Inference of the neutron star matter equation of state: Impact of new data

L. B., W. Weise and N. Kaiser, arXiv:2306.06218 (2023) [accepted by PRD]
L. B., W. Weise and N. Kaiser, Phys. Rev. D 107, 014001 (2023)

MENU conference 2023

Len Brandes

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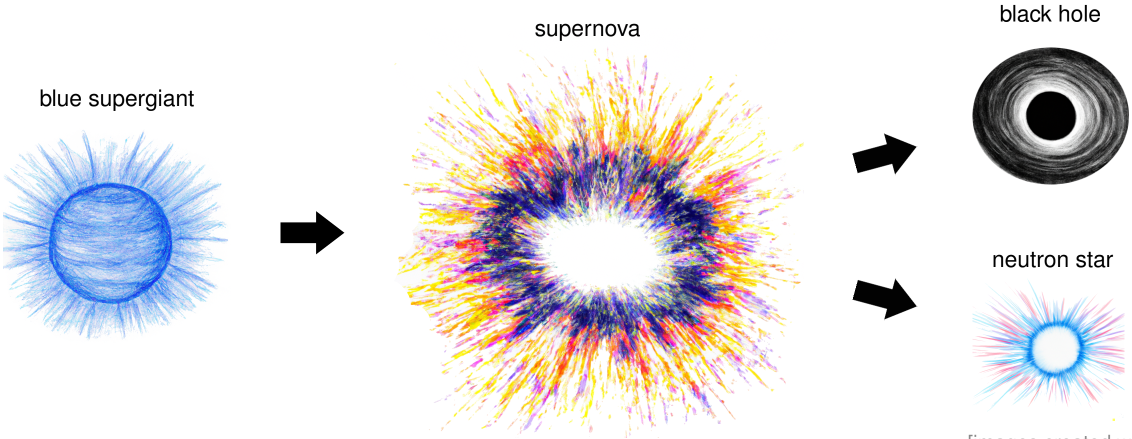
What are neutron stars?

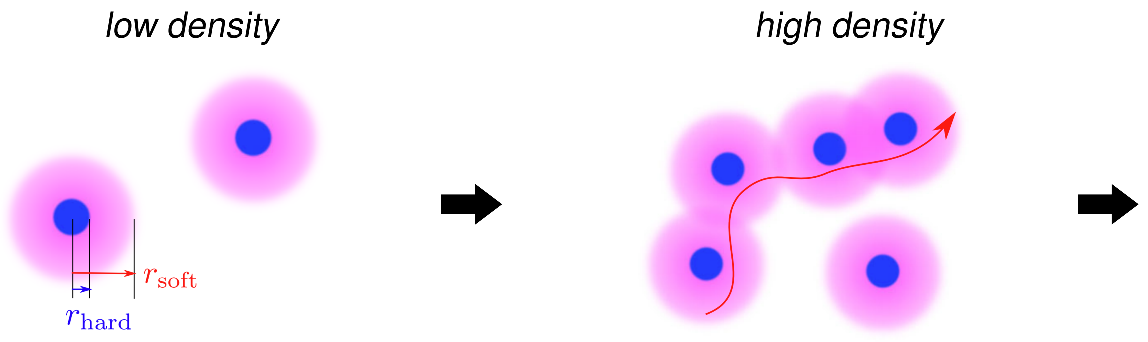


[images created with DALL·E, openAI]

- Stars remain stable by fusing light elements to heavier elements
- At some point no light elements left in core

  → Resulting implosion leads to supernova
- Collapsed core forms **neutron star**

Why are neutron stars interesting?



[Fukushima, Kojo and Weise, Phys. Rev. D 102 (2020)]

- Masses $M \sim 1 - 2\,M_\odot$, radii $R \sim 11 - 13\,\mathrm{km}$
- **High baryon densities** in core, beyond terrestrial experiments
- Transition from nucleonic to quark-gluon matter at high density?

  → Nature and location of possible **phase transition(s)**?

Description of neutron stars

- Internal structure described by Tolman-Oppenheimer-Volkoff (TOV) equations

$$\frac{\partial P(r)}{\partial r} = -\frac{G_N}{r^2}\big(\varepsilon(r)+P(r)\big)\big(m(r)+4\pi r^3 P(r)\big)\left(1-\frac{2G_N m(r)}{r}\right)^{-1},$$

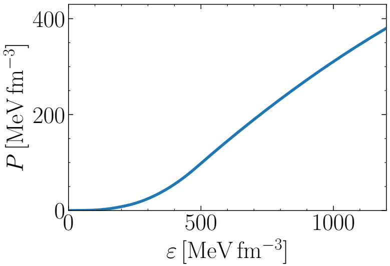
$$\frac{\partial m(r)}{\partial r} = 4\pi r^2 \varepsilon(r)$$

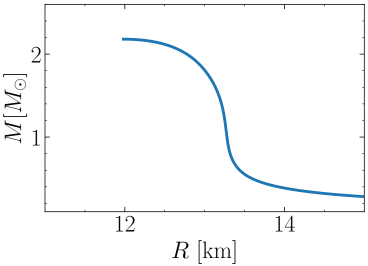
[Tolman, Phys. Rev. 55 (1939)] [Oppenheimer and Volkoff, Phys. Rev. 55 (1939)]

- Solved given **equation of state** (EoS) $P(\varepsilon)$ and central energy density $\varepsilon(r=0)=\varepsilon_c$

  $\rightarrow$ Solution for different ε_c yields (M,R)-relation

- Each EoS has maximum density $\varepsilon_{c,\max}$ corresponding to **maximum supported mass** M_{max}





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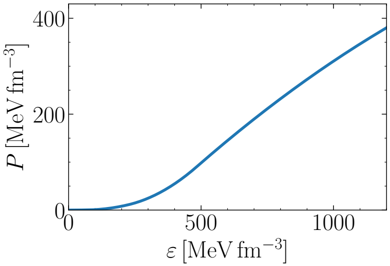
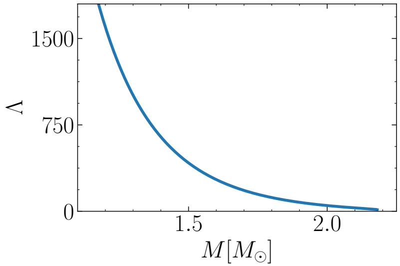
- Solved given **equation of state** (EoS) $P(\varepsilon)$ and central energy density $\varepsilon(r=0)=\varepsilon_c$
- Simultaneously solve for **tidal deformability** Λ

→ Relevant for neutron stars in binary systems





Speed of sound

- Determine EoS from **speed of sound**

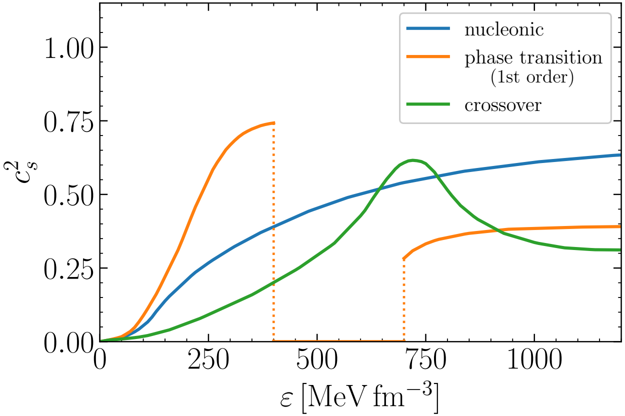
$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

- Causality & thermodynamic stability: $0 \le c_s \le 1$

- Measure of coupling strength in matter

$\rightarrow$ Characteristic signature of phase structure:

  - **Nucleonic**: monotonously rising sound speed
  - **First-order phase transition**: coexistence interval with zero sound speed $c_s^2 \sim 0$
  - **Crossover**: peaked behaviour



[McLerran and Reddy, Phys. Rev. Lett. 122 (2019)]

Parametrization

- Introduce **general parametrization** by segment-wise linear interpolations

$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}$$

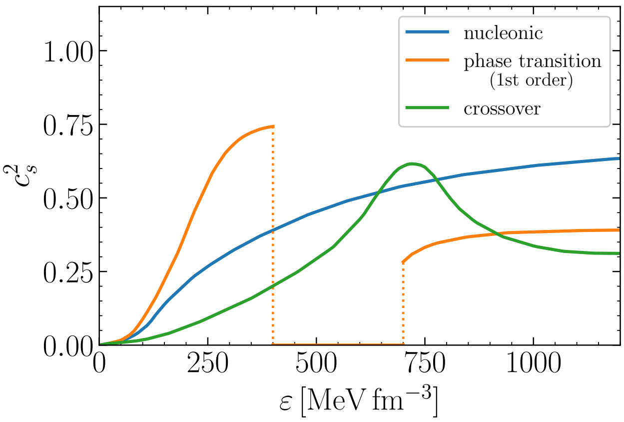
[Annala *et al.*, Nature Phys. 16, 907 (2020)]

- Can describe wide range of possible phase transitions and crossovers

- Previous analyses: similar results to non-parametric representations [Annala *et al.*, arXiv:2303.11356 (2023)]

- Constrain parameters $\theta = (\varepsilon_i, c_{s,i}^2)$ based on **available data**

→ Analyse for signatures of **possible phase transitions**



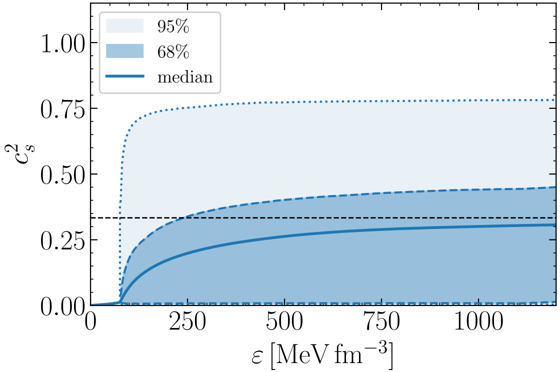
Bayesian inference

- Constrain parameters of $c_s^2(\varepsilon, \theta)$ via **Bayesian inference** based on data $\mathscr{D}$

$$\Pr(\theta|\mathscr{D}) \propto \Pr(\mathscr{D}|\theta)\,\Pr(\theta)$$

- Compute posterior probability $\Pr(\theta|\mathscr{D})$ for parameters θ:
  - Compute likelihood $\Pr(\mathscr{D}|\theta)$ for each data $\mathscr{D}$
  - Choose prior ranges for parameters $\Pr(\theta)$
- Compute median and **credible bands** at 68% or 95% level

→ Here: more prior support at small sound speeds to analyse phase transitions [LB, Weise and Kaiser, Phys. Rev. D 107 (2023)]



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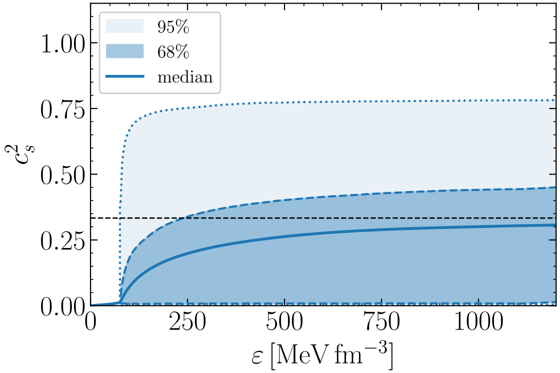


[LB, Weise and Kaiser, arXiv:2306.06218 (2023)]

- Quantify evidence for hypothesis H_0 vs. H_1 with **Bayes factors**

$$\mathscr{B}^{H_1}_{H_0} = \frac{\Pr(\mathscr{D}|H_1)}{\Pr(\mathscr{D}|H_0)}$$

→ Comparison to classification scheme for statistical conclusions

[Lee and Wagenmakers, *Bayesian Cognitive Modeling* (Cambridge University Press, 2014)]
[Jeffreys, *Theory of Probability* (Oxford University Press, 1961)]

Perturbative QCD

- Strong coupling decreases at **high densities**

  → Perturbative QCD calculations in terms of quark and gluon degrees of freedom

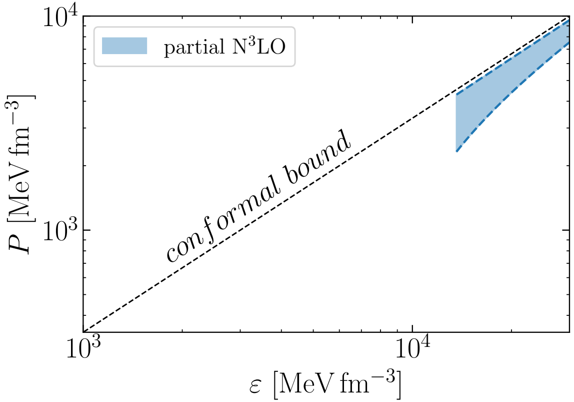
- Asymptotic boundary condition at $n \gtrsim 40\,n_0$ (with $n_0 = 0.16\,\mathrm{fm}^{-3}$)

- Speed of sound reaches **conformal limit** $c_s^2 = 1/3$ from below

  → Interpolation to asymptotic pQCD with $0 \leq c_s \leq 1$ constrains EoS at smaller densities

[Komoltsev and Kurkela, Phys. Rev. Lett. 128 (2022)]

- Exclude EoS where **matching** to asymptotic pQCD is not possible



[Gorda *et al.*, Phys. Rev. Lett. 127 (2021)]

Chiral effective field theory

- ChEFT: systematic expansion of nuclear forces at low momenta with controlled uncertainties

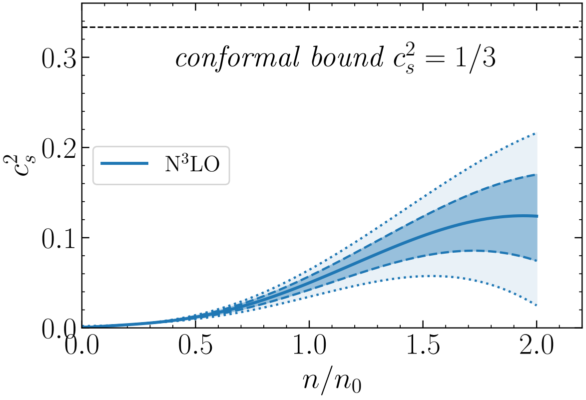
- Extended to finite densities using many-body methods

  → Good description of **low-density** nuclear phenomenology
  [Wellenhofer, Holt and Kaiser, Phys. Rev. C 92 (2015)]

- Combined uncertainties from many-body approximations and convergence errors [Drischler *et al.*, Phys. Rev. C 103 (2021)]

- $\mathrm{N}^3\mathrm{LO}$ sound speed at $n \sim 2\,n_0$ **softer** compared to $\mathrm{N}^2\mathrm{LO}$

  → Employ only up to $n \leq 1.3\,n_0$ [Essick *et al.*, Phys. Rev. C 102 (2020)]



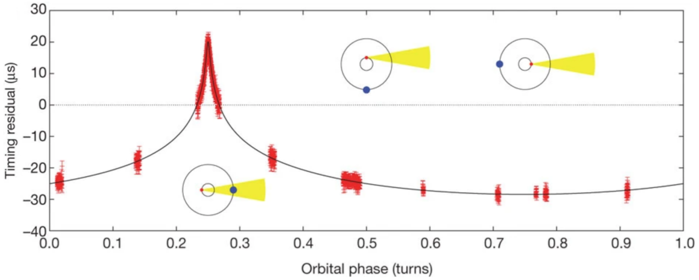
[Drischler, Han and Reddy, Phys. Rev. C 105 (2022)]

Shapiro time delays

- Neutron stars with strong magnetic fields emit synchroton radiation
- If magnetic and rotation axis not aligned, double cone of radiation rotates (→ pulsars)
- Binary systems: gravitational field of **companion changes pulsar signal**
- Extract **neutron star masses** with high precision (68% level):

PSR J0348+0432	$M = 2.01 \pm 0.04\, M_\odot$	[Antoniadis *et al.*, Science 340 (2013)]
PSR J0740+6620	$M = 2.08 \pm 0.07\, M_\odot$	[Fonseca *et al.*, Astrophys. J. Lett. 915 (2021)]

→ Matter must be sufficiently stiff to support such high masses



[Demorest *et al.*, Nature 467 (2010)]

Pulse profile modelling

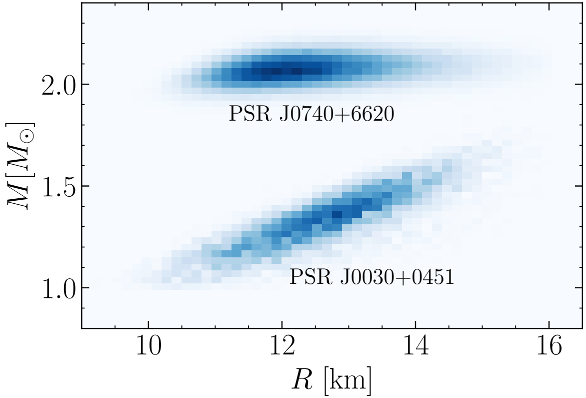
- **Hot spots** form on magnetic polar caps of rapidly rotating neutron stars

- Thermal X-ray emission modulated by gravitational effects

  → Measured by **NICER** telescope on ISS

- Model hot spots and neutron star atmosphere

  → Infer **mass and radius** from X-ray measurements (68% level):



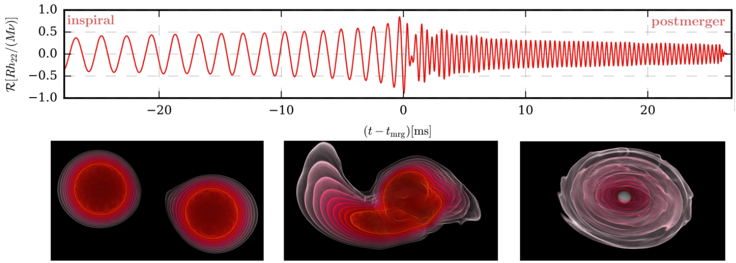
PSR J0030+0451	$R = 12.71^{+1.14}_{-1.19}\,\mathrm{km}$	$M = 1.34^{+0.15}_{-0.16}\,M_\odot$	[Riley *et al.*, Astrophys. J. Lett. 887 (2019)]
PSR J0740+6620	$R = 12.39^{+1.30}_{-0.98}\,\mathrm{km}$	$M = 2.072^{+0.067}_{-0.066}\,M_\odot$	[Riley *et al.*, Astrophys. J. Lett. 918 (2021)]

→ Very similar radii for 1.34 and 2.07 $M_\odot$ neutron stars

Neutron star mergers

- Binary neutron star mergers produce **gravitational waves**

- Compare observed LIGO and Virgo signal to waveform models



[Dietrich, Hinderer and Samajdar, Gen. Rel. Grav. 53 (2021)]

- Waveform depends on M_2/M_1 and combination of **tidal deformabilities**

$$\bar{\Lambda} = \frac{16}{13}\frac{(M_1 + 12M_2)M_1^4\Lambda_1 + (M_2 + 12M_1)M_2^4\Lambda_2}{(M_1 + M_2)^5}$$

- Two binary neutron star mergers detected so far (90% level):

GW170817 $\bar{\Lambda} = 320^{+420}_{-230}$ [Abbott *et al.* (LIGO and Virgo Collaborations), Phys. Rev. X 9 (2019)]

GW190425 $\bar{\Lambda} \leq 600$ [Abbott *et al.* (LIGO and Virgo Collaborations), Astrophys. J. Lett. 892 (2020)]

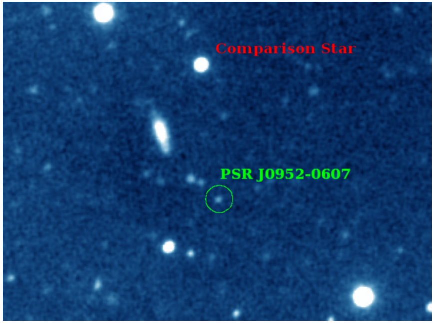
New data: black widow pulsar

- Black widow pulsars accrete most of mass from companion

  → Determine mass via observation of companion

- PSR J0952-0607 **heaviest neutron star** observed so far

  $M = 2.35 \pm 0.17\, M_\odot$ [Romani *et al.*, ApJL 934 (2022)]

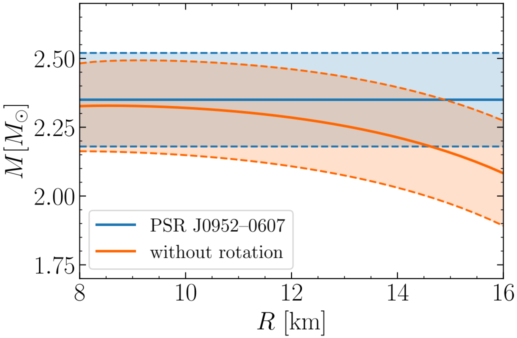


[W.M. Keck Observatory, Roger W. Romani, Alex Filippenko]

- Simpler heating model compared to other black widows

- Second fastest known pulsar $T = 1.41\,\mathrm{ms}$

  → **Rotation correction** via empirical formula
  [Konstantinou and Morsink, Astrophys. J. 934, 139 (2022)]

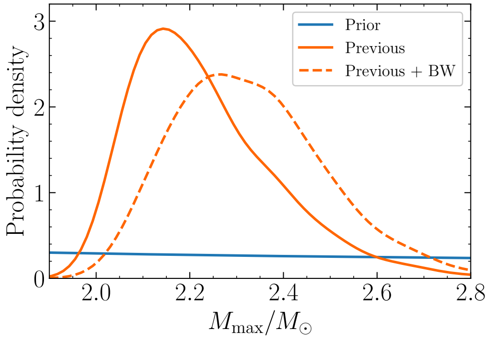


Impact of PSR J0952-0607

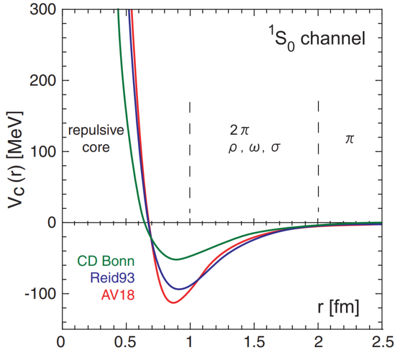
- Analyse impact of new **black widow (BW)** pulsar compared to **Previous** data
- Maximum mass increased (68%): $M_{\max} = 2.31^{+0.14}_{-0.17}\, M_\odot$
- **Central densities** of heavy neutron stars **reduced**:

$$n_c(2.1\,M_\odot) = 4.1^{+0.8}_{-0.9}\, n_0 \quad \rightarrow \quad n_c(2.1\,M_\odot) = 3.6 \pm 0.7\, n_0 \qquad n_c(2.3\,M_\odot) = 3.8^{+0.7}_{-0.8}\, n_0$$

→ Average distance between baryons: $d > 1.0\,\mathrm{fm} \gg r_{\text{hard-core}} \sim 0.5\,\mathrm{fm}$



[LB, Weise and Kaiser, arXiv:2306.06218 (2023)]

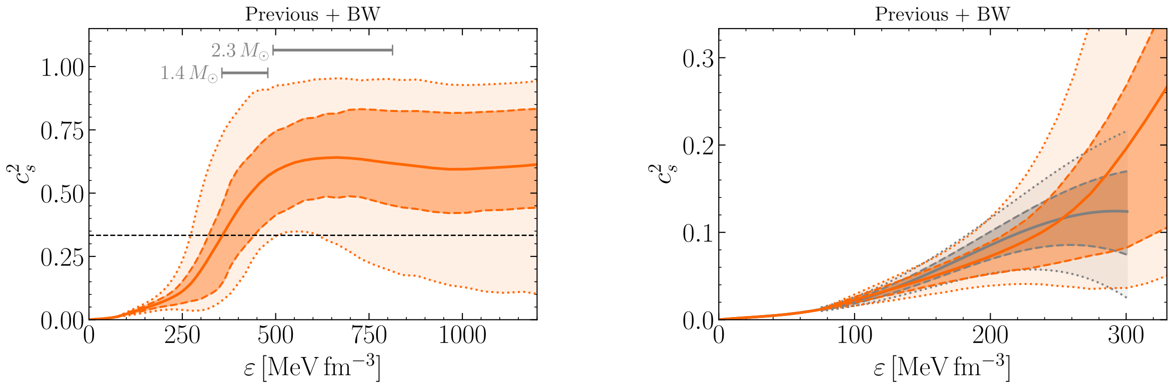


[Ishii, Aoki and Hatsuda, Phys. Rev. Lett. 99 (2007)]

Speed of sound

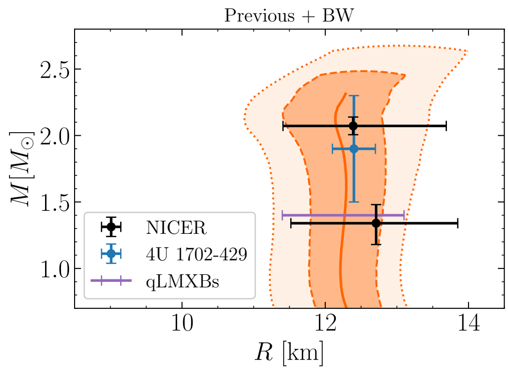
- Sound speed becomes **stiffer** by inclusion of heavy mass measurement
- Conformal bound $c_s^2 \leq 1/3$ exceeded inside neutron stars [Altiparmak, Ecker and Rezzolla, Astrophys. J. Lett. 939 (2022)] [Legred, Chatziioannou, Essick, Han and Landry, Phys. Rev. D 104 (2021)]

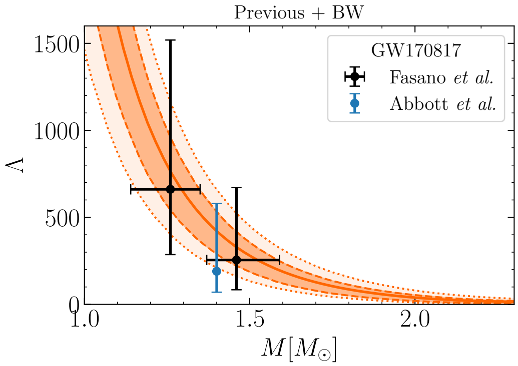
  → Strongly repulsive correlations at high densities
- Slight **tension** between ChEFT at $n \simeq 2\,n_0$ and astro data [Essick *et al.*, Phys. Rev. C 102 (2020)]



Mass-radius & tidal deformability

- Good agreement with data **not included** in Bayesian analysis:
  - Thermonuclear burster 4U 1702-429
    [Nättilä *et al.*, Astron. & Astrophys. 608 (2017)]
  - $R(M = 1.4\,M_\odot)$ from quiescent low mass X-ray binaries (qLMXBs)
    [Steiner *et al.*, Mon. Not. Roy. Astron. Soc. 476 (2018)]
- Median with **almost constant radius** $R \sim 12.3\,\mathrm{km}$
- Good agreement with other GW170817 analyses:
  - Masses and tidal deformabilities of two neutron stars
    [Fasano, Abdelsalhin, Maselli, and Ferrari, Phys. Rev. Lett. 123 (2019)]
  - $\Lambda(M = 1.4\,M_\odot)$ from universal relations
    [Abbott *et al.* (LIGO and Virgo Collaborations), Phys. Rev. Lett. 121 (2018)]





Pressure & coexistence interval

- Compare pressure credible bands to APR EoS
  [Akmal, Pandharipande and Ravenhall, Phys. Rev. C 58 (1998)]

- Maxwell construction of first-order phase transition: constant pressure in **coexistence interval**

  → Width Δn measure of phase transition 'strength'

- **Maximum possible interval** within posterior credible band

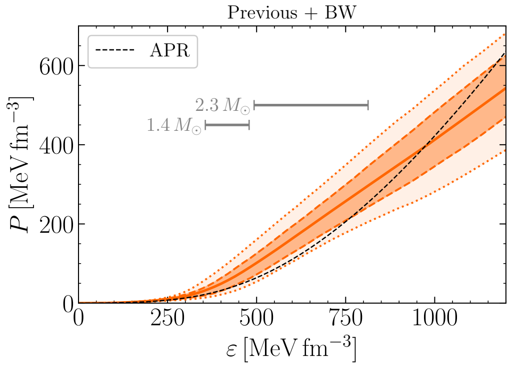
$$\left(\frac{\Delta n}{n}\right)_{\max} \leq 0.2 \qquad \text{at 68\% level}$$

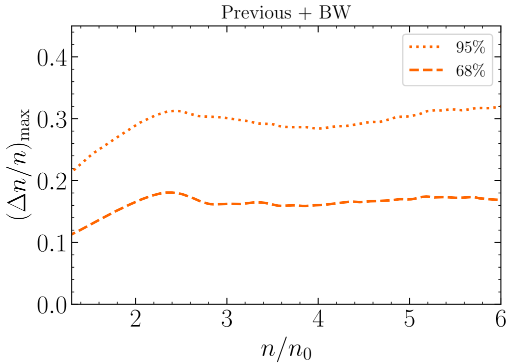
- Compare to 'strong' nuclear liquid-gas phase transition

$$\frac{\Delta n}{n} > 1$$

[Fiorilla, Kaiser and Weise, Nucl. Phys. A 880 (2012)]

→ Only **weak first-order phase transitions** possible





Small sound speeds

- Quantify **evidence of small sound speeds** inside neutron star cores with Bayes factor

$$\mathscr{B}^{c^2_{s,\mathrm{min}}>0.1}_{c^2_{s,\mathrm{min}}\leq 0.1}$$

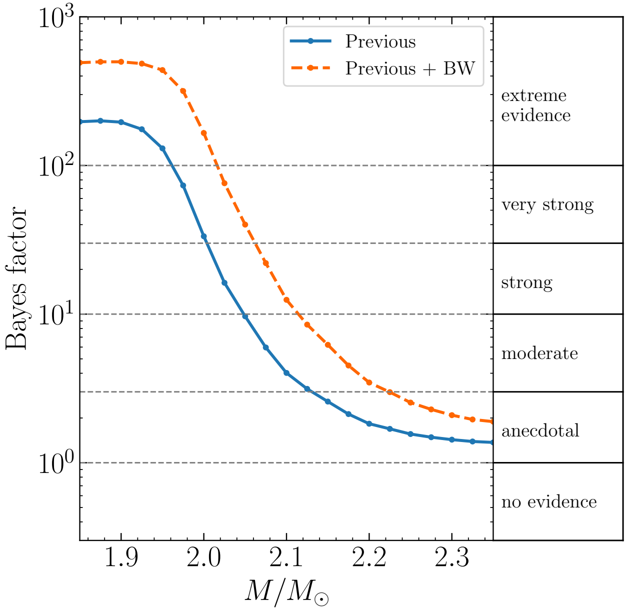
→ $c^2_{s,\mathrm{min}} \leq 0.1$ perquisite for first-order phase transition

- Previous analyses: $c_s^2 > 0.1$ in neutron stars with $M = 2\,M_\odot$
[Ecker and Rezzolla, Astrophys. J. Lett. 939 (2022)] [Annala *et al.*, arXiv:2303.11356 (2023)]

- **Heavy mass measurement** increases Bayes factor

- Strong evidence against $c^2_{s,\mathrm{min}} \leq 0.1$ in cores of neutron stars with $M \leq 2.1\,M_\odot$

→ **Strong first-order phase transitions unlikely**



Possible impact of HESS J1731-347

- Central compact object within supernova remnant HESS J1731-347:

$$M = 0.77^{+0.20}_{-0.17}\,M_\odot$$
$$R = 10.4^{+0.86}_{-0.78}\,\mathrm{km}$$

[Doroshenko *et al.*, Nat. Astron. 6 (2022)]

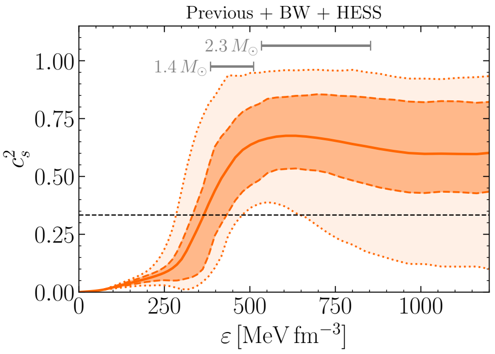
- Unusually **light neutron star** with very low radius

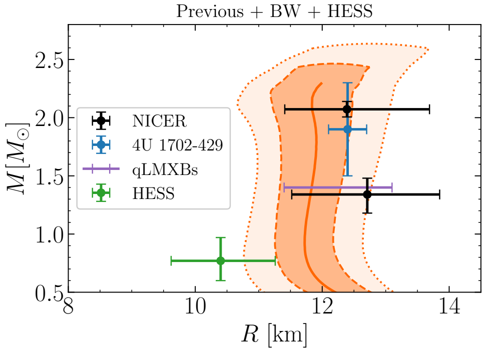
  → Neutron star mass $M < 1.17\,M_\odot$ in contradiction with known formation mechanisms [Suwa *et al.*, MNRAS 481 (2018)]

  → Strange star?

- Systematic uncertainty: larger masses and radii might be possible
  [Alford and Halpern, Astrophys. J. 944 (2023)]

- **Tension** between HESS and current astrophysical data
  [Jiang, Ecker and Rezzolla, arXiv:2211.00018 (2022)]





Summary

- **Bayesian inference** of sound speed in neutron star matter based on:
  - Shapiro time-delays
  - NICER X-ray measurements
  - Gravitational waves from binary neutron star mergers
  - ChEFT results at small densities
  - Perturbative QCD calculations at asymptotically high densities
  - (New) black widow pulsar $M = 2.35 \pm 0.17\, M_\odot$
- **Black widow** heavy mass measurement further stiffens EoS

  → Central densities of neutron stars reduced: $n_c < 5\, n_0$ for $M \leq 2.3\, M_\odot$

  → Slight tension with ChEFT at $n \sim 2.0\, n_0$
- Maximum possible **coexistence interval** $(\Delta n/n)_{\max} \leq 0.2$

  → Only weak phase transitions possible within posterior credible band
- Strong evidence against $c^2_{s,\min} \leq 0.1$ in cores of neutron stars with $M \leq 2.1\, M_\odot$

  → **Strong first-order phase transitions unlikely**

Supplementary material

Outlook

- Fourth observation run of LIGO, Virgo and KAGRA started on May 4th
- Four more objects set to be measured by NICER telescope [Greif *et al.*, MNRAS 485 (2019)]
- Moment-of-inertia measurement of PSR J0737-3039 in next few years [Landry and Kumar, Astrophys. J. 868 (2018)]

→ **Many more future measurements** will put even tighter constraints on $c_s^2(\varepsilon)$

→ More information about high density phase structure

nature

NEWS FEATURE | 04 March 2020

The golden age of neutron-star physics has arrived

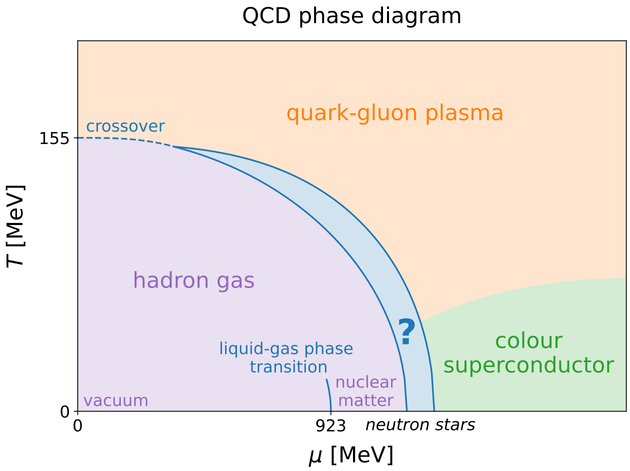
These stellar remnants are some of the Universe's most enigmatic objects – and they are finally starting to give up their secrets.

Adam Mann

Phases of strongly interacting matter

- Crossover at $T \sim 155\,\mathrm{MeV}$ from lattice QCD and heavy ion collisions [Bazavov *et al.*, Phys. Rev. D 90 (2014)] [Andronic, Braun-Munzinger, Redlich and Stachel, Nature 561 (2018)]
- **Sign problem** at large $\mu \rightarrow$ no lattice QCD
- Nuclear liquid-gas phase transition at $\mu = 923\,\mathrm{MeV}$ [Elliott, Lake, Moretto and Phair, Phys. Rev. C 87 (2013)]
- Colour superconducting phase at asymptotic densities [Schäfer and Wilczek, Phys. Rev. D 60 (1999)]
- **Unknown transition** from nuclear to quark matter

$\rightarrow$ High densities and low temperatures realized in neutron stars



Phases of strongly interacting matter

- Effective models find **first-order phase transition**
  [Buballa, Phys. Rep. 407 (2005)]

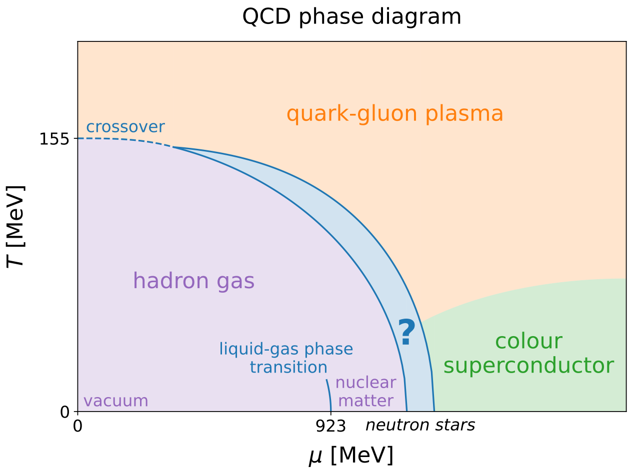
  → Fluctuations beyond mean-field important
  [Brandes, Kaiser and Weise, Eur. Phys. J. A 57 (2021)]

- **Continuous crossover** from hadronic to quark matter [Baym *et al.*, Rep. Prog. Phys. 81 (2018)]

  → Intermediate phase of quarkyonic matter?
  [Fukushima and Hatsuda, Rep. Prog. Phys. 74 (2011)]

- Hyperons energetically favourable at high densities?

  → Might be inhibited by repulsive three-body forces
  [Gerstung, Kaiser and Weise, Eur. Phys. J. A 56 (2020)]



General EoS parametrization

- Determine EoS from **speed of sound**

$$c_s^2(\varepsilon) = \frac{\partial P(\varepsilon)}{\partial \varepsilon}$$

- Parametrize by **segment-wise linear interpolations**

$$c_s^2(\varepsilon, \theta) = \frac{(\varepsilon_{i+1} - \varepsilon) c_{s,i}^2 + (\varepsilon - \varepsilon_i) c_{s,i+1}^2}{\varepsilon_{i+1} - \varepsilon_i}$$

[Annala *et al.*, Nature Phys. 16, 907 (2020)]

- Matching to BPS crust at low densities $(c_{s,0}^2, \varepsilon_0) = (c_{s,\mathrm{crust}}^2, \varepsilon_{\mathrm{crust}})$ [G. Baym, C. Pethick, and P. Sutherland, Astrophys. J. 170 (1971)]
- Constant speed of sound $c_s^2(\varepsilon, \theta) = c_{s,N}^2$ beyond last point $\varepsilon > \varepsilon_N$
- Choose $N = 5$ corresponding to 7 segments and 10 free parameters
- **Priors** sampled logarithmically

$$c_{s,i}^2 \in [0, 1] \quad \varepsilon_i \in [\varepsilon_{crust}, 4\,\mathrm{GeV\,fm^{-3}}]$$

- Parametrizations with only 4 segments leads to comparable results as non-parametric Gaussian process

[Annala *et al.*, arXiv:2303.11356 (2023)]

Bayesian inference

- Bayes theorem:

$$\text{Pr}(\theta|\mathscr{D},\mathscr{M}) = \frac{\text{Pr}(\mathscr{D}|\theta,\mathscr{M})\,\text{Pr}(\theta|\mathscr{M})}{\text{Pr}(\mathscr{D}|\mathscr{M})}$$

- Choose **Priors** for parameters $\text{Pr}(\theta|\mathscr{M})$

- **Likelihood** $\text{Pr}(\mathscr{D}|\theta,\mathscr{M})$: probability of data $\mathscr{D}$ to occur for θ and model $\mathscr{M}$

- (M,R,Λ) can be deterministically determined for θ

$$\text{Pr}(\mathscr{D}|\theta,\mathscr{M}) = \text{Pr}(\mathscr{D}|M,R,\Lambda,\mathscr{M})$$

→ For computational feasibility assume (valid for flat Priors in (M,R,Λ))

$$\text{Pr}(\mathscr{D}|M,R,\Lambda,\mathscr{M}) \propto \text{Pr}(M,R,\Lambda|\mathscr{D},\mathscr{M})$$

[Riley, Raaijmakers and Watts, Mon. Not. Roy. Astron. Soc. 478 (2018)] [Raaijmakers *et al.*, ApJL 918 (2021)]

Bayesian inference

- Bayes theorem:

$$\Pr(\theta|\mathscr{D},\mathscr{M}) = \frac{\Pr(\mathscr{D}|\theta,\mathscr{M})\Pr(\theta|\mathscr{M})}{\Pr(\mathscr{D}|\mathscr{M})}$$

- **Evidence** $\Pr(\mathscr{D}|\mathscr{M})$: determined via normalization of the posterior

$$\Pr(\mathscr{D}|\mathscr{M}) = \int d\theta\ \Pr(\mathscr{D}|\theta,\mathscr{M})\Pr(\theta|\mathscr{M})$$

→ High-dimensional integral, use sampling techniques

- **Credible bands**: determine $P(\varepsilon_i,\theta)$ on grid $\{\varepsilon_i\}$ for posterior samples to get $\Pr\left(P\middle|\varepsilon_i,\mathscr{D},\mathscr{M}\right)$

→ Compute credible interval $[a,b]$ with probability α at ε_i

$$\alpha = \int_a^b dP\ \Pr\left(P\middle|\varepsilon_i,\mathscr{D},\mathscr{M}\right)$$

→ Combine credible intervals at all ε_i to posterior credible band $P(\varepsilon)$

Trace anomaly measure

- Trace anomaly measure as signature of **conformality**

$$\Delta = \frac{g_{\mu\nu} T^{\mu\nu}}{3\varepsilon} = \frac{1}{3} - \frac{P}{\varepsilon}$$

[Fujimoto, Fukushima, McLerran and Praszałowicz, Phys. Rev. Lett. 129 (2022)]

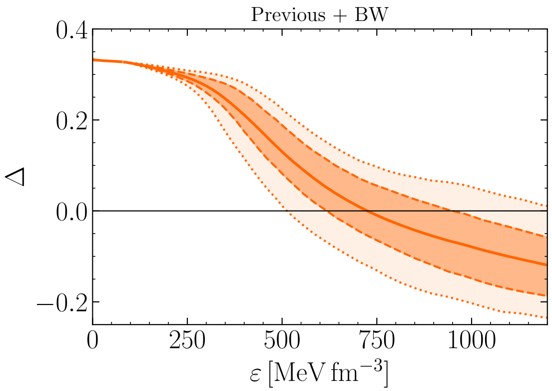
- Median becomes negative around $\varepsilon \sim 700\,\mathrm{MeV\,fm^{-3}}$

→ Moderate evidence for Δ turning **negative** inside neutron stars

Bayes factor $\mathscr{B}^{\Delta<0}_{\Delta\geq 0} = 8.11$

[Ecker and Rezzolla, Astrophys. J. Lett. 939 (2022)] [Annala *et al.*, arXiv:2303.11356 (2023)]
[Marczenko, McLerran, Redlich and Sasaki, Phys. Rev. C 107 (2023)]

- At higher energy densities again positive Δ to reach asymptotic pQCD limit



Twin stars

- Strong phase transitions can lead to mass-radius relations with **multiple stable branches** ('twin stars')

- Bayes factor gives **extreme evidence** against multiple stable branches
[Gorda *et al.*, arXiv:2212.10576 (2022)]

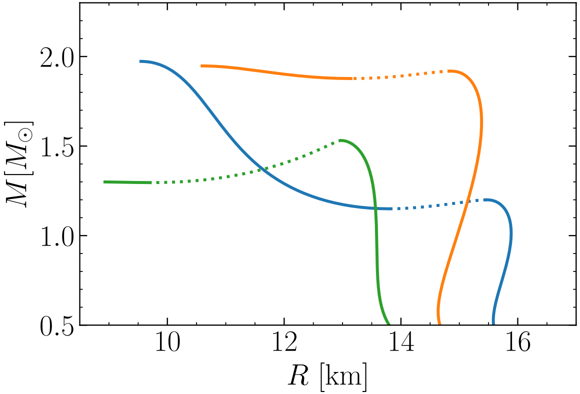
- Without likelihood from **ChEFT** 'only' strong evidence:

$$\mathscr{B}^{N_{\text{branches}}=1}_{N_{\text{branches}}>1} = 12.97$$

[Essick, Legred, Chatziioannou, Han and Landry, arXiv:2305.07411 (2023)]

- Disconnection takes place at $M \sim 0.8\,M_\odot$

→ Unlikely based on nuclear phenomenology



Impact of pQCD

- Matching to pQCD at $n_{c,\mathrm{max}}$ has only **negligible impact**

  [Somasundaram, Tews and Margueron, arXiv:2204.14039 (2022)]

- **Change matching** to asymptotic pQCD from $n_{c,max}$ to $10\,n_0$

  → Much smaller c_s^2 at high energy densities

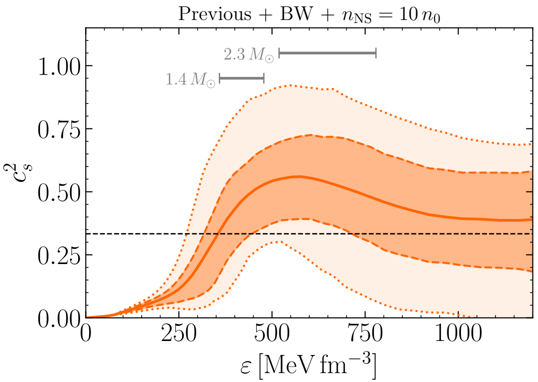
  [Gorda, Komoltsev, and Kurkela, arXiv:2204.11877 (2022)]
  [Komoltsev and Kurkela, Phys. Rev. Lett. 128 (2022)]

  → Few changes in mass-radius, properties of $2.3\,M_\odot$ neutron star change only slightly

- EoS beyond $n_{c,\mathrm{max}}$ no longer constrained by astrophysical data

  → Impact depends **unconstrained interpolation** to high densities

  [Essick, Legred, Chatziioannou, Han and Landry, arXiv:2305.07411 (2023)]



Perturbative QCD

- Connection of $(\mu_{\rm NS}, n_{\rm NS}, P_{\rm NS})(\theta)$ to $(\mu_{\rm pQCD}, n_{\rm pQCD}, P_{\rm pQCD})$

$$\int_{\mu_{\rm NS}}^{\mu_{\rm pQCD}} d\mu \; n(\mu) = P_{\rm pQCD} - P_{\rm NS} = \Delta P$$

- Causality and thermodynamic stability imply minimum and maximum values

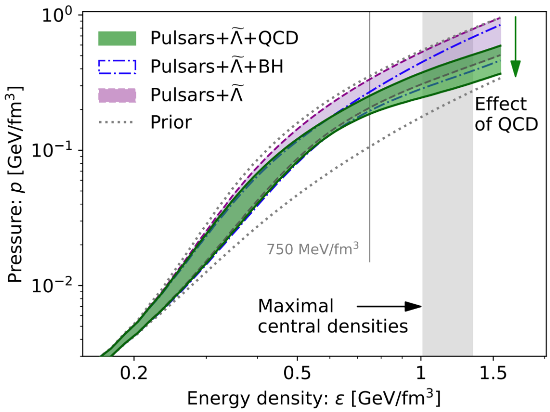
$$\Delta P_{\min} = \frac{\mu_{\rm pQCD}^2 - \mu_{\rm NS}^2}{2\mu_{\rm NS}} n_{\rm NS} \qquad \Delta P_{\max} = \frac{\mu_{\rm pQCD}^2 - \mu_{\rm NS}^2}{2\mu_{\rm pQCD}} n_{\rm pQCD}$$

- **Likelihood**

$$\Pr\left(\mathscr{D}_{\rm pQCD} \middle| \Delta P(\theta), \mathscr{M}\right) = \begin{cases} 1 & \text{if } \Delta P(\theta) \in [\Delta P_{\min}(\theta), \Delta P_{\max}(\theta)] \\ 0 & \text{else} \end{cases}$$

→ Take logarithmic average over renormalisation scale $X \in [1/2, 2]$

[Komoltsev and Kurkela, Phys. Rev. Lett. 128 (2022)]



[Gorda, Komoltsev, and Kurkela, arXiv:2204.11877 (2022)]

Likelihoods

- EoS supports masses between $M_{\rm min}$ and $M_{\rm max}(\theta)$
- Choose flat **mass prior** and $M_{\rm min} = 0.5\,M_\odot$

$$\mathrm{Pr}(M(\theta)) = \begin{cases} \frac{1}{M_{\rm max}(\theta) - M_{\rm min}} & \text{if } M \in [M_{\rm min}, M_{\rm max}(\theta)] \\ 0 & \text{else} \end{cases}$$

[Landry, Essick and Chatziioannou, Phys. Rev D 101 (2020)]

- When number of data increases incorporate mass population

→ Wrong population model causes a bias [Mandel, Farr and Gair, Mon. Not. Roy. Astron. Soc. 486 (2019)]

- Assume **Shapiro** mass measurements Gaussian to compute likelihood

$$\begin{aligned}\mathrm{Pr}\left(M(\theta) \middle| \mathscr{D}_{\text{Shapiro}}, \mathscr{M}\right) &= \int_{M_{\rm min}}^{M_{\rm max}(\theta)} \mathrm{d}M \ \mathscr{N}\left(M, \langle M \rangle, \sigma_M\right) \mathrm{Pr}(M(\theta)) \\ &\approx \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{M_{\rm max}(\theta) - \langle M \rangle}{\sqrt{2}\sigma_M}\right)\right] \mathrm{Pr}(M(\theta))\end{aligned}$$

Likelihoods

- Data available as samples, approximate underlying probability with Kernel Density Estimation (KDE)
- Solve TOV equations to obtain $R(M, \theta)$ and $\Lambda(M, \theta)$
- **NICER likelihood**:

$$\mathrm{Pr}\big((M,R)(\theta)\big|\mathcal{D}_{\mathrm{NICER}},\mathcal{M}\big) = \int_{M_{\min}}^{M_{\max}(\theta)} \mathrm{d}M\ \mathrm{KDE}\big(M, R(M,\theta)\big)\mathrm{Pr}(M(\theta))$$

- **GW likelihood**:

$$\mathrm{Pr}\big((M,\Lambda)(\theta)\big|\mathcal{D}_{\mathrm{GW}},\mathcal{M}\big) = \int \mathrm{d}M_1 \int \mathrm{d}M_2\ \mathrm{KDE}\big(M_1, M_2, \Lambda(M_1,\theta), \Lambda(M_2,\theta)\big)$$

- Do not assume neutron star-neutron star merger events

  → GW likelihood not weighted by mass prior and $\Lambda(M) = 0$ for black holes

- Do not assumed fixed chirp mass $M_{\mathrm{chirp}} = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}$

Conformal limit

- Derived from naive dimensional analysis and asymptotic limit

$$\mu \gg \Lambda_{\text{QCD}} \qquad \Longrightarrow \qquad \begin{aligned} &P \propto \mu^{d+1} \\ &c_s^2 = \frac{\partial P}{\partial \varepsilon} \sim \frac{1}{d} \end{aligned}$$

[Hippert, Fraga and Noronha, Phys. Rev. D 104 (2021)]

- Expected to hold in all **conformal field theories** [Bedaque and Steiner, Phys. Rev. Lett. 114 (2015)]

- Recent Bayesian analyses found speeds of sound $c_s^2 > 1/3$ inside neutron stars

[Landry, Essick and Chatziioannou, Phys. Rev. D 101 (2020)] [Gorda, Komoltsev, and Kurkela, arXiv:2204.11877 (2022)]
[Altiparmak, Ecker, and Rezzolla, arXiv:2203.14974 (2022)] [Leonhardt *et al.*, Phys. Rev. Lett. 125 (2020)]

→ Also $c_s^2 > 1/3$ in recent $N_C = 2$ **lattice QCD** [Iida and Itou, PTEP 2022 (2022)]

- Hard Dense Loop resummation methods: conformal limit may be approached asymptotically from above

[Fujimoto and Fukushima, Phys. Rev. D 105 (2022)]

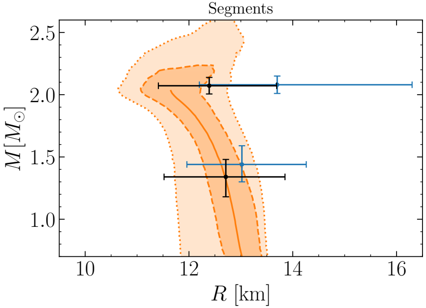
Parametrization dependence

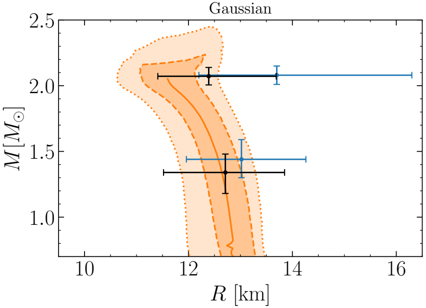
- 'Old' segment-wise parametrisation: different ChEFT constraint, $c_s^2 = 1/3$ reached asymptotically from below
- Compared to skewed Gaussian plus logistic function to reach asymptotic limit $c_s^2 = 1/3$

$$c_s^2(x,\theta) = a_1 \exp\left[-\frac{1}{2}\frac{(x-a_2)^2}{a_3^2}\right]\left(1+\mathrm{erf}\left[\frac{a_6}{\sqrt{2}}\frac{x-a_2}{a_3}\right]\right) + \frac{1/3 - a_7}{1+\exp\left[-a_5(x-a_4)\right]} + a_7$$

[Greif *et al*,, MNRAS 485, 5363 (2019)] [Tews, Margueron and Reddy, EPJA 55, 97 (2019)]

- Very similar findings, results **robust against change** of parametrization and Prior





[LB, Weise and Kaiser, Phys. Rev. D 107 (2023)]

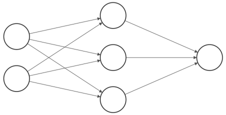
Machine learning

- Sparse amount of data: Bayesian results depend on prior choices and implicit assumptions

  → **Complimentary** machine learning analysis

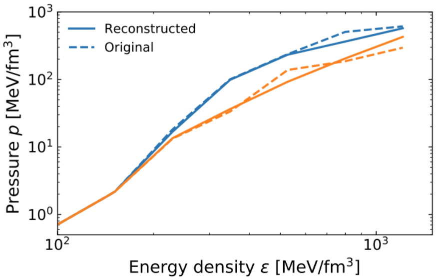
- Train neural network to learn EoS from measurements based on **simulated data**

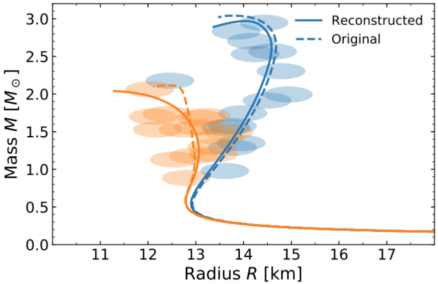
$$\begin{pmatrix} M \\ \Delta M \\ R \\ \Delta R \end{pmatrix}$$



θ (EoS parameters)

- Limitations: observations assumed to be gaussian, simple EoS model, only mass-radius data





[Fujimoto, Fukushima and Murase, Phys. Rev. D 101 (2020)]

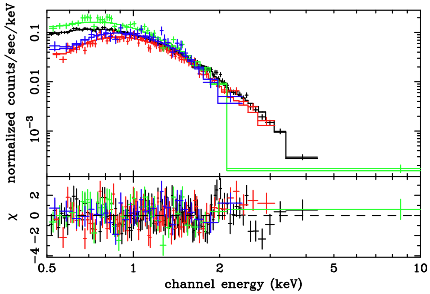
Simulation based inference

- Train neural density estimator (normalizing flow) to **approximate likelihood**

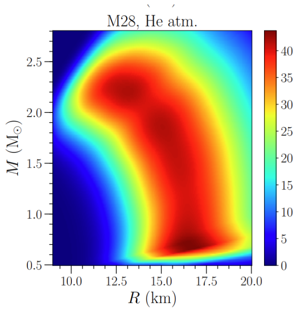
$$q_\phi(\mathscr{D}|\theta) \approx \mathrm{Pr}(\mathscr{D}|\theta)$$

[Papamakarios, Sterratt and Murray, AISTAT (2019)]

1. Sample θ from prior $\mathrm{Pr}(\theta)$
2. Solve TOV equations for $R(M)$, sample M and determine $R = R(M)$
3. **Simulate spectrum** x for given (M, R)
4. Train neural **density estimator** $q_\phi(x|\theta)$ with (x, θ)
5. Insert real data x_0 into $q_\phi(x_0|\theta) \approx \mathrm{Pr}(x_0|\theta)$ and combine with likelihoods from other sources



[Heinke, Rybicki, Narayan and Grindlay, Astrophys. J. 644 (2006)]



[Steiner *et al.*, Mon. Not. Roy. Astron. Soc. 476 (2018)]

Likelihood $\mathrm{Pr}(\mathscr{D}|\theta)$

Simulation based inference

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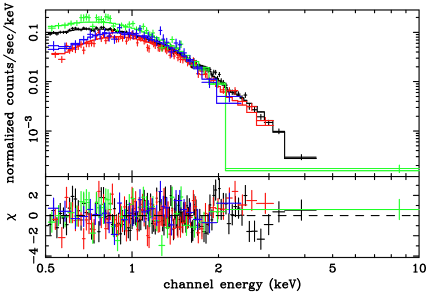
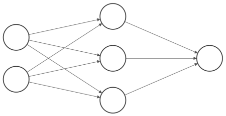
EoS parameters θ



[Heinke, Rybicki, Narayan and Grindlay, Astrophys. J. 644 (2006)]

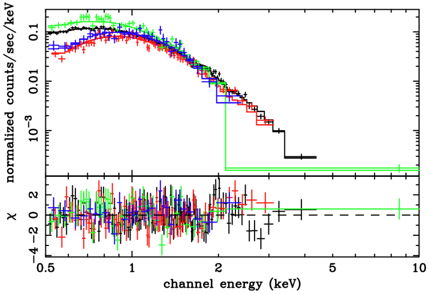


Likelihood $q_\phi(\mathscr{D}|\theta)$

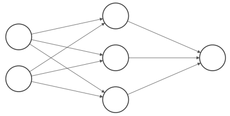
Simulation based inference

- Similar approach already used for inference of gravitational waves [Dax *et al.*, Phys. Rev. Lett. 127 (2021)]
- Easy to implement with `sbi` package [Tejero-Cantero *et al.*, Journal of Open Source Software (2020)]
- Infer likelihood directly from **detector output** [Farrell et al., arXiv:2305.07442 (2023)]
- Learn likelihood instead of posterior, possible to combine with other data
- **Amortization**: easy to include new measurements, analyse impact of future measurements
- Might be necessary to first learn a simpler model

EoS parameters θ



[Heinke, Rybicki, Narayan and Grindlay, Astrophys. J. 644 (2006)]



Likelihood $q_\phi(\mathscr{D}|\theta)$

Neural Likelihood Estimation

- **Sample** (θ_i, x_i) from $\Pr(\theta, x)$

$$\theta_i \sim \Pr(\theta)$$
$$x_i \sim \Pr(x|\theta)$$

- **Training** q_ϕ based on (θ_i, x_i) equals maximizing $\sum_i \log q_\phi(x_i|\theta_i)$

$$\mathbb{E}_{\Pr(\theta,x)}\left[\log q_\phi(x|\theta)\right] = -\mathbb{E}_{\Pr(\theta)}\left[D_{KL}\left(\Pr(x|\theta)||q_\phi(x|\theta)\right)\right] + \text{const}$$

$\rightarrow$ Kullback-Leibler divergence zero for $q_\phi(x|\theta) = \Pr(x|\theta)$

- Insert **observation** x_0

$$\Pr(x_0|\theta) \approx q_\phi(x|\theta)$$

- Sequential NLE: Change proposal $\Pr(\theta)$ to $q_\phi(\theta|x_0)$ for faster convergence [Papamakarios, Sterratt and Murray, AISTAT (2019)]

Normalizing Flows

- Represent **probability distribution** $\Pr(x)$ via *invertible* and *differentiable* transformation f of base distribution $\pi(u)$

- Sample easily:

$$x = f(u) \qquad u \sim \pi(u)$$

- Choose simple **base distribution** $\pi(u)$, i.e. standard Gaussian

- Compute probability density:

$$\Pr(x) = \pi\left(f^{-1}(x)\right) \left|\det\left(\frac{\partial f^{-1}}{\partial x}\right)\right|$$

→ **Transformation** f should be easy to invert and determinant of Jacobian must be easy to compute

[Kingma and Dhariwal, NeurIPS (2018)]