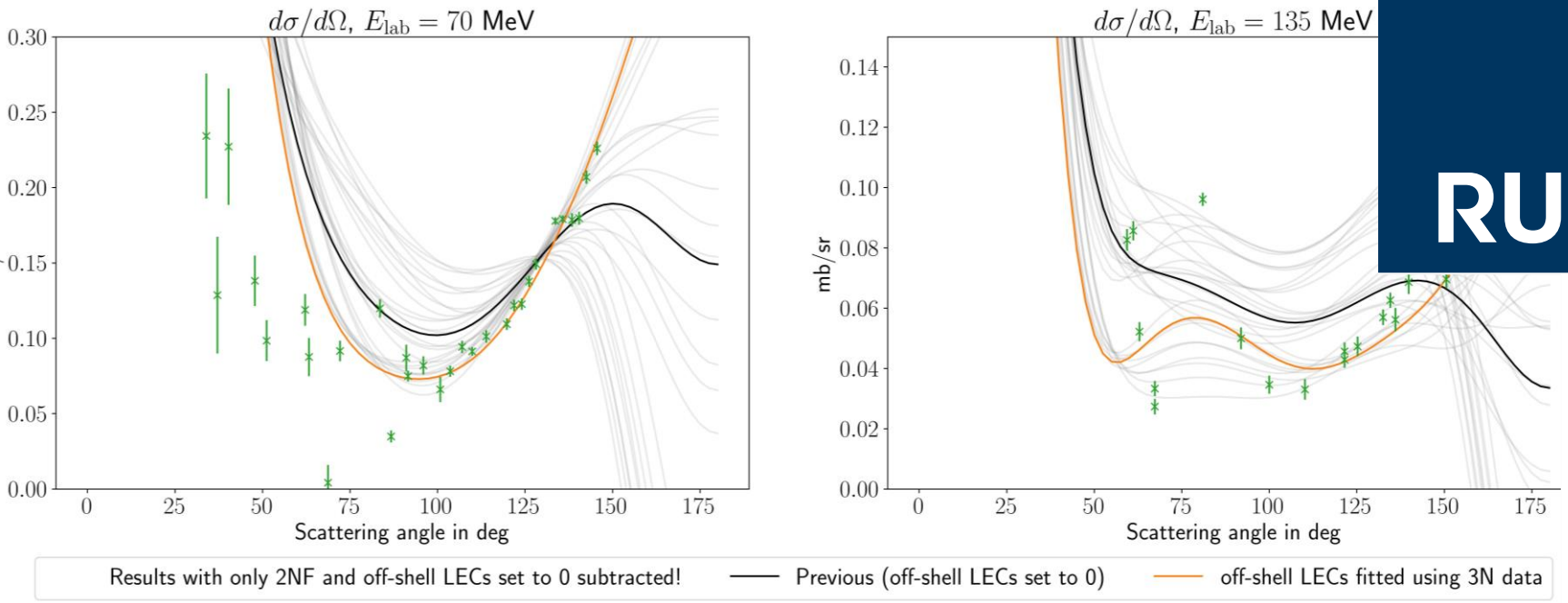

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CONSTRAINING THE TWO-NUCLEON FORCE IN CHIRAL EFT FROM THREE-NUCLEON DATA

Observing the unobservable off-shell behavior?

Sven Heihoff, Evgeny Epelbaum

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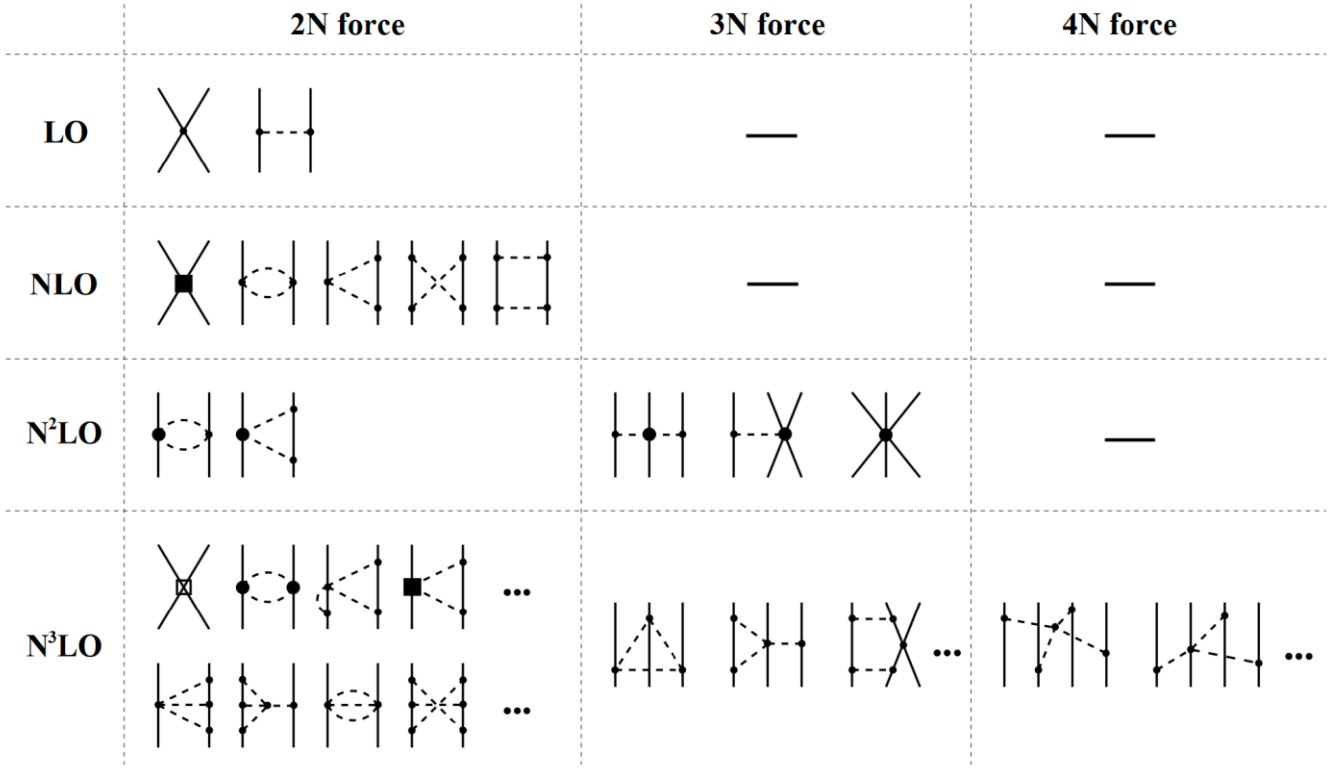
Introduction

- Chiral EFT is a low-energy effective theory of QCD
- Degrees of freedom: Pions and Nucleons
- Uses momenta and pion mass divided by the breakdown scale as expansion parameter



E. Epelbaum, Nuclear Forces from Chiral Effective Field Theory: A Primer (2010)

Introduction

- Chiral EFT potential comes with a-priori unknown Low-Energy-Constants (LECs)
  - Usually determined by fits to experimental data
  - At N^3LO three redundant LECs (so called off-shell LECs) appear in the 2N potential

- 2N potential in chiral EFT leads to high-precision description of 2N data
  (P. Reinert, et al., Eur. Phys. J. A 54, 86 (2018))

- 3N observables are not described precisely (large χ^2 for some observables and/or energies)
  - 2 Nucleon force makes up most of the 3N scattering amplitude
  - Need to push chiral expansion of three nucleon forces (3NF) to higher orders

- **In this talk:** Calculating 3NFs without calculating 3NFs and determining redundant LECs.

Off-shell Low-Energy Constants (off-shell LECs)

- 2N potential in chiral EFT in the order N^{3}LO in the $^1S_0$ partial wave:

$$\langle\, {}^1S_0, p' | V^{(4)} |\, {}^1S_0, p \rangle = D_{1S0}\, p^2\, p'^2 + D_{1S0}^{\text{off}} \left(p^2 - p'^2\right)^2 + \ \ldots$$

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- Off-shell LECs are somewhat similar to transformation angles γ_i of a unitary transformation

$$\hat{U} = \exp\left(\gamma_1 \hat{T}_1 + \gamma_2 \hat{T}_2 + \ \gamma_3 \hat{T}_3\right), \qquad \langle \vec{p}\,' | \hat{T}_1 | \vec{p} \rangle = \frac{m_\text{N}}{2\Lambda_\text{b}^4} \left(\vec{p}\,'^2 - \vec{p}^{\,2}\right)/2$$

$$\langle \vec{p}\,' | \delta \hat{H} | \vec{p} \rangle = \langle \vec{p}\,' | \hat{U}^\dagger \hat{H}^{(0)} \hat{U} - \hat{H}^{(0)} \, | \vec{p} \rangle = \Sigma_i \, \gamma_i \left\langle \vec{p}\,' \left| \left[\hat{H}_\text{kin}^{(0)}, \hat{T}_i \right] \right| \vec{p} \right\rangle + \cdots = \gamma_1 \frac{1}{\Lambda_\text{b}^4} \left(\left(\vec{p}\,'^2 - \vec{p}^{\,2}\right)/2 \right)^2 + \cdots$$

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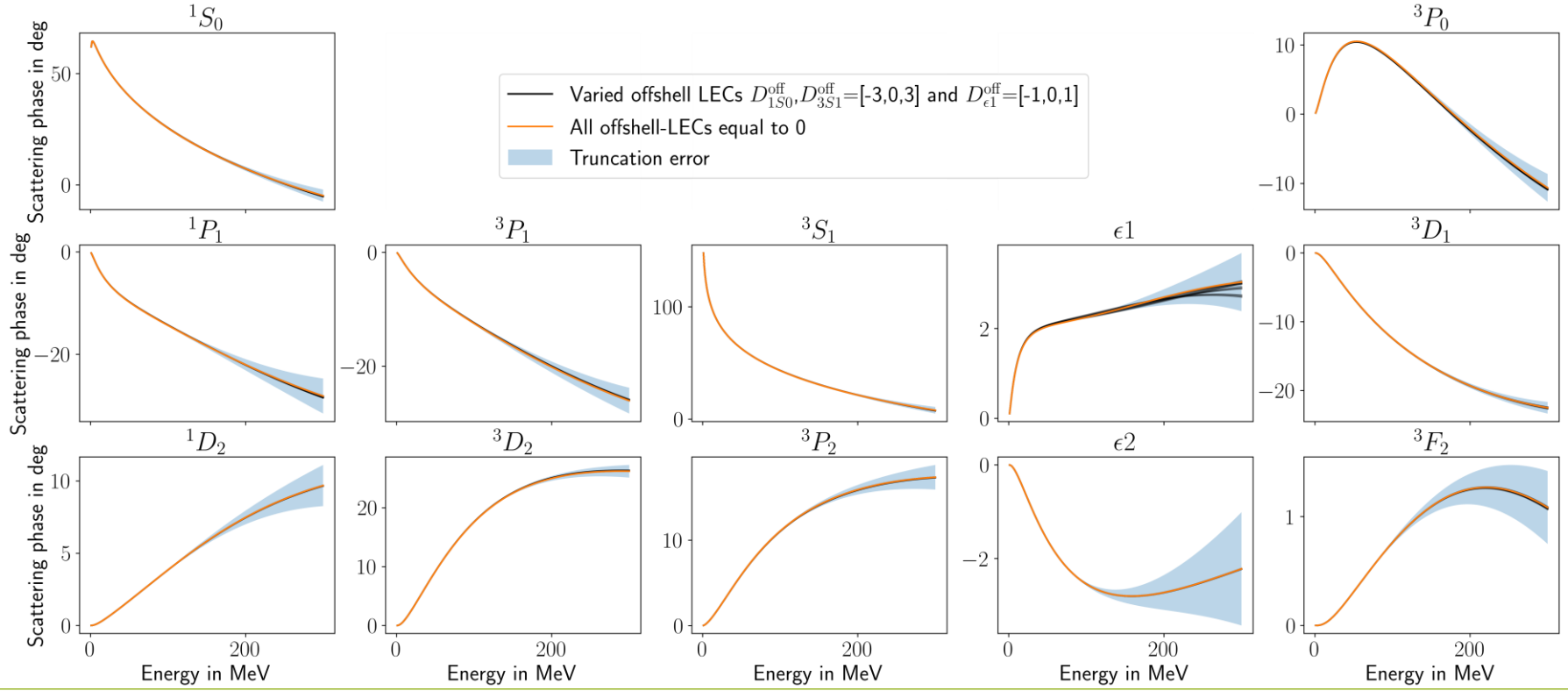
- **Conclusion:** Fixing off-shell LECs is equivalent to fixing arbitrary transformation angles!

Off-shell LECs do not affect 2N observables



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Unitary Transformation in 3N Systems

- 3N forces are induced by the unitary transformation $\hat{U} = \exp\left(\gamma_1 \hat{T}_1 + \gamma_2 \hat{T}_2 + \gamma_3 \hat{T}_3\right)$

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$$= \gamma_1 \frac{\mathrm{m_N}}{4\Lambda_{\mathrm{b}}^4} |\vec{p}_3' - \vec{p}_3|^2 \left(C_S + C_T \vec{\sigma}_1 \vec{\sigma}_2\right) + \text{permutations} + \cdots$$

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- This is observable → γ_{i} can be determined → off-shell LECs can be determined!
- Induced 3N forces appear at N^{3}LO and there are similar terms in N^{4}LO
  - Determination necessary for complete N^{3}LO calculation
  - Going to infinite chiral order → all off-shell effects stay unobservable
- More generally: “Different off-shell extensions of two-body forces can be equivalently realized as three-body interactions.” (W. N. Polyzou and W. Glöckle, Few-Body Syst. 9, 97 (1990))

Emulator for the 3N Scattering Amplitude

- To obtain 3N scattering observables, Faddeev equation must be iterated

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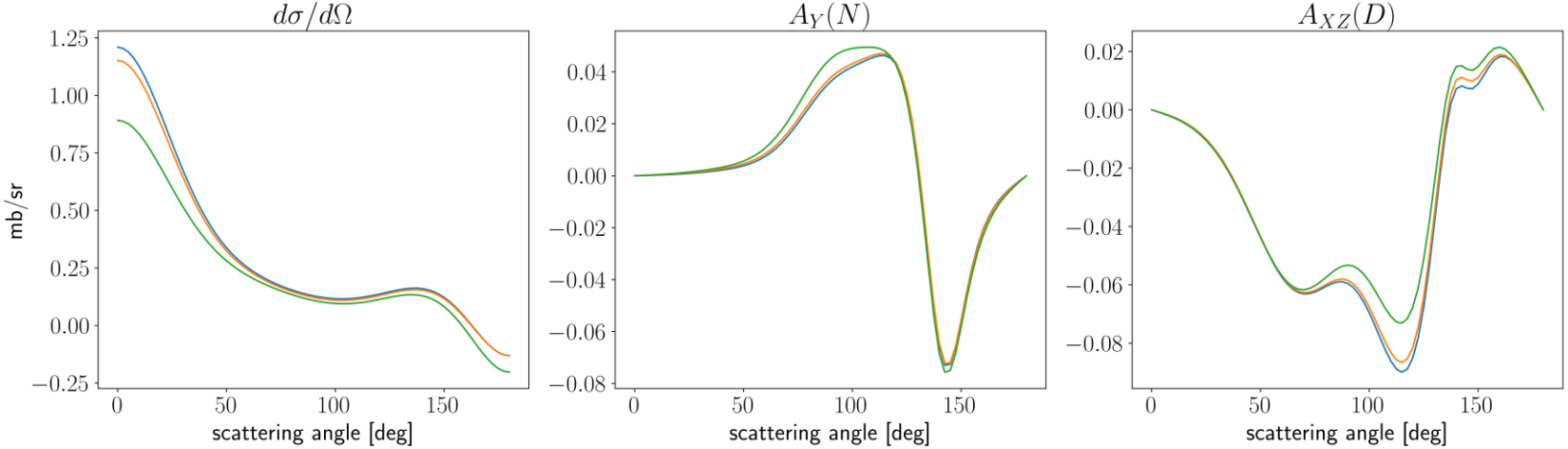
- Solution: use an emulator
  - Fitting off-shell LECs takes ~ 1 min instead of ~ 1 week
  - Cost: on average 3% error
  - Algorithm: radial basis function interpolation (RBF)
  - LECs sampled at 135 different combinations used as the basis mesh for interpolation
  - $c_D \in \{-5,-3,-1,2,5\}$, $D_{1S0}^{\mathrm{off}}, D_{3S1}^{\mathrm{off}} \in \{-3,0,3\}$ and $D_{\epsilon 1}^{\mathrm{off}} \in \{-1,0,1\}$
  - Same procedure at four different energies (10, 70, 135 and 200 MeV)

Precision of the Emulator

- Emulator tested for seven different sets of randomly chosen LEC combinations

  → Averaged error for the differential cross-section is ~ 2%

- For comparison: linear interpolator ~ 15% error



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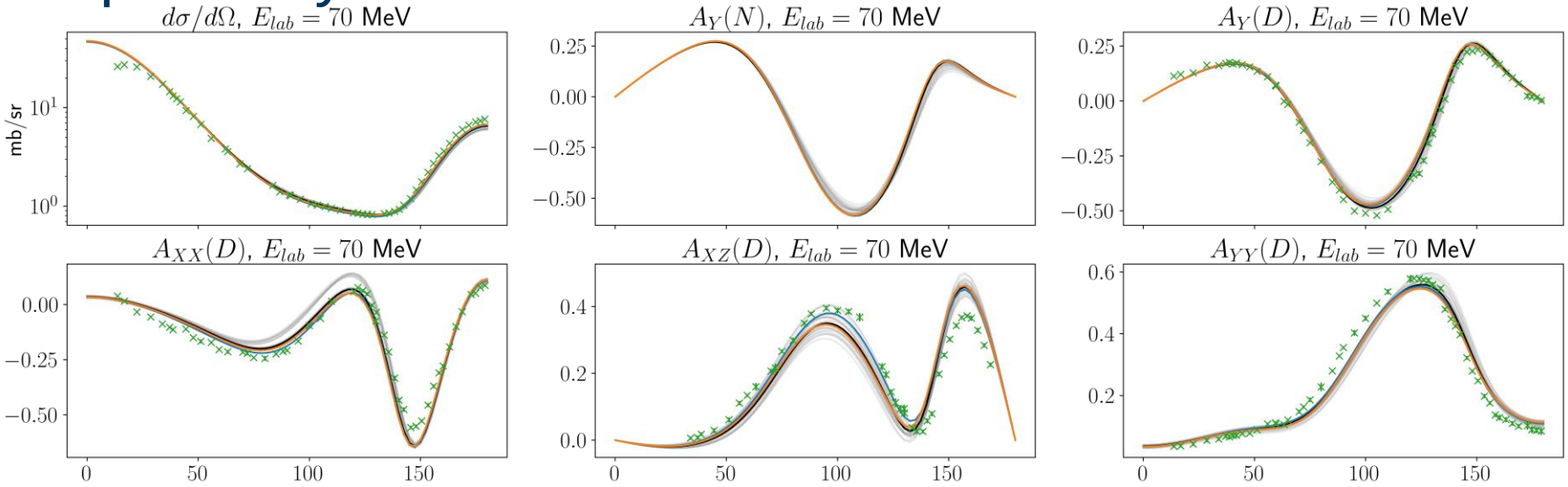
Fitting Procedure

- 2N potential at order N^4LO+ and 3N potential at order N^2LO

- 4 LECs to be fitted in total
  - 3 off-shell LECs from the 2N potential
  - 2 LECs (c_D and c_E) from the 3N potential, c_E is determined indirectly from Triton binding energy

- Experimental data from 3N scattering experiments at 10, 70, 135 and 200 MeV
  - Scattering angle > 40° to keep Coulomb effects small

Exploratory fits of off-shell LECs to 3N data



Varying off-shell LECs in natural range

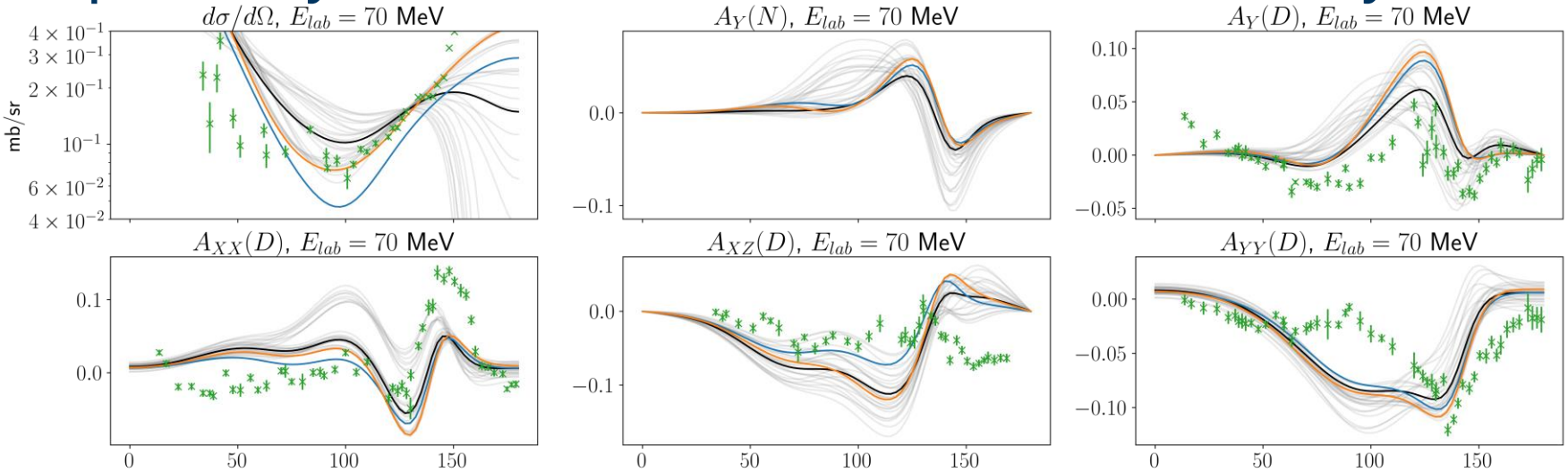
2NF off.-LECs=0 with 3NF, red. $\chi^2 \approx 116$

Fit to complete experimental database, LECs: D^{off}_{1S0} =0.1, D^{off}_{3S1} =2.11, $D^{\text{off}}_{\epsilon 1}$ =-0.97, c_D =-0.84, red. $\chi^2 \approx 84$

Fit only to all cross-sec. data, LECs: D^{off}_{1S0} =3.0, D^{off}_{3S1} =3.0, $D^{\text{off}}_{\epsilon 1}$ =-0.46, c_D =2.04, red. $\chi^2 \approx 96$

Experimental data from K. Sekiguchi et. al, Phys. Rev. C 65, 034003 (2002), Phys. Rev. C 70, 014001 (2004) and Phys. Rev. Lett. 95, 162301 (2005)

Exploratory fits – subtract 2NF for better visibility



Results with 2NF with off-shell LECs=0 subtracted!

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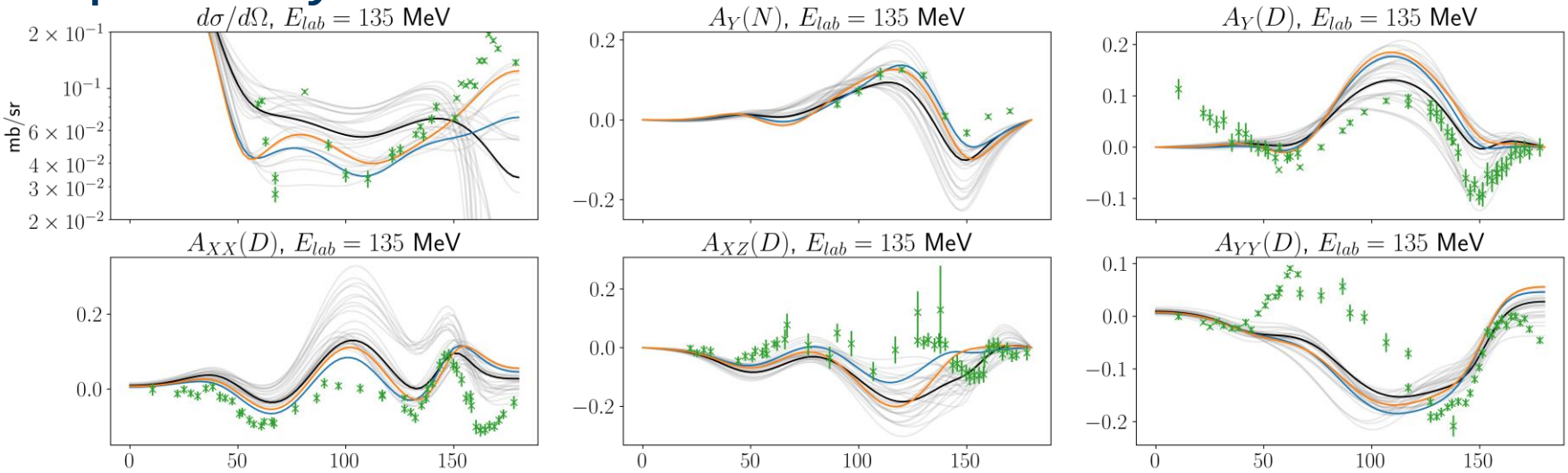
Exploratory fits 135MeV



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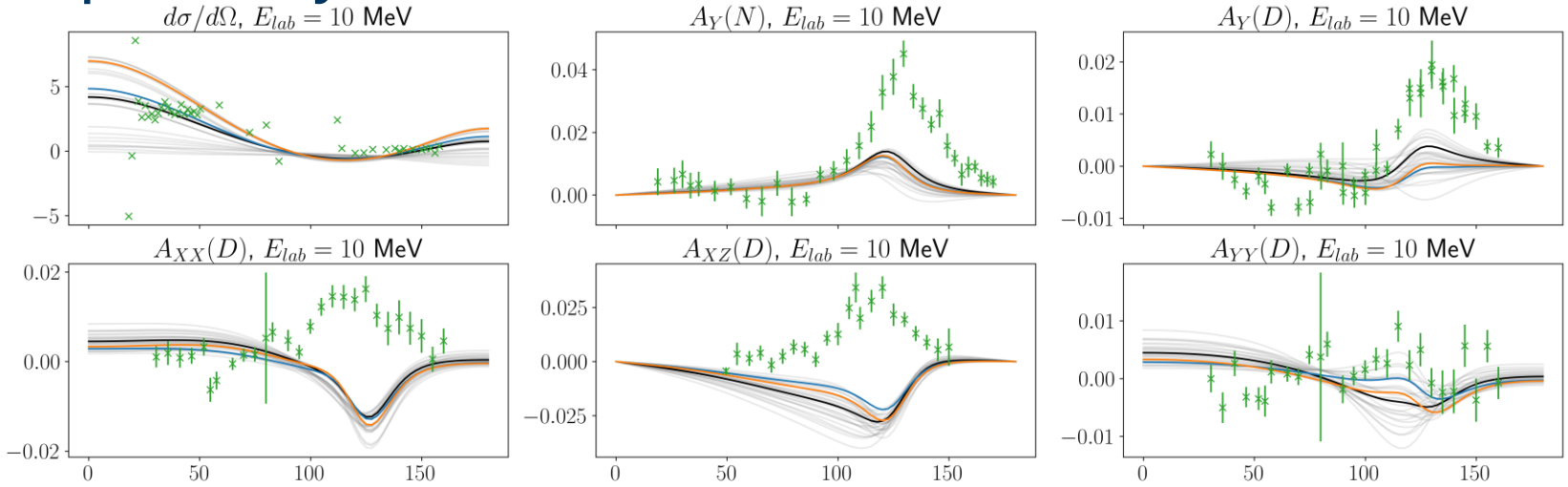
Conclusion and Outlook

- Radial basis function interpolation is an efficient way to compute 3N scattering observables for LEC-fitting
- Tuning the off-shell LECs of the 2N potential improves description of 3N data
  - 3N data is not yet fully described → need to increase chiral order of 3NF
- Well prepared to do full N^3LO calculation of 3N observables → LENPIC
- Fits to 3N data can be extended
  - Including other data than scattering data (e.g. Triton beta decay) or theoretical uncertainties
- There are further generators of the unitary transformation, which vanish in 2N c.o.m frame (Girlanda et al. Phys. Rev. C 102, 064003(2020))

Backup slides
More results

Exploratory fits 10MeV



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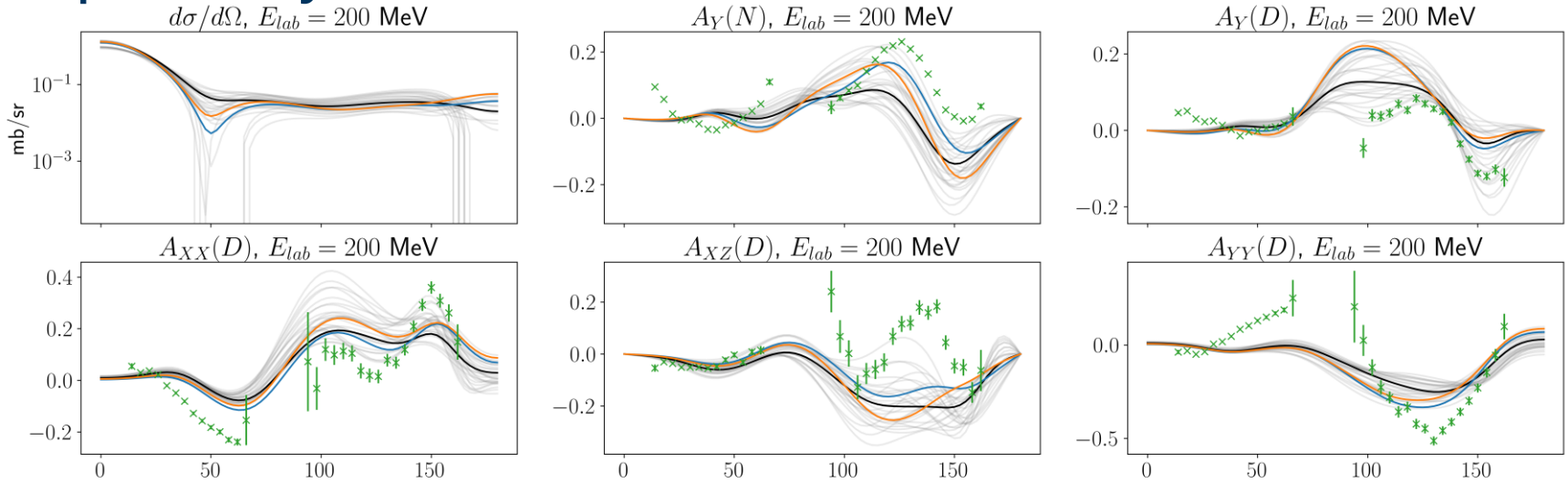
Exploratory fits 200MeV



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- B.v.Przewoski et al. PhysRevC.74.064003