# Hadron Spectroscopy with Lattice QCD: challenges and opportunities

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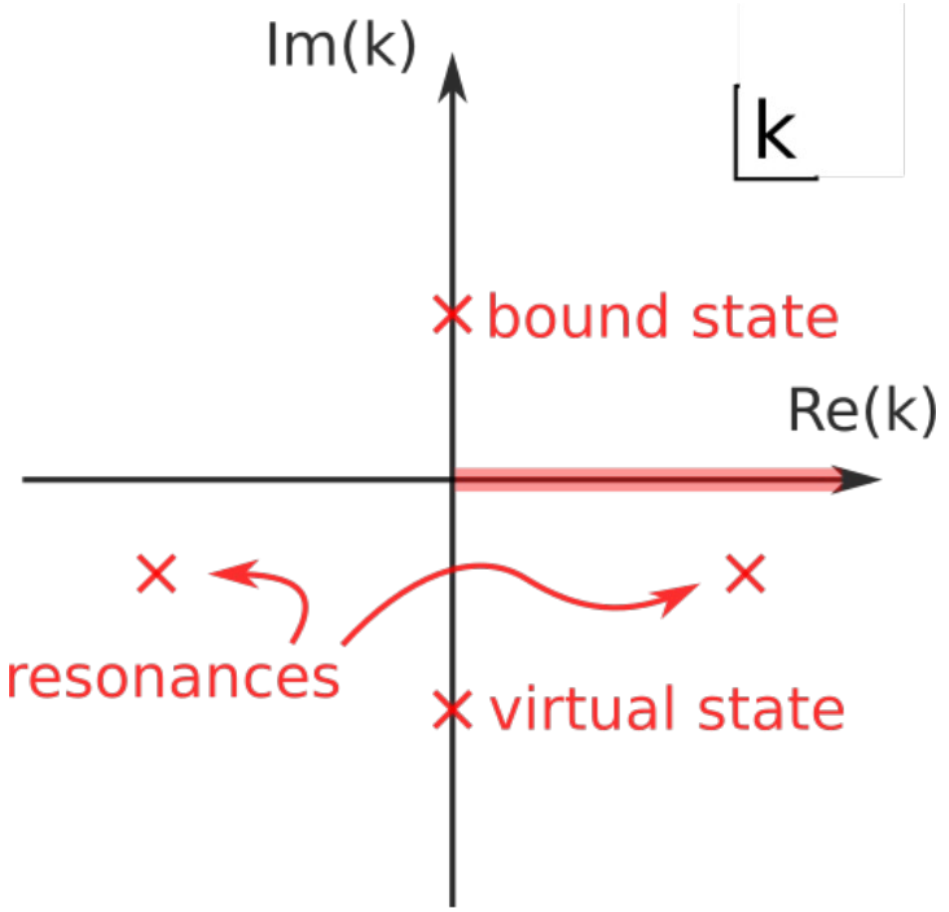
RUB

MENU 2023 Mainz, DE Oct. 16-20th, 2023

Introduction:

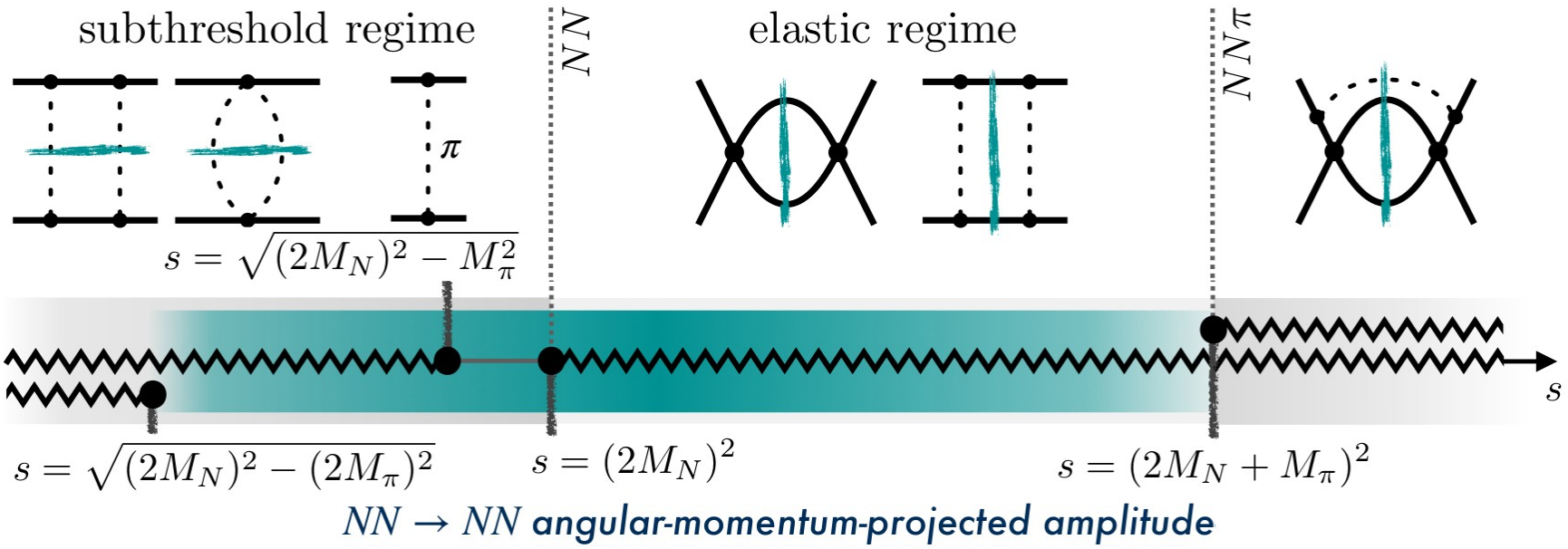
- Resonances and near-threshold bound-states
- Poles of scattering amplitude in the complex plane
- data on the positive real axis used for analytic continuation




from I. Matuschek, V. Baru, F.-K. Guo, C. Hanhart
Eur.Phys.J.A 57 (2021) 3, 101

Nearby non-analyticities must be treated:

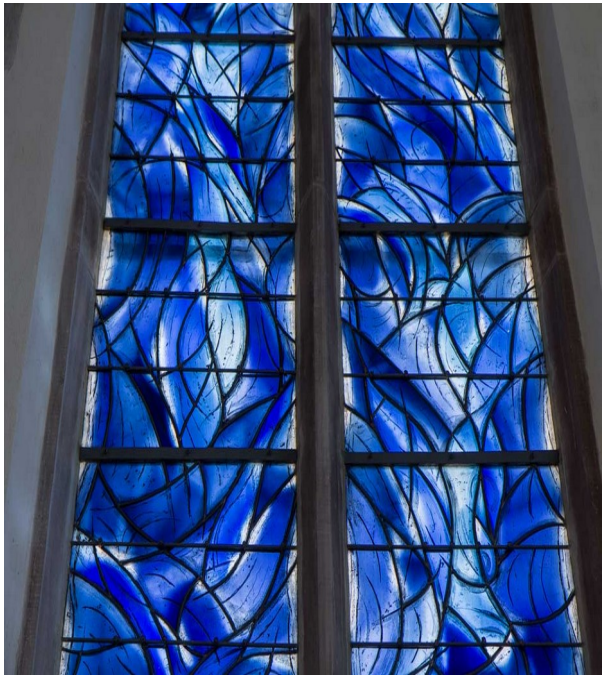
- Right-hand (threshold) cuts
- Left-hand (cross-channel) cuts
- …




A. Baião Raposo, Lattice ‘23

Lattice QCD:

- Sources of error:
  - Monte Carlo statistics
  - Finite volume and lattice spacing



Lattice QCD by M. Chagall :-P
(St. Stephen's Church, Mainz)

- Imaginary time (Wick rotation): $t \to i\tau$

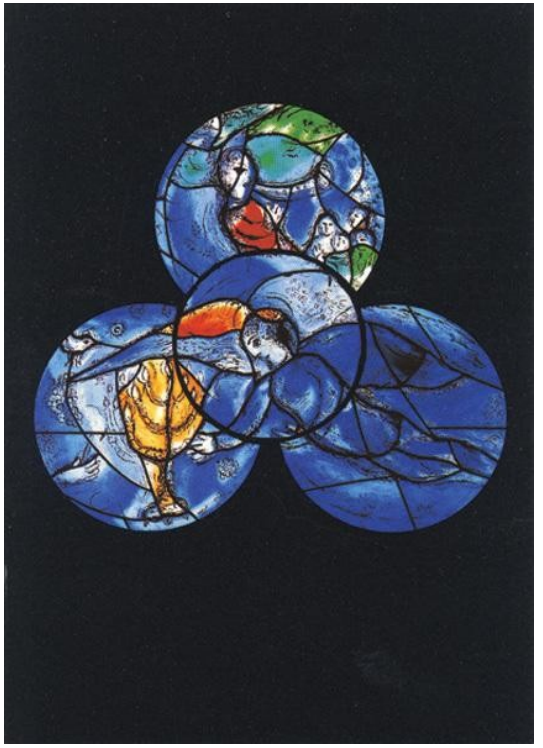
- Energies/matrix elements from large-time limit of correlation functions:

$$C^{\mathrm{2pt}}(\tau) = \sum_n |\langle 0|\hat{O}|n\rangle|^2 \, \mathrm{e}^{-E_n \tau}$$

$$\lim_{\tau\to\infty} C^{\mathrm{2pt}}(\tau) = |\langle 0|\hat{O}|1\rangle|^2 \, \mathrm{e}^{-E_1 \tau}$$

# Lattice QCD computations of scattering amplitudes: potential advantages

- Channels difficult to realize experimentally
  - Large strangeness, charm, or beauty
  - Three-to-three scattering



3-hadron scattering by M. Chagall :-P
(St. Stephen's Church, Mainz)

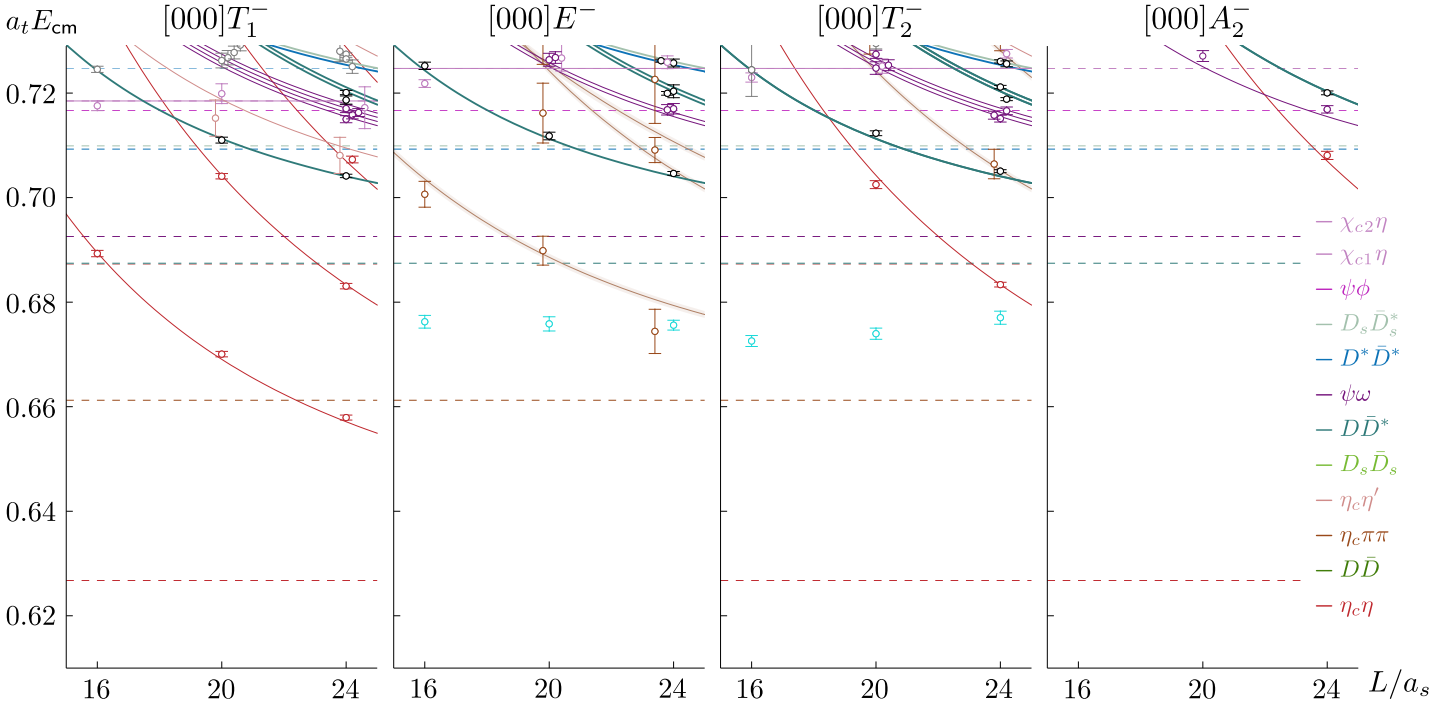
- Vary the physical parameters
  - $m_{\mathrm{u}} = m_{\mathrm{d}}$ and turn off electroweak interaction
  - $m_{\mathrm{u,d}} \lesssim m_{\mathrm{u,d}}^{\mathrm{phys}}$ (chiral EFT)
  - $m_{\mathrm{u}} = m_{\mathrm{d}} = m_{\mathrm{s}}$ (SU(3) flavor symmetry)
  - $m_{\mathrm{c,b}} \to \infty$ (HQET, NRQCD)
  - $N_{\mathrm{color}} \to \infty$ (large-Nc QCD)
  - …

https://bistummainz.de/pfarrei/mainz-st-stephan/chagall-fenster/die-fenster-von-marc-chagall/

# Lattice QCD computations of scattering amplitudes: ingredients

1) Finite-volume multi-hadron energies. Ex: hidden charm (without annihilation)

$$N_{\rm f} = 2 + 1, \quad m_\pi \approx 391\,{\rm MeV}, \quad a_{\rm s} = 0.12{\rm fm} = 3.5 a_{\rm t}$$




D. J. Wilson, C. E. Thomas, J. J. Dudek, R. G. Edwards (Hadron Spectrum Coll.), 2309.14071 [hep-lat]
see talk from D. J. Wilson, Mon. 4:20pm, Spectroscopy

2) Determinant condition relating energies to amplitudes

3) Amplitude parametrizations/fits

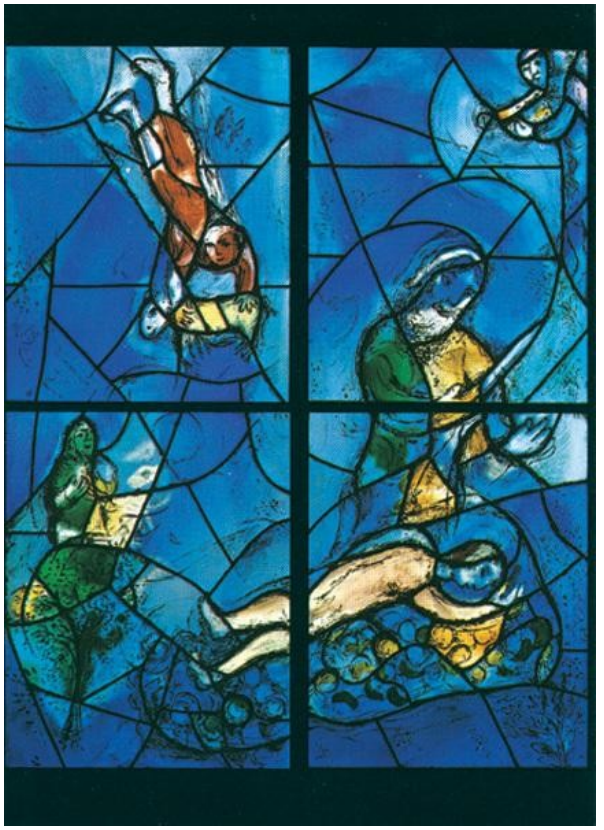
4) Analytic continuation → pole positions/residues

# 2) Amplitudes from finite-volume energies

$$\det[K^{-1}(E_{\rm cm}^{\rm FV}) - B(\boldsymbol{p}_{\rm cm}^{\rm FV})] = 0$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531;...

- Determinant over partial waves and channels
- Direct info below threshold (!) if $E_{\rm cm}^{\rm FV} < E_{\rm thresh}$
- Neglects:
  - Partial waves above $\ell_{\rm max}$
    C. Morningstar, *et al*., Nucl.Phys.B 924 (2017) 477-507
  - Right-hand cuts due to 3+ particles
    Z. Draper, *et al*., JHEP 07 (2023) 226
    S. Dawid, *et al*., Phys.Rev.D 108 (2023) 3, 034016; ...
  - Left-hand cuts A. Baiao Raposo, M. T. Hansen, Lattice '22,'23
    M. Habib E Islam, *et al*., Lattice '23
    S. Sharpe, *et al*., Lattice '23
    M.-L. Du, *et al*., 2303.09441 [hep-ph]
  - Exponential finite-volume effects



n-body in a finite volume by M. Chagall :-P
(St. Stephen's Church, Mainz)

3) Amplitude parametrizations/fits:

- Typically a variant of the effective range expansion (ERE):

$$p_{\rm cm}^{2\ell+1}\cot\delta_\ell = \frac{1}{a} + \frac{r}{2}p_{\rm cm}^2 + \dots$$

4) Analytic continuation: s-wave pole occurs if

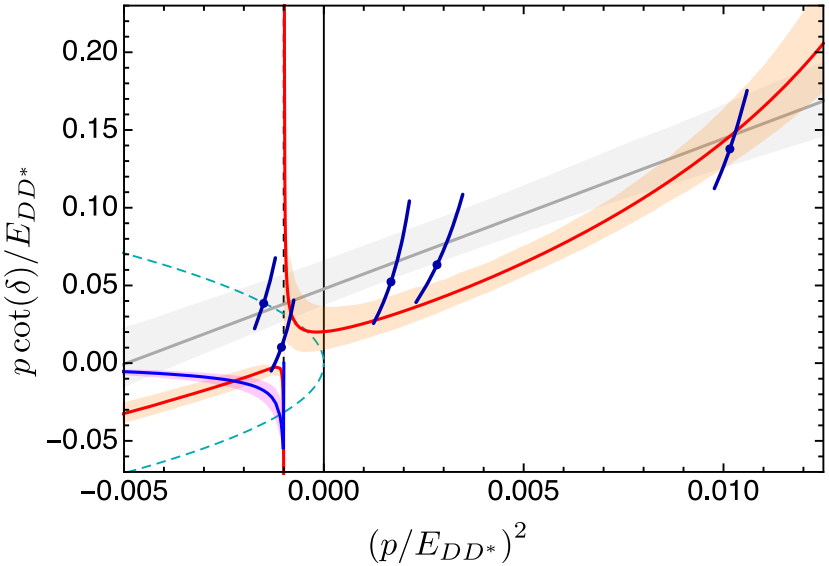
$$p_{\rm cm}\cot\delta_0 - ip_{\rm cm} = 0$$

- Radius of convergence limited by nearest cut
  - Ex: $T_{cc}(3875)^+$ in *DD*$^*$-scattering

    M.-L. Du, et al., 2303.09441 [hep-ph]
  - Points from lattice QCD at $m_\pi = 280\text{MeV}$, gray band is ERE fit

    S. Prelovsek, M. Padmanath, Phys. Rev. Lett. 129, 032002 (2022);
    See also: S. Chen et al., PLB 833, 137391 (2022); Y. Lyu et al., 2302.04505
  - Left hand cut invalidates naive FV formalism

    J. R. Green, et al., Phys.Rev.Lett. 127 (2021) 24, 242003
  - Near left-hand cuts, pole positions from ERE not trustworthy




from M.-L. Du, et al., 2303.09441 [hep-ph]

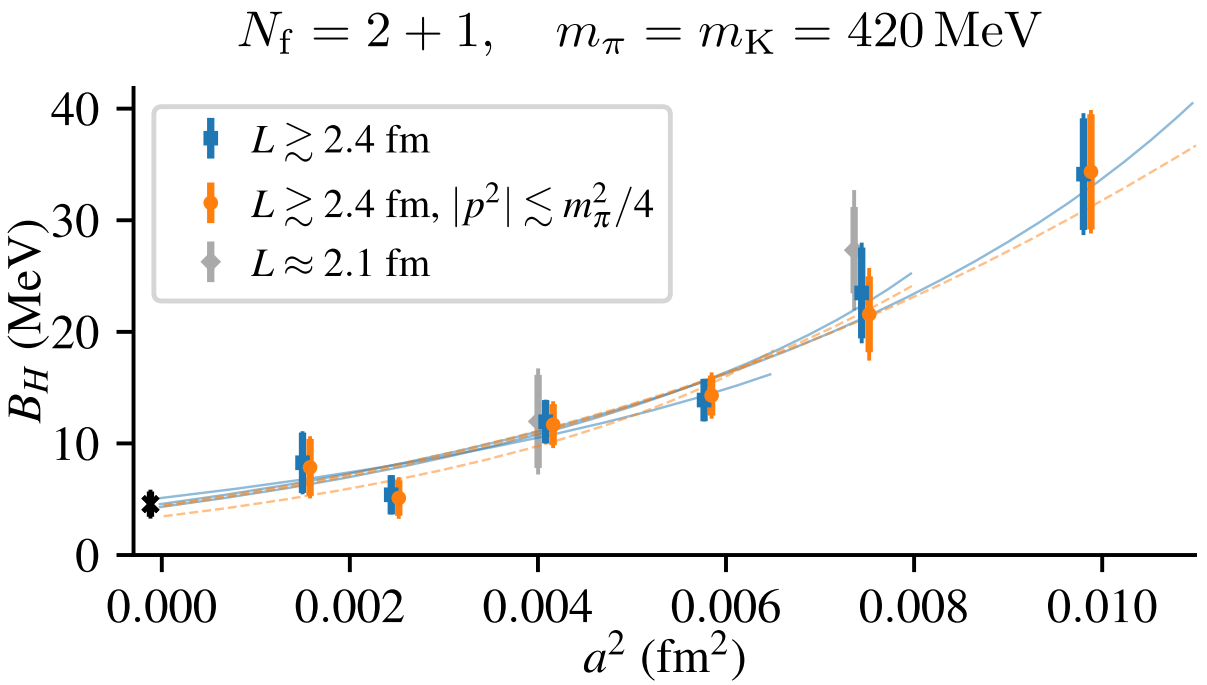
## Another challenge: cutoff effects

- Finite-volume energies have asymptotic $O(a^2)$ cutoff effects
  (log corrections may be significant)

N. Husung, Eur.Phys.J.C 83 (2023) 2, 142

- Significant cutoff effects observed for $\Lambda\Lambda$ -scattering (*H*-dibaryon)

J. Green, A. Hanlon, P. Junnarkar, H. Wittig, Phys.Rev.Lett. 127 (2021) 24, 242003




- Also observed for DD$^*$ scattering (preliminary)

J. Green, A. Hanlon, H. Wittig, M. Padmanath, R. J. Hudspith, S. Paul, Lattice '23

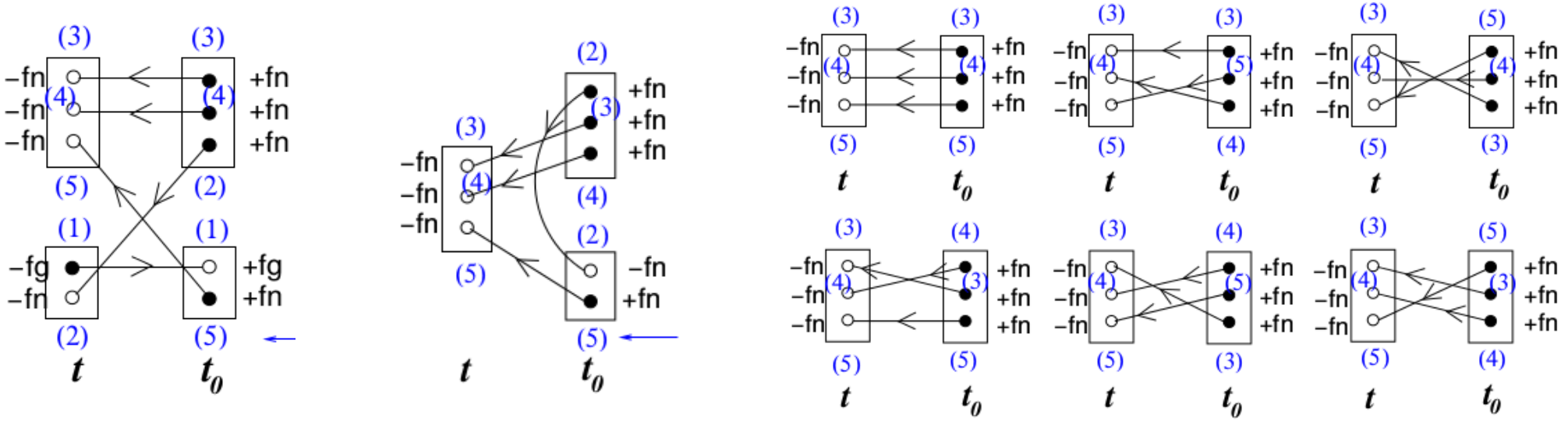
# Extended example: recent lattice computation of the Lambda(1405)

JB, B. Cid-Mora, A. Hanlon, B. Hoerz, D. Mohler, C. Morningstar, J. Moscoso, A. Nicholson, F. Romero-Lopez, A. Walker-Loud
(For the Baryon Scattering (BaSc) Collaboration)

- CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV}$$

- Correlation functions from tensor contraction:




- Factorization enabled by the distillation/stochastic LapH algorithms for quark propagation

M. Peardon et al. Phys.Rev.D 80 (2009) 054506; C. Morningstar et al. Phys.Rev.D 83 (2011) 114505

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CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV}$$

Part of a comprehensive study of many channels

| Flavor channel | Number of Correlators |
|---|---|
| $I = 0,\ S = 0,\ NN$ | 8357 |
| $I = 0,\ S = -1,\ \Lambda, N\overline{K}, \Sigma\pi$ | 8143 |
| $I = \frac{1}{2},\ S = 0,\ N\pi$ | 696 |
| $I = \frac{1}{2},\ S = -1,\ N\Lambda, N\Sigma$ | 17816 |
| $I = 1,\ S = 0,\ NN$ | 7945 |
| $I = \frac{3}{2},\ S = 0,\ \Delta, N\pi$ | 3218 |
| $I = \frac{3}{2},\ S = -1,\ N\Sigma$ | 23748 |
| $I = 0,\ S = -2,\ \Lambda\Lambda, N\Xi, \Sigma\Sigma$ | 16086 |
| $I = 2,\ S = -2,\ \Sigma\Sigma$ | 4589 |
| Single hadrons (SH) | 33 |

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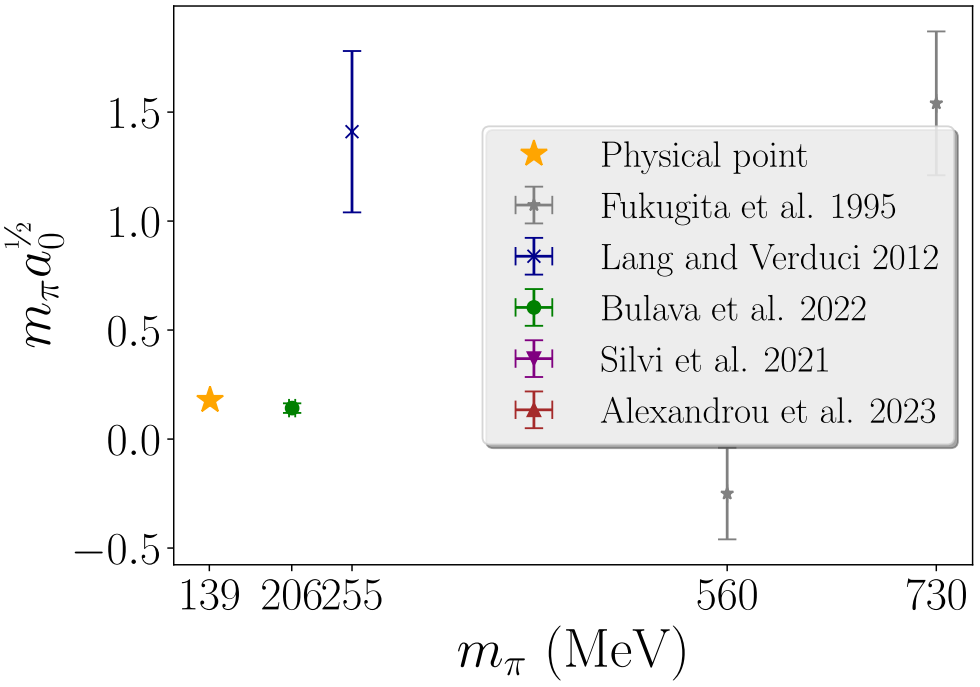
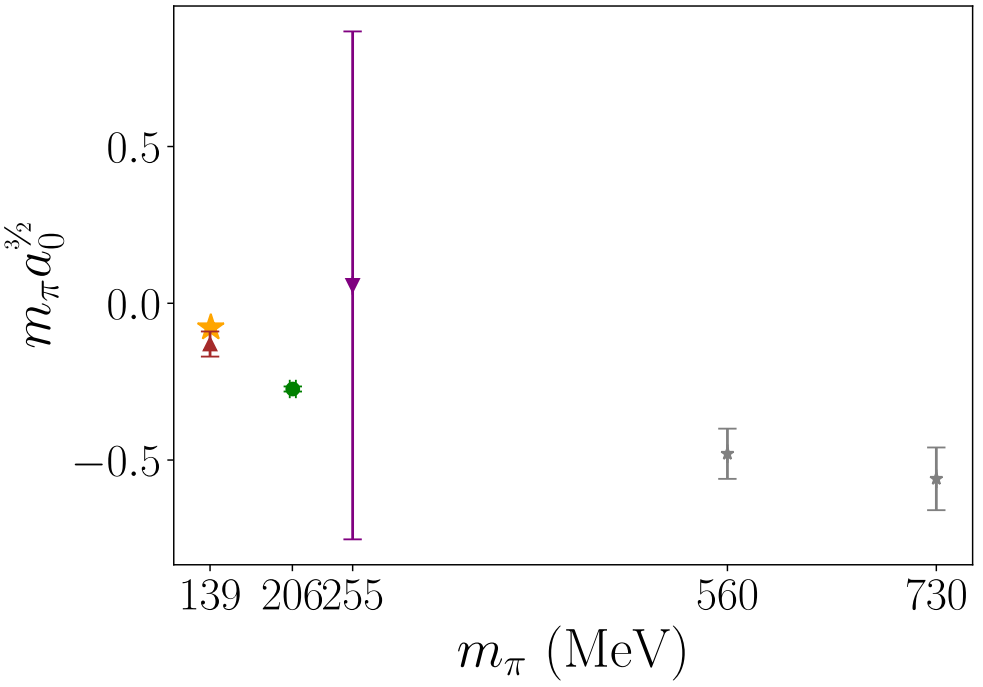
(For the Baryon Scattering (BaSc) Collaboration): 2307.13471 [hep-lat] and 2307.10413 [hep-lat]

## CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV}$$

## First results on nucleon-pion scattering:




JB, A. Hanlon, B. Hoerz, C. Morningstar, A. Nicholson, F. Romero-Lopez, S. Skinner, P. Vranas, A. Walker-Loud
Nucl.Phys.B 987 (2023) 116105

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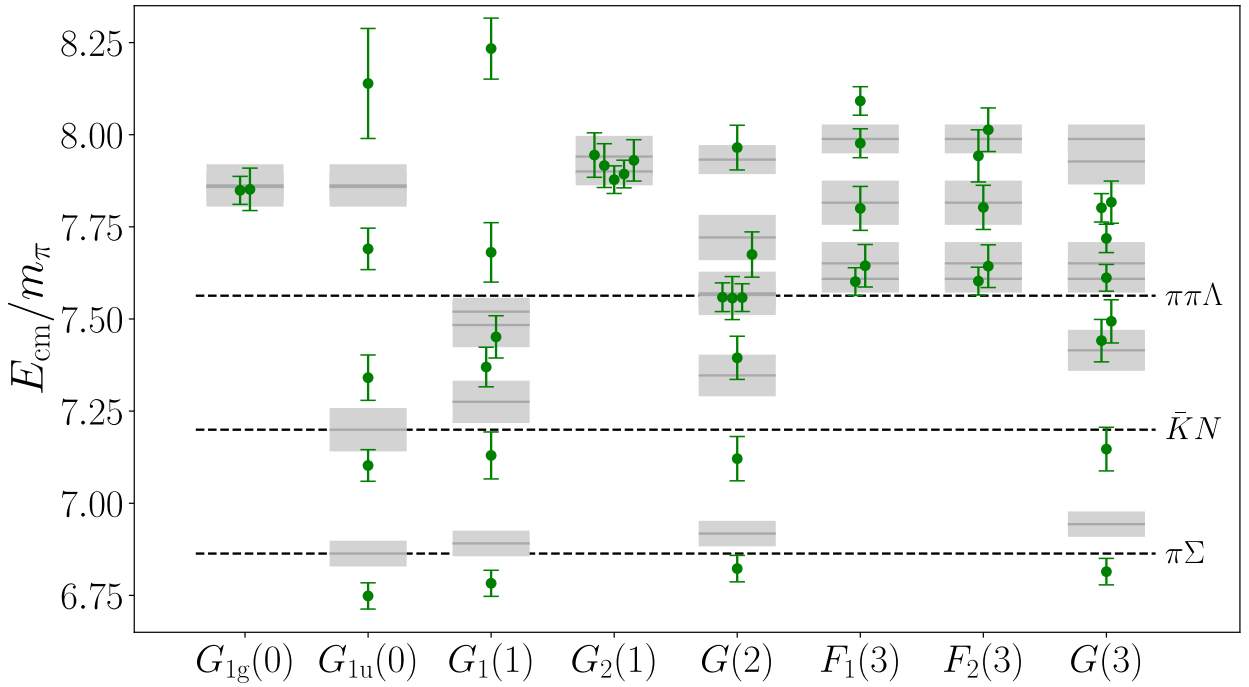
## CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV}$$

## 1) Finite-volume energies:




More details in talk of B. Cid-Mora Mon. 4:50pm, Hadron Spectroscopy

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CLS (D200) lattice:

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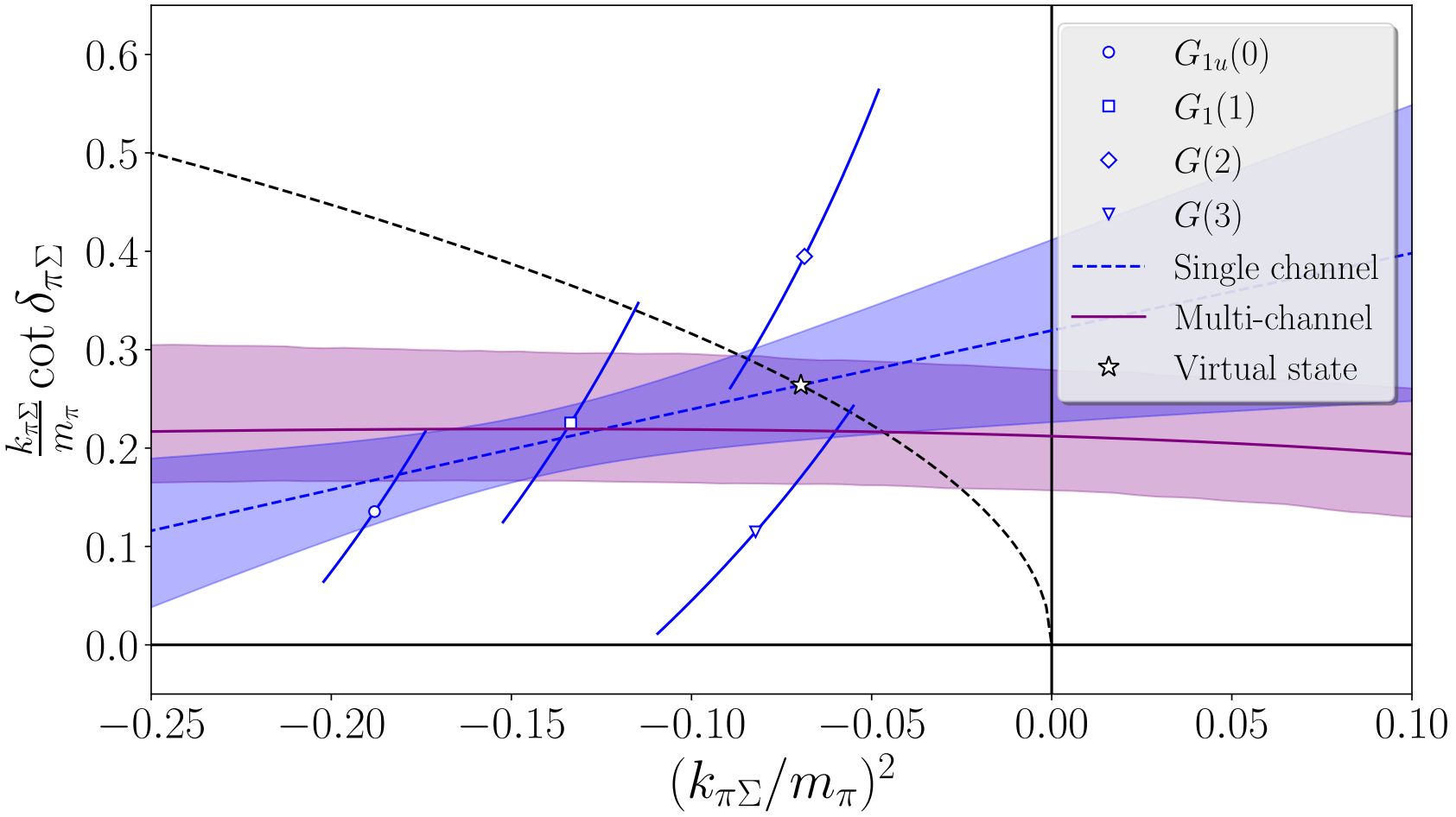
2) Quantization condition: leading partial wave approximation



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CLS (D200) lattice:

$$64^3 \times 128,\ a = 0.064\text{fm},\ m_\pi = 200\text{MeV}$$

3) Amplitude parametrization

Variants of:

$$K_{ij}^{-1} = A_{ij} + B_{ij}\Delta(E_{\text{cm}})$$

as well as for *K* and Blatt-Biedenharn

4) Analytic continuation: find zeroes of

$$t^{-1} = K^{-1} - i\hat{k}$$

No nearby left hand/circular cuts!

# Extended example: recent lattice computation of the Lambda(1405)

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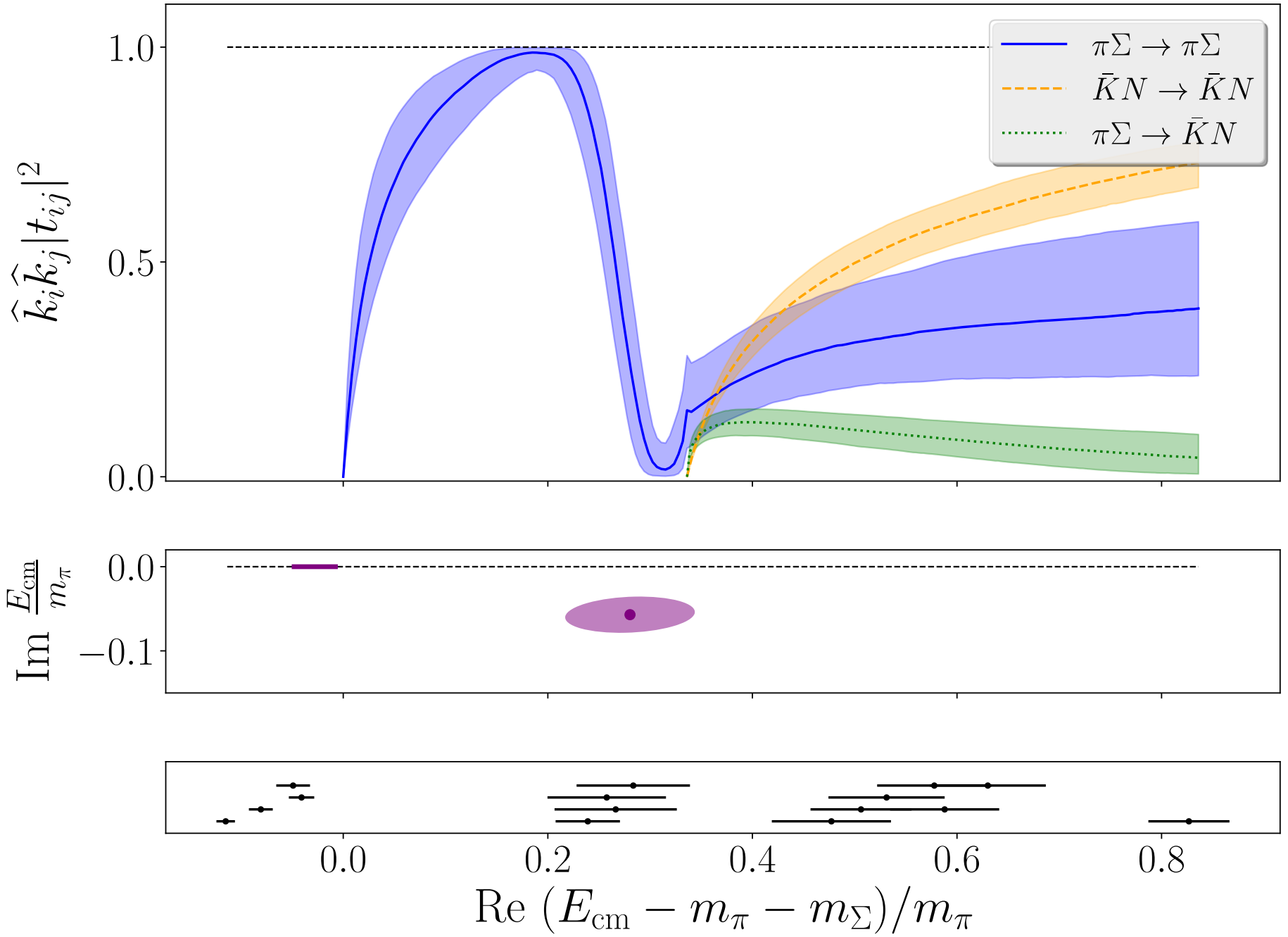

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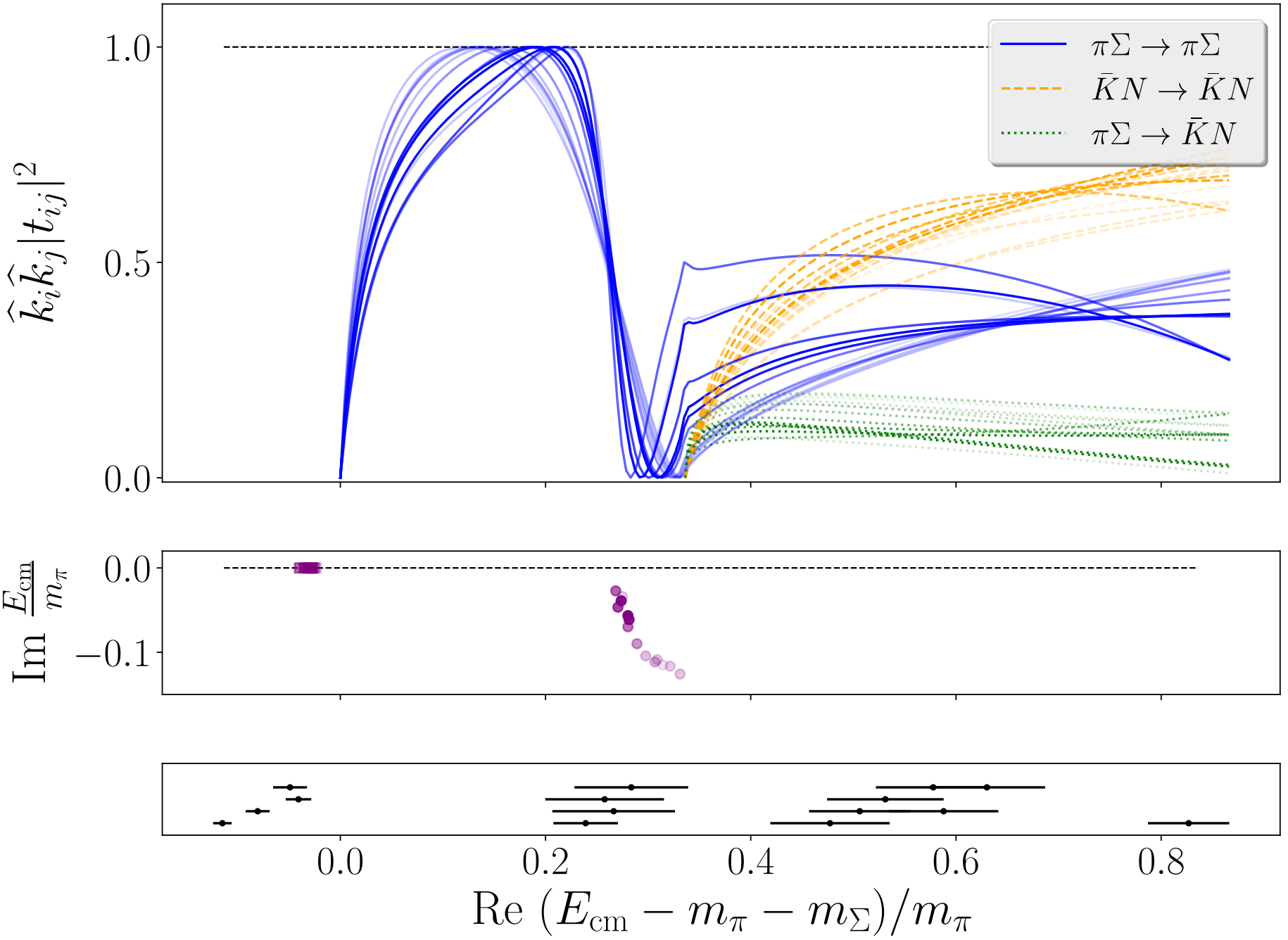
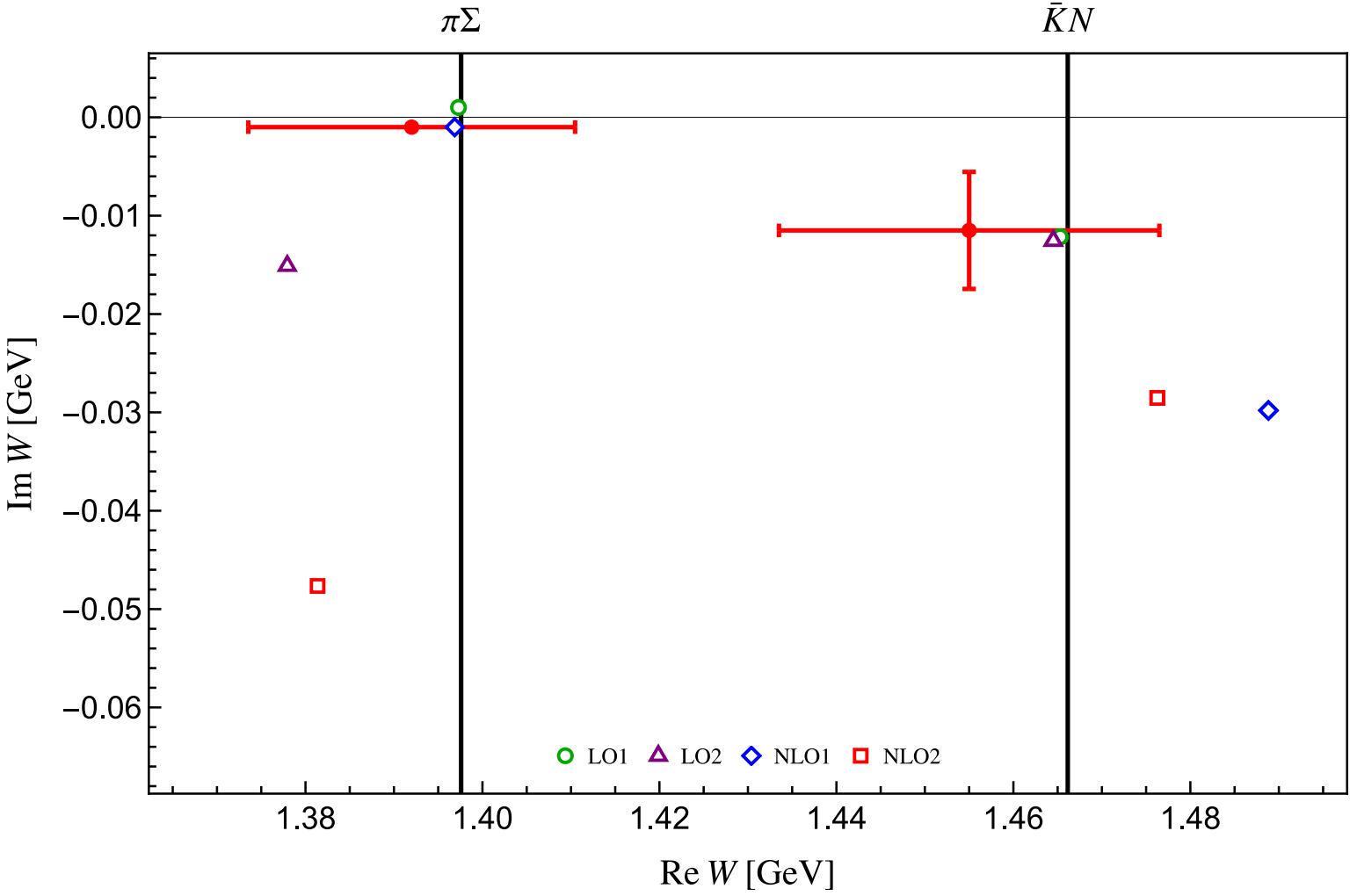


# Extended example: recent lattice computation of the Lambda(1405)

## Recent ChEFT analysis: consistent pole locations

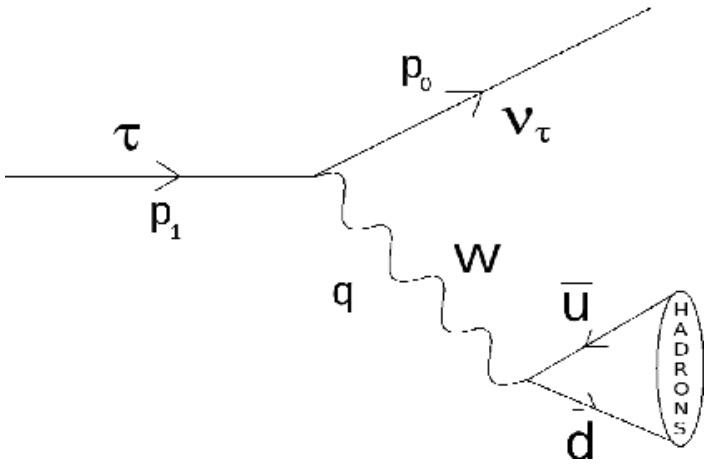
F.-K. Guo, Y. Kamiya, M. Mai, U.-G. Meissner, 2308.07658 [hep-ph]

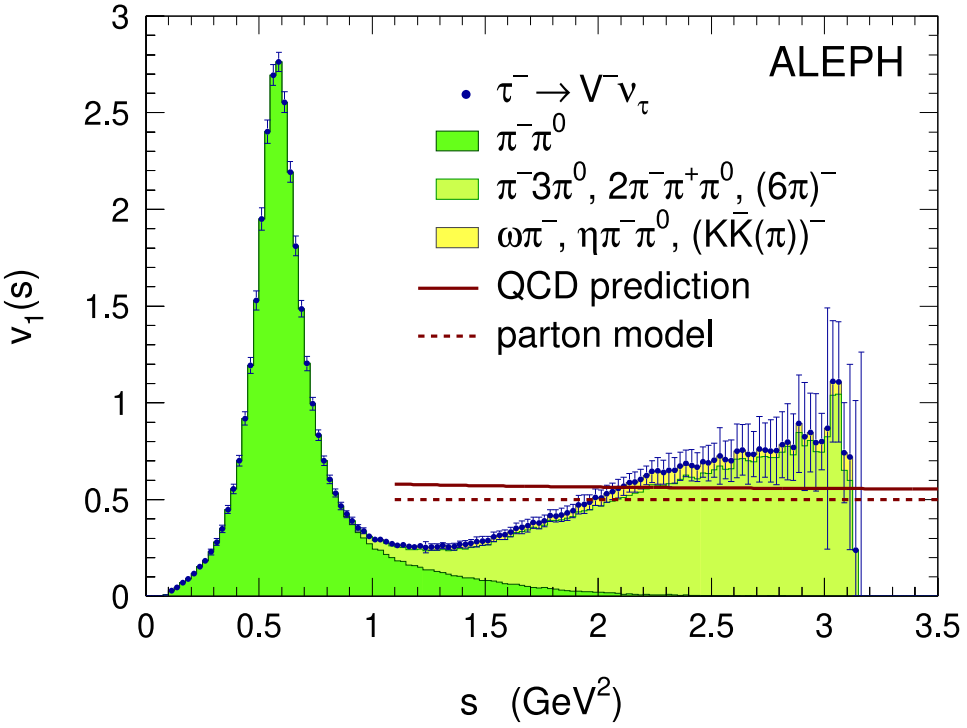



See also L.-S. Geng, Tue. 12:30pm (Spectroscopy); Xiu-Lei Ren Mon. 5:10pm (Spectroscopy); M. F. M. Lutz, Mon. 5:10pm, (Low energy Nucleon Structure)

# Inclusive processes in lattice QCD

Hadronic Tau decays:



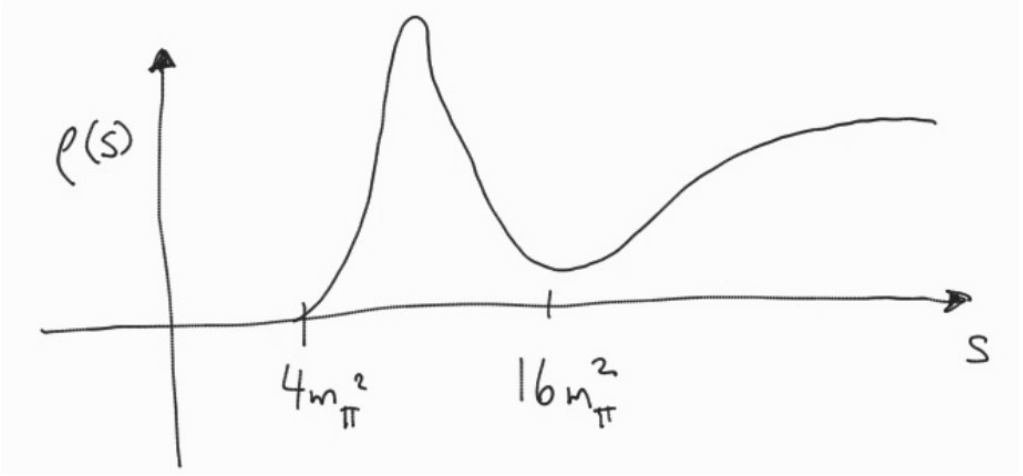

Decay rate from spectral density:

$$C(t) = \int d^3\boldsymbol{x}\, \langle \Omega | \hat{V}_z^{\mathrm{cc}}(\boldsymbol{x})\, \mathrm{e}^{-\hat{H}t}\, \hat{V}_z^{\mathrm{cc}}(0)^\dagger | \Omega \rangle$$

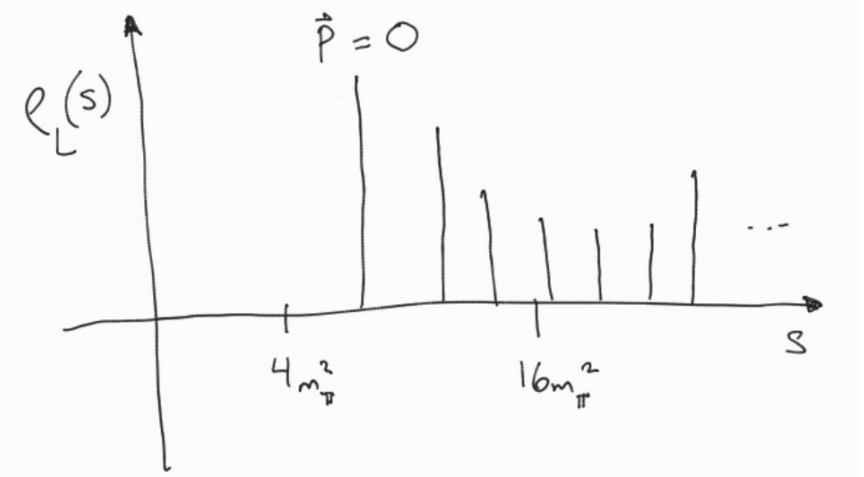
$$\propto \int_0^\infty d\omega\, \omega^2 v_1(\omega^2)\, \mathrm{e}^{-\omega t}$$

# Finite vs. infinite volume

Infinite volume: continuous




Finite volume:  sum of Dirac-delta peaks.




Not ‘close’ to infinite volume at finite L!

# Masterfield lattice QCD

- Large volumes needed to saturate ordered double limit:

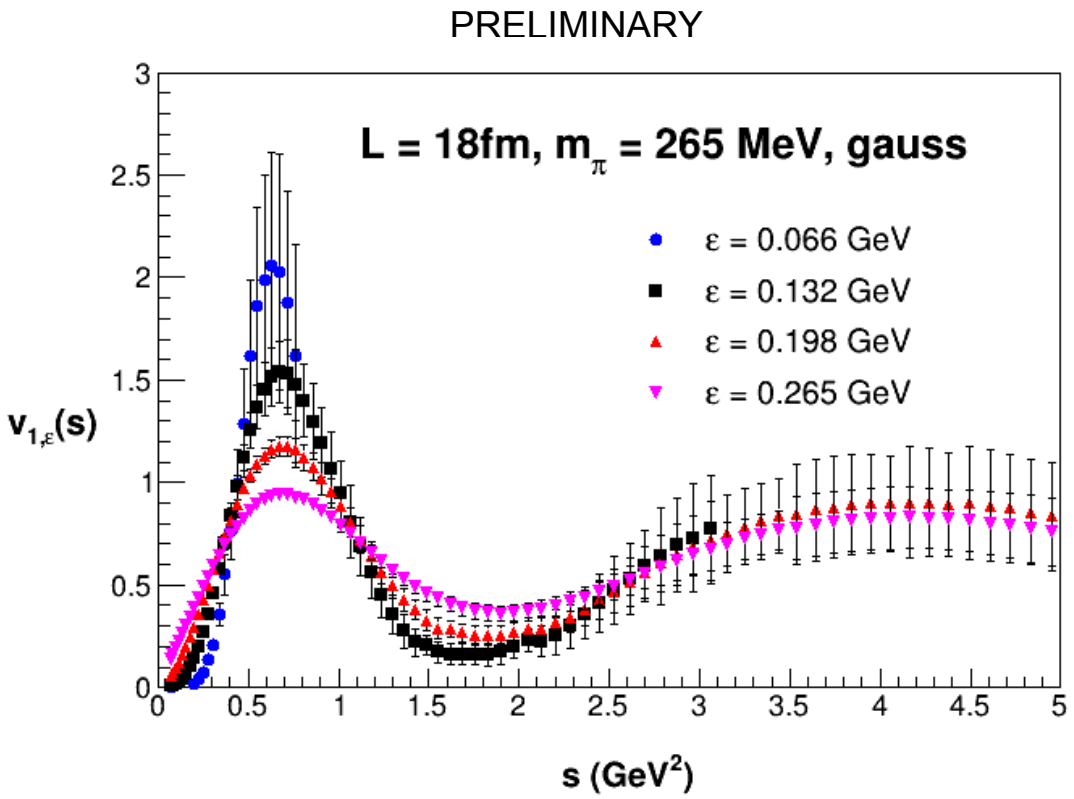
$$v_1(s) = \lim_{\epsilon \to 0^+} \lim_{L \to \infty} v_{1,\epsilon}^{\mathrm{g}}(s), \quad v_{1,\epsilon}^{\mathrm{g}}(s) = \int \mathrm{d}\omega \, \frac{\mathrm{e}^{-\frac{(\omega - \sqrt{s})^2}{2\epsilon^2}}}{\sqrt{2\pi}\epsilon} v_1(\omega^2)$$

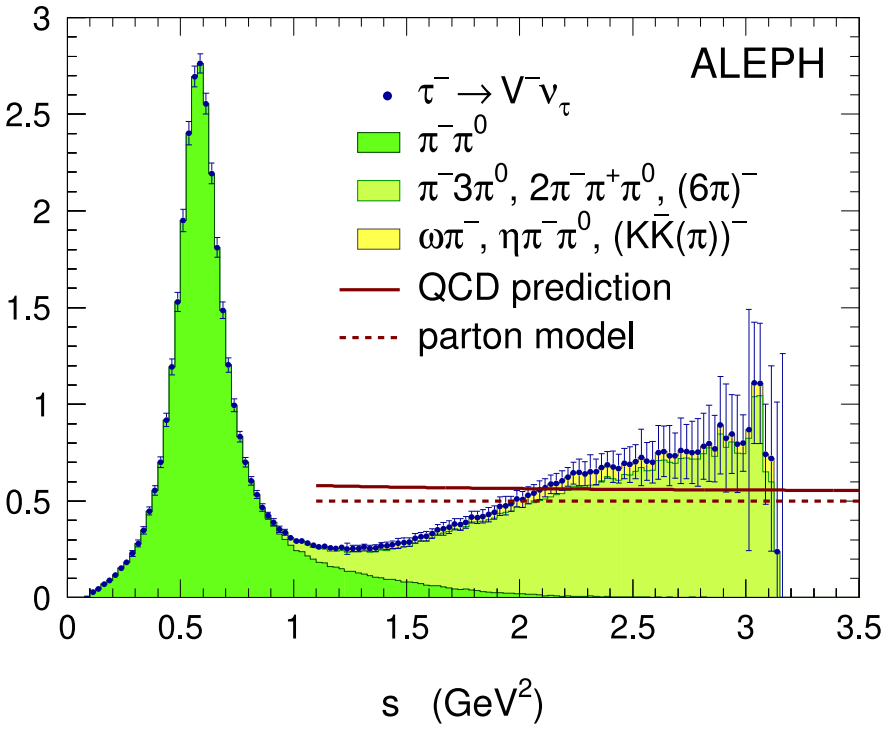
- Relevant idea: masterfield simulation paradigm M. Lüscher, `17
  - ➔ Only a few gauge configurations
  - ➔ Accrue statistics from separate space-time regions:
    - ➔ O(1000) gauge configs = 6^4 space time regions of size $m_\pi L \approx 3$
- Preliminary application: isovector (axial)vector correlators at

$$N_{\mathrm{f}} = 2 + 1, \quad L = 18\mathrm{fm},$$
$$a = 0.09\mathrm{fm}, \quad m_\pi = 265\mathrm{MeV}$$

M. Bruno, JB, A. Francis, P. Fritzsch, J. Green, M. T. Hansen, M. Lüscher, A. Patella, A. Rago, *in prep.*

# Preliminary results

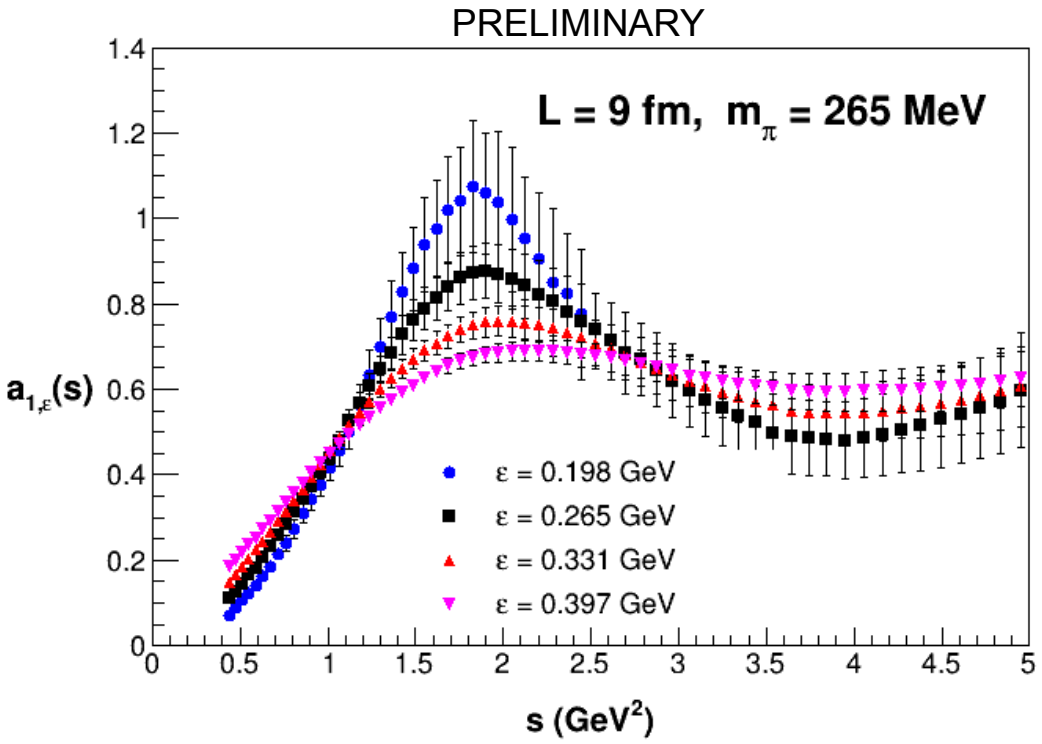


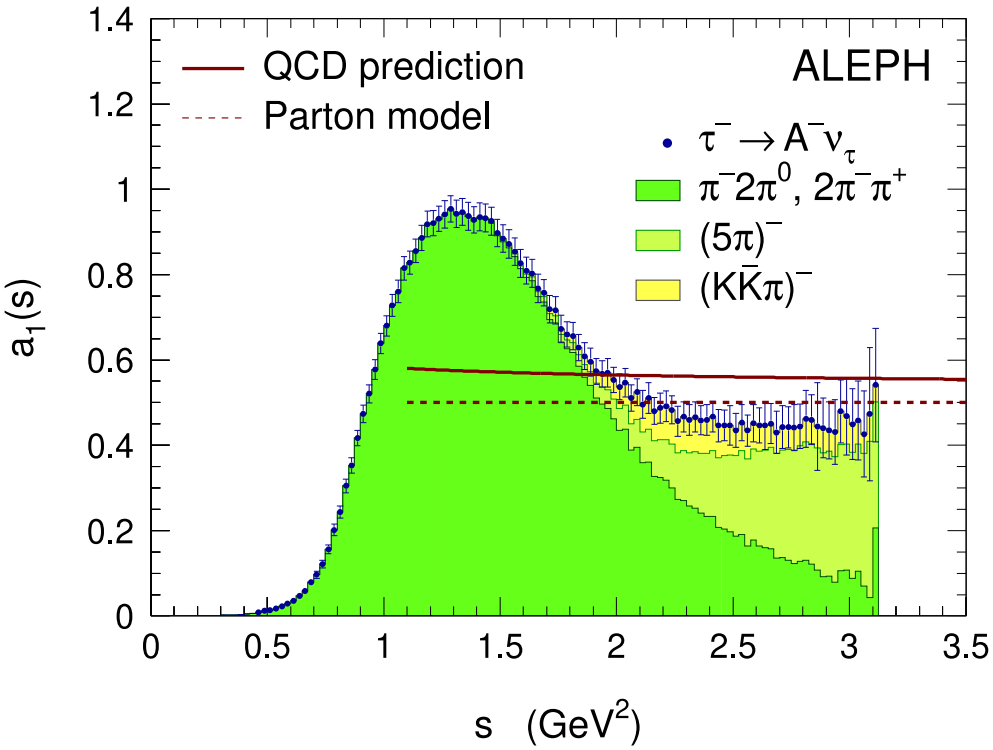




- Comparison to hadronic tau-decay (right) ALEPH collaboration `05
- No extrapolation to zero-width yet
- Mild indication of four-particle effects.

M. Bruno, JB, A. Francis, P. Fritzsch, J. Green, M. T. Hansen, M. Lüscher, A. Patella, A. Rago, *in prep.*

# Preliminary results




- Comparison to hadronic tau-decay (right)
- No extrapolation to zero-width yet
- Bump from $a_1(1260)$, indication of five pions

ALEPH collaboration `05

M. Bruno, JB, A. Francis, P. Fritzsch, J. Green, M. T. Hansen, M. Lüscher, A. Patella, A. Rago, *in prep.*

# Conclusions

- Near-threshold scattering amplitudes from lattice QCD via finite-volume approach: challenges
  - ➔ Left/right hand cuts
  - ➔ Cutoff effects
  - ➔ Signal-to-noise (statistical errors)

- Opportunities for low-lying meson-baryon resonances at the physical point are good:
  - ➔ Tractable statistical errors
  - ➔ A few coupled channels (e.g. Lambda(1405)
  - ➔ Left-hand cuts less important

- Opportunities for (hidden) charm exotics hampered by cutoff effects, and left hand cuts

- First lattice QCD computations of inclusive amplitudes are possible!
  - ➔ Large volumes needed for good energy resolution
  - ➔ Masterfield simulation paradigm effective
  - ➔ Exclusive amplitudes possible in principle

JB, M. T. Hansen, Phys.Rev.D 100 (2019) 3, 034521