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## Combined analysis of $K^-p$ reactions and $\pi\Sigma$ photoproduction data

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- ① Chirally motivated  $\pi\Sigma - \bar{K}N$  interactions (brief overview)
- ②  $\pi\Sigma$  photoproduction mass spectra
  - Formalism
  - Results
- ③ Summary

collaboration with P. C. Bruns, partly with M. Mai (Bonn) and J. Smejkal (Prague)

# Chirally motivated $\bar{K}N$ interactions

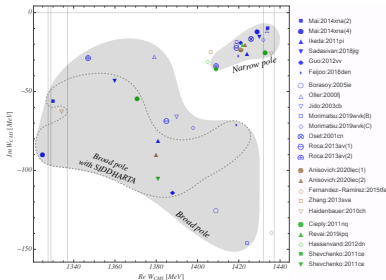
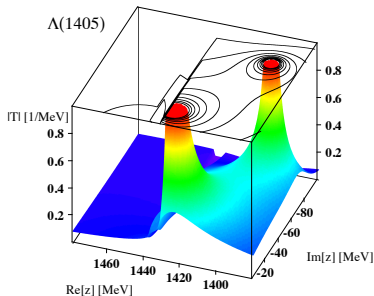
$\bar{K}N - \pi\Sigma$  system (+ add-ons, mostly more  $MB$ )  
meson octet - baryon octet coupled channels interactions

involved channels	$\pi\Lambda$	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$\eta\Sigma$	$K\Xi$
thresholds (MeV)	1250	1330	1435	1660	1740	1810

- strongly interacting multichannel system with an s-wave resonance, the  $\Lambda(1405)$ , just below the  $K^-p$  threshold
- modern theoretical treatment based on **effective chiral Lagrangians**: effective potentials constructed to match the chiral meson-baryon amplitudes up to LO or NLO order
- Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series
- **unitary coupled channels approaches** based on effective chiral Lagrangian generate **two poles related to  $\Lambda(1405)$**  (Oller, Meißner in 2001)

$K^-p$  data fits: low energy x-sections, threshold BRs, kaonic hydrogen 1s level

# Model predictions - $\Lambda(1405)$ resonance



Hyodo, Jido - Prog. Part. Nucl. Phys. 67 (2012) 55

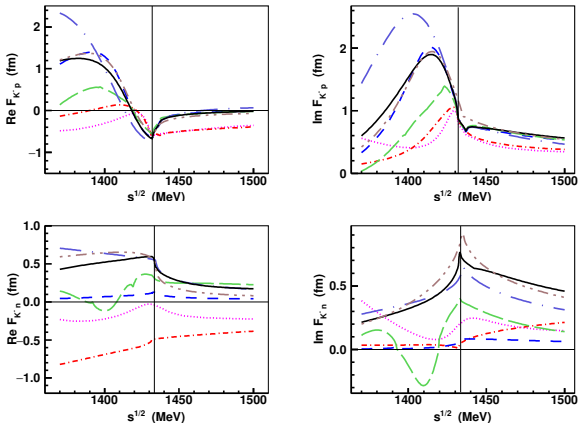
all recent (year  $\geq 2000$ ) predictions

M. Mai - Eur. Phys. J. Spec. Top. 230 (2021) 6, 1593

- the *higher* pole around 1425 MeV couples more strongly to  $\bar{K}N$ , the *lower* pole is much further from the real axis and has larger coupling to  $\pi\Sigma$
- most common interpretation -  $\bar{K}N$  quasi-bound state submerged in  $\pi\Sigma$  continuum, a result of coupled channels  $\pi\Sigma - \bar{K}N$  dynamics
- all models tend to agree on the position of the  $\bar{K}N$  related pole
- the  $K^-p$  reactions data are not very sensitive to the position of the  $\pi\Sigma$  related pole

# Model predictions - $K^-N$ amplitudes

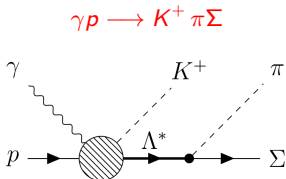
$K^-p$  and  $K^-n$  elastic amplitudes



models: Kyoto-Munich, Prague, Bonn, Murcia, Barcelona  
updated, original in A.C., M.Mai, U.G.Meißner, J.Smejkal - NPA 954 (2016) 17

# Chiral models in need of more data

- Model parameters fitted to experimental data available at energies from  $K^-p$  threshold up, provide varied theoretical predictions for subthreshold energies and in the isovector sector.
- In the  $K^-p$  reactions data, the  $\Lambda(1405)$  is hidden below the threshold. The resonance can be seen in processes, where  $\pi\Sigma$  re-scatter in the final state, e.g.



In this **two meson protoproduction reaction** the  $K^+$  meson carries away momentum, enabling a scan in the invariant mass of the  $\pi\Sigma$  system down to its production threshold.

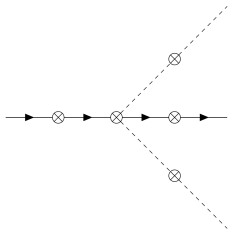
# $\pi\Sigma$ photoproduction: Formalism

formalism outlined in: P. C. Bruns - arXiv:2012.11298 [nucl-th] (2020)

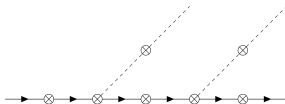
application to  $\pi\Sigma$  mass spectra predictions:

P. C. Bruns, A. C., M. Mai - Phys. Rev. D 106 (2022) 7, 074017

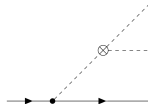
leading-order **B $\chi$ PT** used to derive expressions for the photoproduction amplitude  $\mathcal{M}$ , constructed from tree level graphs:



Weinberg-Tomozawa (WT)



Born term (BT) - B1, B2

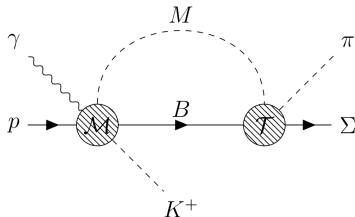


anomalous (AN)

lines: directed - baryons, dashed - pseudoscalar mesons;  $\otimes$  symbols - photon insertions  
 $5 + (2 \times 7) + 1 = 20$  tree graphs, 16 independent  $\mathcal{M}_j$  structure functions

# $\pi\Sigma$ photoproduction: Formalism

Final state interaction of the  $MB$  pair needs to be accounted for:



$\pi\Sigma - \bar{K}N$  coupled channels models provide the  $f_{\ell\pm}^{c',c}(M_{\pi\Sigma})$  amplitudes, that describe the scattering from channel  $c$  to channel  $c'$  ( $c, c' = \pi\Lambda, \pi\Sigma, \bar{K}N, \eta\Lambda, \dots$ )

unitarized amplitudes for  $\gamma p \rightarrow K^+ MB$  will be taken as the coupled-channel vector

$$[\mathcal{A}_{0+}^i] = [\mathcal{A}_{0+}^{i(\text{tree})}] + [f_{0+}] [8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})] [\mathcal{A}_{0+}^{i(\text{tree})}]$$

The second term represents the final-state  $MB$  rescattering and  $G(M_{\pi\Sigma})$  is a diagonal channel-space matrix with entries given by regularized loop integrals

## $\pi\Sigma$ photoproduction: Formalism

There are four independent structure functions  $\mathcal{A}_{0+}^i(s, M_{\pi\Sigma}^2, t_K)$  constructed from  $\mathcal{M}_j$ , projected on s-wave and satisfying the partial-wave unitarity relation

$$\text{Im}(\mathcal{A}_{0+}^i) = (f_{0+})^\dagger(|\vec{p}^*|)(\mathcal{A}_{0+}^i), \quad i = 1, \dots, 4.$$

Neglecting  $\ell > 0$  contributions, we get

$$\frac{d^2\sigma}{d\Omega_K dM_{\pi\Sigma}} = \frac{|\vec{q}_K||\vec{p}_\Sigma^*|}{(4\pi)^4 s |\vec{k}|} |\mathcal{A}|^2,$$

$$\begin{aligned} 4|\mathcal{A}|^2 = & (1-z_K) \left| \mathcal{A}_{0+}^1 + \mathcal{A}_{0+}^2 \right|^2 + (1+z_K) \left| \mathcal{A}_{0+}^1 - \mathcal{A}_{0+}^2 \right|^2 \\ & + (1-z_K) \left| \mathcal{A}_{0+}^1 + \mathcal{A}_{0+}^2 + \frac{2|\vec{q}_K|(1+z_K)}{M_K^2 - t_K} ((\sqrt{s} + m_N)\mathcal{A}_{0+}^3 + (\sqrt{s} - m_N)\mathcal{A}_{0+}^4) \right|^2 \\ & + (1+z_K) \left| \mathcal{A}_{0+}^1 - \mathcal{A}_{0+}^2 - \frac{2|\vec{q}_K|(1-z_K)}{M_K^2 - t_K} ((\sqrt{s} + m_N)\mathcal{A}_{0+}^3 - (\sqrt{s} - m_N)\mathcal{A}_{0+}^4) \right|^2, \end{aligned}$$

with  $z_K \equiv \cos\theta_K$ ,  $\theta_K$  being the angle between  $\vec{q}_K$  and  $\vec{k}$  in the overall c.m. frame.

# $\pi\Sigma$ photoproduction: Formalism

**two coupled channels approaches to generate the  $f_{0+}$  amplitudes:**

**Bonn** B2, B4 models - M.Mai, U.-G.Meißner, Eur. Phys. J. A 51 (2015) 30

BW model - D.Sadasivan, M.Mai, M.Döring, Phys. Lett. B 789 (2019) 329–335  
dimensional regularization used in  $G(M_{\pi\Sigma})$ , mass scales  $\mu_c$

**Prague** P model - P.C.Bruns, A.C., Nucl. Phys. A 1019 (2022) 122378

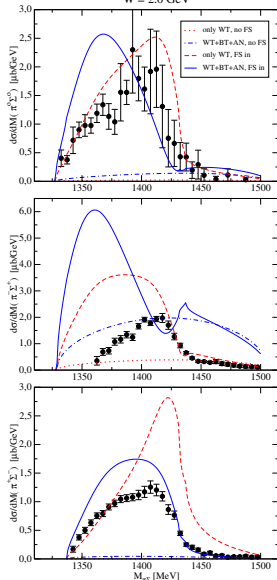
Yamaguchi form factors used in  $G(M_{\pi\Sigma})$ , inverse ranges  $\alpha_c$

**CLAS data:** K. Moriya et al. - Phys. Rev. C 87 (2013) 035206

**Result:** different models provide varied predictions of the  $\pi\Sigma$  mass spectra

# $\pi\Sigma$ photoproduction: FSI impact

$W = 2.0 \text{ GeV}$



CLAS data (2013) by Moriya et al.

c.m. energy  $W = \sqrt{s} = 2.0 \text{ GeV}$

P model used for the  $MB$  amplitudes

- **only WT, no FSI:**  
small (or zero for  $\pi^+\Sigma^-$ ) cross sections
- **WT+BT+AN, no FSI:**  
the cross sections remain flat, the  $\pi^-\Sigma^+$  one reaches magnitude comparable with the data
- **addition of FSI:**  
 **$MB$  rescattering is responsible for the peak structure**  
 $\pi^0\Sigma^0$  and  $\pi^+\Sigma^-$  reproduced rather well  
Born terms move the peak to lower energies

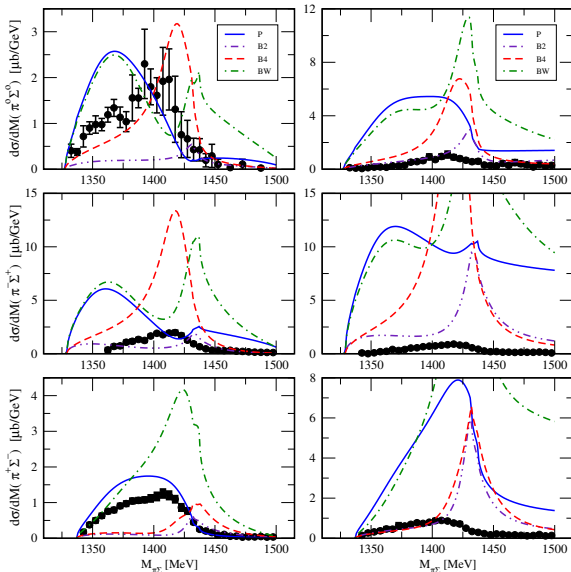
**parameter-free predictions!**

no adjustment to the  $f_{0+}$  amplitudes

# $\pi\Sigma$ mass spectra - model dependence

W = 2.0 GeV

W = 2.4 GeV



## New results - FSI not fixed

First time fits to combined  $K^-p$  reactions and  $\pi\Sigma$  photoproduction data with the FSI no longer fixed at a particular  $\pi\Sigma - \bar{K}N$  model parameters setup.

A. C., P. C. Bruns - submitted to NPA, arXiv:2305.06205

- The P model used for the  $MB \rightarrow \pi\Sigma$  amplitude represented by the  $MB$  rescattering  $\mathcal{T}$  bubble:  
6 regularization scales  $\alpha_c$ , 6 NLO couplings ( $b_0, b_F, d_{1...4}$ )
- Tree level photoproduction amplitudes representing the  $\mathcal{M}$  bubble multiplied by  $MB$  and  $K^+$  form factors: 6 regularization scales  $\beta_c$  (3 fixed), and  $\beta_K$
- Yamaguchi forms adopted for all form factors  $g_c(k^*) = 1/[1 + (k^*/\alpha_c)^2]$  etc.
- fitted data:** kaonic hydrogen characteristics,  $K^-p$  threshold branching ratios,  $K^-p$  reaction cross sections,  $\pi\Sigma$  photoproduction mass distributions at  $\sqrt{s} = 2.1$  GeV

CLAS data			$K^-p$ threshold		$K^-p$ cross sections								all
$\pi^0\Sigma^0$	$\pi^+\Sigma^-$	$\pi^-\Sigma^+$	atom	BRs	$\pi^0\Lambda$	$\pi^0\Sigma^0$	$\pi^+\Sigma^-$	$\pi^-\Sigma^+$	$K^-p$	$\bar{K}^0n$	$\eta\Lambda$	$\eta\Sigma^0$	total
34	30	32	2	3	4	4	31	32	27	22	24	7	252

$$\chi^2/\text{dof} = \frac{\sum_i N_i}{N_{\text{obs}}(\sum_i N_i - N_{\text{par}})} \sum_i \frac{\chi_i^2}{N_i}, \quad N_{\text{obs}} = 13, \quad N_{\text{par}} = 16$$

# New results - FSI not fixed

results for 4 selected solutions (local  $\chi^2$  minima):

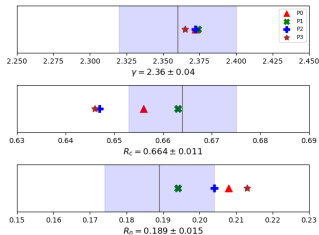
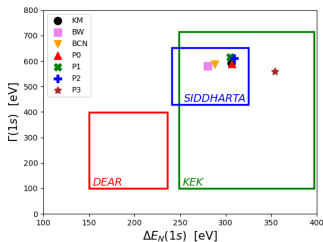
**P0**  $\chi^2/\text{dof} \approx 5.40$ , MB FSI sector fixed to the P model setting, 4 parameters

**P1**  $\chi^2/\text{dof} \approx 3.34$ , both, FSI and tree level photoproduction sectors varied, 16 par.

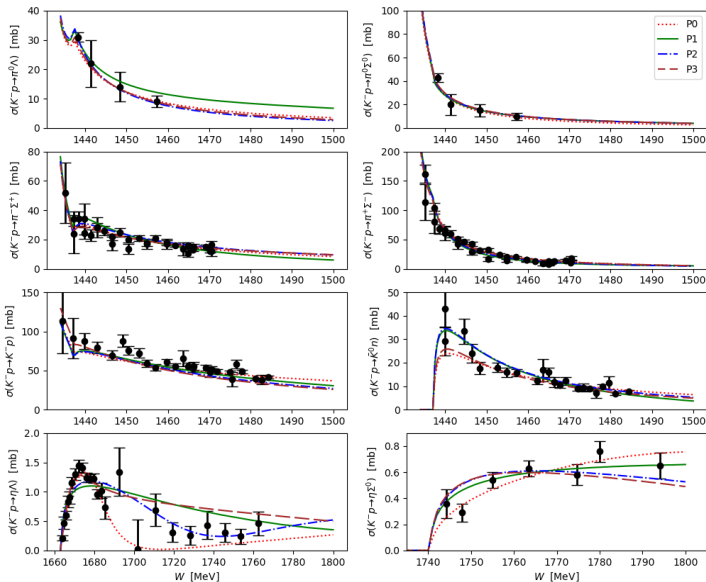
**P2**  $\chi^2/\text{dof} \approx 4.41$ , same as for P1

**P3**  $\chi^2/\text{dof} \approx 4.72$ , same as for P1

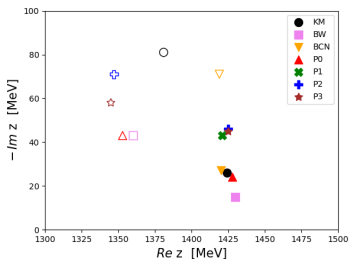
$K^- p$  threshold data:



# $K^-p \rightarrow MB$ total cross sections



# $\Lambda(1405)$ poles predictions

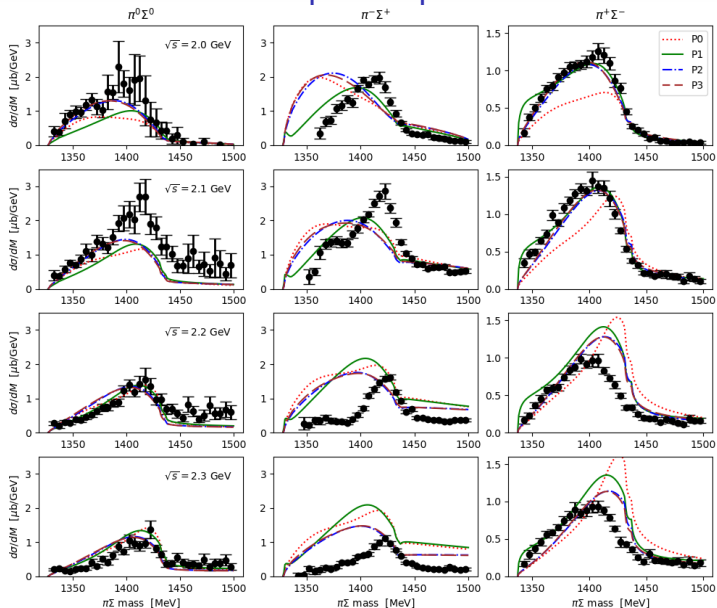


$\Lambda(1405) - z_1$  and  $z_2$ ;  $\Lambda(1670) - z_3$

model	$z_1$ [MeV]	$z_2$ [MeV]	$z_3$ [MeV]
P0	(1353,-43)	(1428,-24)	(1677,-14)
P1	—	(1421,-43)	—
P2	(1347,-71)	(1425,-46)	(1725,-57)
P3	(1345,-58)	(1425,-45)	(1665,-7.1)

- The **P1 model provides the best  $\chi^2$**  but was found **unphysical** due to generating an extremely narrow  $I = 1$  resonance close to  $\pi\Sigma$  threshold. It is also missing the *lower mass*  $\Lambda(1405)$  and the  $\Lambda(1670)$  poles.
- Apparently, the  $K^-p$  reaction (and to some extend threshold) **data are not very sensitive to the pole positions**. Similar observations by **Revai** (with a non-chiral model), **Shevchenko** (phenomenology), or **Anisovich** (partial wave analysis), all indicating **good reproduction of the  $K^-p$  data with one-pole  $\Lambda(1405)$  models**.
- All our models agree on the position of the **higher mass  $\Lambda(1405)$  pole with  $\text{Im } z_2$  much larger than in the fits based merely on the  $K^-p$  reactions data**. The *lower mass*  $\Lambda(1405)$  pole seems to be constrained around  $z_1 \approx (1350, -60)$  MeV. **Model dependence should be checked!**

# $\pi\Sigma$ mass spectra predictions



## $\pi\Sigma$ mass spectra predictions

- Despite being rather simple, the model reproduces reasonably well the  $\pi^0\Sigma^0$  and  $\pi^+\Sigma^-$  mass distributions.
- The energy dependence seems to be under control thanks to introducing the  $g_{K^+}$  form-factor in the tree level photoproduction amplitudes.
- Our model fails to reproduce the  $\pi^-\Sigma^+$  mass distributions.
- The applied  $\pi\Sigma$  photoproduction formalism may not be realistic: *p-waves, vector meson contributions?*. Thus, the results should be taken with caution and may change once a more advanced photo-kernel is implemented.
- our current work: implementing vector meson ( $K^*$ ) contributions in the photoproduction kernel (preliminary results seem promising)
- Much better agreement with CLAS photoproduction data can be achieved either by an ad-hoc modelling of the photoproduction dynamics (Roca, Oset, Mai, Meißner) or by employing much more parameters, and keeping some of them energy dependent, e.g. Nakamura, Jido (2014) have 20 subtraction constants, 15 energy dependent complex couplings plus common form factor scale  $\beta_c = \beta_{K^+} = \Lambda$  (51 real parameters in total!)

# Summary

- The up-to-date (NLO) chirally motivated  $\pi\Sigma - \bar{K}N$  models provide very different predictions for the  $MB$  amplitudes at energies below  $\bar{K}N$  threshold. The positions of the  $\Lambda(1405)$  poles are also not much restricted by the  $K^-p$  reactions data.
- Our approach to the two-meson photoproduction implements **coupled-channel unitarity, low-energy theorems from ChPT and gauge invariance**. We have revealed **large variations when different models for the  $MB$  amplitudes are adopted**.
- Our new results of **fits that combine the  $K^-p$  reactions data with those on the  $\pi\Sigma$  photoproduction** can achieve a reasonable  $\chi^2/\text{dof}$  but are not satisfactory especially for the  $\pi^-\Sigma^+$  mass distributions.
- The presented models tend to limit the mass of the *lower mass*  **$\Lambda(1405)$**  pole and yield a **larger width of the pole that couples more strongly to  $\bar{K}N$** . A possible relation to a large absorption width found for the  $\bar{K}NN$  bound state at J-PARC?
- **We need a more precise experimental data on low energy  $K^-p$  reactions** as the current ones are not too restrictive on the theoretical models. **Our model** (the photoproduction kernel) **should also be improved by implementing the vector mesons** (work in progress).

# EXTRA: Comparison with other approaches

our photoproduction amplitude constructed from four  $\mathcal{A}_{0+}^i$  amplitudes

$$[\mathcal{A}_{0+}^i(s, M_{\pi\Sigma})] = [\mathcal{A}_{0+}^{i(\text{tree})}(s, M_{\pi\Sigma})] + [f_{0+}(M_{\pi\Sigma})] [8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})] [\mathcal{A}_{0+}^{i(\text{tree})}(s, M_{\pi\Sigma})]$$

L. Roca, E. Oset - Phys. Rev. C 87 (2013) 055201

M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30

makeshift photoproduction amplitude  $[\mathcal{A}] = [f_{0+}] [8\pi M_{\pi\Sigma} G(M_{\pi\Sigma})] [C(\sqrt{s})]$

S.X. Nakamura, D. Jido - PTEP 2014 (2014) 023D01

similar to our approach with some non-relativistic simplifications, additional contributions from  $K^*$  exchange, phenomenological energy dependent contact terms, and adjustments to the first loop function and to the photoproduction vertex

E. Wang et al. - Phys. Rev. C 95 (2017) 015205

focus on triangle singularity contribution  $\gamma p \rightarrow N^*(2030) \rightarrow K^* \Sigma \rightarrow K^+ \Lambda(1405)$   
combined with  $K$ ,  $K^*$  meson exchanges and a contact term

$$\mathcal{A} = \mathcal{A}^{\text{tree}} = a t_{\text{triangle}} + b t_{K \text{ exchange}} + c t_{K^* \text{ exchange}} + d t_{\text{contact}}$$

All these fit a good number of model parameters to reproduce the CLAS data.

In contrast, we just demonstrated what can be achieved with **just four new parameters** (originally parameter-free!) approach based on (unitarized) ChPT.