \mathrm{fm}^2$$

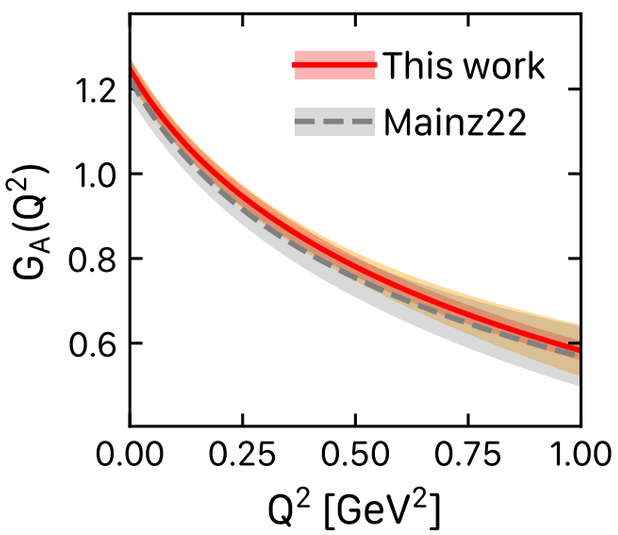
Low energy constant

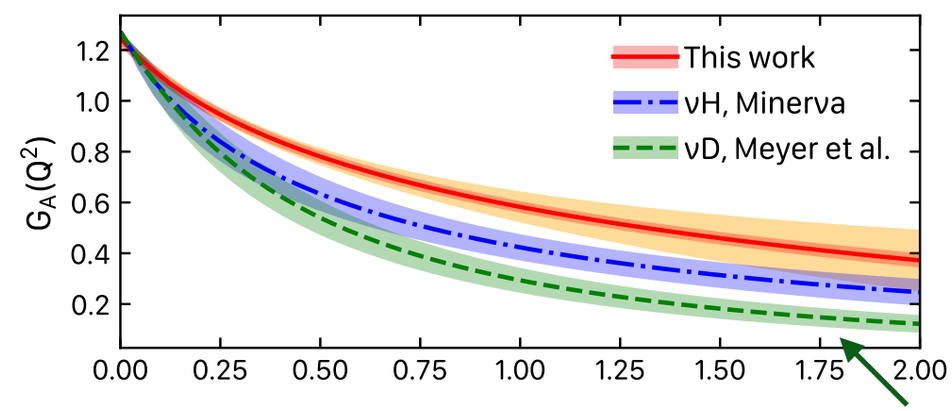
Recent results on $G_A(Q^2)$ and $G_P(Q^2)$



Systematic error due to excited states







deuterium bubble-chamber data, Phys. Rev. D 93, 113015 (2016)

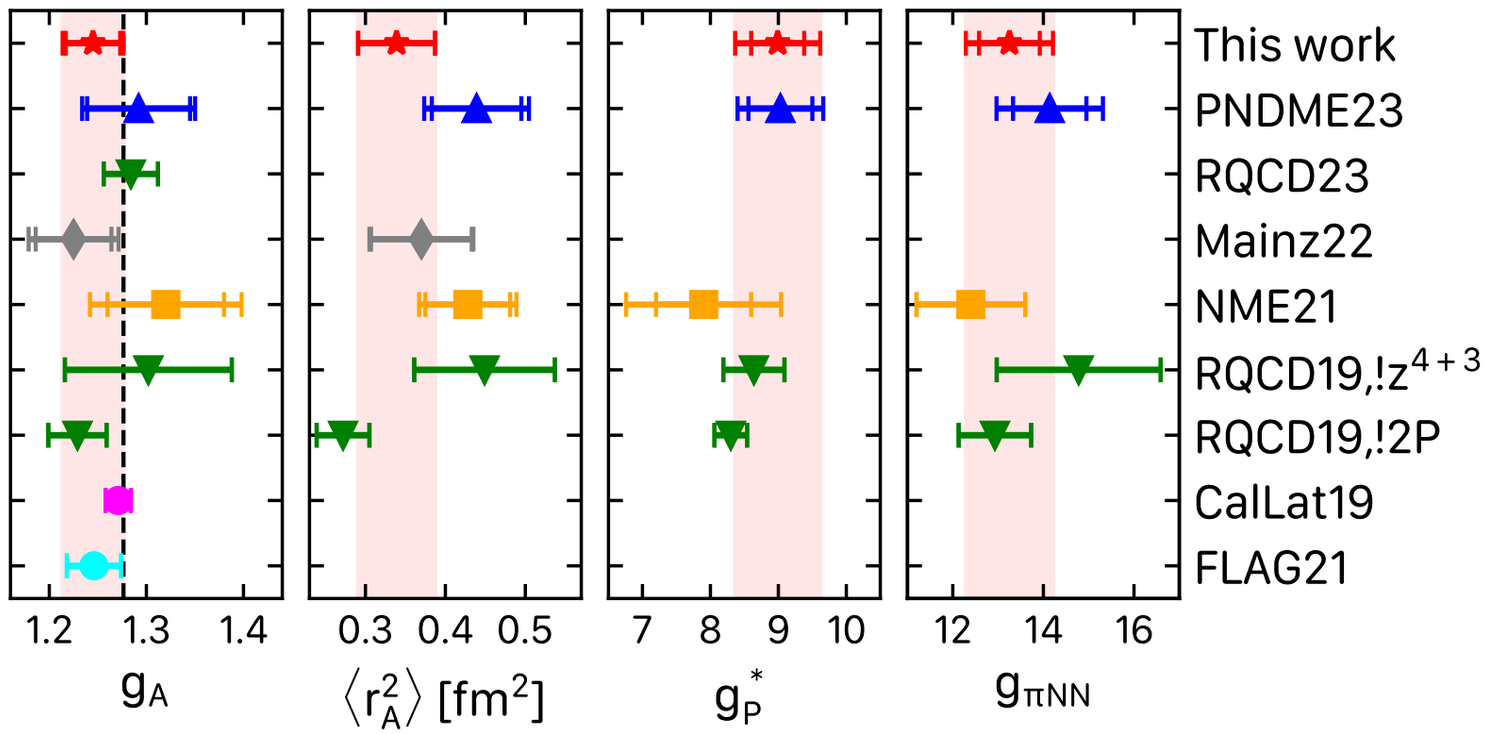
- PCAC satisfied in the continuum limit
- Pion pole dominance satisfied for induced pseudoscalar
- Lattice QCD results closer to the new Minerva antineutrino-hydrogen data

T. Cai *et al.*, Nature 614, 48 (2023)

- Agreement between our results and those of Mainz

D. Djukanovic et al. PRD 106, 074503 (2022), arXiv: 2207.03440

Comparison



❋Very good agreement among lattice QCD results

C.A., S. Bacchio, M. Constantinou, J. Finkenrath, R. Frezzotti, B. Kostrzewa, G. Koutsou, G. Spanoudes, C. Urbach, arXiv:2309.05774

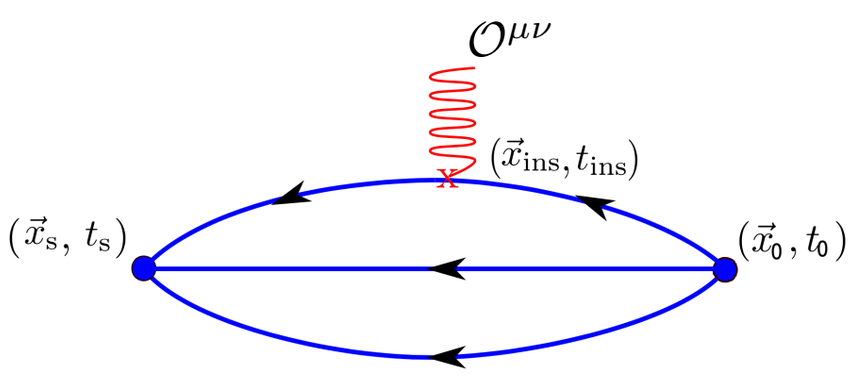
Second Mellin moments

- Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

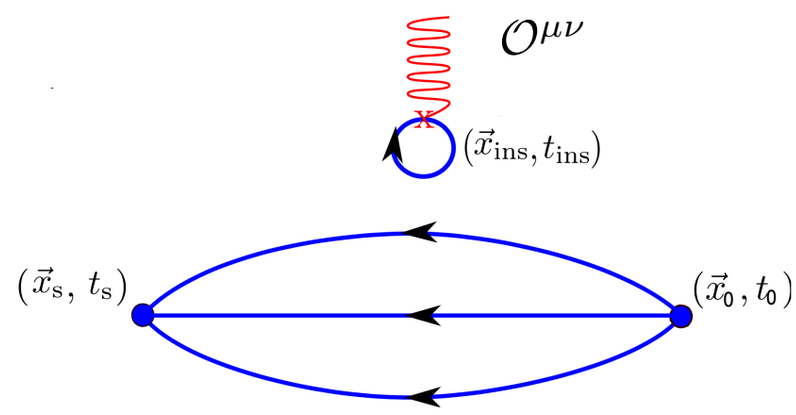
$$\langle N(p',s')|\mathcal{O}^{\mu\nu,q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A^q_{20}(q^2)\gamma^{\{\mu P^\nu\}} + B^q_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C^q_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

$$\langle x\rangle_q = A^q_{20}(0) \qquad J_q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

Momentum fraction carried by quark - best measured



+



Second Mellin moments

- Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu} iD^{\nu\}} q$

- Gluon unpolarised moment $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho} F_{\rho}^{\nu\}}$ Field strength tensor

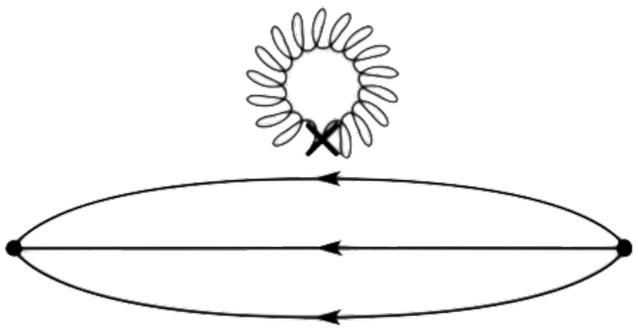
$$\langle N(p',s')|\mathcal{O}^{\mu\nu,q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^q(q^2)\gamma^{\{\mu P^{\nu\}}} + B_{20}^q(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

Momentum fraction carried by quark - best measured

- Equivalent expression for gluon

$$\langle x\rangle_g = A_{20}^g(0) \qquad J_g = \frac{1}{2}\left[A_{20}^g(0) + B_{20}^g(0)\right]$$



- Momentum sum: $\sum_q \langle x\rangle_q + \langle x\rangle_g = 1$
- Spin sum: $\sum_q \left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}$

J_q

- Matrix elements of helicity and transversity one derivative operators yield $\langle x\rangle_{\Delta q}$, $\langle x\rangle_{\delta q}$

Second Mellin moments

- Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu} iD^{\nu\}} q$

- Gluon unpolarised moment $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho} F_{\rho}^{\nu\}}$ ← Field strength tensor

$$\langle N(p',s')|\mathcal{O}^{\mu\nu,q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^q(q^2)\gamma^{\{\mu P^{\nu}\}} + B_{20}^q(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

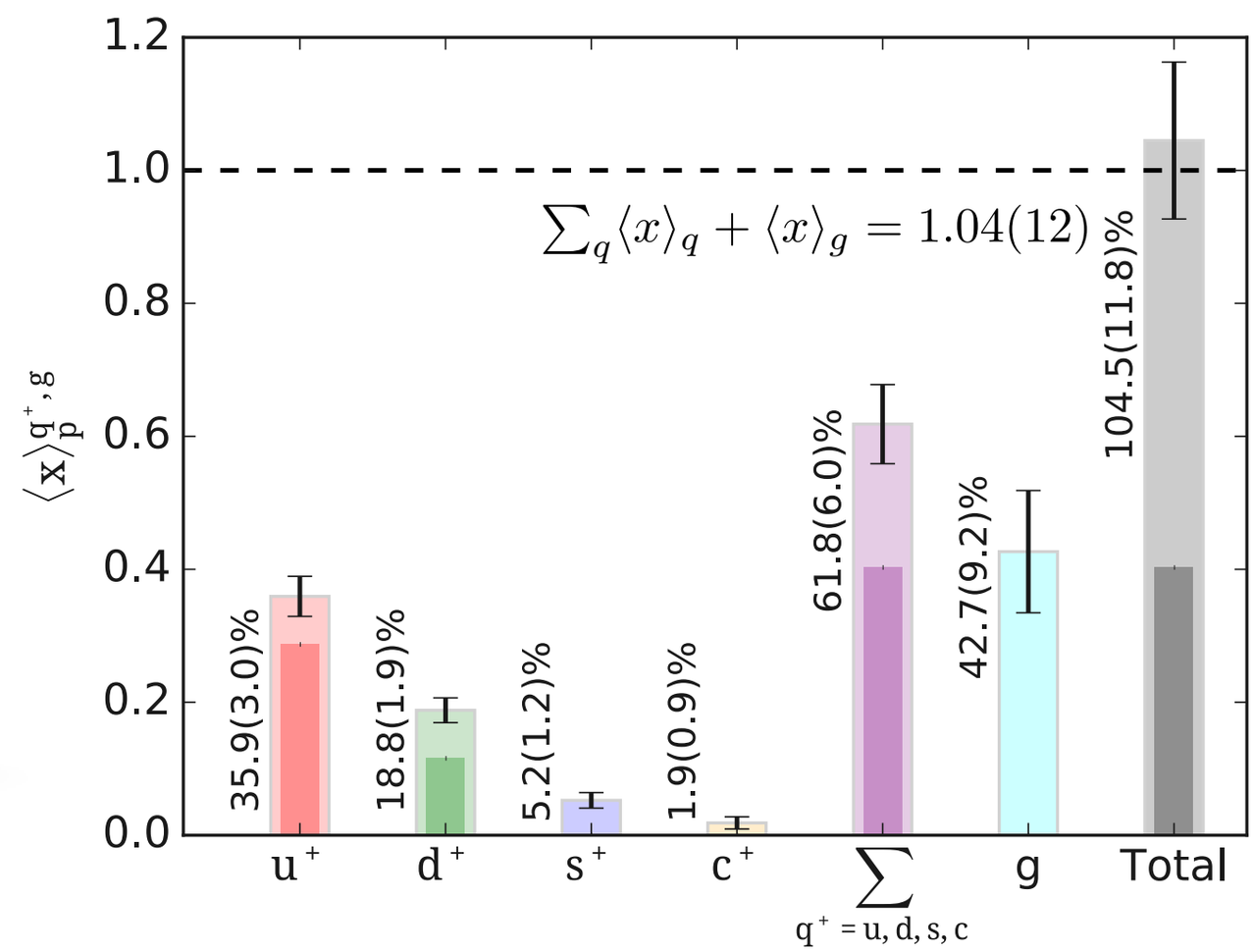
$$\langle x\rangle_q = A_{20}^q(0) \qquad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$$

↑ Momentum fraction carried by quark - best measured

- Equivalent expression for gluon

$$\langle x\rangle_g = A_{20}^g(0)$$

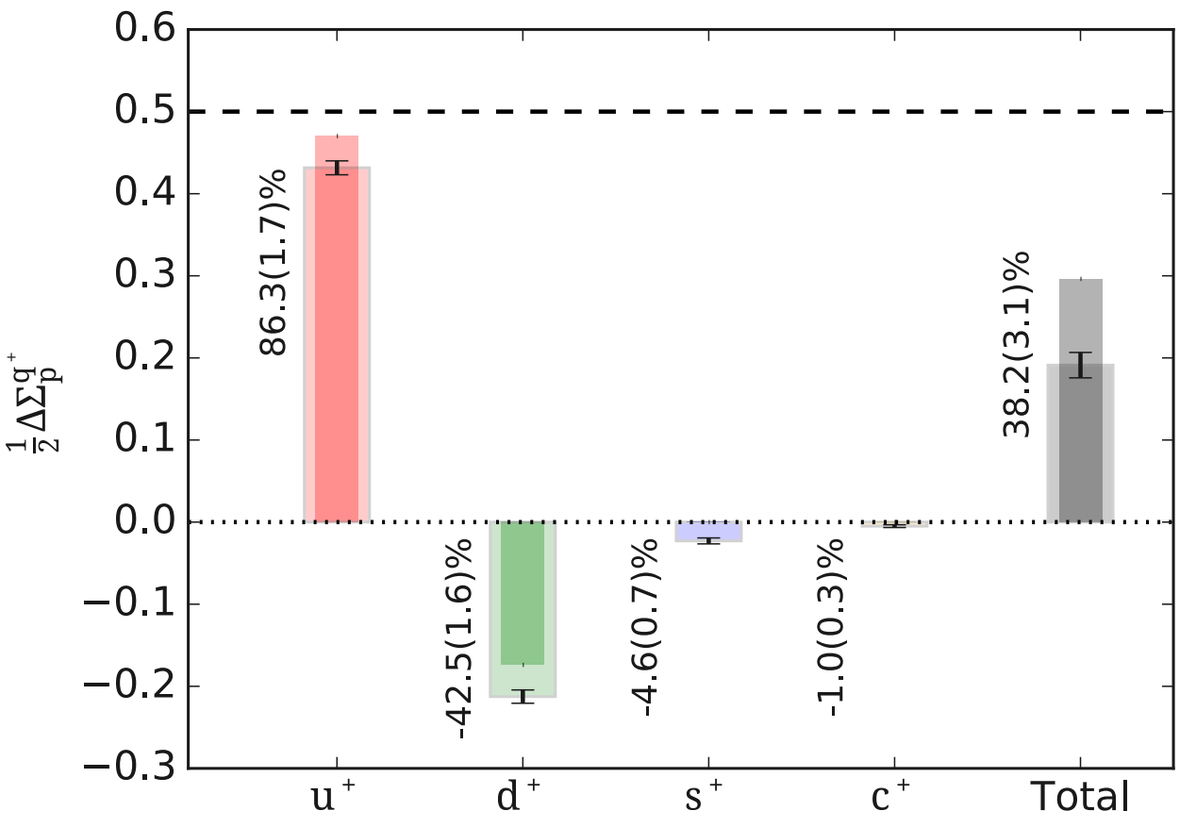
→ Nucleon momentum sum verified



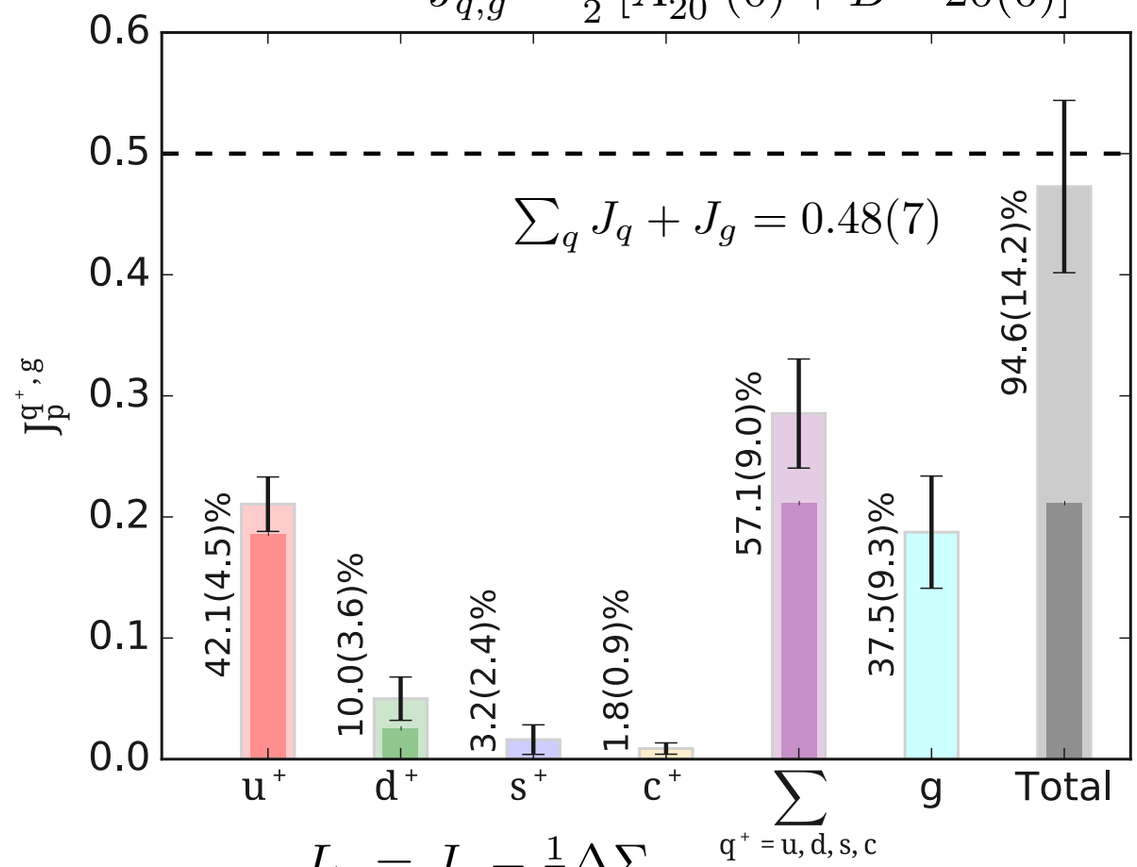
Momentum and spin sums

- Axial charge determines intrinsic spin carried by each quark

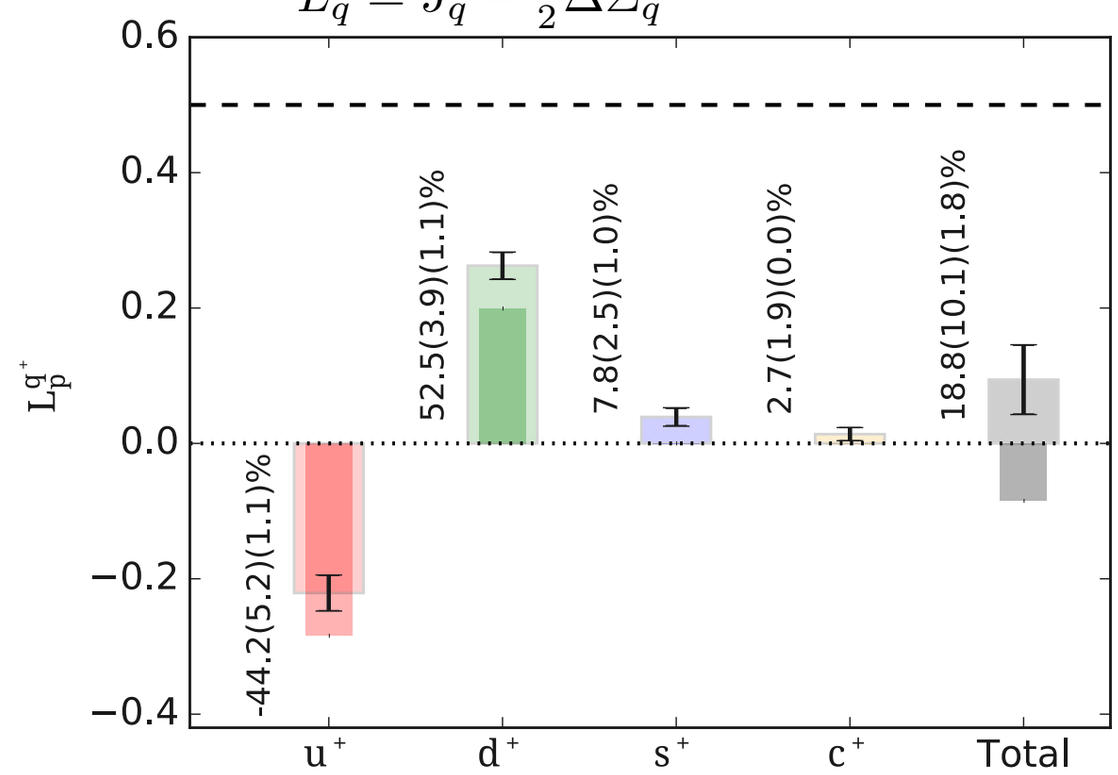
$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$



$$J_{q,g} = \frac{1}{2}\left[A_{20}^{q,g}(0) + B^{q,g}20(0)\right]$$



$$L_q = J_q - \frac{1}{2}\Delta\Sigma_q$$



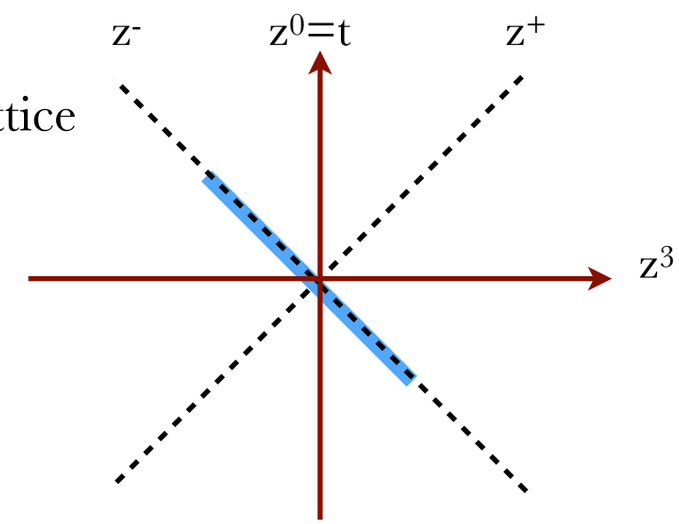
Nucleon spin sum verified - lattice QCD solves a 30 year puzzle

C. A. *et al.* (ETMC) Phys. Rev. Lett. **119**, 142002, 1909.00485
C. A. *et al.* (ETMC) Phys.Rev.D **101** (2020) 9, 094513, 2003.08486

Direct computation of parton distributions

- PDFs light-cone correlation matrix elements - cannot be computed on a Euclidean lattice

$$F_\Gamma(x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle N(p)|\bar{\psi}(-z/2)\Gamma W(-z/2, z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$

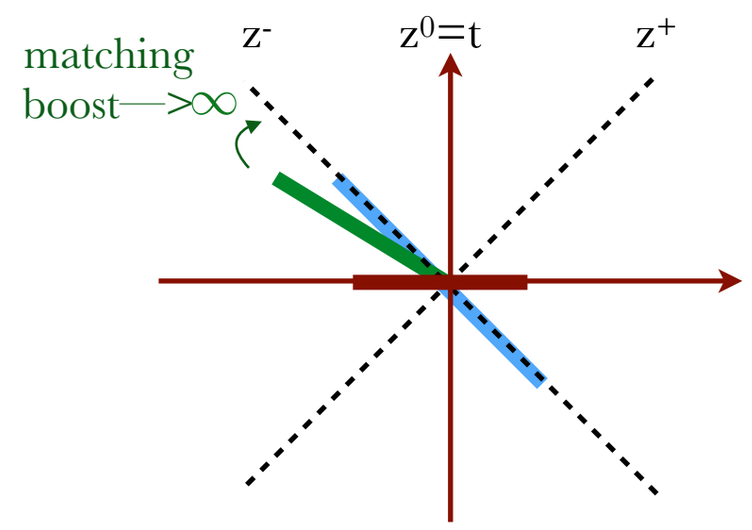


- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (large momentum effective theory - LaMET)

- Allow momentum transfer —> generalised parton distributions



Computation of quasi-PDFs

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

Renormalise non-perturbatively, $\mathcal{Z}(z,\mu)$

Need to eliminate both UV and exponential divergences

- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

Direct computation of PDFs (and GPDs)

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

Renormalise non-perturbatively, $\mathcal{Z}(z,\mu)$

Need to eliminate both UV and exponential divergences

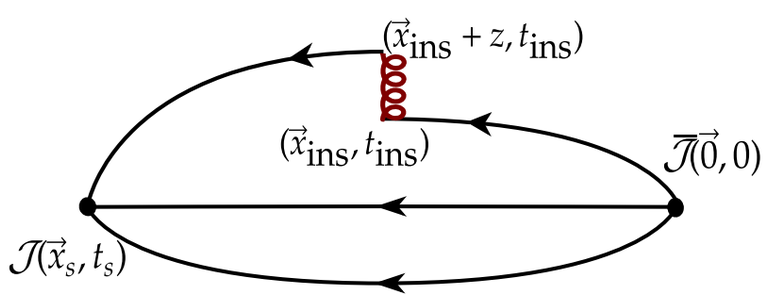
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y,\mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\rm QCD}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

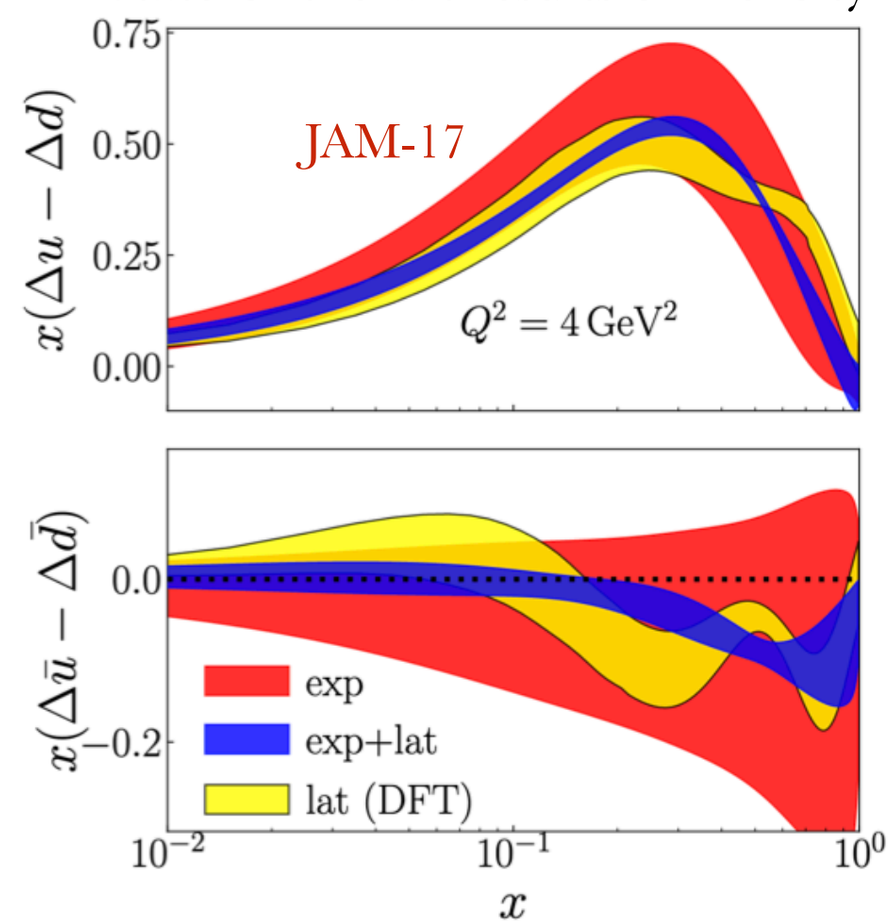
Isovector (u-d)



$\Gamma =$	γ_0	unpolarised
	$\gamma_5\gamma_3$	helicity
	$\sigma_{3i}, i = 1,2$	transversity

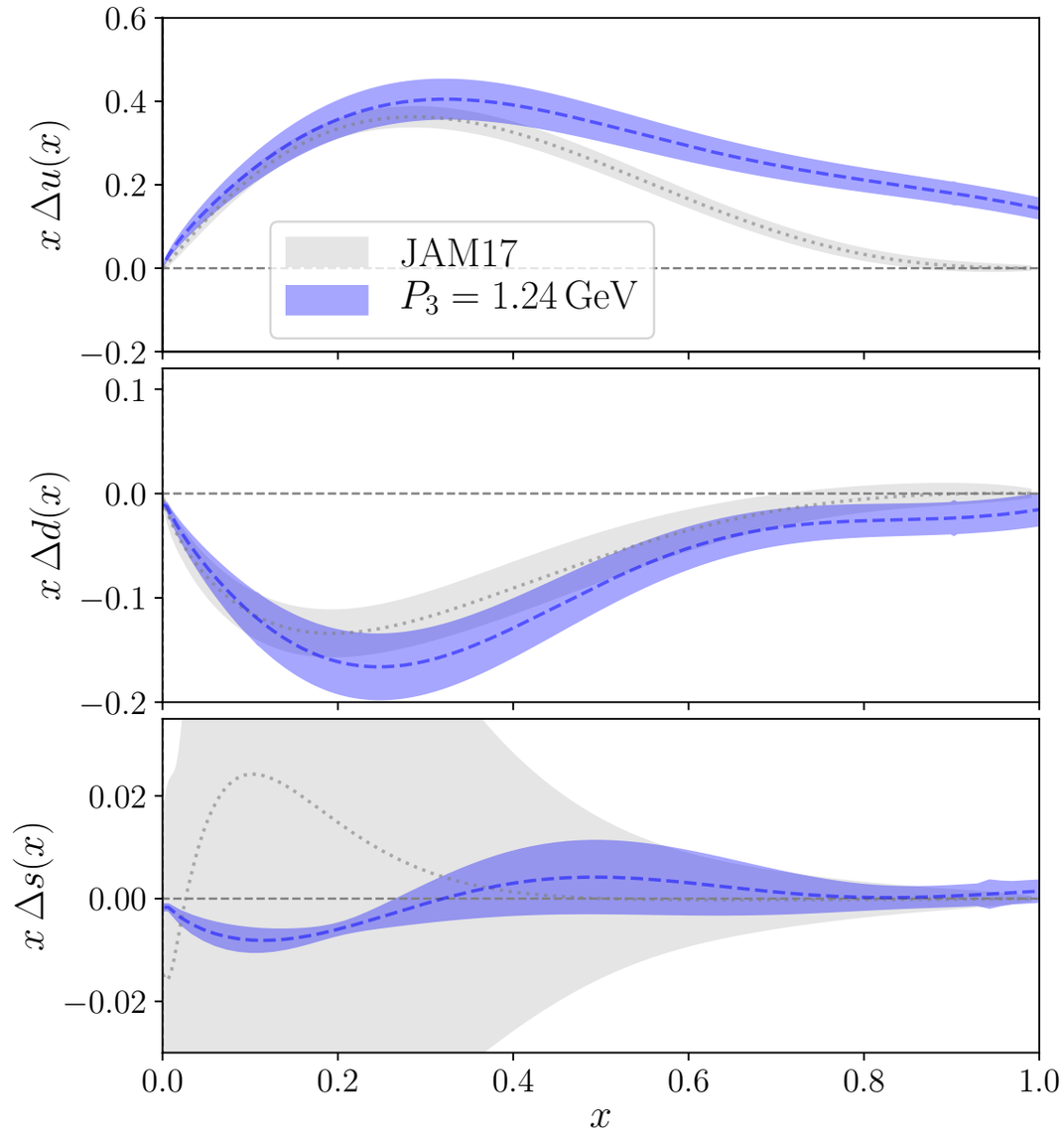
C.A. et al. (ETMC) Phys. Rev. Lett. **121**, 112001 (2018)

State-of-the-art results on helicity



Parton distribution functions can be computed directly in lattice QCD

Helicity distributions

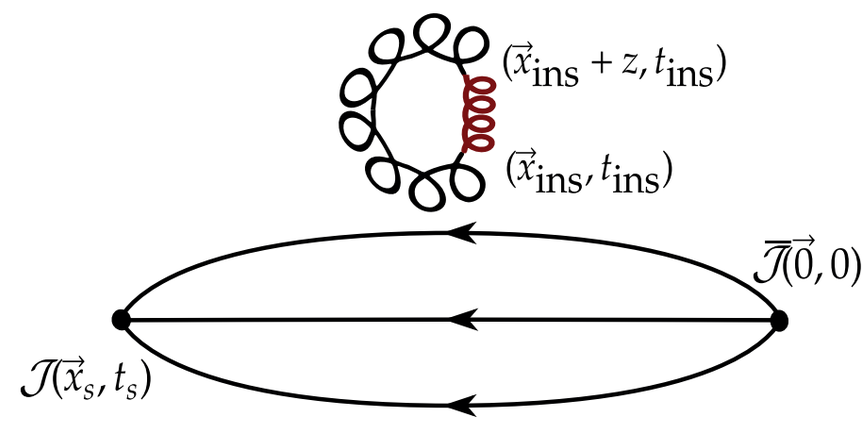


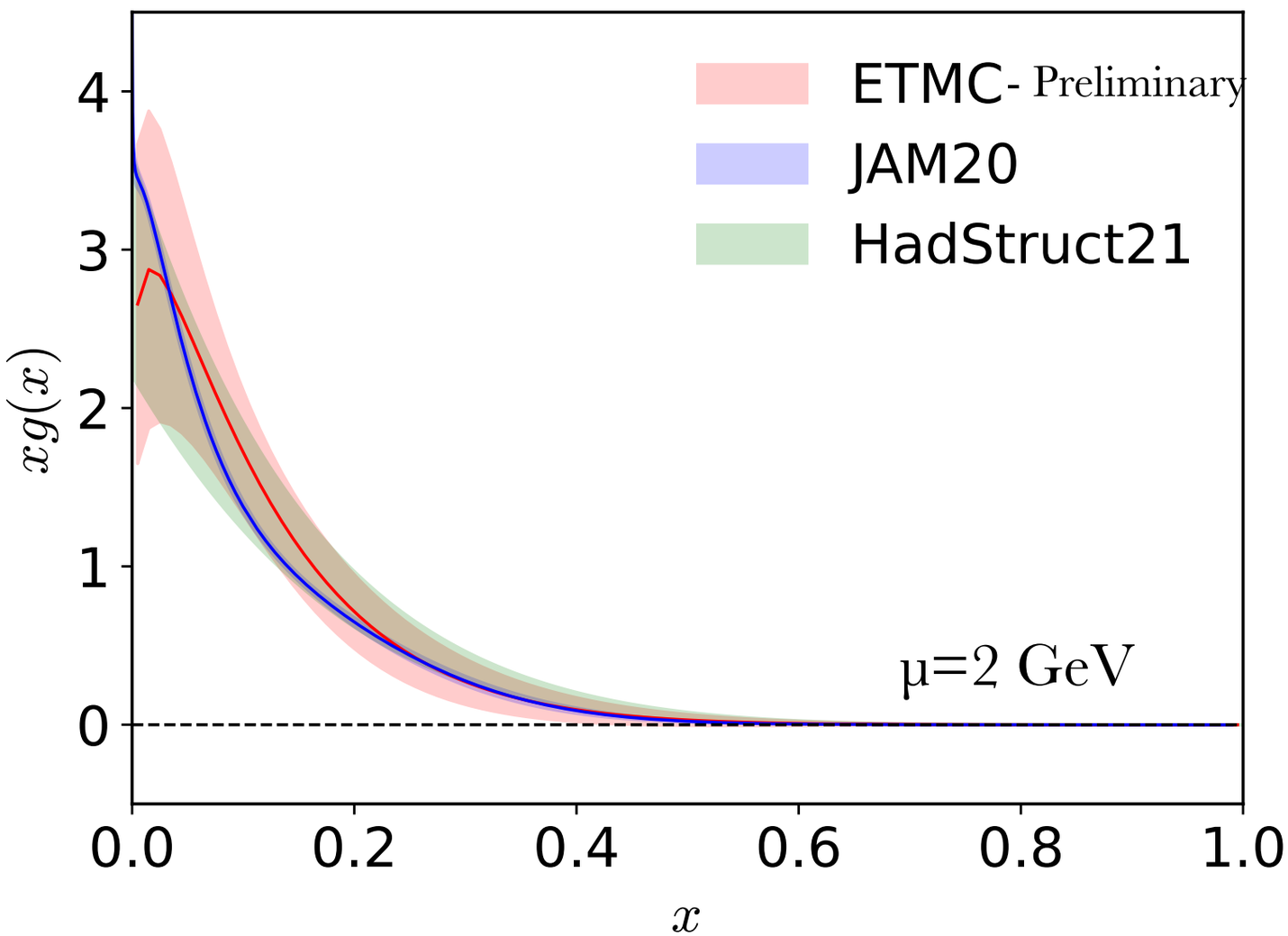
C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.13061
C.A., G. Iannelli, K. Jansen, F. Manigrasso, Phys. Rev. D 102 (2020) 9, 094508, arXiv:2007.13800

- Computation at the physical point is currently on-going

Unpolarized gluon PDF

- Calculate the matrix elements of a spin-averaged nucleon for two gluon fields connected by a Wilson line
- Use Wilson flow to reduce ultraviolet fluctuations
- Pseudo-PDF approach with pion mass 358 MeV





T. Khan, *et al.* (HadStruc Collaboration) Phys. Rev. D 101 (2021) 094516, 2107.08960

Conclusions

(1) Lattice QCD results produces known experimental values of e.g. nucleon axial charge, EM form factors, etc —> predict tensor charge, axial form factors

(2) Lattice QCD provides accurate results on second Mellin moments that probe the distribution of spin among the quarks and gluons

(3) Direct computation of PDFs, GPDs and TMDs providing a more complete picture of hadron structure is a very active field

Extended Twisted Mass Collaboration

Computational resources