Scalar and tensor charmonium resonances from lattice QCD

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hadspec.org

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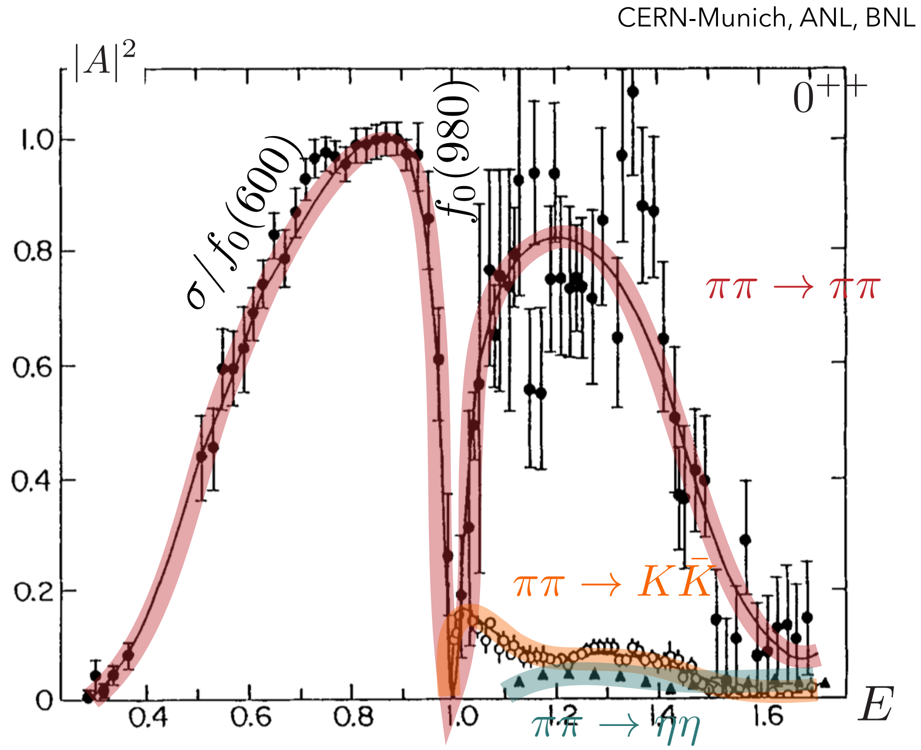
based on work:
arXiv: 2309.14070 (7 pages)
arXiv: 2309.14071 (55 pages)

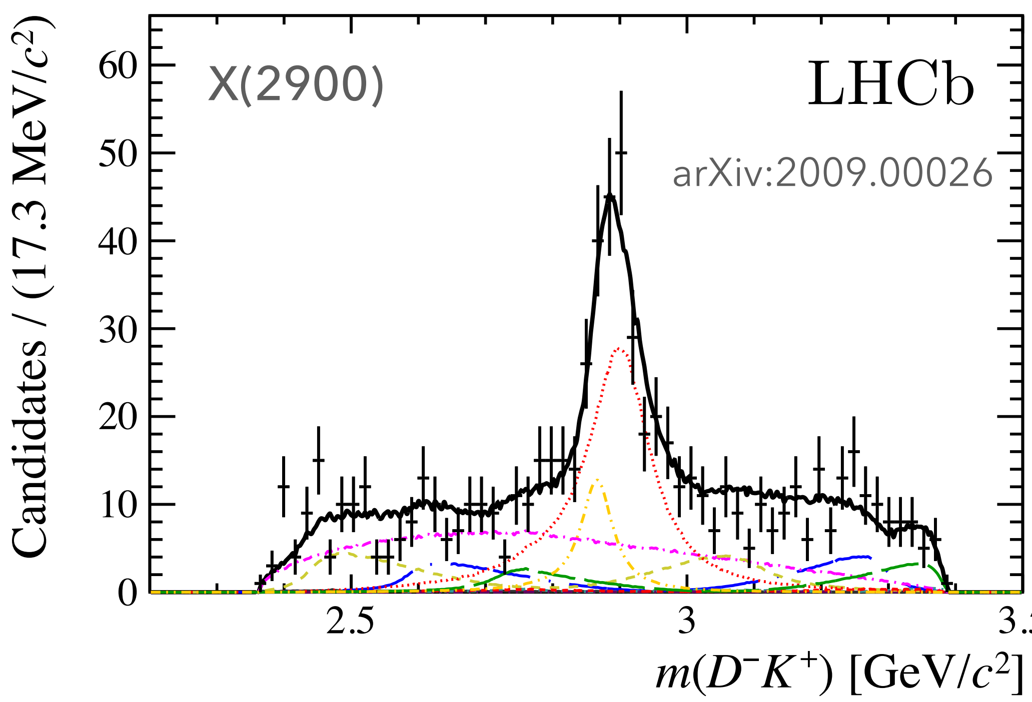
UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

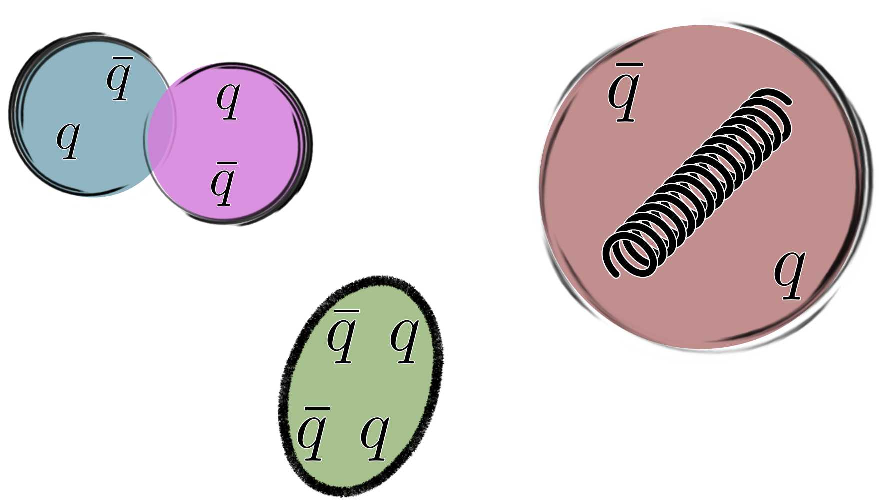
spectroscopy from first-principles is a hard problem



many puzzles - new and old



the quark model is a good guide for low-lying states



models are useful, but what does **QCD** say?

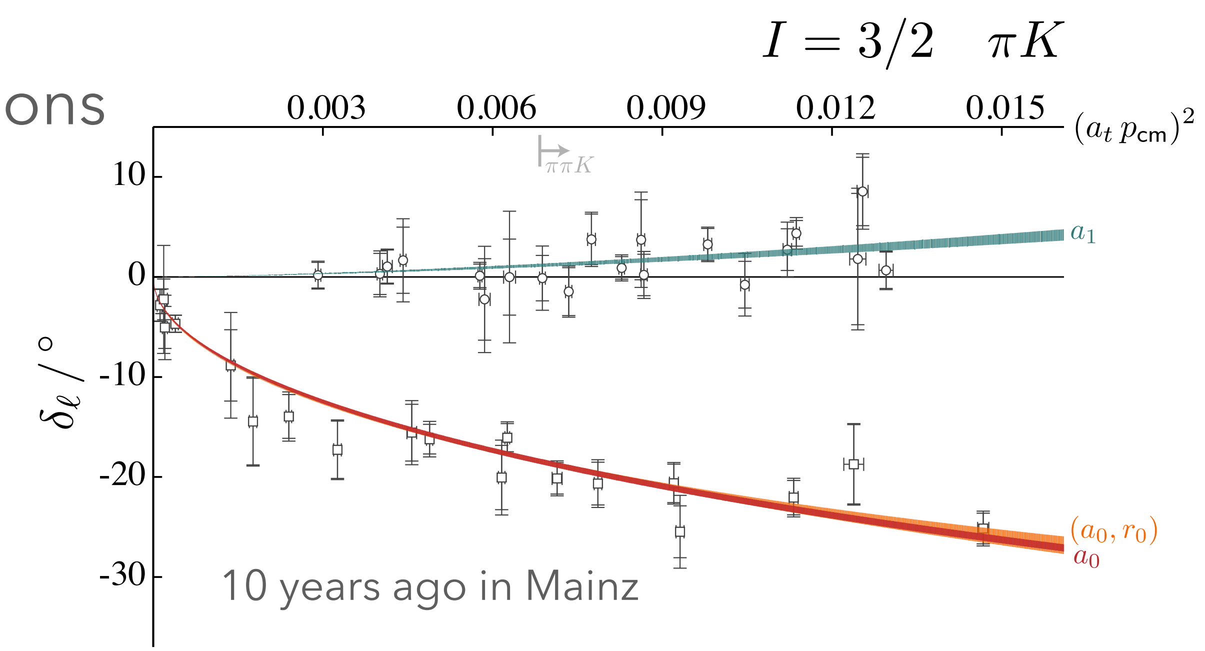
Lattice QCD provides a rigorous approach to hadron spectroscopy

- as **rigorous** as possible
- **all** necessary **QCD** diagrams are computed
- **excited states** appear as **unstable resonances** in a scattering amplitude

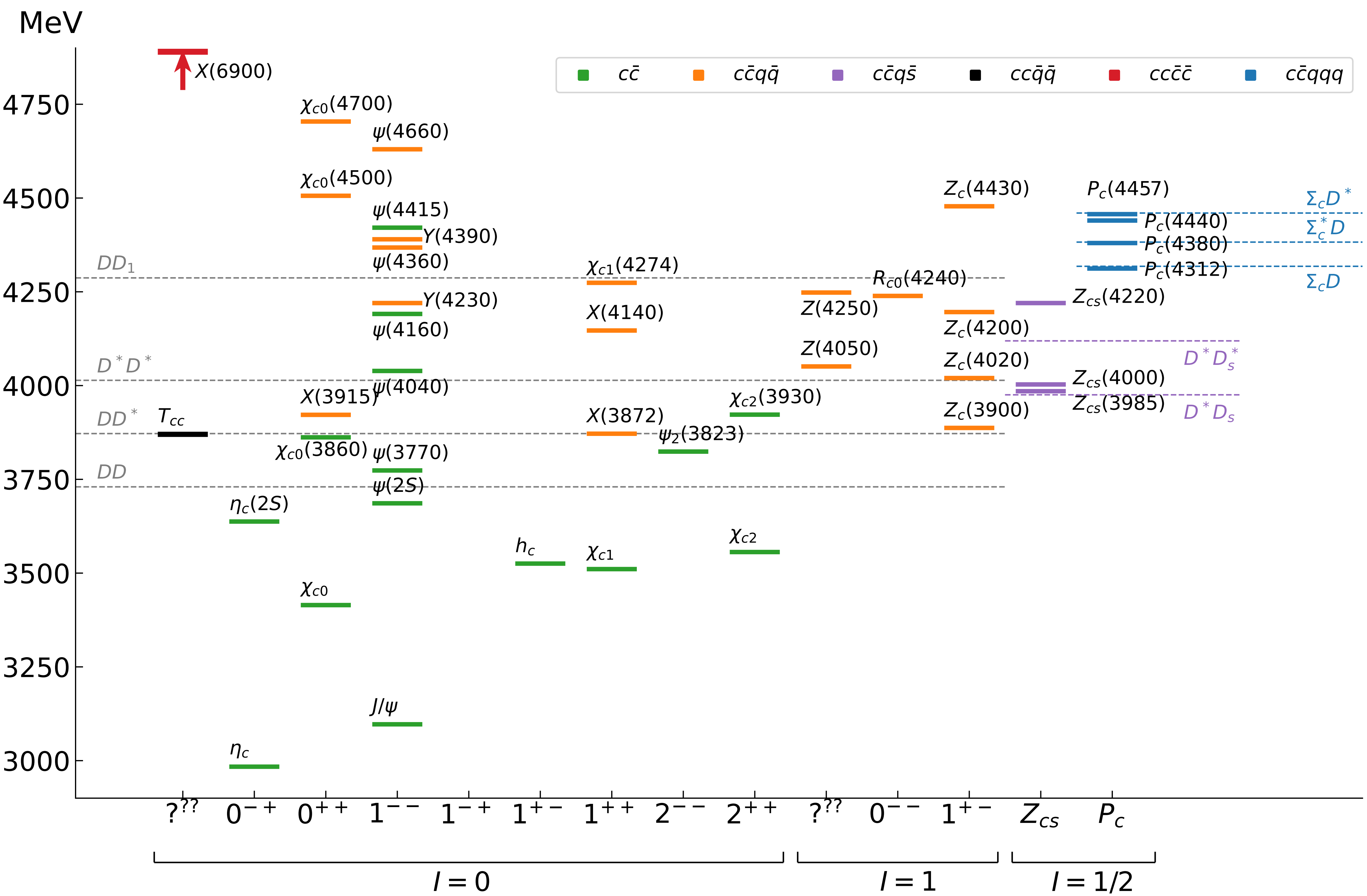
tremendous progress in recent years
but not yet ready for precision comparisons

- physical pions are very light
- most interesting states can decay to **many** pions
- control of light-quark mass is a useful tool
- small effects not considered in general:

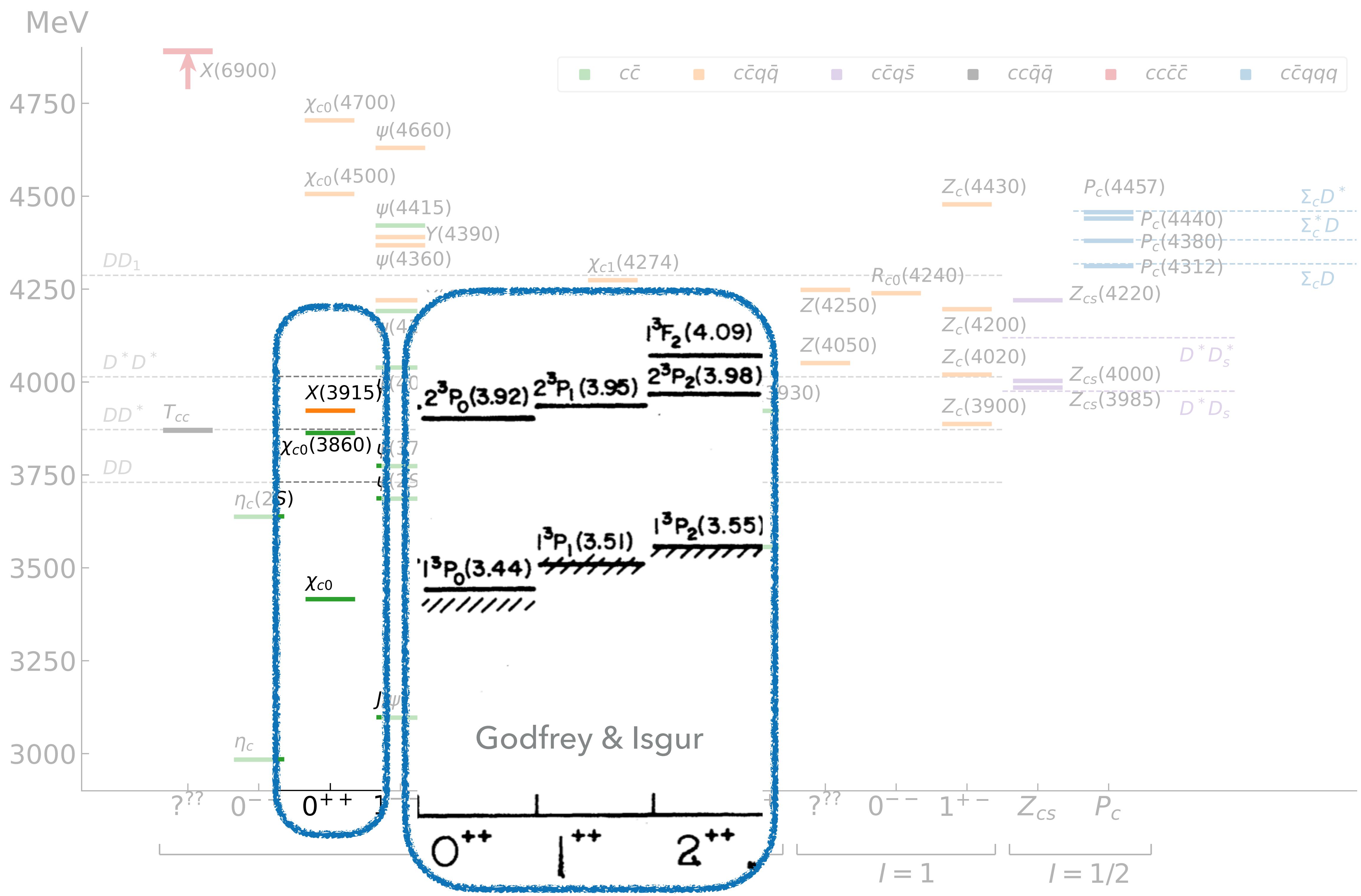
finite lattice spacing, isospin breaking, EM interactions



goal: what does **QCD** say about the excited hadron spectrum?



JPAC arXiv:2112.13436



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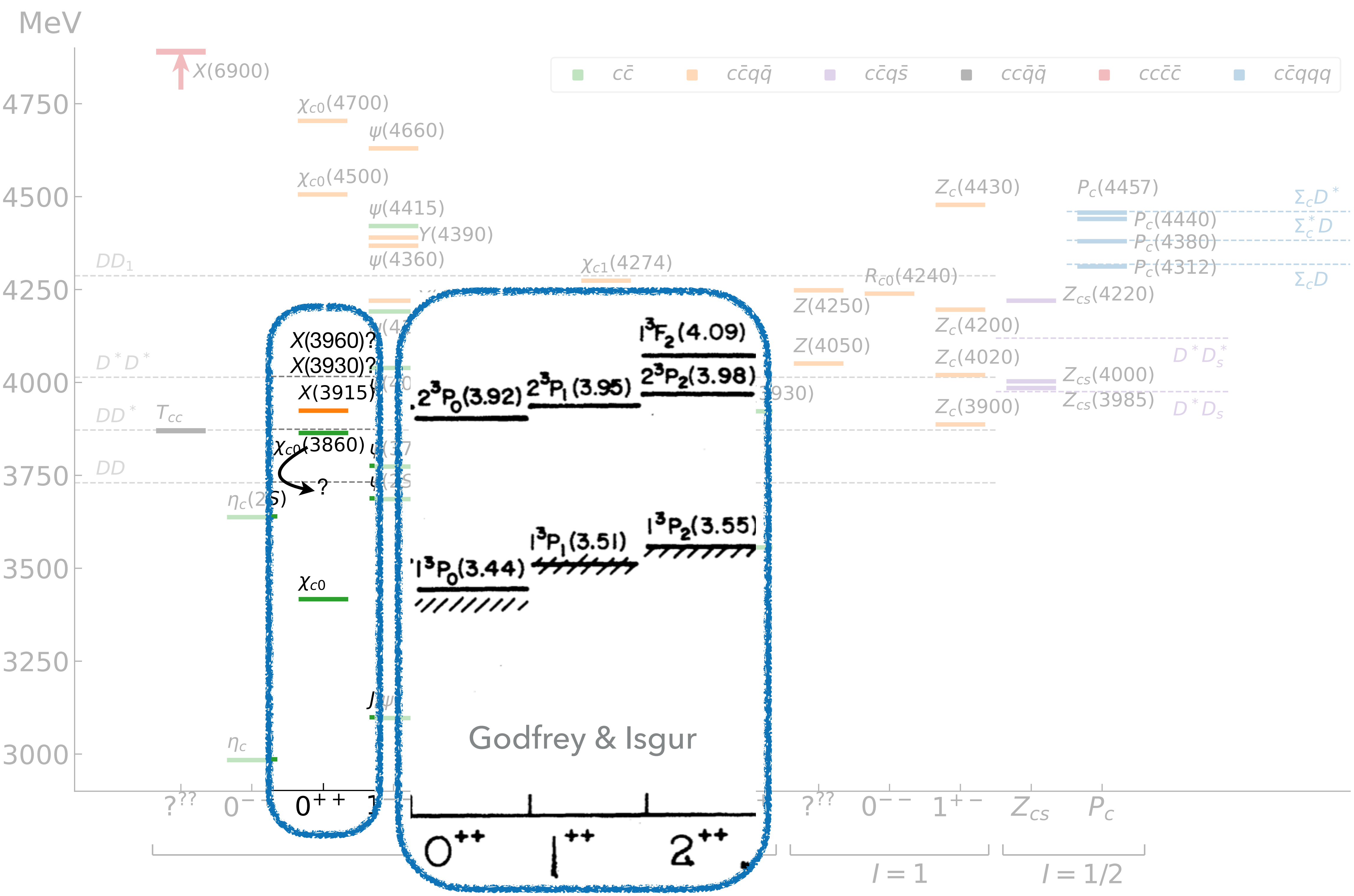
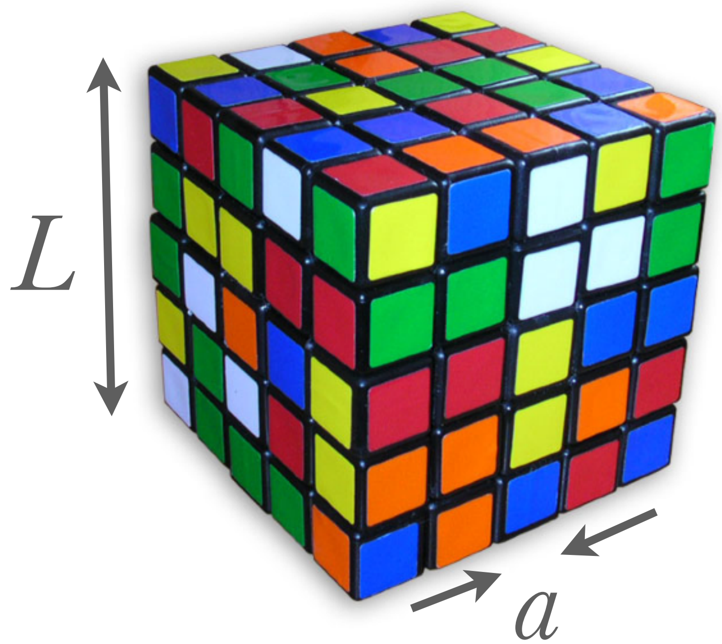


JPAC arXiv:2112.13436



anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16, 20, 24
m_π = 391 MeV

rest and moving frames

N_f = 2+1 flavours
all light+strange annihilations included
no charm annihilation

operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$ local qq-like constructions

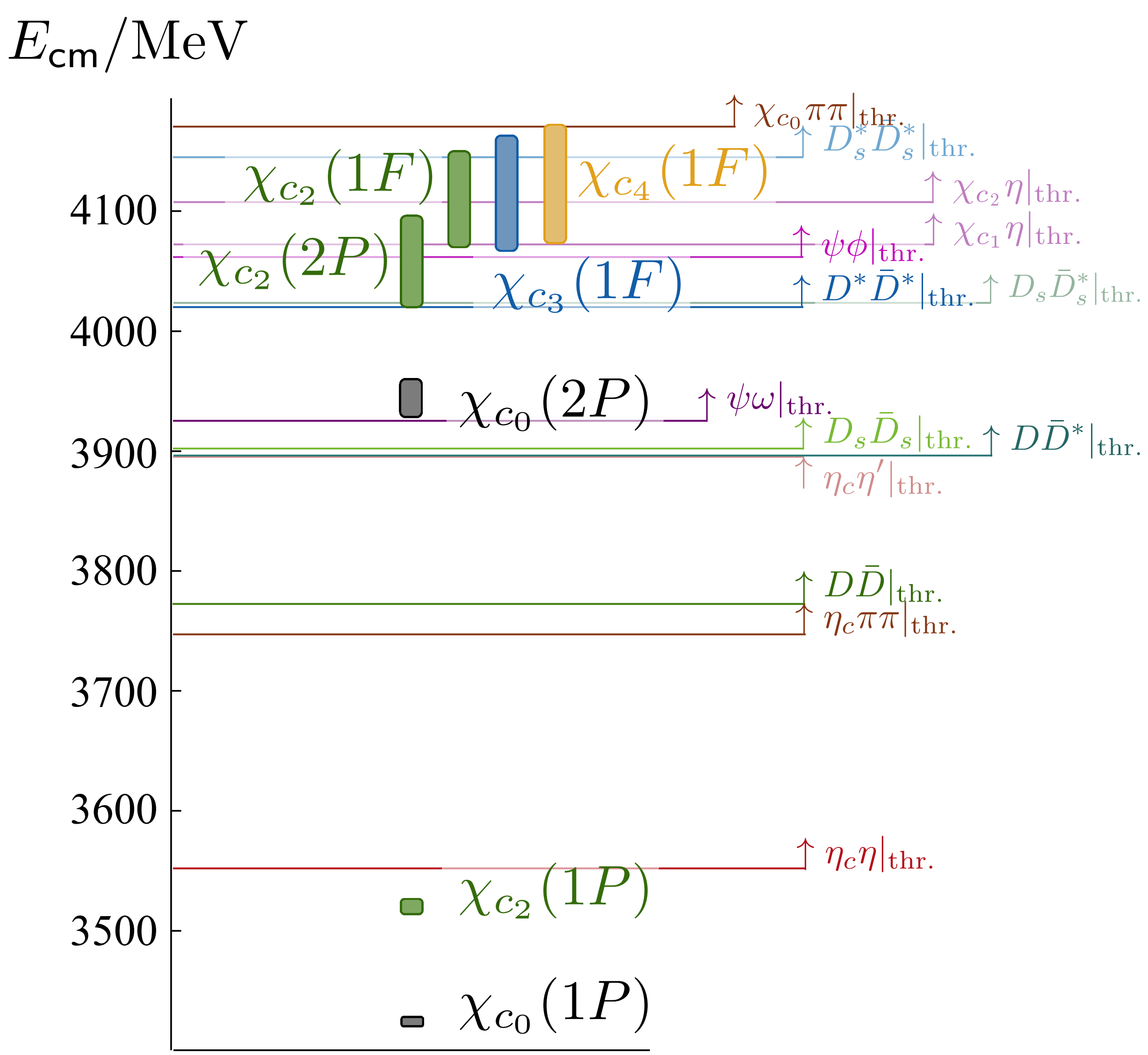
$$\sum_{\vec{p}_1+\vec{p}_2 \in \vec{p}} C(\vec{p}_1, \vec{p}_2; \vec{p}) \Omega_\pi(\vec{p}_1)\, \Omega_\pi(\vec{p}_2)$$ 2&3-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$ uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
many channels, many wick contractions

- compute a large correlation matrix
- solve generalised eigenvalue problem to extract energies

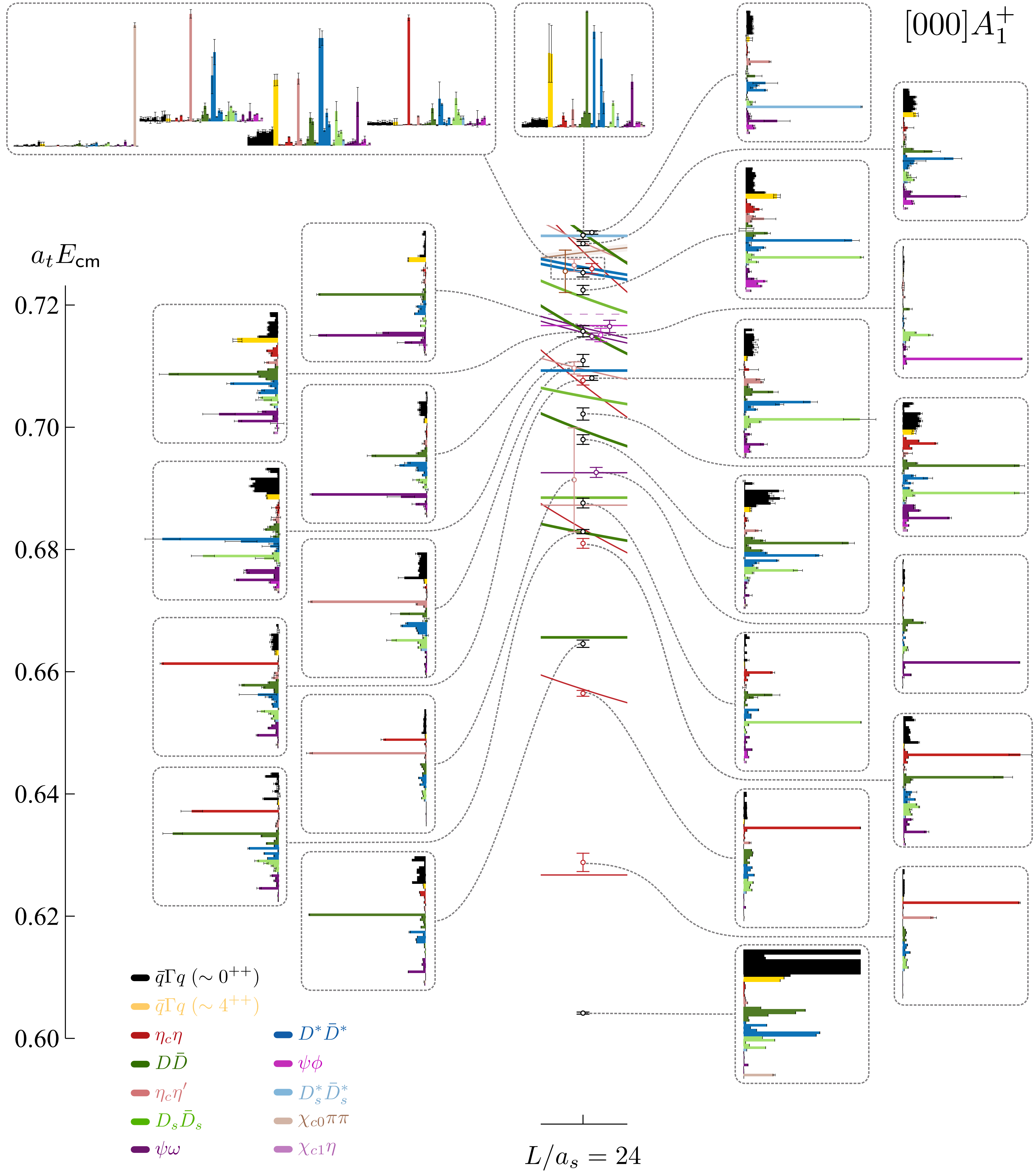
χ_{c0} & χ_{c2}

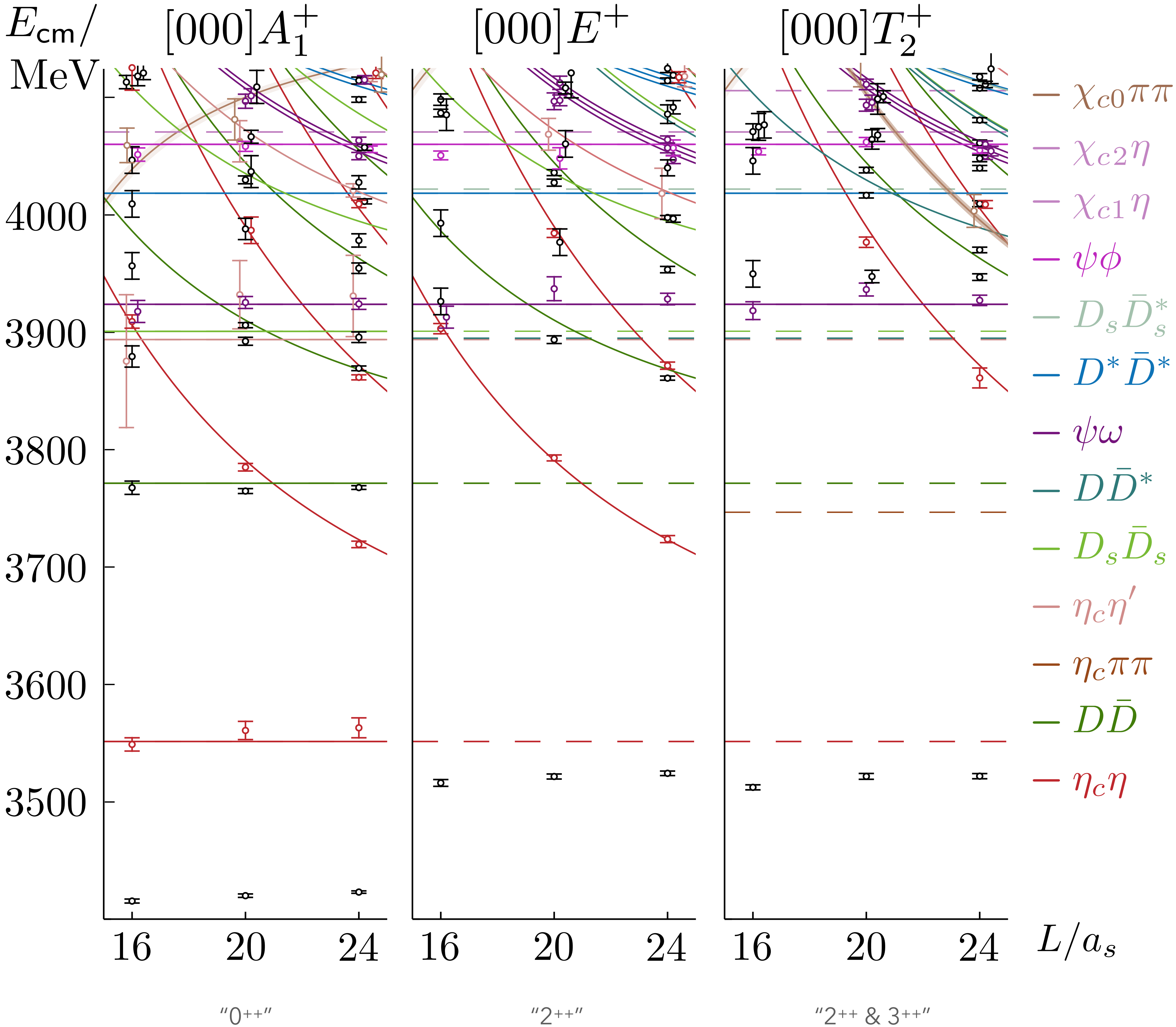


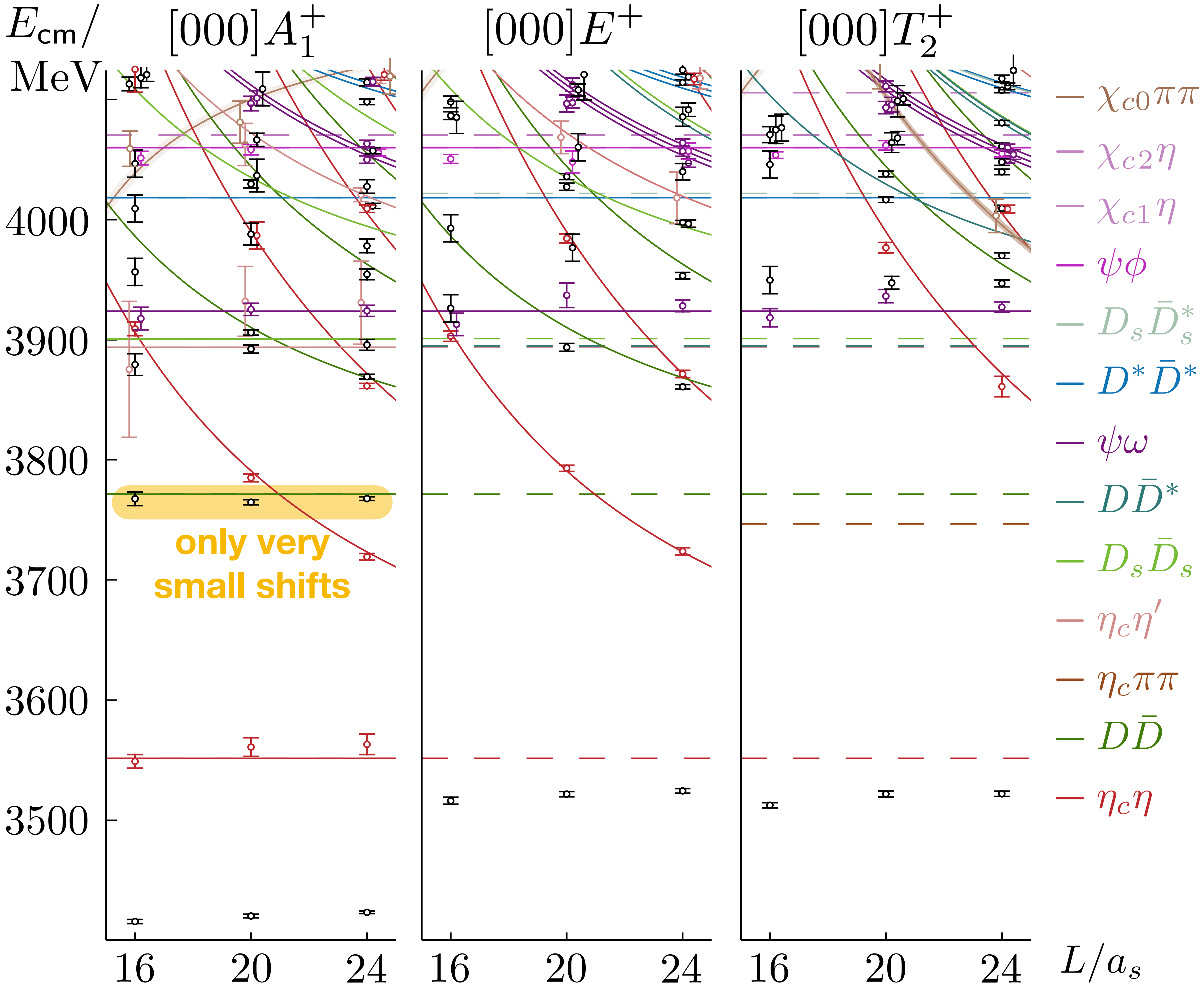
- spectra from qqbar operators only, Liu et al JHEP 1207 (2012) 126

- indicates energy regions where resonance effects are likely

- add meson-meson operators

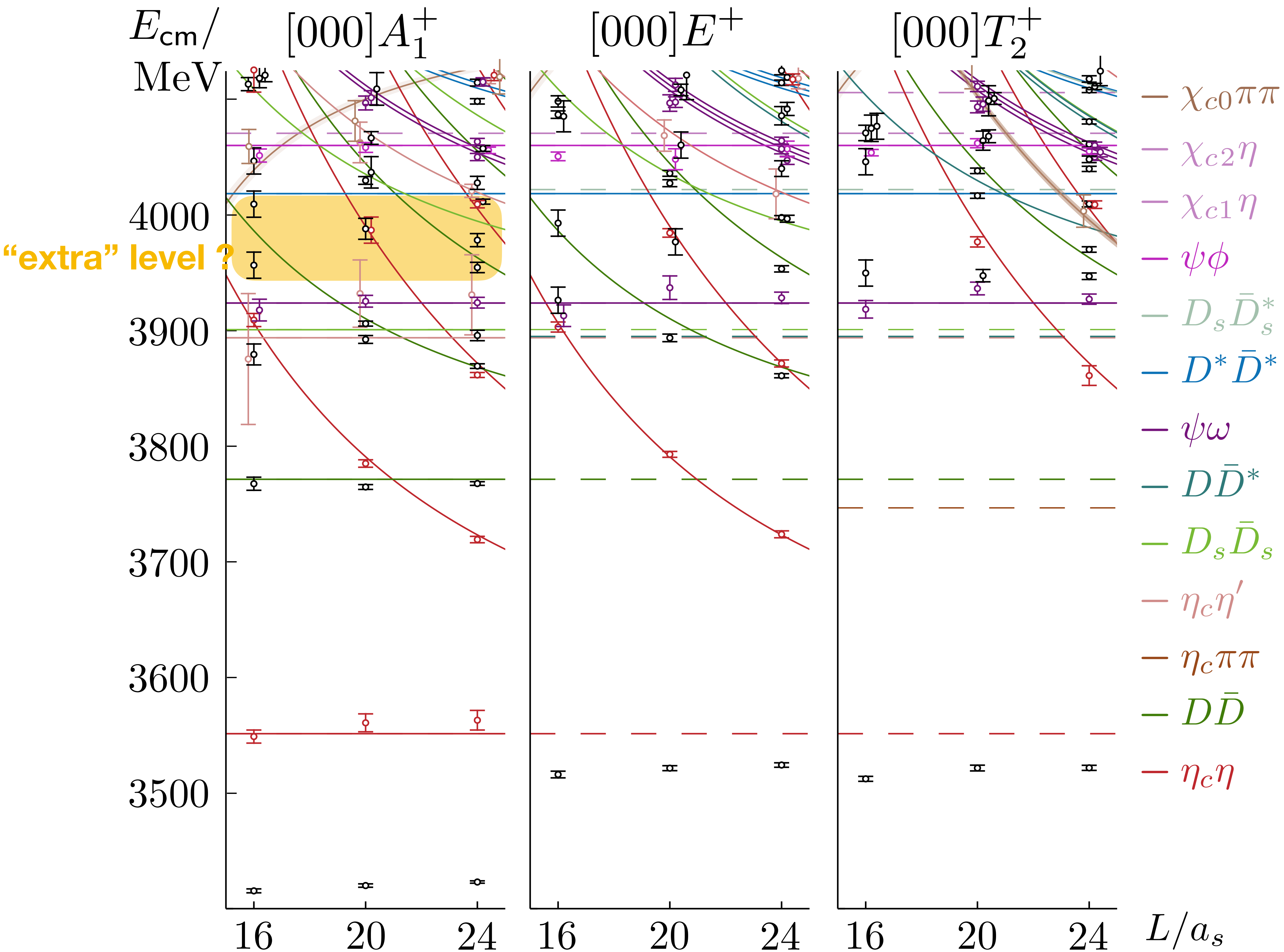

$[000]A_1^+$
$a_t E_{\mathsf{cm}}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$
$L/a_s = 24$


$E_{\mathsf{cm}}/$ MeV
$[000]\,A_1^+$
$[000]\,E^+$
$[000]\,T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



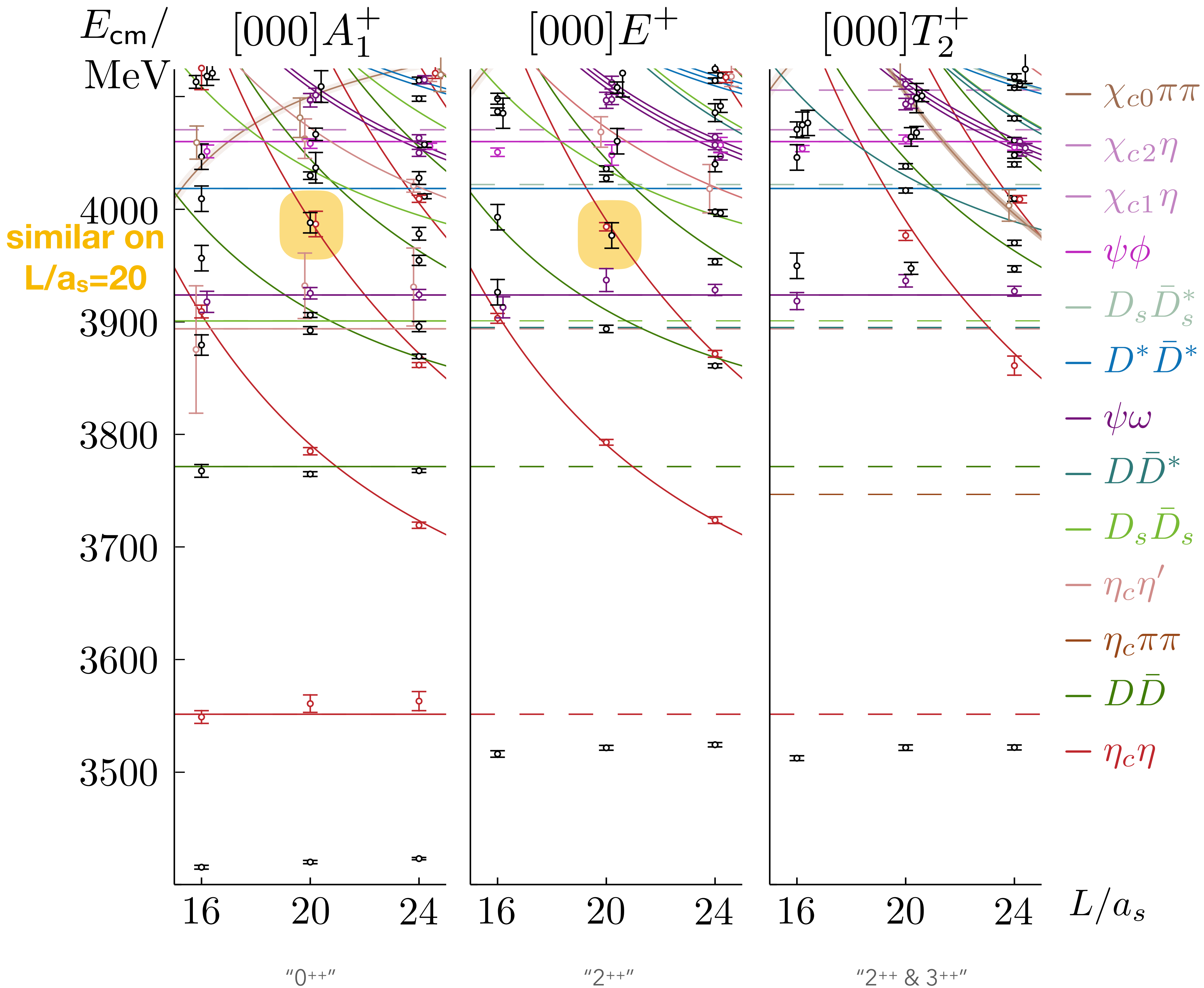
E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
only very small shifts

"0++"   "2++"   "2++ & 3++"

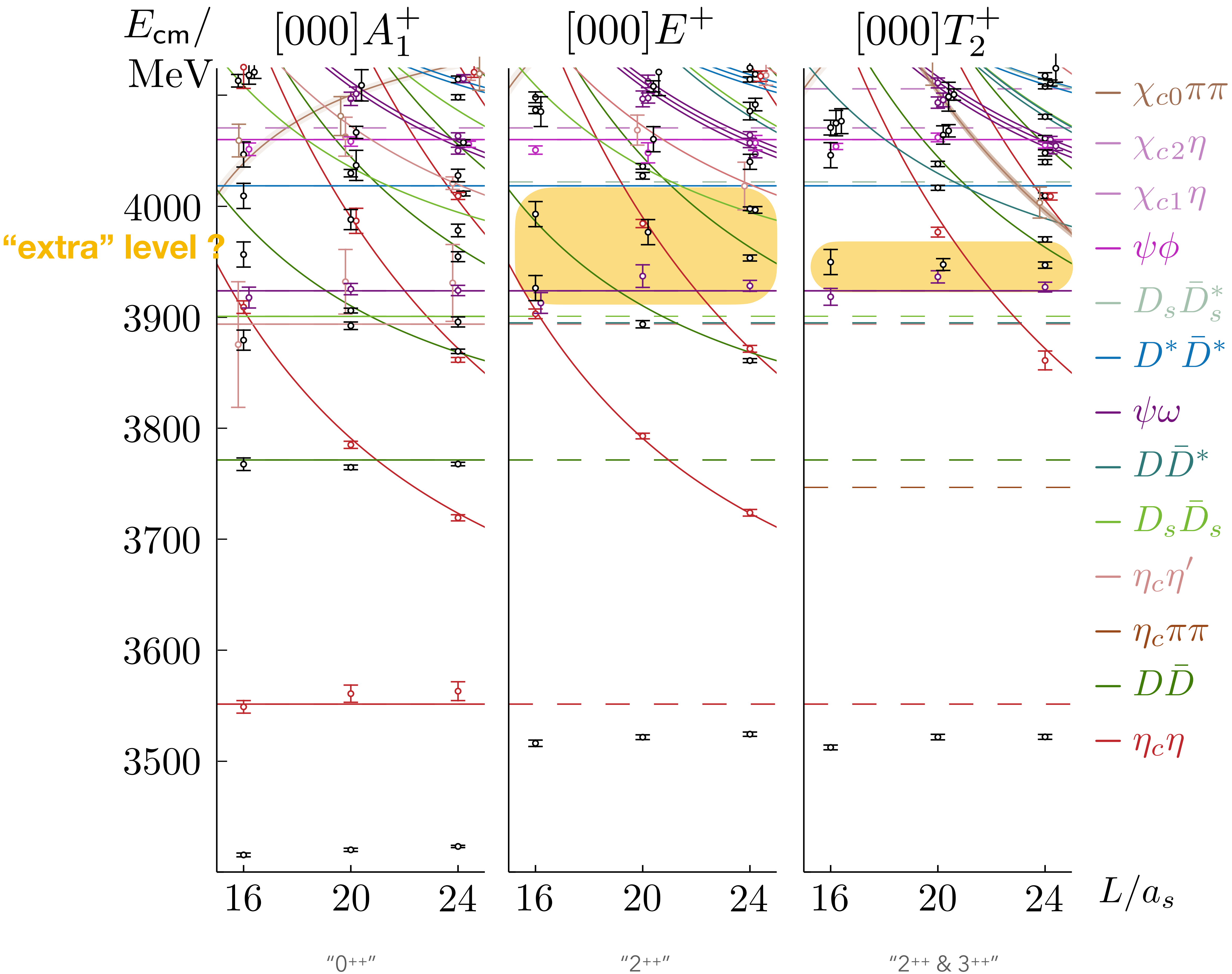


$E_{\mathsf{cm}}/$ MeV
$[000]\,A_1^+$
$[000]\,E^+$
$[000]\,T_2^+$
"extra" level ?
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

"0++" "2++" "2++ & 3++"



$E_{\mathsf{cm}}/\mathrm{MeV}$
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
similar on L/a$_s$=20
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



$E_{\mathrm{cm}}/\mathrm{MeV}$
$[000]\,A_1^+$
$[000]\,E^+$
$[000]\,T_2^+$
"extra" level ?
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"

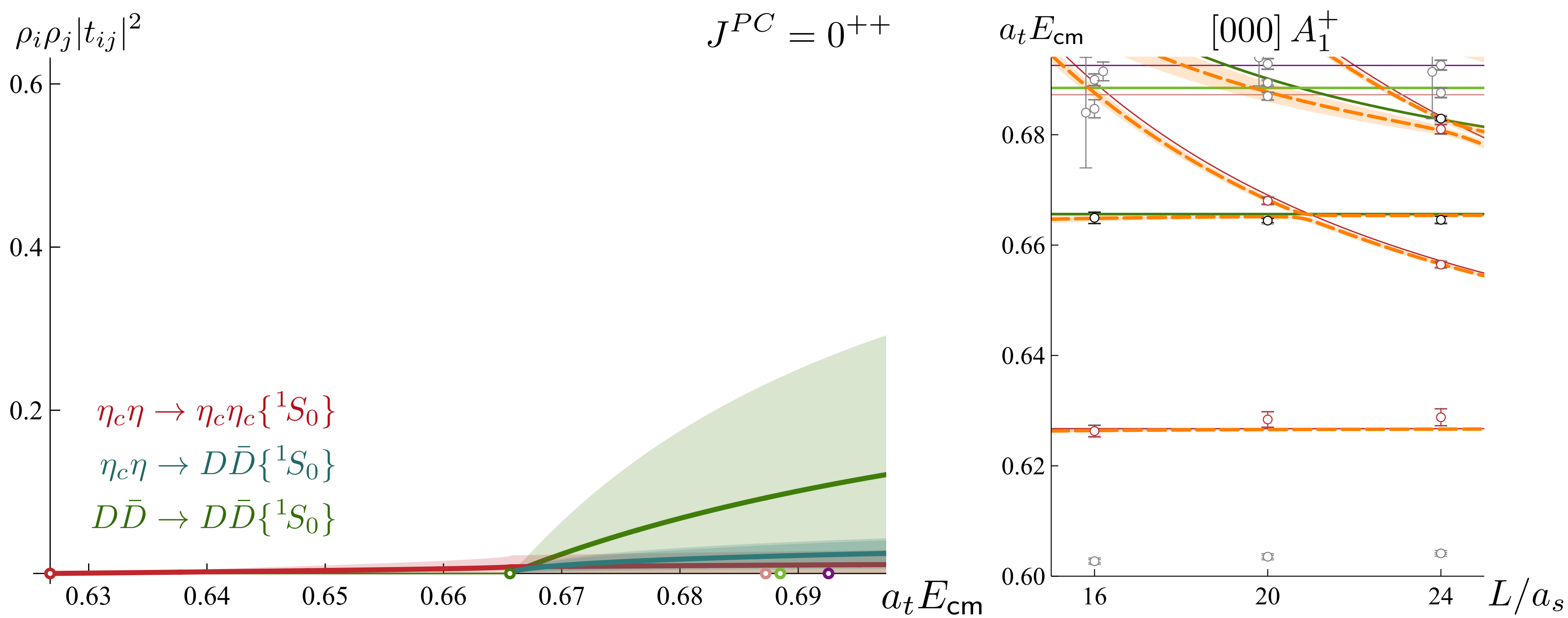
$$\boldsymbol{S} = \mathbf{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}} \cdot \boldsymbol{t} \cdot \boldsymbol{\rho}^{\frac{1}{2}}$$

$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\mathbf{1} + i\boldsymbol{\rho} \cdot \boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

$$\boldsymbol{K} = \begin{bmatrix} \gamma_{\eta_c\eta\to\eta_c\eta} & \gamma_{\eta_c\eta\to D\bar{D}} \\ \gamma_{\eta_c\eta\to D\bar{D}} & \gamma_{D\bar{D}\to D\bar{D}} \end{bmatrix}$$



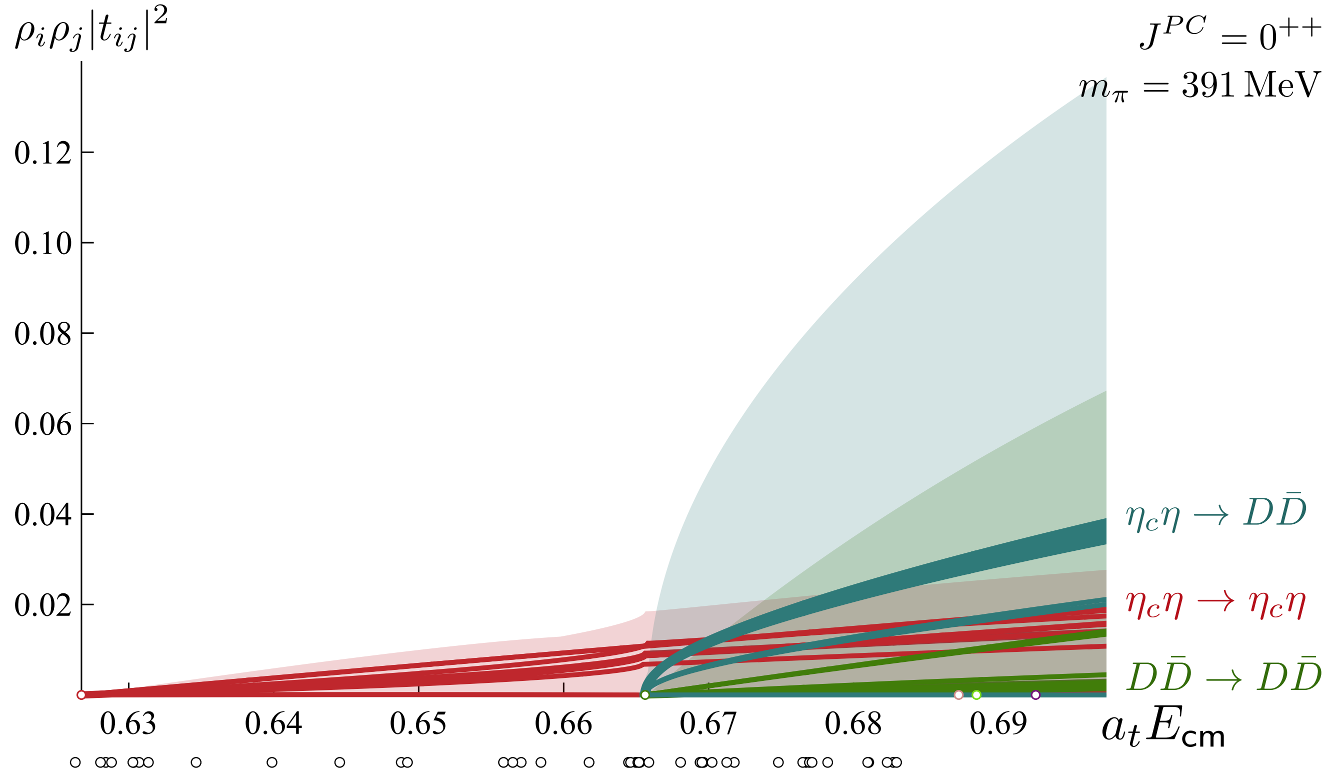
using rest-frame only

$$\begin{aligned} \gamma_{\eta_c\eta\to\eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta\to D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D}\to D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \qquad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

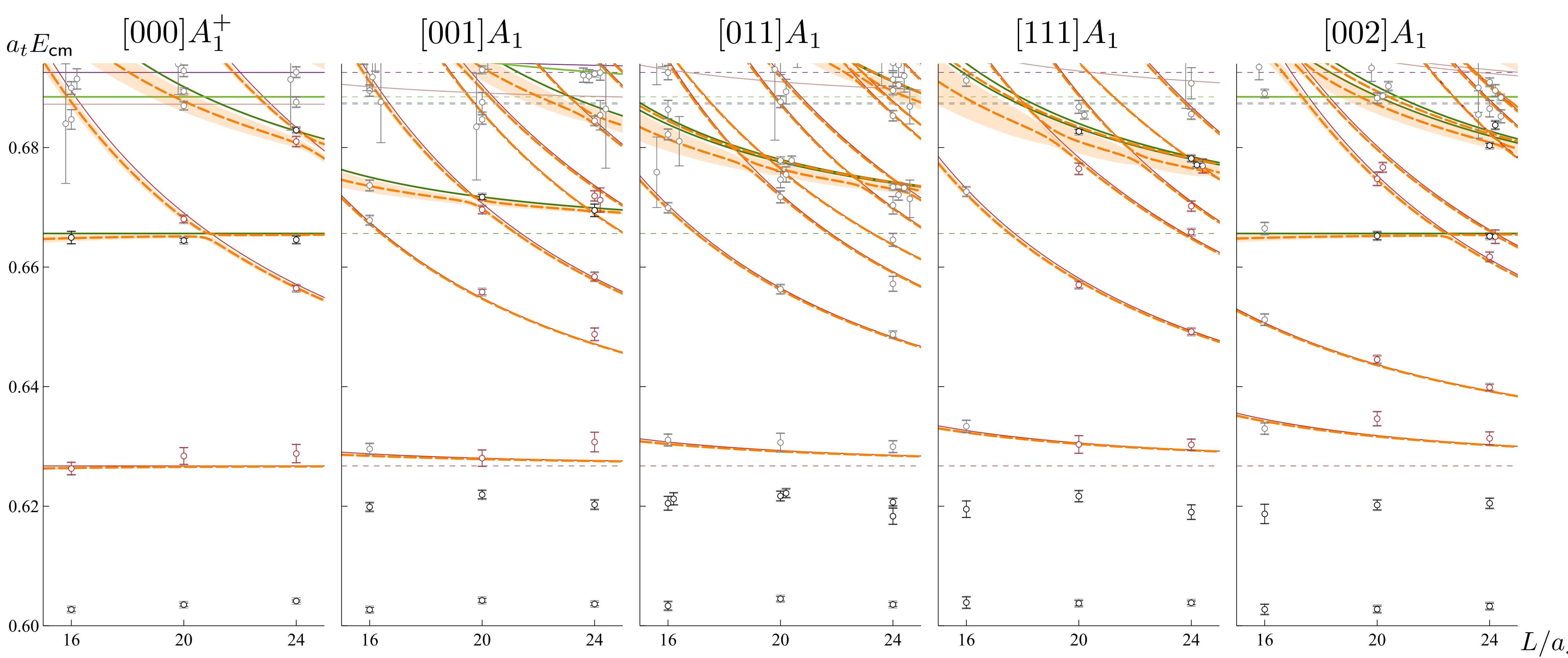
$$\chi^2/N_{\mathrm{dof}} = \frac{5.65}{10-3} = 0.81$$

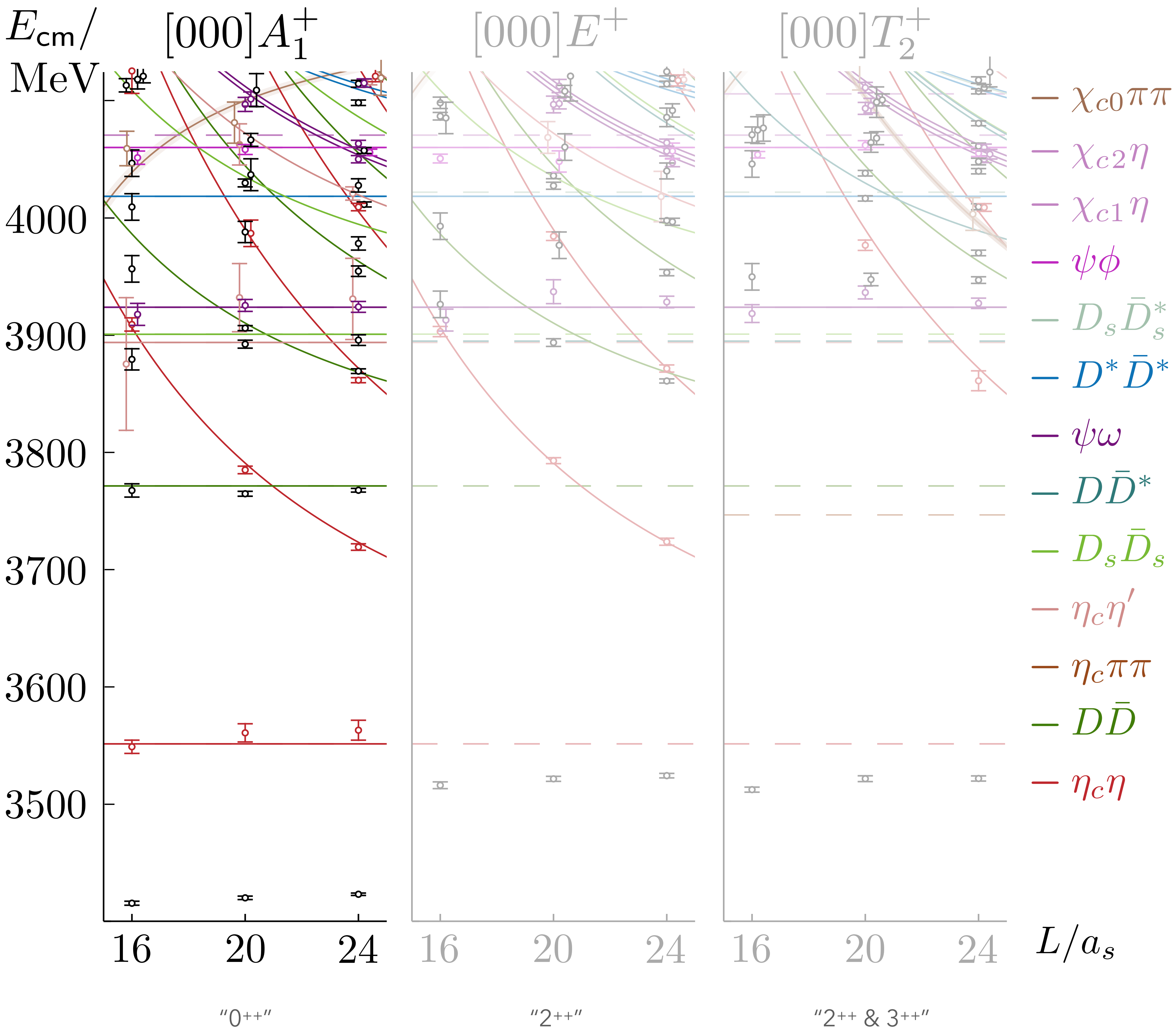
what's going on a DDbar threshold?

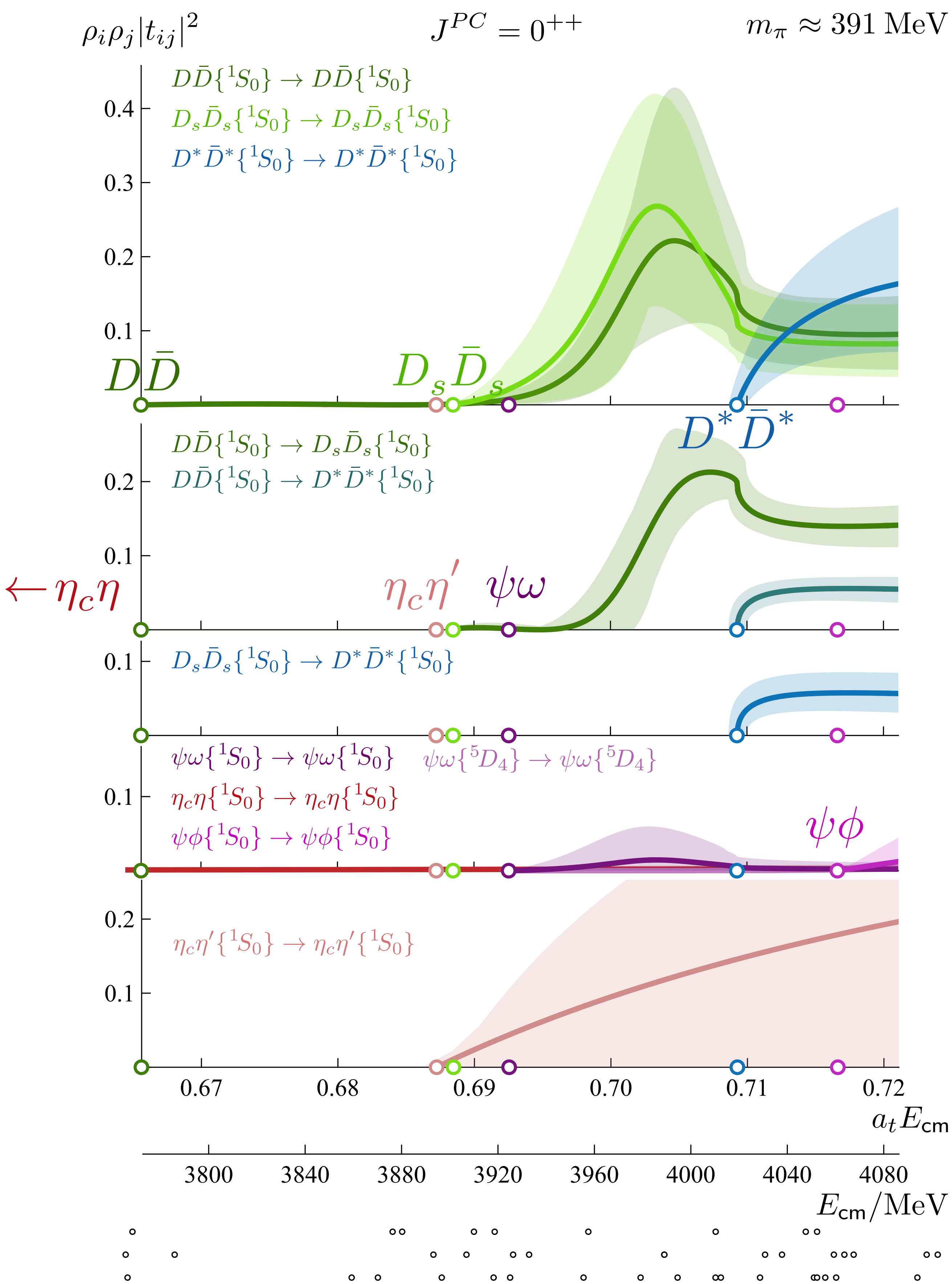
(smaller range)



using zero and non-zero total momentum




$E_{\mathsf{cm}}/$ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



three channels open close together: $\eta_c\eta'$, $D_s\bar{D}_s$, $\psi\omega$

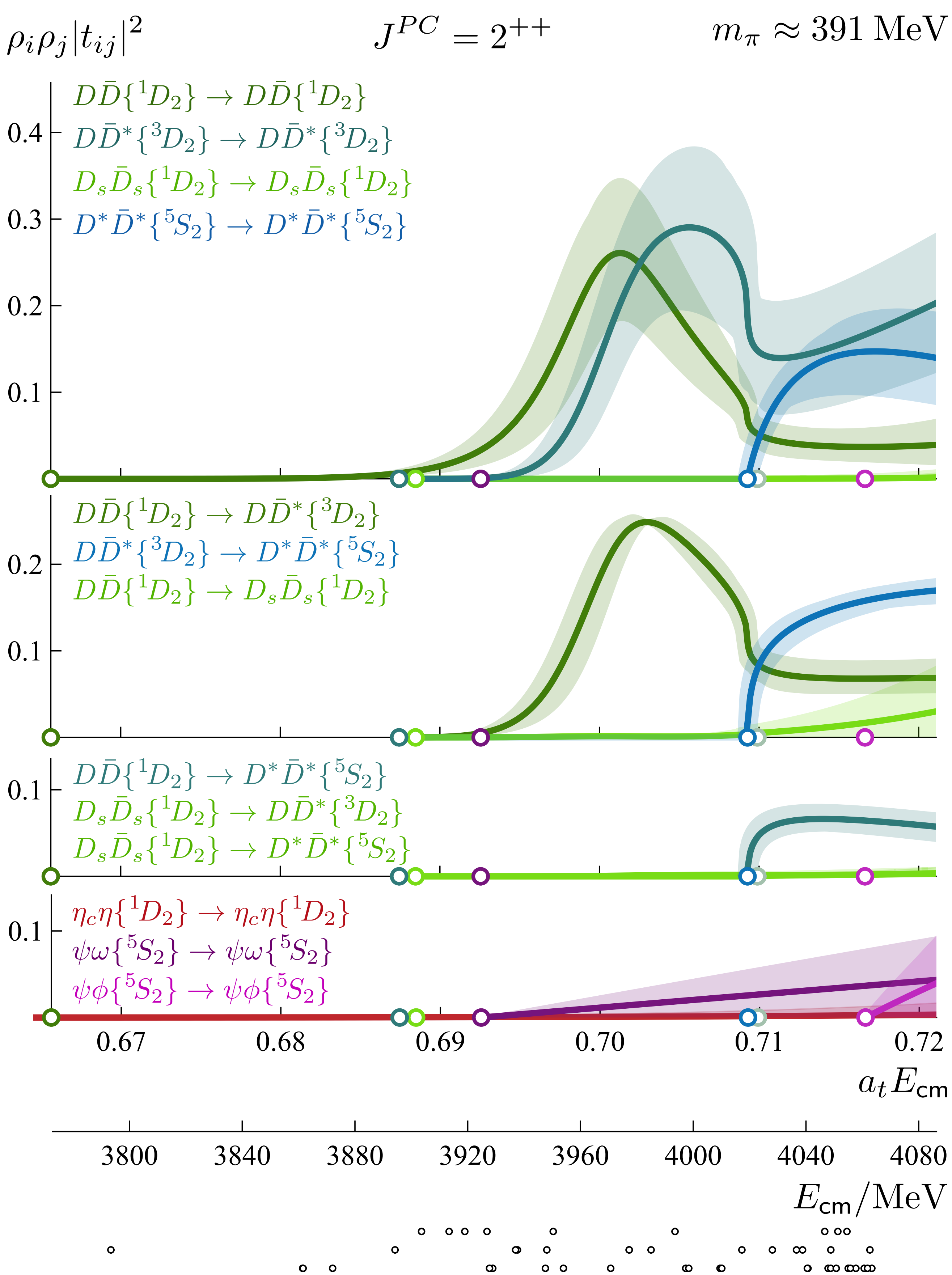
operator overlaps suggest $D^*\bar{D}^*$ is important

$\psi\phi$ has been seen to be important in some places

consider 7-channel system

$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

K-matrix pole terms become necessary to obtain a good quality of fit



7-channels, mixture of *S* and *D*

$D\bar{D}, D_s\bar{D}_s\{^1D_2\}$ $D\bar{D}^*\{^3D_2\}$ $D^*\bar{D}^*\{^5S_2\}$

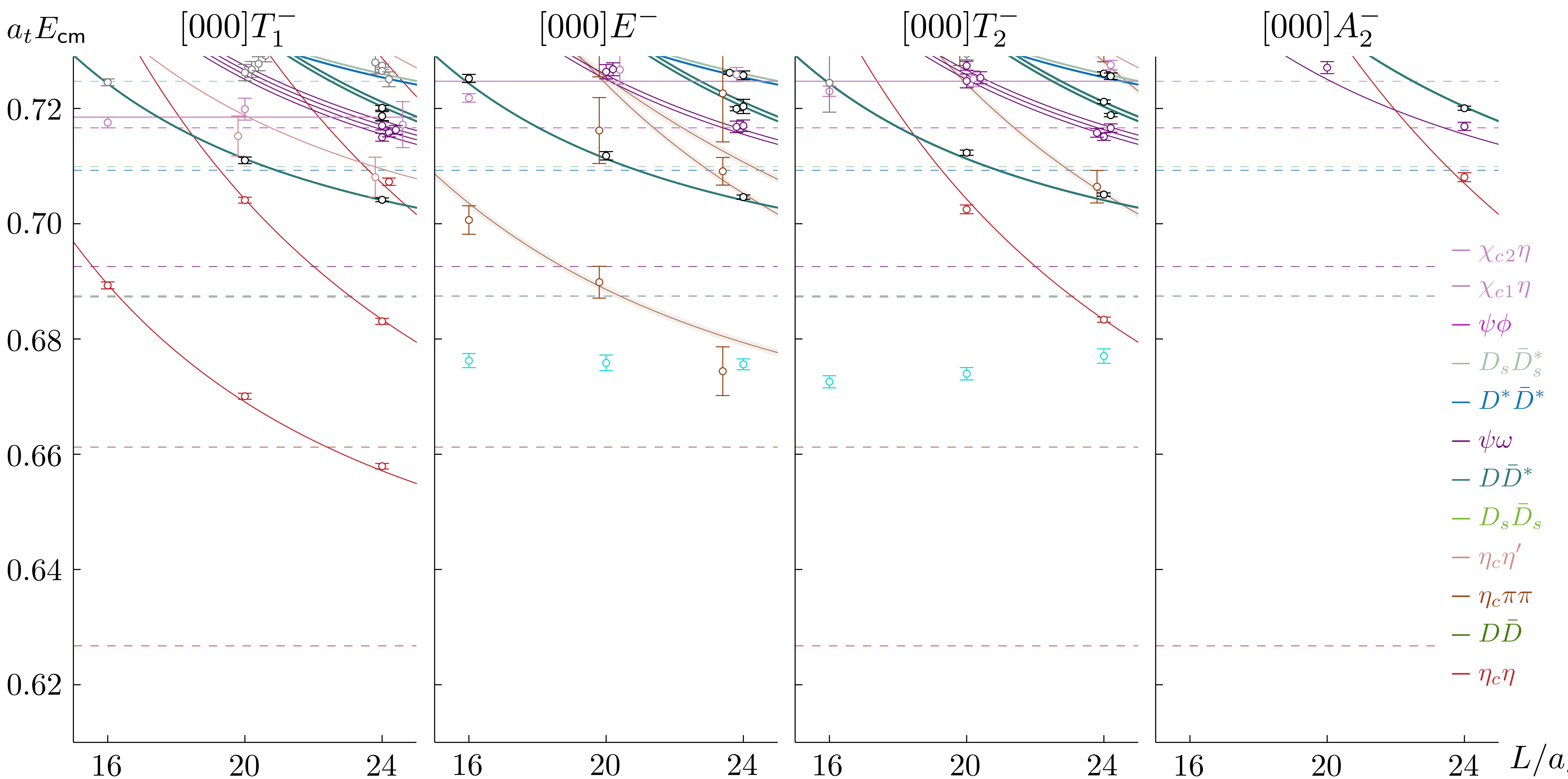
$\eta_c\eta\{^1D_2\}$ $\psi\omega, \psi\phi\{^5S_2\}$

peaks at a similar energy

very small DsDs amplitudes -
some phase space suppression

DD* is large -
similar phase space to DsDs

"background" waves - P=-

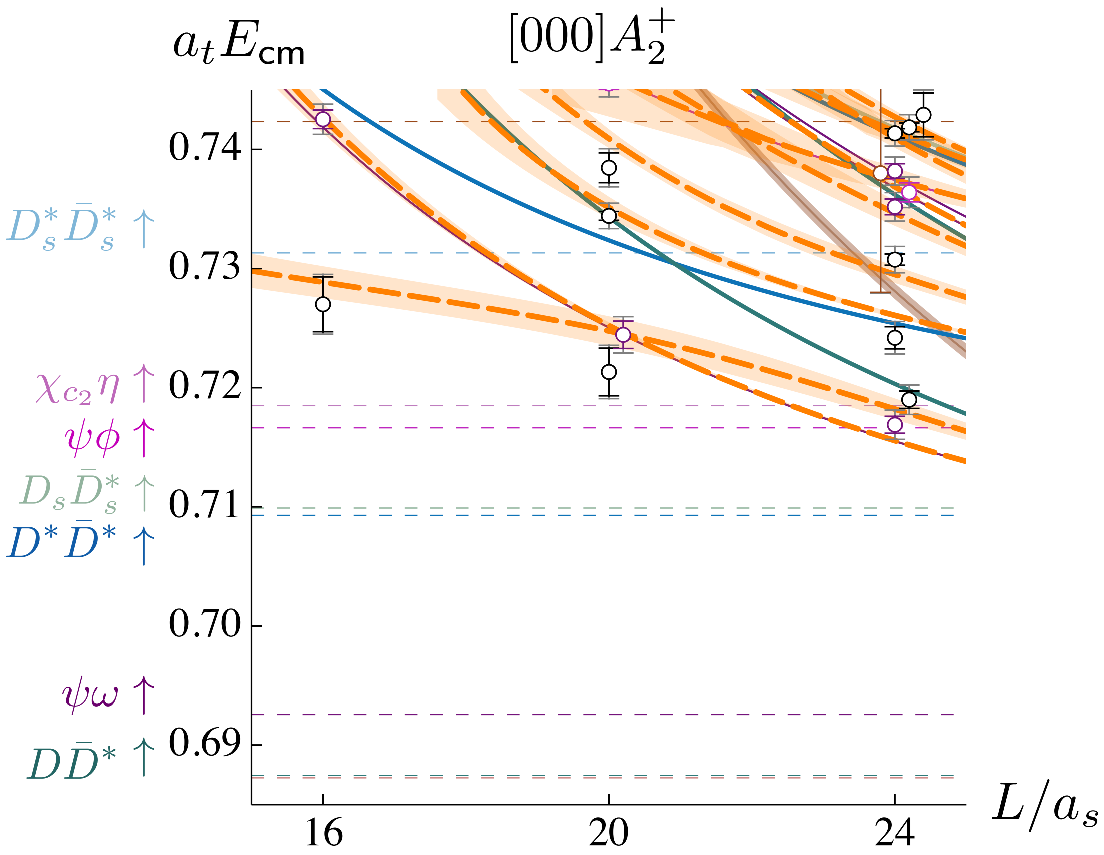


we also computed lattice irreps
with non-zero total momentum

P=- partial waves can then contribute

very little going on

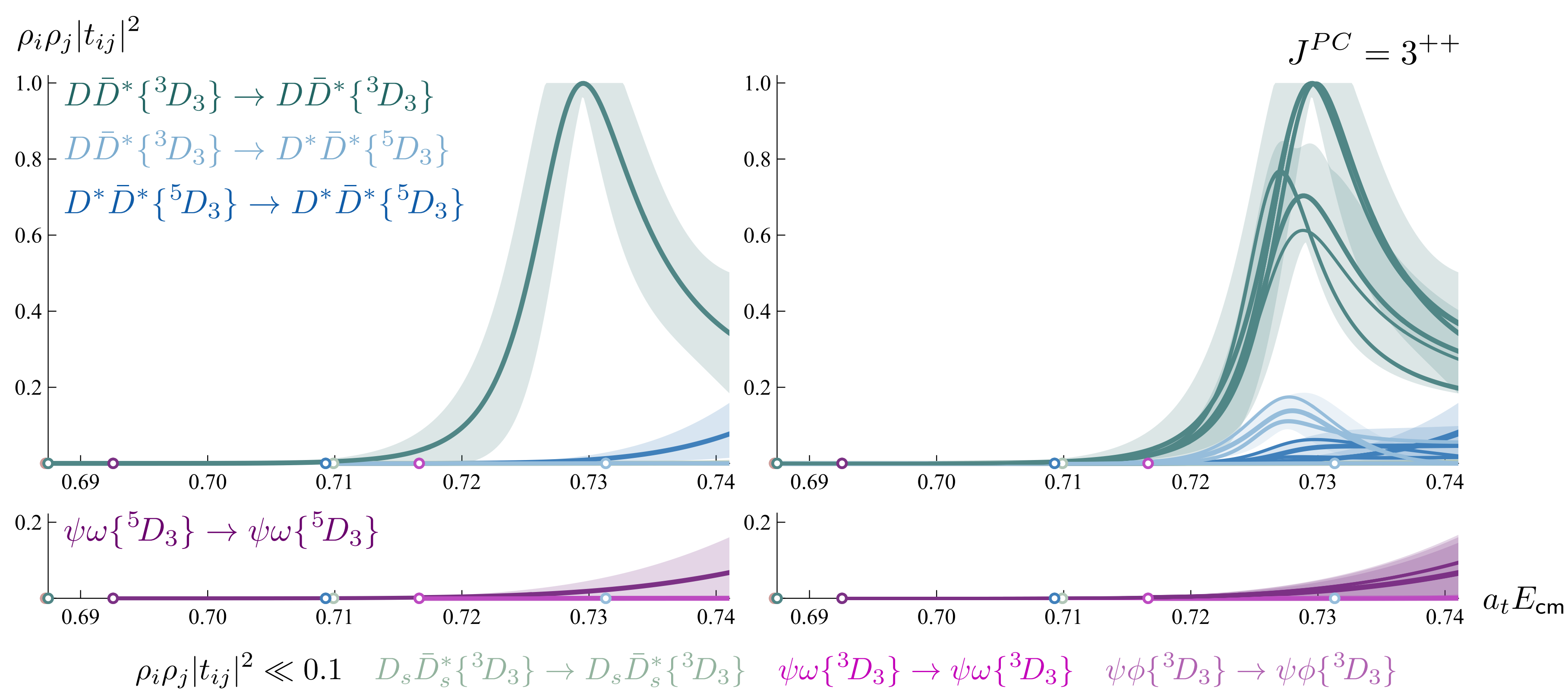
an η_{c2} 2^{-+} state arises below DD*



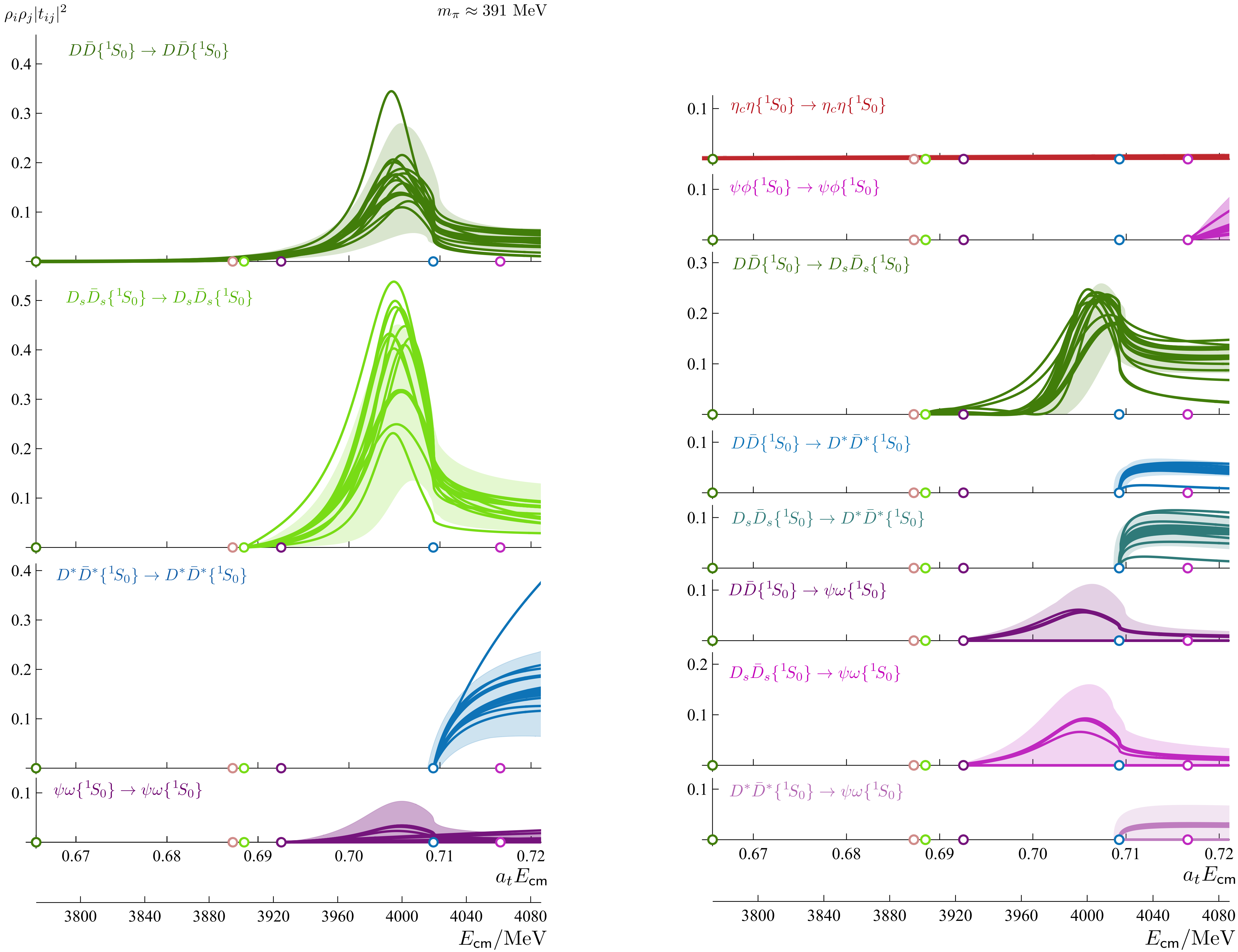
extra level and resonance higher up

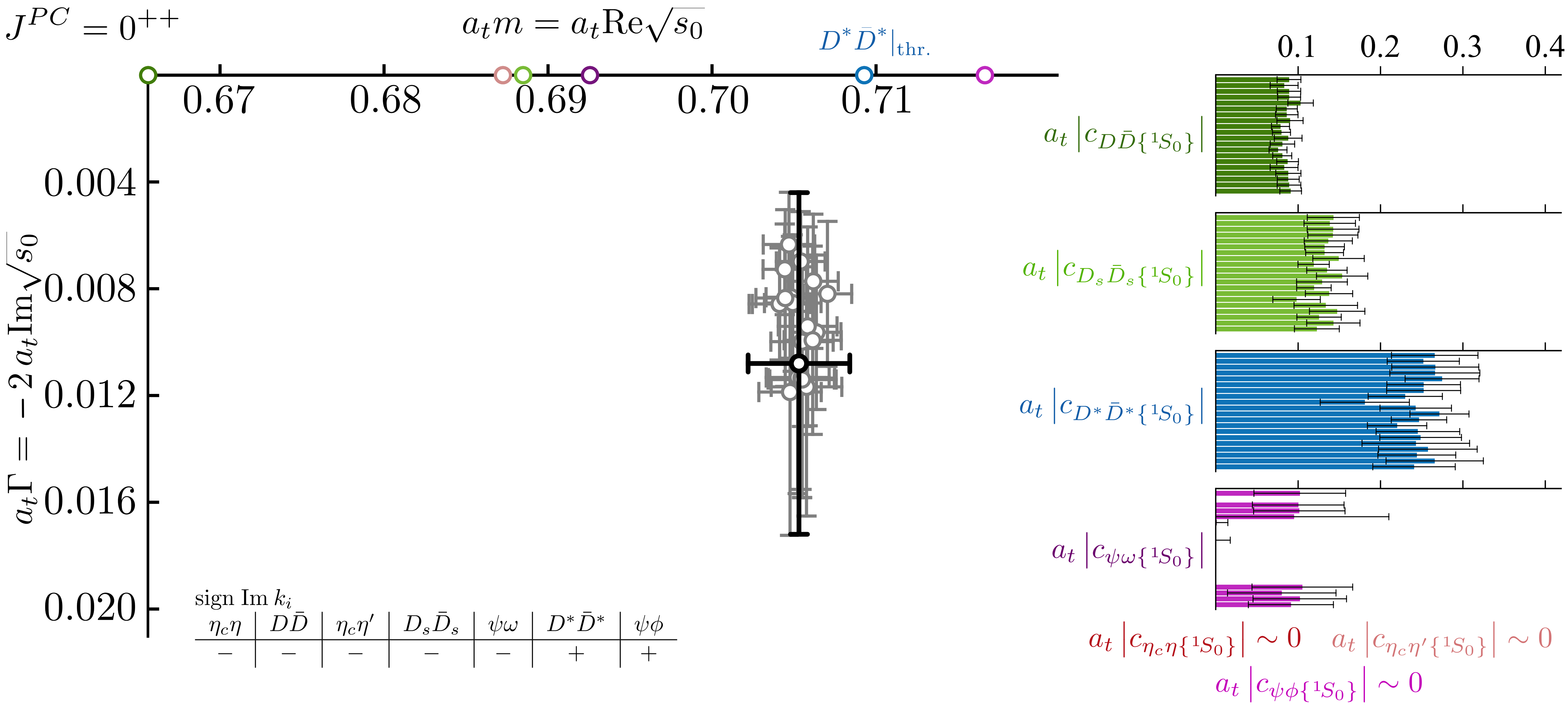
two classes of amplitudes were found:

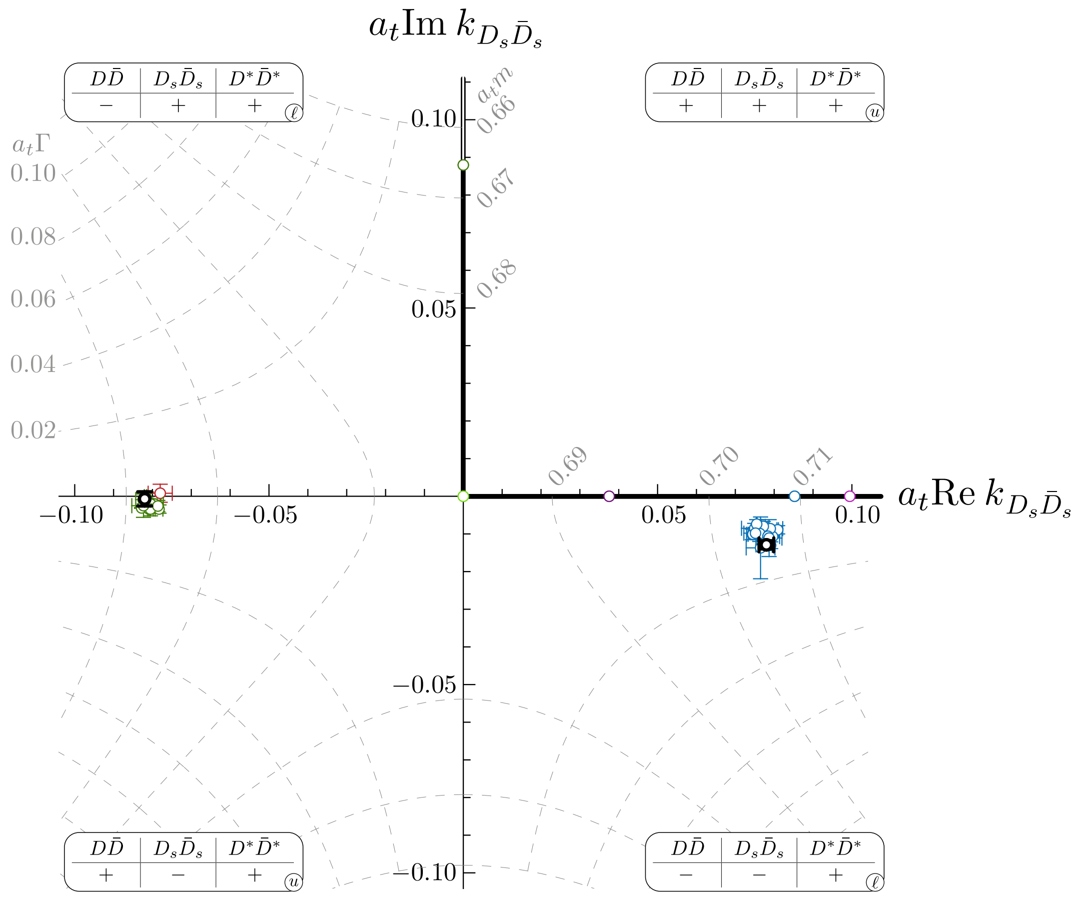
- zero D*D* coupling
- finite D*D* coupling
- all had significant DD* coupling
- amps very small below 4050 MeV ($a_t E_{cm}$=0.715)



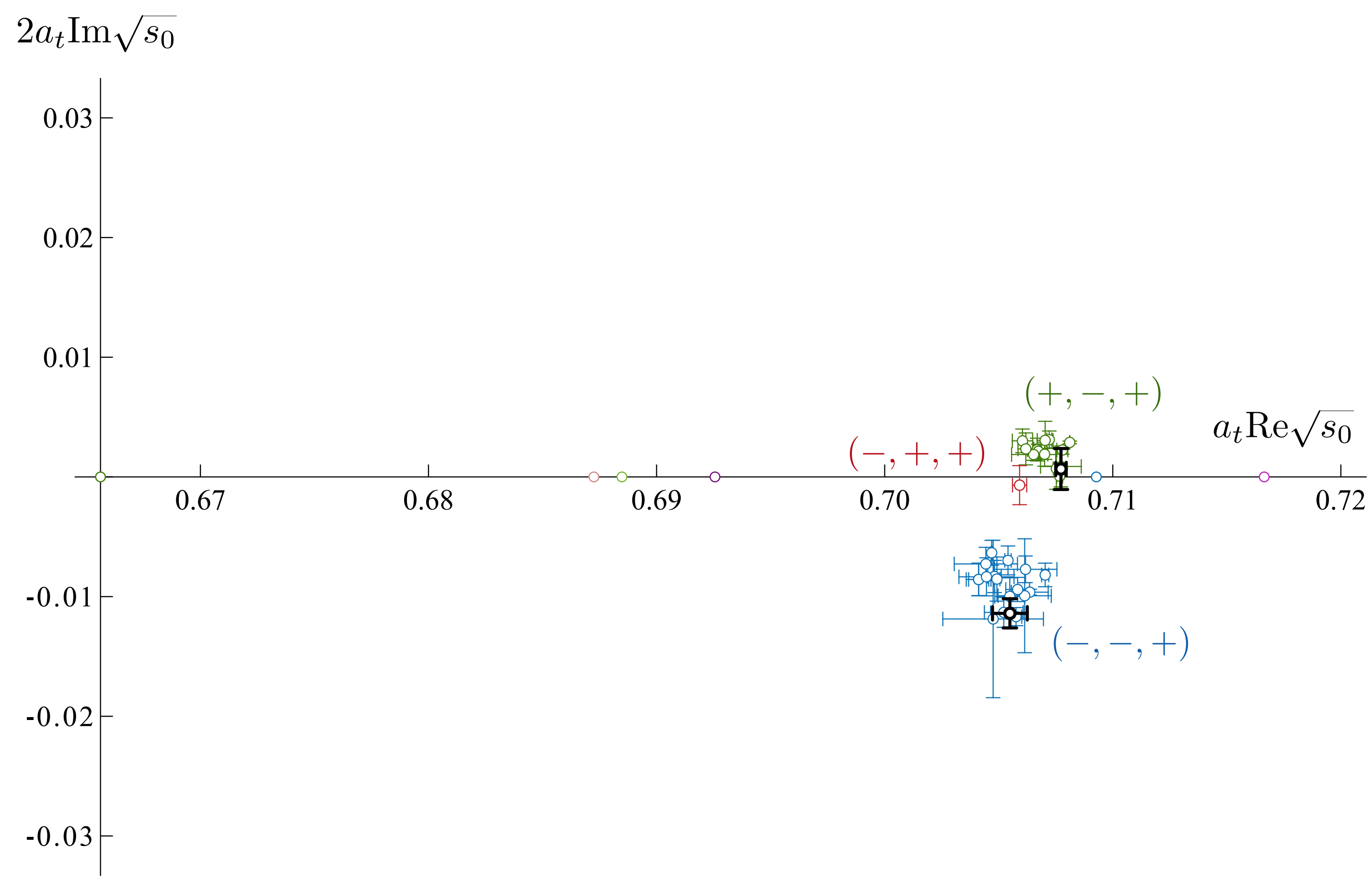
amplitude variations - scalar



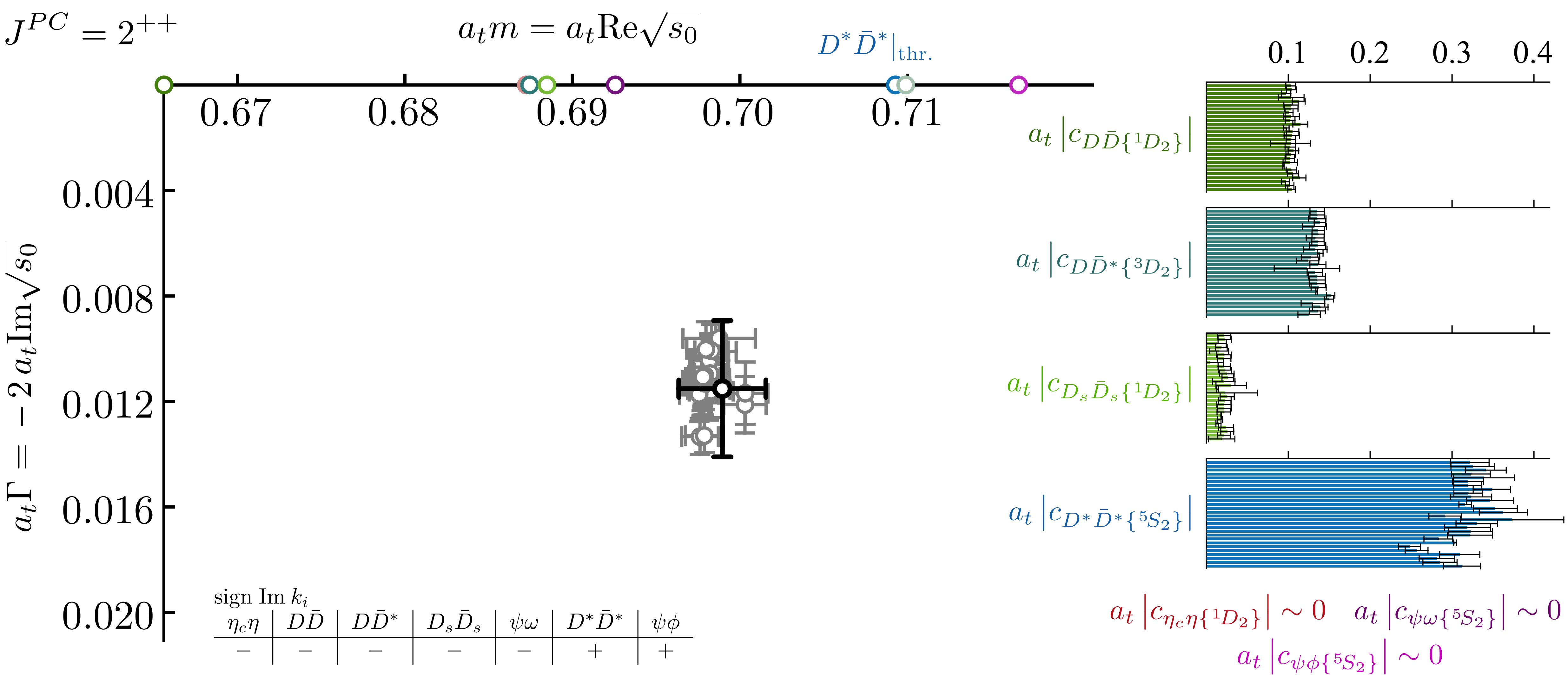

$J^{PC} = 0^{++}$
$a_t m = a_t \mathrm{Re}\sqrt{s_0}$
$D^*\bar{D}^*|_{\mathrm{thr.}}$
0.67
0.68
0.69
0.70
0.71
$a_t\Gamma = -2\,a_t \mathrm{Im}\sqrt{s_0}$
0.004
0.008
0.012
0.016
0.020
sign Im k_i
$\eta_c\eta$ | $D\bar{D}$ | $\eta_c\eta'$ | $D_s\bar{D}_s$ | $\psi\omega$ | $D^*\bar{D}^*$ | $\psi\phi$
$-$ | $-$ | $-$ | $-$ | $-$ | $+$ | $+$
0.1
0.2
0.3
0.4
$a_t\,|c_{D\bar{D}\{^1S_0\}}|$
$a_t\,|c_{D_s\bar{D}_s\{^1S_0\}}|$
$a_t\,|c_{D^*\bar{D}^*\{^1S_0\}}|$
$a_t\,|c_{\psi\omega\{^1S_0\}}|$
$a_t\,|c_{\eta_c\eta\{^1S_0\}}| \sim 0$
$a_t\,|c_{\eta_c\eta'\{^1S_0\}}| \sim 0$
$a_t\,|c_{\psi\phi\{^1S_0\}}| \sim 0$



$a_t \mathrm{Im}\, k_{D_s \bar{D}_s}$
$a_t \mathrm{Re}\, k_{D_s \bar{D}_s}$
$D\bar{D}$ | $D_s\bar{D}_s$ | $D^*\bar{D}^*$
$-$ $+$ $+$ ℓ
$+$ $+$ $+$ u
$+$ $-$ $+$ u
$-$ $-$ $+$ ℓ
$a_t\Gamma$
$a_t m$
0.66
0.67
0.68
0.69
0.70
0.71



$2a_t \mathrm{Im}\sqrt{s_0}$
$a_t \mathrm{Re}\sqrt{s_0}$
$(+,-,+)$
$(-,+,+)$
$(-,-,+)$

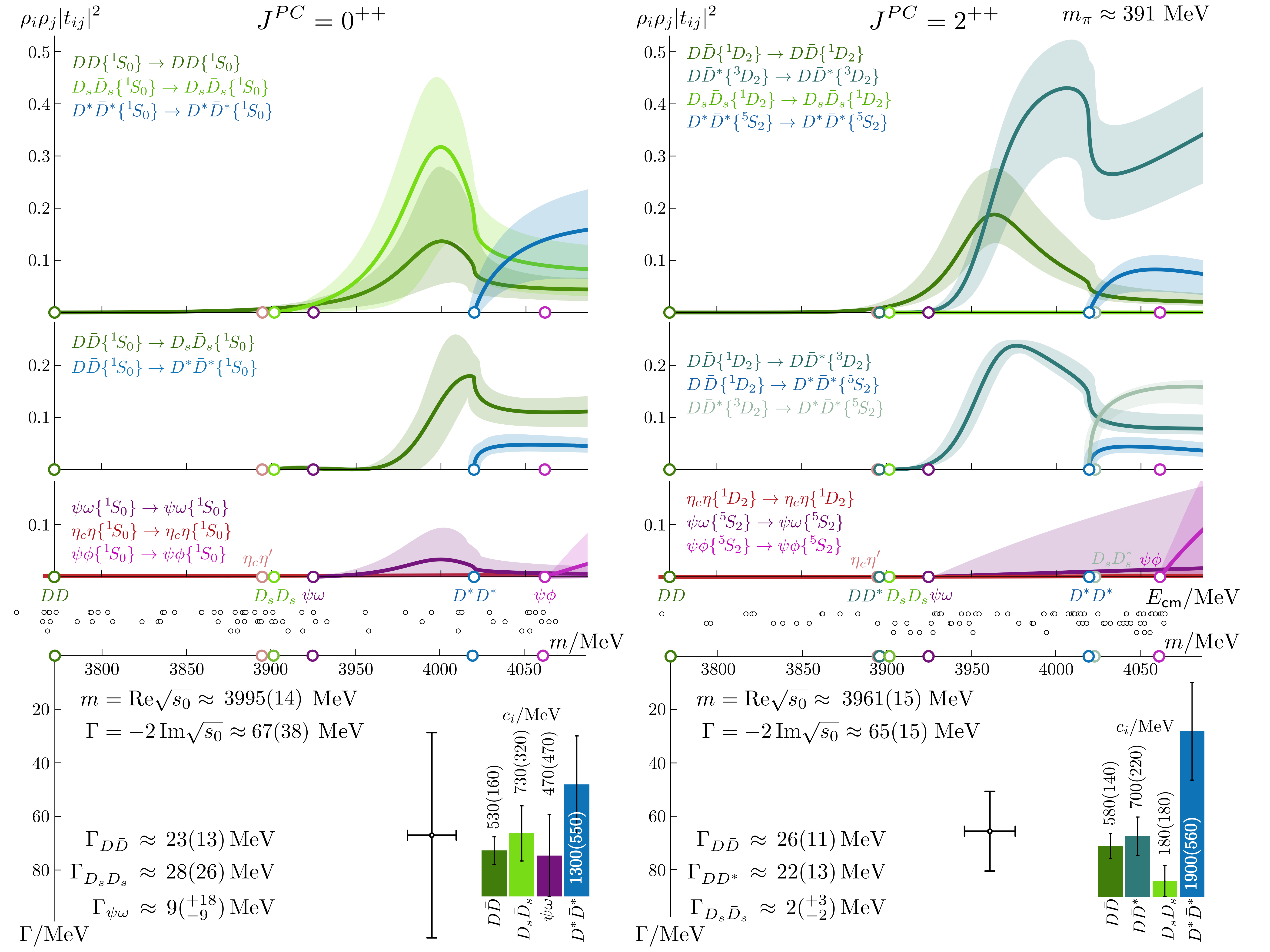


additional poles were found

- don’t appear to be important

“coupling-ratio” phenomena seen in K-matrix pole parameters

- possible to rescale K-matrix g_i factors and obtain similar amplitudes
- t-matrix couplings are found to be well-determined

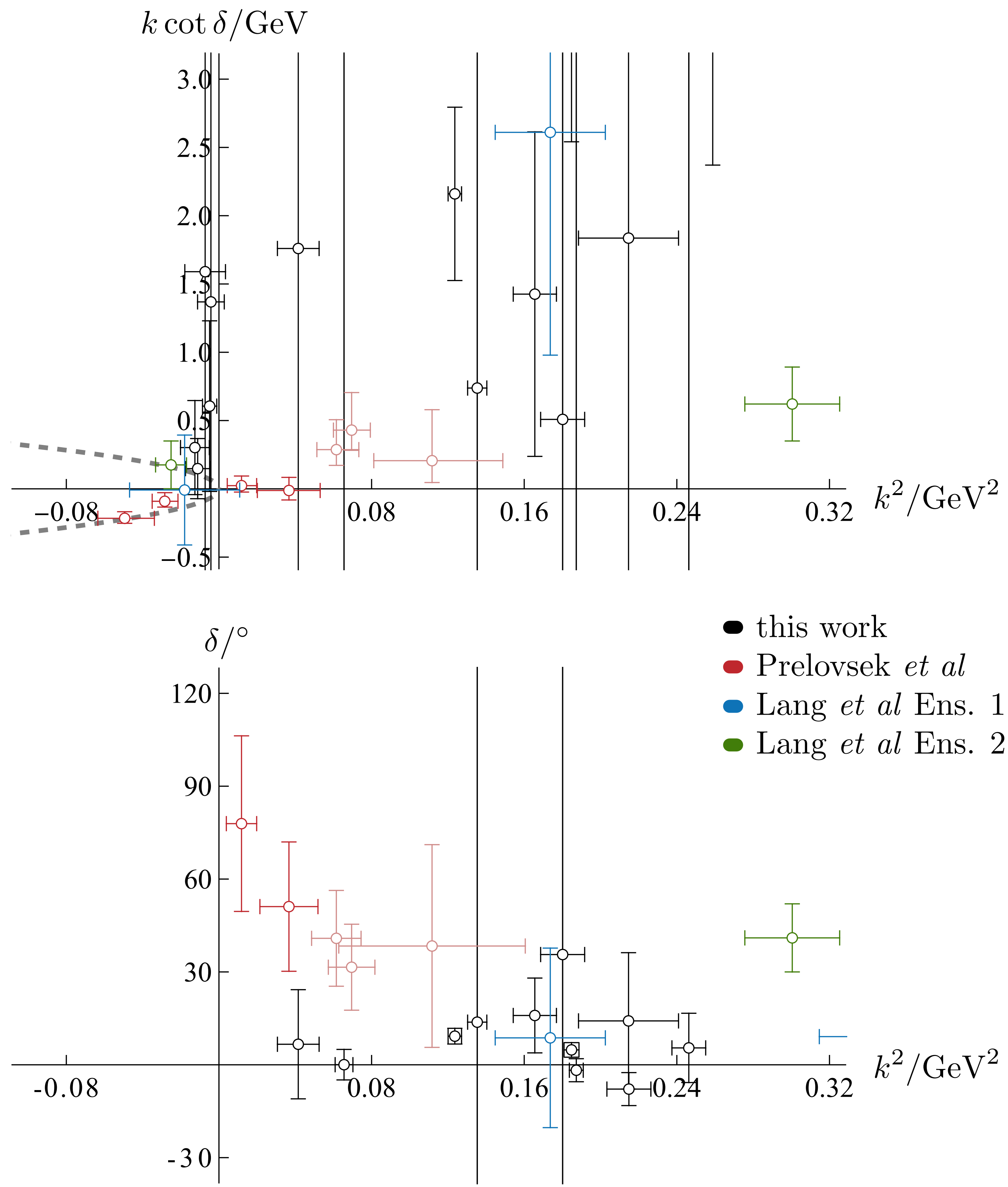


$\rho_i\rho_j|t_{ij}|^2$
$J^{PC}=0^{++}$
$D\bar{D}\{^1S_0\}\to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\}\to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\}\to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\}\to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\}\to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\}\to\psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\}\to\eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\}\to\psi\phi\{^1S_0\}$
$\eta_c\eta'$
$D\bar{D}$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$ $\psi\phi$
m/MeV
$m=\mathrm{Re}\sqrt{s_0}\approx 3995(14)$ MeV
$\Gamma=-2\,\mathrm{Im}\sqrt{s_0}\approx 67(38)$ MeV
$\Gamma_{D\bar{D}}\approx 23(13)$ MeV
$\Gamma_{D_s\bar{D}_s}\approx 28(26)$ MeV
$\Gamma_{\psi\omega}\approx 9(^{+18}_{-9})$ MeV
c_i/MeV
530(160) 730(320) 470(470) 1300(550)
Γ/MeV
$J^{PC}=2^{++}$
$m_\pi\approx 391$ MeV
$D\bar{D}\{^1D_2\}\to D\bar{D}\{^1D_2\}$
$D\bar{D}^*\{^3D_2\}\to D\bar{D}^*\{^3D_2\}$
$D_s\bar{D}_s\{^1D_2\}\to D_s\bar{D}_s\{^1D_2\}$
$D^*\bar{D}^*\{^5S_2\}\to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}\{^1D_2\}\to D\bar{D}^*\{^3D_2\}$
$D\bar{D}\{^1D_2\}\to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}^*\{^3D_2\}\to D^*\bar{D}^*\{^5S_2\}$
$\eta_c\eta\{^1D_2\}\to\eta_c\eta\{^1D_2\}$
$\psi\omega\{^5S_2\}\to\psi\omega\{^5S_2\}$
$\psi\phi\{^5S_2\}\to\psi\phi\{^5S_2\}$
$D_sD_s^*$ $\psi\phi$
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $\psi\omega$ $D^*\bar{D}^*$
$E_{\mathrm{cm}}/\mathrm{MeV}$
$m=\mathrm{Re}\sqrt{s_0}\approx 3961(15)$ MeV
$\Gamma=-2\,\mathrm{Im}\sqrt{s_0}\approx 65(15)$ MeV
$\Gamma_{D\bar{D}}\approx 26(11)$ MeV
$\Gamma_{D\bar{D}^*}\approx 22(13)$ MeV
$\Gamma_{D_s\bar{D}_s}\approx 2(^{+3}_{-2})$ MeV
580(140) 700(220) 180(180) 1900(560)

Results from Prelovsek et al, suggest effects at DDbar and DsDsbar thresholds

- pion mass ~ 280 MeV
- light quark heavier than physical, strange quark lighter than physical

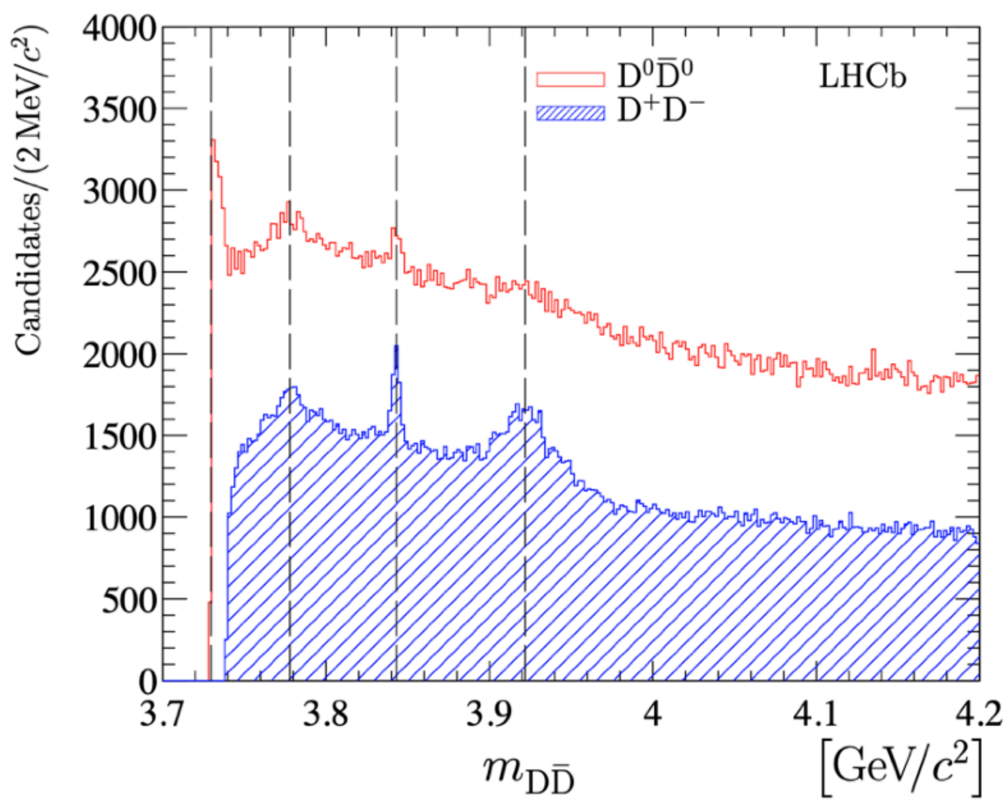
hard to justify such a large change due to the light quark mass (no one-pion-exchange term)



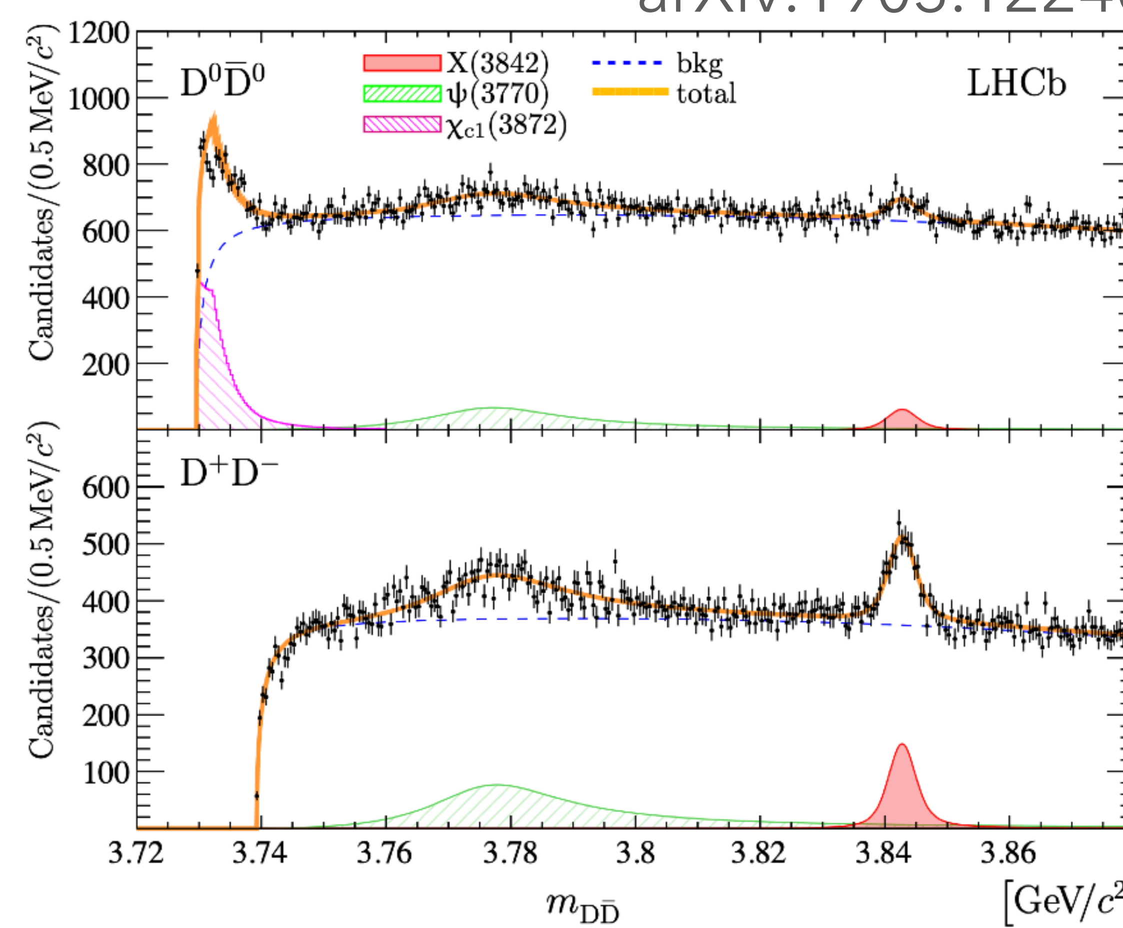
Many models with meson-meson components find strong effects in S-wave DDbar

Several suggestions of a near-threshold state in DDbar scattering
- $\gamma\gamma$ to DDbar (BaBar, Belle)
- near threshold structure partly due to Born/t-channel photon exchange
- see e.g. Guo & Meißner 2012, Wang et al 2021, Deineka et al 2022

Recent LHCb analyses find a peak at DDbar threshold but attribute this to "feed-down" from X(3872) decays



arXiv:1903.12240



Lattice QCD provides a **first-principles** tool to do **hadron spectroscopy**

Scalar and tensor scattering amplitudes in the charmonium energy region have been determined

- at m_π=391 MeV, the level counting is not obviously different from the quark model
- large coupled-channel effects in OZI connected D-meson channels
- OZI disconnected channels look small everywhere
- also found a 3++ resonance

Charmonium systems are difficult, but achievable

- overlapping effects in several J^{PC}
- many open channels
- quark mass dependence is readily accessible

These methods are widely applicable

- baryons (see John Bulava's plenary this morning, and the next talk)
- doubly-charmed systems, b-quarks
- form factors, radiative transitions (incl. resonances)

…

Control of 3+ body effects needed for

- lighter pion masses
- higher resonances