Deutsche Forschungsgemeinschaft
FOR5327

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Towards data-driven evaluation of the nucleon polarizability effects contributing to the Lamb shift

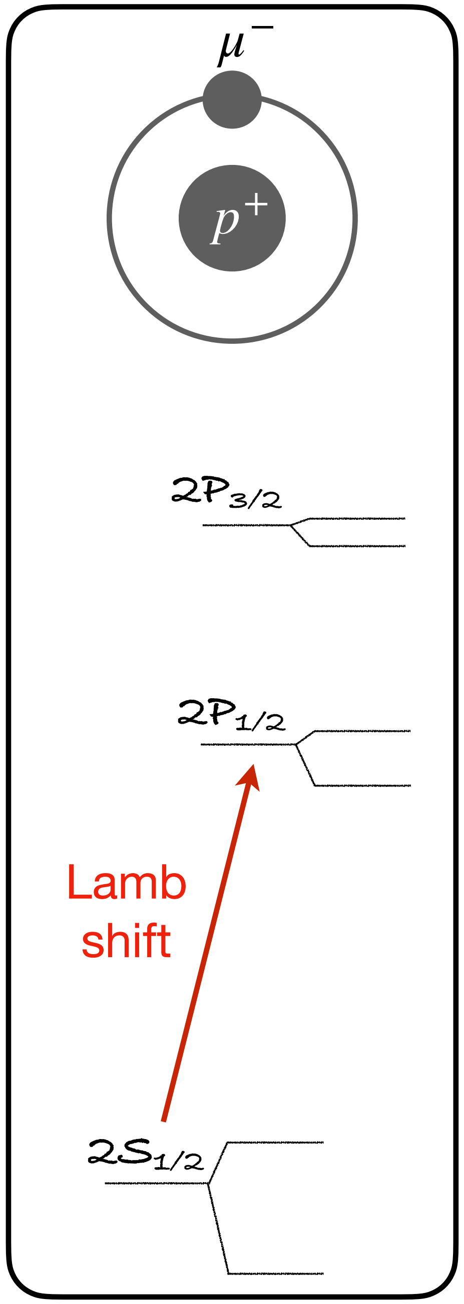
arXiv:2305.08814 [hep-ph]

Volodymyr Biloshytskyi, I. Ciobotaru-Hriscu, F. Hagelstein, V. Lensky and V. Pascalutsa

MENU 2023

The 16th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon, Mainz, 15-20 October 2023

Lamb shift in μH



$$E^{\rm exp}_{2S-2P}(\mu{\rm H}) = 202.3706(23)\,{\rm meV}$$

$$E^{\rm th}_{2P-2S}(\mu{\rm H}) = \Big[\underbrace{205.0074}_{\text{Uehling}} + \underbrace{1.0153}_{r_p\text{ indep.}} + \underbrace{0.0114(3)}_{\text{hVP}} + \underbrace{0.0006(1) - 5.2275(10)\left(\frac{r_p}{\rm fm}\right)^2}_{\text{f.s. corr.}} - \underbrace{E^{\{2\gamma\}}_{2S}}_{2\gamma\text{ exch.}}\Big]\,{\rm meV}$$

Table 1 Forward 2γ-exchange contributions to the 2S-shift in μH, in units of μeV.

Reference	$E^{\rm (subt)}_{2S}$	$E^{\rm (inel)}_{2S}$	$E^{\rm (pol)}_{2S}$	$E^{\rm (el)}_{2S}$	$E^{\langle 2\gamma\rangle}_{2S}$
DATA-DRIVEN					
(73) Pachucki '99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(74) Martynenko '06	2.3	−16.1	−13.8(2.9)		
(75) Carlson *et al.* '11	5.3(1.9)	−12.7(5)	−7.4(2.0)		
(76) Birse and McGovern '12	4.2(1.0)	−12.7(5)	−8.5(1.1)	−24.7(1.6)	−33(2)
(77) Gorchtein et al.'13 [a]	−2.3(4.6)	−13.0(6)	−15.3(4.6)	−24.5(1.2)	−39.8(4.8)
(78) Hill and Paz '16					−30(13)
(79) Tomalak'18	2.3(1.3)		−10.3(1.4)	−18.6(1.6)	−29.0(2.1)
LEADING-ORDER BχPT					
(80) Alarcòn *et al.* '14			$-9.6^{+1.4}_{-2.9}$		
(81) Lensky *et al.* '17 [b]	$3.5^{+0.5}_{-1.9}$	−12.1(1.8)	$-8.6^{+1.3}_{-5.2}$		
LATTICE QCD					
(82) Fu *et al.* '22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the Δ(1232)-isobar contribution.

A. Antognini et al.,
Ann.Rev.Nucl.Part.Sci. (2022)

Leading polarizability contribution to the Lamb shift

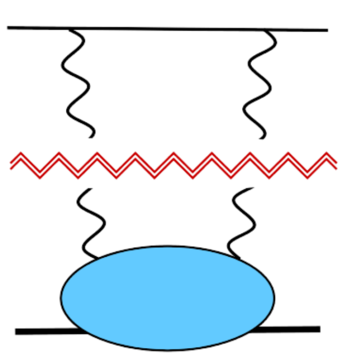
Doubly-virtual Compton scattering tensor:

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu,Q^2)$$

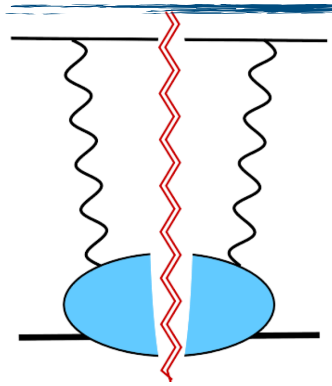
Spin-independent part

$$+\frac{i}{m}\epsilon^{\mu\nu\alpha\beta}q_\alpha s_\beta S_1(\nu,Q^2) + \frac{i}{m^3}\epsilon^{\mu\nu\alpha\beta}q_\alpha\left(p\cdot q s_\beta - s\cdot q p_\beta\right) S_2(\nu,Q^2)$$

Spin-dependent part



$\sim (Z\alpha)^6 \log Z\alpha$



$\sim (Z\alpha)^5$

The leading two-photon contribution to the Lamb shift in a hydrogen-like atom:

$$\Delta E(nS) = 8\pi\alpha m\phi_n^2 \frac{1}{i}\int_{-\infty}^{\infty}\frac{d\nu}{2\pi}\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{(Q^2-2\nu^2)T_1(\nu,Q^2)-(Q^2+\nu^2)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

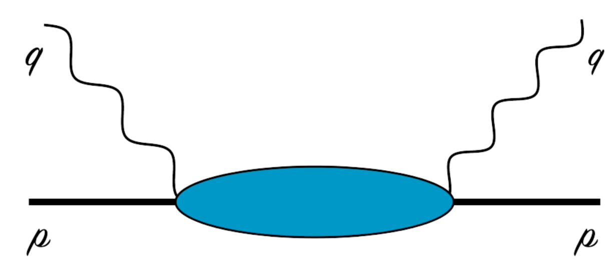
A. Antognini et al., Ann.Rev.Nucl.Part.Sci. (2022)

$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi Z^2\alpha_{\rm em}M\nu^2}{Q^4}\int_0^1 dx\frac{x\,F_1(x,Q^2)}{1-x^2(\nu/\nu_{\rm el})^2 - i0^+} \qquad T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha_{\rm em}M}{Q^2}\int_0^1 dx\frac{F_2(x,Q^2)}{1-x^2(\nu/\nu_{\rm el})^2 - i0^+}$$

$\nu = p\cdot q, \quad q^2 = -Q^2, \quad p^2 = M^2, \quad \nu_{\rm el} = Q^2/2M$

$F_1(x,Q^2)$, $F_2(x,Q^2)$ are the proton structure functions

$x = Q^2/2M\nu$ is a Bjorken variable



Accessing the subtraction function $T_1(0,Q^2)$

Constraints: $\lim_{Q^2\to 0} T_1^{\text{non-Born}}(0,Q^2) = 4\pi\beta_{M1}Q^2 + \mathcal{O}(Q^4)$, $\lim_{Q^2\to\infty} T_1^{\text{non-Born}}(0,Q^2) = \mathcal{O}(Q^{-2})$

Covariant baryon χPT

Lensky et al., PRD (2018)

Alarcón et al., PRD (2020)

Different types of models

e.g., heavy-baryon χPT+ high-Q^2 model

Birse & McGovern, EPJA (2012)

$$T_1^{\text{non-Born}} = 4\pi\beta_{M1}\frac{Q^2}{\left(1+\frac{Q^2}{\Lambda^2}\right)^4}$$

$T_1(0,Q^2)$

Data-driven evaluation

- Can be extracted from the dilepton electroproduction on the nucleon
  Pauk, Carlson, Vanderhaeghen, PRC (2020)
- Can be obtained via another sum rule
  arXiv:2305.08814 (2023)

Prospective Lattice QCD calculation at the subtraction point

$\nu = iQ$

Hagelstein & Pascalutsa, Nucl. Phys. A (2021)

Sum rule for the subtraction function

1. Introduce the Compton helicity amplitude with two longitudinally polarized photons

$$T_L(\nu, Q^2) = \left(1 + \frac{\nu^2}{Q^2}\right) T_2(\nu, Q^2) - T_1(\nu, Q^2)$$

2. Assume that $T_L(\nu, Q^2)$ obeys unsubtracted dispersion relation

Dispersion relation for T_L

$$T_L(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_0}^{\infty} d\nu' \nu'^2 \frac{\sigma_L(\nu', Q^2)}{\nu'^2 - \nu^2}$$

Dispersion relation for T_1

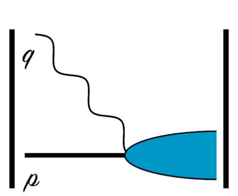
$$T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{2\nu^2}{\pi} \int_{\nu_{\text{el}}}^{\infty} d\nu' \frac{\sigma_T(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+}$$

Equalling the amplitudes at the Siegert point $\nu = iQ$

$$T_1(\nu = iQ, Q^2) = -T_L(\nu = iQ, Q^2)$$

$$T_1(0, Q^2) = \frac{2}{\pi} Q^2 \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2 + Q^2} \left[\sigma_T - \frac{\nu^2}{Q^2} \sigma_L\right] (\nu, Q^2)\,.$$

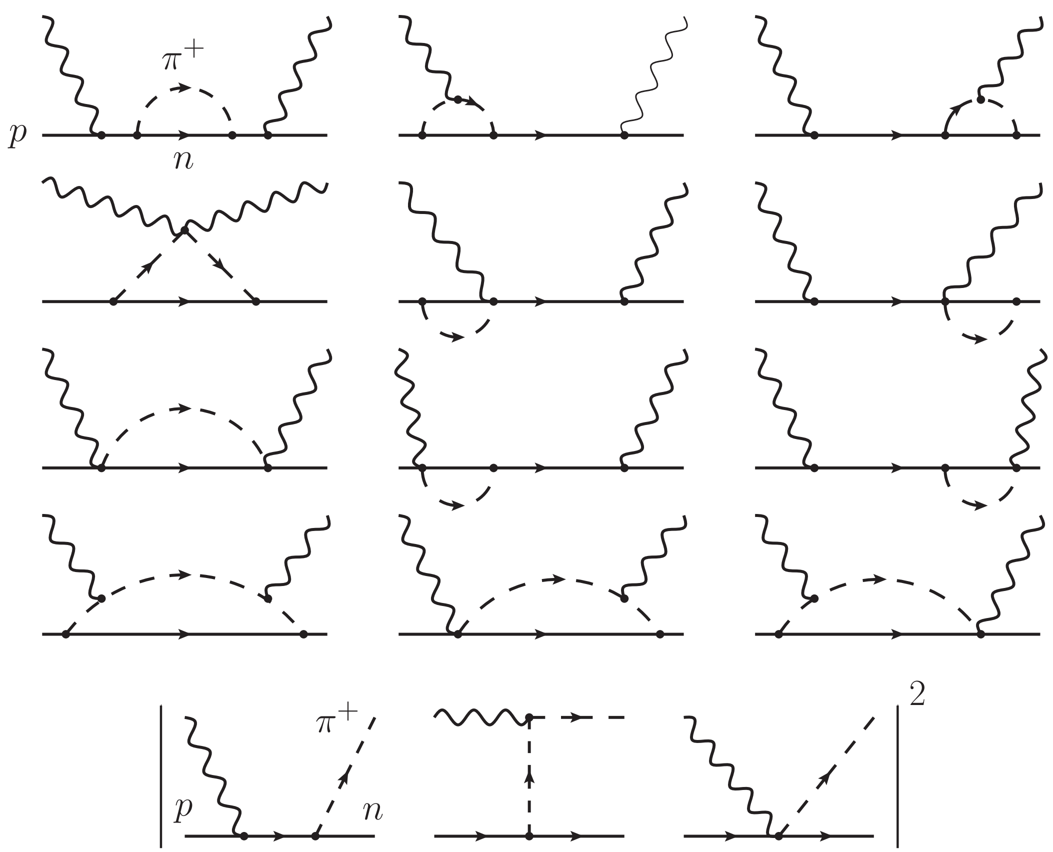
σ_L, σ_T are polarized photoabsorption cross sections



Verification of the sum rule for subtraction function

arXiv:2305.08814 (2023)

$$T_1(0,Q^2) = \frac{2}{\pi} Q^2 \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2 + Q^2} \left[\sigma_T - \frac{\nu^2}{Q^2} \sigma_L \right] (\nu, Q^2) \,.$$



This sum rule is valid in the manifestly covariant baryon χPT for the $\mathcal{O}(p^3)$ contribution to the proton electric polarizability that comes from the <u>charged</u> pion loops.

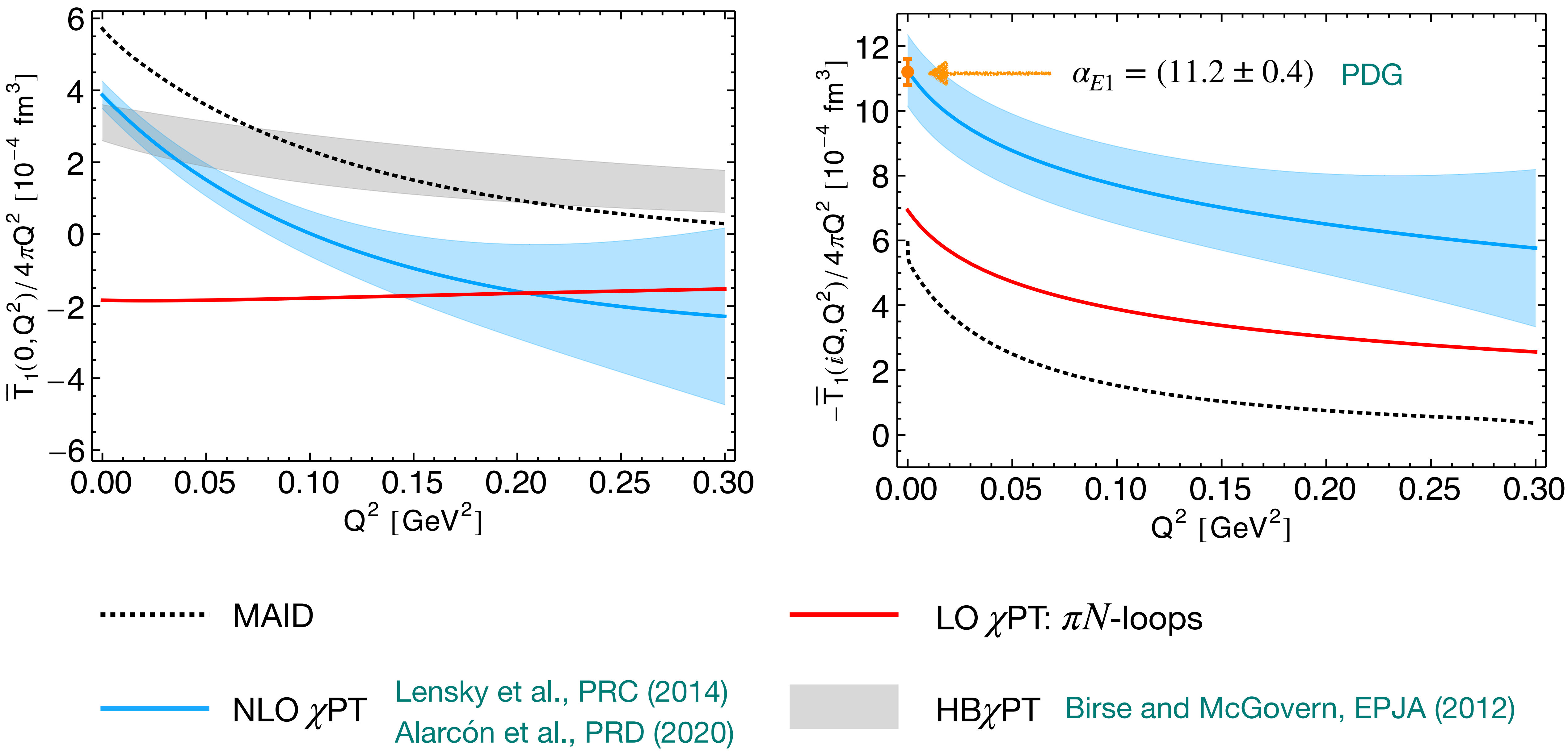
Note that at this order we only verify the polarizability contribution(no contributions from the possible non-pole Born terms)

Non-Born part of the subtraction function

arXiv:2305.08814 (2023)

$$\overline{T}_1(0,Q^2) = -\frac{\pi\alpha_{\rm em}Q^2}{M^3}F_2^2(Q^2) + \frac{2}{\pi}Q^2\int_{\nu_0}^{\infty}\frac{d\nu}{\nu^2+Q^2}\left[\sigma_T - \frac{\nu^2}{Q^2}\sigma_L\right](\nu,Q^2)$$

$$\overline{T}_L(iQ,Q^2) = \frac{\pi\alpha_{\rm em}Q^2}{M^3}F_2^2(Q^2) + \frac{2}{\pi}\int_{\nu_0}^{\infty}d\nu\,\nu^2\,\frac{\sigma_L(\nu,Q^2)}{\nu^2+Q^2} = -\,\overline{T}_1(iQ,Q^2)$$



Sum rules for spin-independent polarizabilities

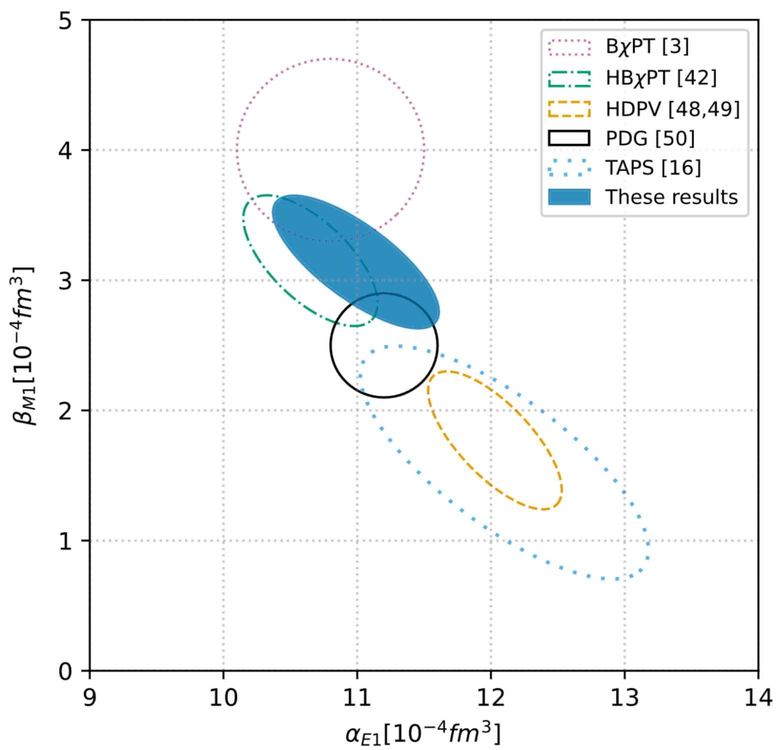
1. The low-energy expansion of $T_1(\nu, Q^2)$: Baldin sum rule
   - a powerful tool for data-driven evaluation of $(\alpha_{E1} + \beta_{M1})$
   - widely used in experimental data analysis

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_0^\infty d\nu \frac{\sigma_T(\nu)}{\nu^2}$$

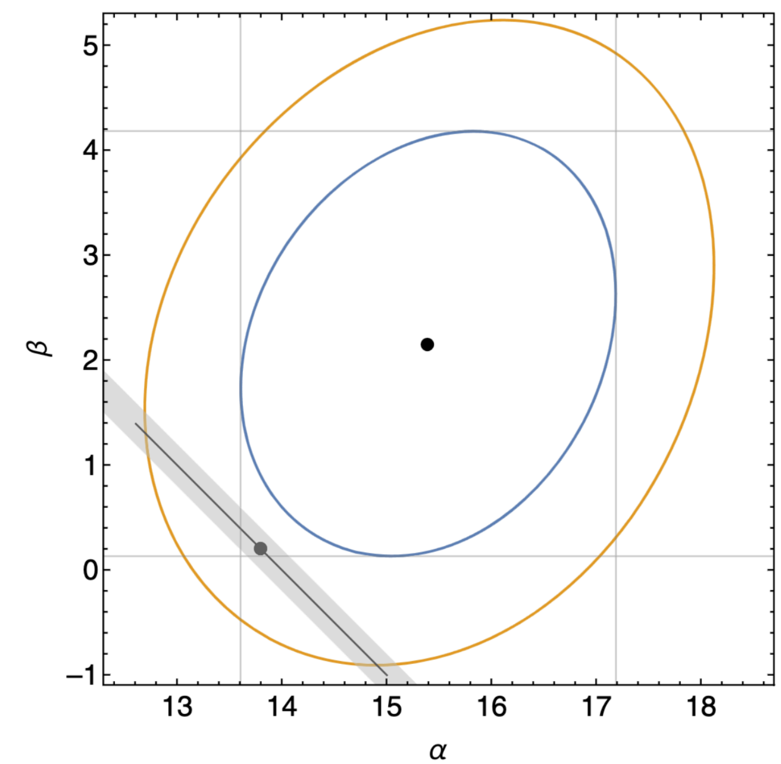
Baldin, Nucl. Phys. 18, 310 (1960)

EXAMPLES:

A2 at MAMI, PRL (2022): Baldin sum rule is treated as an "additional data point"



HIγS, PRL (2022): Baldin sum rule is embedded in the fit function



2. The low-energy expansion of $T_L(\nu, Q^2)$: Bernabéu-Tarrach sum rule
   - a data-driven determination of the electric polarizability alone

$\varkappa$ is the anomalous magnetic moment of the nucleon

$$\alpha_{E1} - \frac{\alpha_{em}\varkappa^2}{4M^3} = \frac{1}{2\pi^2} \int_0^\infty d\nu \left[\frac{\sigma_L(\nu, Q^2)}{Q^2} \right]_{Q^2 \to 0}$$

Is it a valid sum rule?

Bernabéu and Tarrach, PLB (1975)

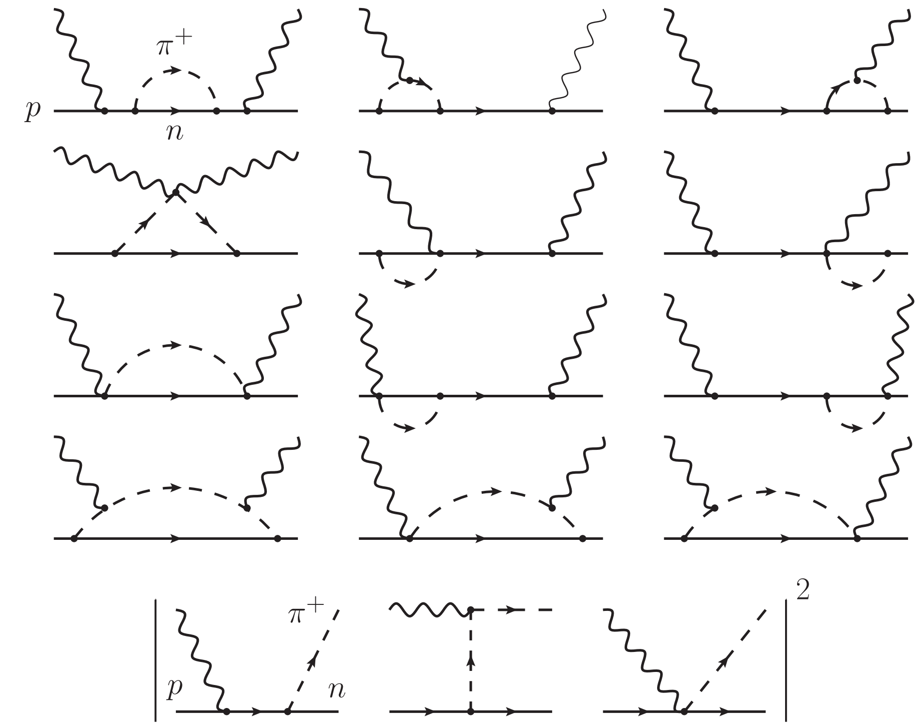
Validation of the Bernabéu-Tarrach sum rule

arXiv:2305.08814 (2023)

We have found cases when the Bernabéu-Tarrach sum rule holds exactly!

1) The sum rule is exactly valid in the manifestly covariant baryon χPT for the $\mathcal{O}(p^3)$ contribution to the proton electric polarizability that comes from the <u>charged</u> pion loops.

The results of [Bernard et al., PRL (1991), NPB (1992)], were reproduced.



2) The sum rule is exactly valid in the naïve parton model

- Callan-Gross relation: $F_2(x, Q^2) = 2xF_1(x, Q^2) \iff \sigma_L = \frac{Q^2}{\nu^2}\sigma_T$

  $x = Q^2/2M\nu$ is a Bjorken variable

- T_L satisfies unsubtracted dispersion relation exactly:

$$T_L(x, Q^2) = T_2(x, Q^2) = \frac{4M\alpha_{\text{em}}}{Q^2}\int_0^1 \frac{d\zeta}{\zeta^2}\frac{x^2F_2(\zeta, Q^2)}{\left(\frac{x}{\zeta}\right)^2 - 1 - i\epsilon}$$

Bernabéu-Tarrach sum rule: (in)validation?

LO QED
[Llanta and Tarrach, PLB (1978)]
- the sum rule converges to the wrong answer

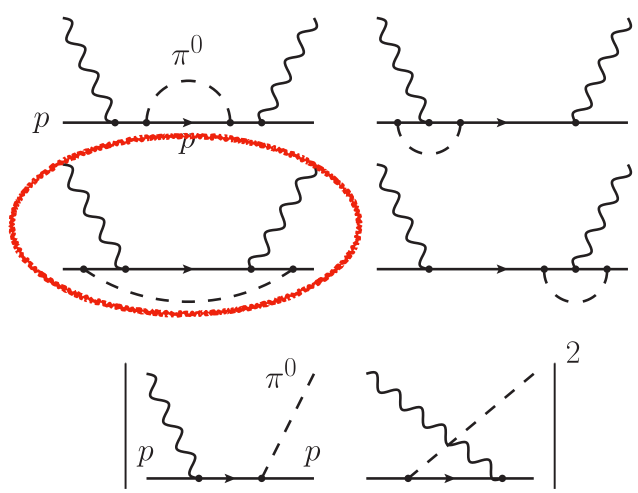
EFT for pions
Llanta and Tarrach, Phys.Lett. (1980)
L’vov, Sov. J. Nucl. Phys. (1981)
- $\alpha_{E1} > 0$ for both π^0 and $\pi^{\pm}$. It violated some of the EFT predictions

Regge theory arguments
[L’vov, NPA (1998)]
- the sum rule is in general invalid since it is divergent without the subtraction.

Our checks in LO covariant baryon χPT

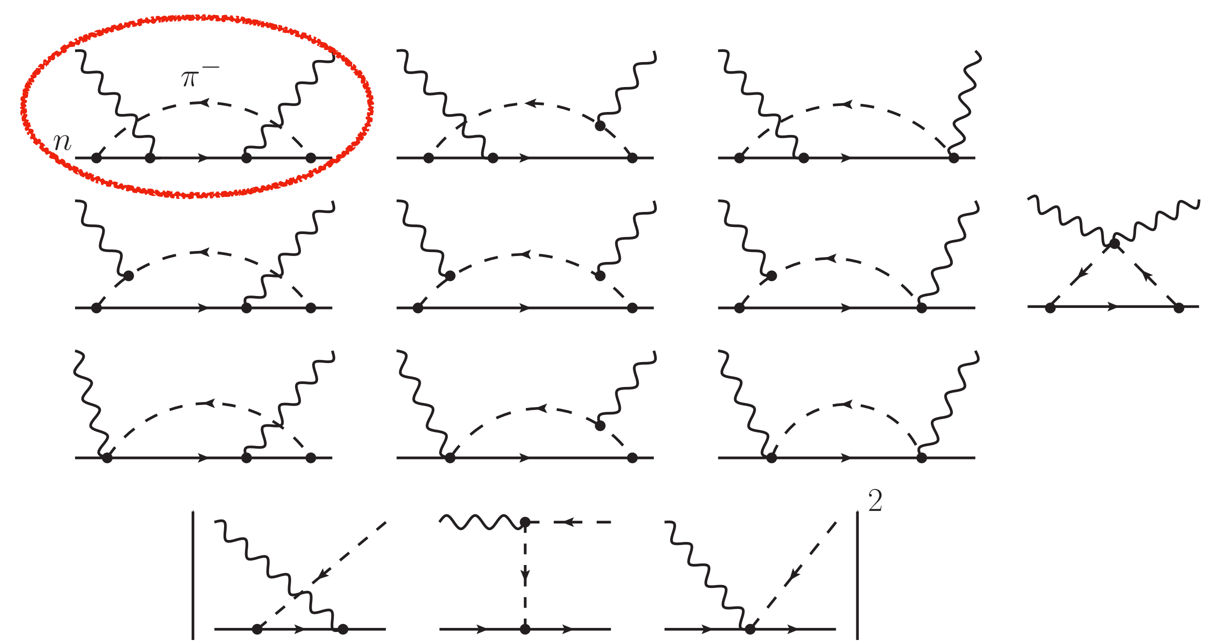
arXiv:2305.08814 (2023)

π^0 contribution to the Compton scattering off the proton



$$T_L^{\pi^0 p-\text{loops}}(\infty, Q^2) = -\frac{\alpha_{\text{em}}}{12\pi}\frac{g_{\pi N}^2}{M^3}Q^2 + \mathcal{O}(Q^4)$$

The Compton scattering off the neutron



$$T_L^{\pi^- n-\text{loops}}(\infty, Q^2) = -\frac{\alpha_{\text{em}}}{6\pi}\frac{g_{\pi N}^2}{M^3}Q^2 + \mathcal{O}(Q^4)$$

➡In these cases the dispersion relation for T_L should be modified as follows:

$$T_L(\nu, Q^2) = T_L(\infty, Q^2) + \frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu' \nu'^2 \frac{\sigma_L(\nu', Q^2)}{\nu'^2 - \nu^2}$$

Sugawara and Kanazawa, PhysRev (1961)

How to deal with asymptotic constants?

Our point: the sum rule is valid if convergent.

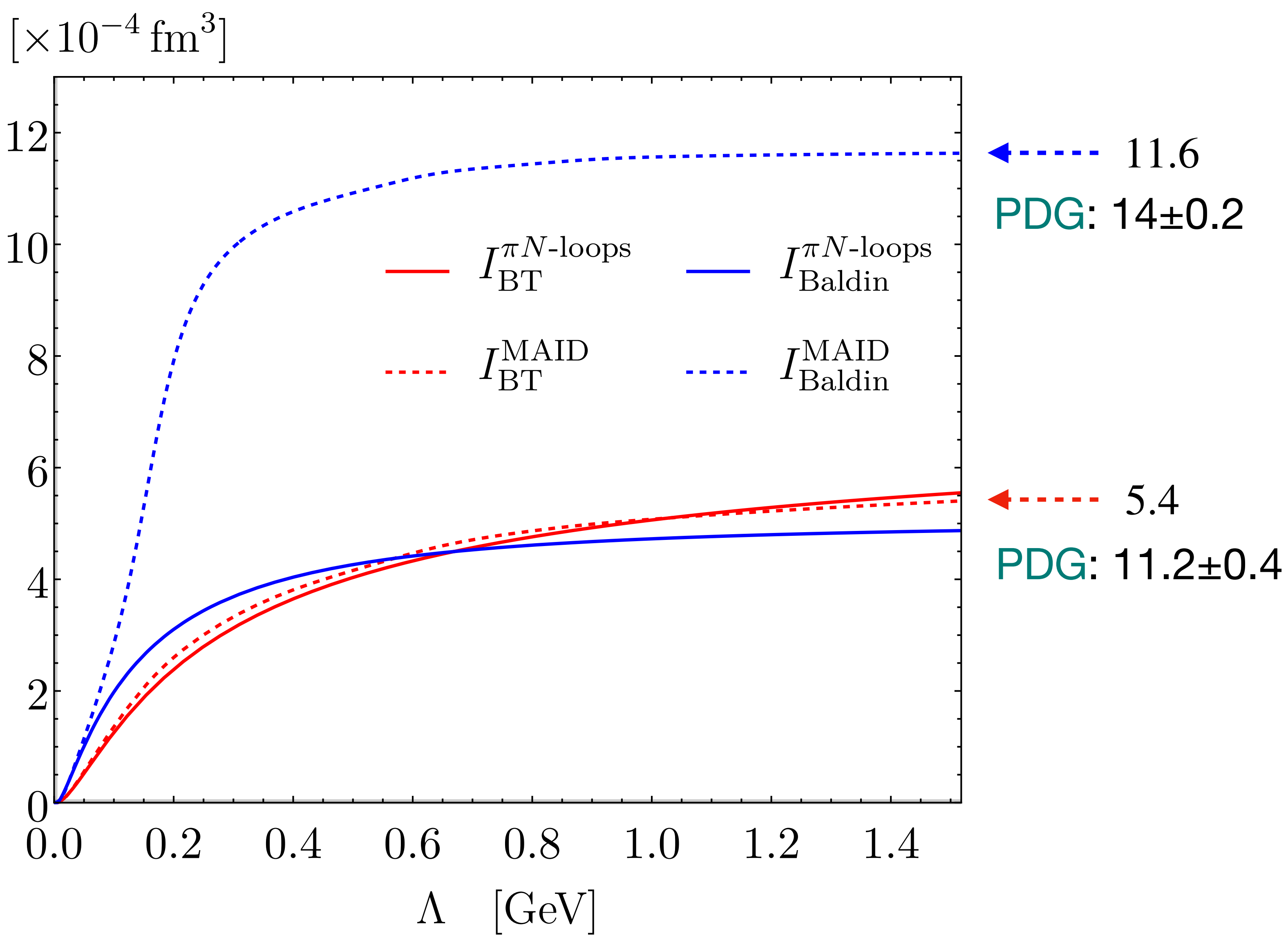
- The low-energy physics should not depend on the behavior at very high energies (i.e. physics at the Plank scale)
- The asymptotic constants are the artifacts of the low-energy theory, which is not valid at high energies.
- With proper ultraviolet completion, a theory does not produce the asymptotic constants in the sum rules.

In conclusion, we ought to treat the Bernabéu-Tarrach sum rule as the valid sum rule

Saturation of the sum rule integral

arXiv:2305.08814 (2023)

$$I_{\mathrm{BT}}(\Lambda) = \frac{1}{2\pi^2}\int_{\nu_0}^{\Lambda} d\nu \left[\frac{\sigma_L(\nu,Q^2)}{Q^2}\right]_{Q^2\to 0} \qquad I_{\mathrm{Baldin}}(\Lambda) = \frac{1}{2\pi^2}\int_{\nu_0}^{\Lambda} d\nu \frac{\sigma_T(\nu,Q^2)}{\nu^2}$$

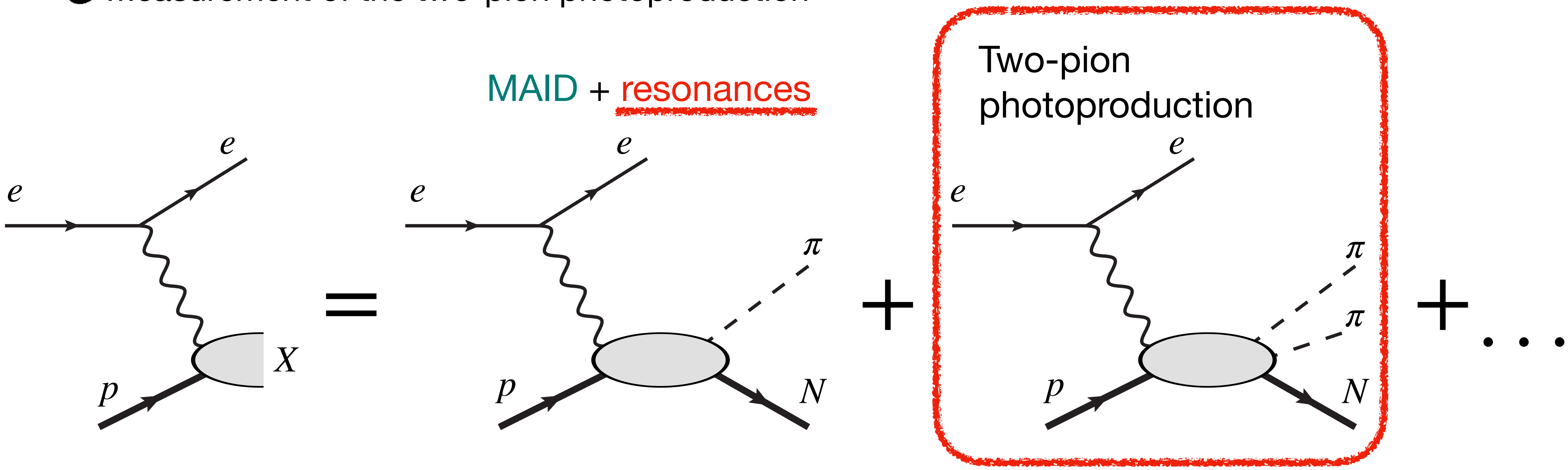


Source	α_{E1} [$\times 10^{-4}\mathrm{fm}^3$]
MAID (extrapolated)	5.4 (~7)
Kappa term	0.5
resonances	0.5-1*
Total (w/o Regge region)	8-8.5*
[PDG]	11.2±0.4

*Currently, we have no parametrization of the existing data, which has a stable behavior within the limit $Q^2 \to 0$

Useful experimental input to σ_L

- Low-energy input:
  - Improve the existing parametrizations of resonance region to make them valid at the limit $Q^2 \to 0$
  - Measurement of the two-pion photoproduction



- Moderate and high-energy input:
  - Detailed measurement of $F_L(x, Q^2)$ in the region $\nu = 20\,..\,20\,000\,\mathrm{GeV}$ at low Q^2

Summary

- The Bernabéu-Tarrach sum rule seems to be as valid as the Baldin sum rule. The dipole polarizabilities can be determined separately within the data-driven approach

- Consequently, the data-driven evaluation of the subtraction function in the proton polarizability contribution to the Lamb shift of hydrogen-like atoms is also possible

- Several experimental inputs on σ_L are needed in order to test the sum rules. At low energies, the most interesting one is the two-pion photoproduction on a proton.

- The high-quality parametrization of the current data on σ_L with the correct limit $Q^2 \to 0$ is highly required!

Thank you for attention!