Generalized Polarizabilities of the proton

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and the Structure of the Nucleon

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Temple
University

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Outline

Introduction to the GPs

Overview / Status & Challenges

Recent results / Jlab & MAMI

Prospects / VCS-II @ Jlab, measuring w positrons, …

Proton Polarizablities

Fundamental structure constants
(such as mass, size, shape, …)

Response of the nucleon to external EM field

Sensitive to the full excitation spectrum

Accessed experimentally through Compton Scattering

RCS: static polarizabilities → net effect on the nucleon

PDG

150 Baryon Summary Table

N BARYONS
($S = 0$, $I = 1/2$)
p, $N^+ = uud$; n, $N^0 = udd$

p $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.00727646681 \pm 0.00000000009$ u
Mass $m = 938.272046 \pm 0.000021$ MeV [a]
$|m_p - m_{\overline{p}}|/m_p < 7 \times 10^{-10}$, CL = 90% [b]
$|\frac{q_{\overline{p}}}{m_{\overline{p}}}|/(\frac{q_p}{m_p}) = 0.99999999991 \pm 0.00000000009$
$|q_p + q_{\overline{p}}|/e < 7 \times 10^{-10}$, CL = 90% [b]
$|q_p + q_e|/e < 1 \times 10^{-21}$ [c]
Magnetic moment $\mu = 2.792847356 \pm 0.000000023\ \mu_N$
$(\mu_p + \mu_{\overline{p}}) / \mu_p = (0 \pm 5) \times 10^{-6}$
Electric dipole moment $d < 0.54 \times 10^{-23}$ e cm
Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4}$ fm^3
Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}$ fm^3 (S = 1.2)
Charge radius, μp Lamb shift = 0.84087 ± 0.00039 fm [d]
Charge radius, ep CODATA value = 0.8775 ± 0.0051 fm [d]
Magnetic radius = 0.777 ± 0.016 fm
Mean life $\tau > 2.1 \times 10^{29}$ years, CL = 90% [e] ($p \rightarrow$ invisible mode)
Mean life $\tau > 10^{31}$ to 10^{33} years [e] (mode dependent)

Virtual Compton Scattering:

Virtuality of photon gives access to the GPs : $\alpha_E(Q^2)$ & $\beta_M(Q^2)$ (+ 4 spin GPs)

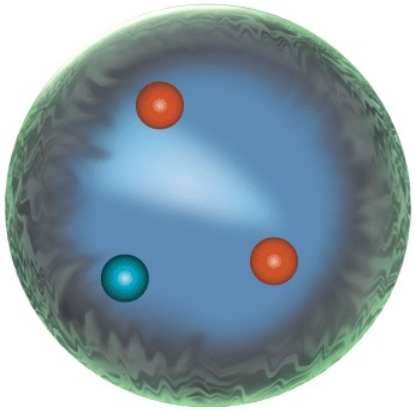
→ mapping out the spatial distribution of the polarization densities

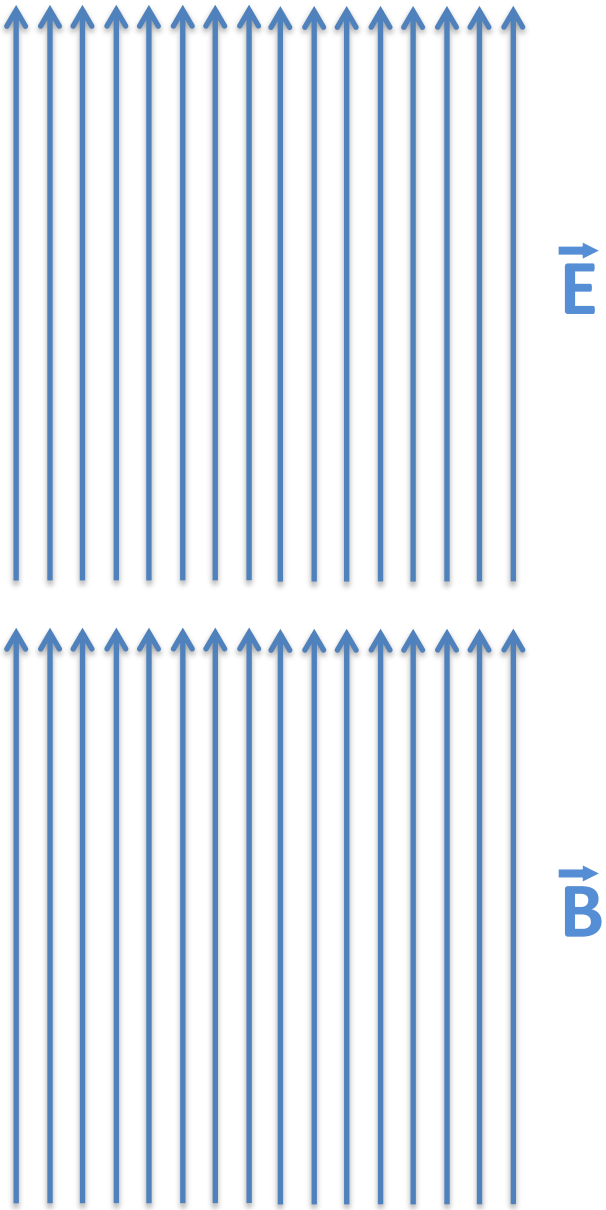
Fourier transform of densities of electric charges and magnetization of a nucleon deformed by an applied EM field

Scalar Polarizablities

Response of internal structure to an applied EM field

Interaction of the EM field with the internal structure of the nucleon





Scalar Polarizablities

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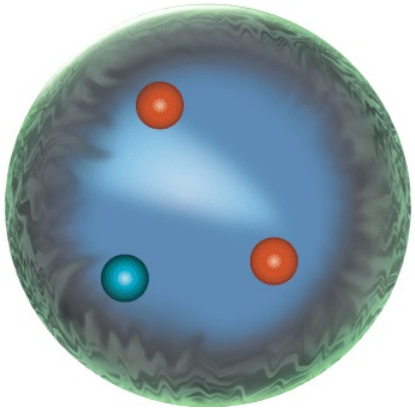
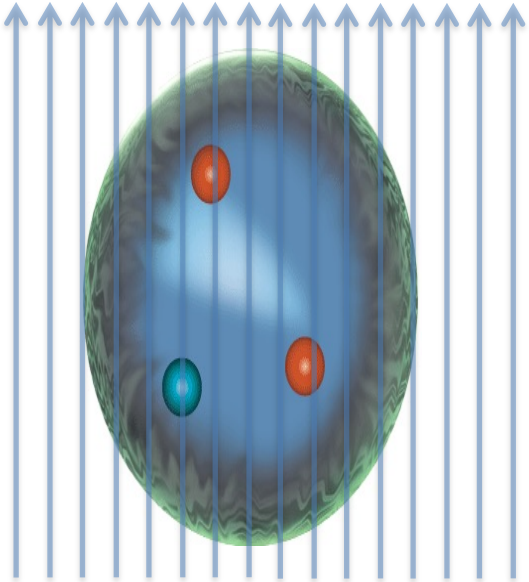


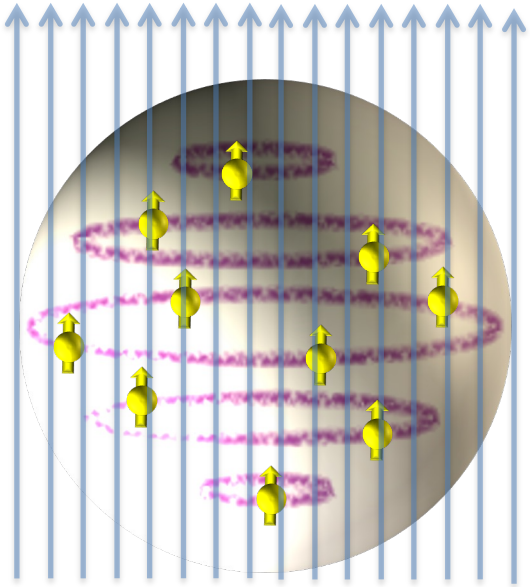


$\vec{E}$

"stretchability"

$\vec{d}_{E\ \text{induced}} \sim \alpha \vec{E}$

External field deforms the charge distribution



$\vec{B}$

"alignability"

$\vec{d}_{M\ \text{induced}} \sim \beta \vec{B}$

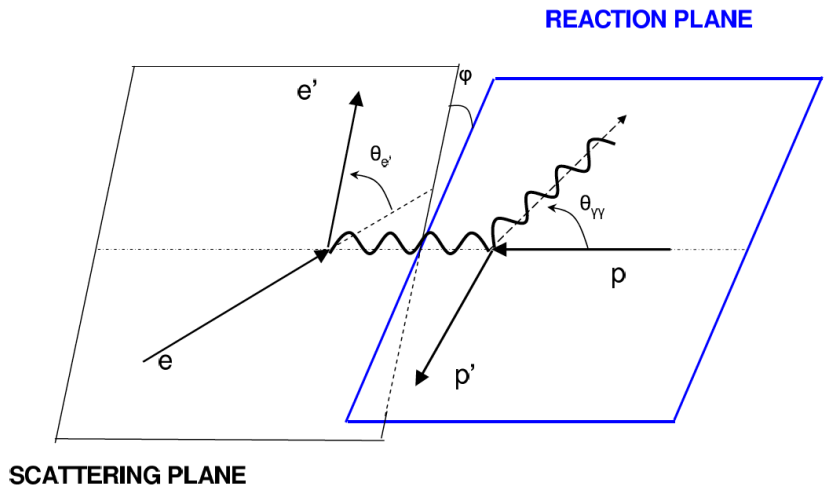
$\beta_{para} > 0$

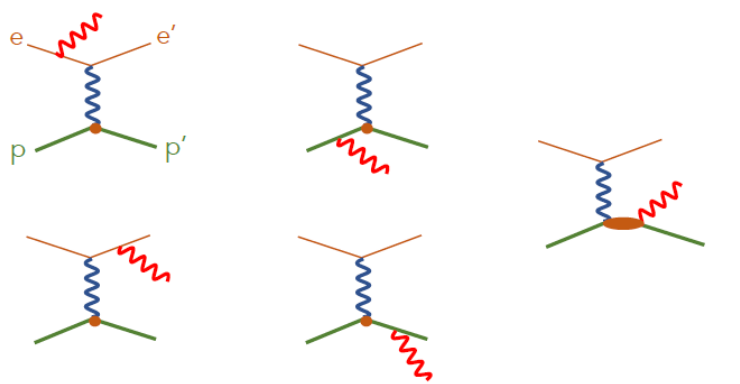
$\beta_{diam} < 0$

Paramagnetic: proton spin aligns with the external magnetic field

Diamagnetic: π-cloud induction produces field counter to the external perturbation

Virtual Compton Scattering





Elastic FFs

GPs

Virtual Compton Scattering

DR

valid below & above
Pion threshold

Dispersive integrals
for Non Born amplitudes

Spin GPs are fixed

Scalar GPs have
an unconstrained part

Fit to the experimental
cross sections at each Q^2

LEX

valid only below
Pion threshold

Response functions

$$d^5\sigma \;=\; d^5\sigma^{BH+Born} \;+\; q'_{cm}\cdot\phi\cdot\Psi_0 \;+\; \mathcal{O}(q'^2_{cm})$$

$$\Psi_0 \;=\; v_1\cdot(P_{LL}-\frac{1}{\epsilon}P_{TT}) \;+\; v_2\cdot P_{LT}$$

Subtract the spin part

$$P_{TT} \;=\; [P_{TT\ spin}]$$

$$P_{LT} \;=\; -\frac{2M}{\alpha_{em}}\sqrt{\frac{q^2_{cm}}{Q^2}}\cdot G^p_E(Q^2)\cdot\beta_M(Q^2)+[P_{LT\ spin}]$$

utilize DR

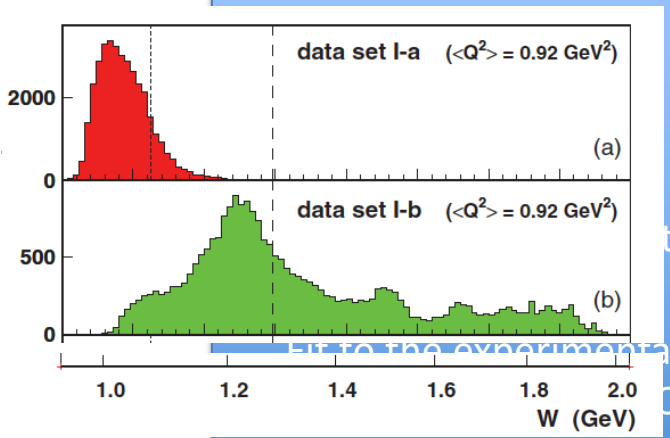
scalar GPs α_E and β_M

Virtual Compton Scattering

DR

valid below & above Pion threshold

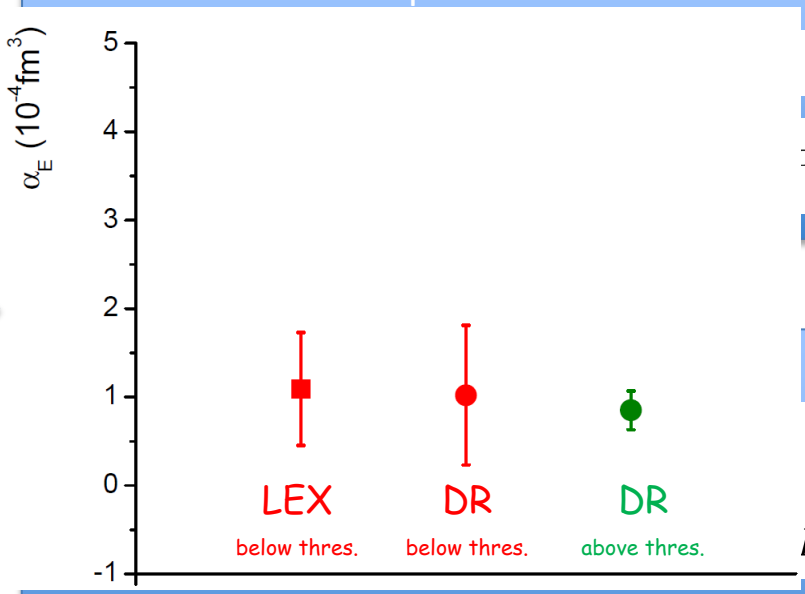
Dispersive integrals for Non Born amplitudes



LEX

valid only below Pion threshold

Response functions



$\mathcal{O}(q_{cm}'^2)$

P_{LT}

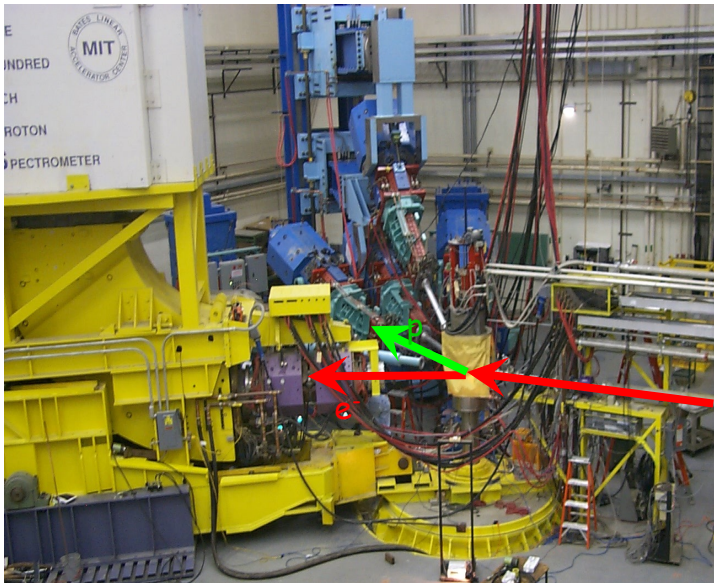
$P_{LT\ spin}]$

utilize DR

scalar GPs α_E and β_M

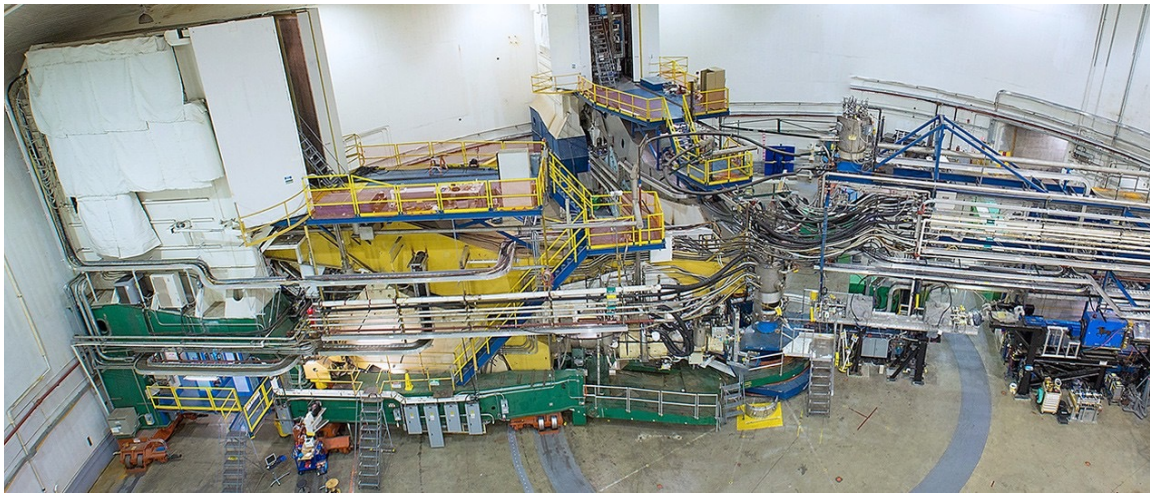
Early Experiments

MIT-Bates @ Q^2=0.06 GeV^2



MAMI-A1 @ Q^2=0.33 GeV^2

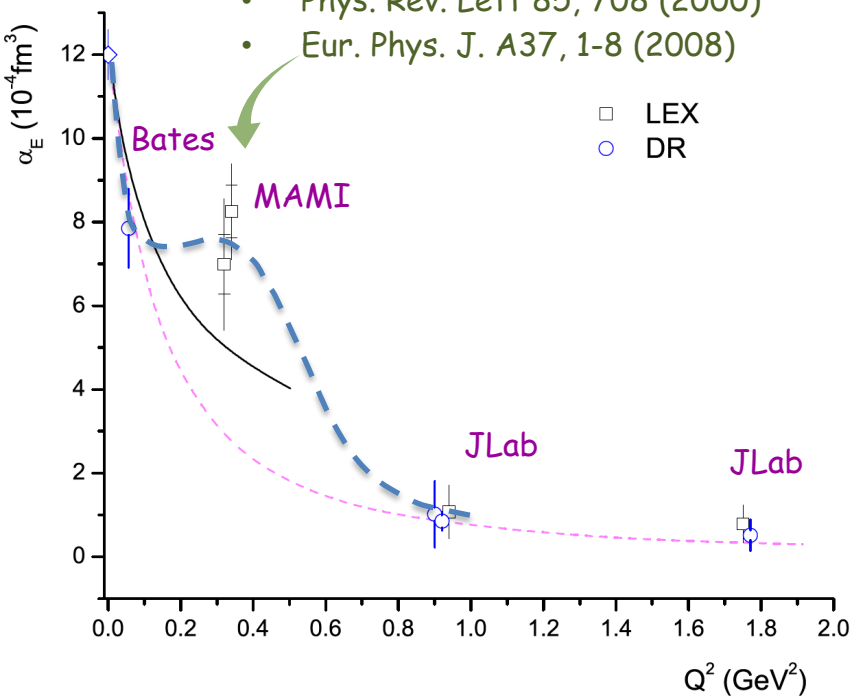
Jlab-Hall A @ Q^2=0.9 & 1.8 GeV^2

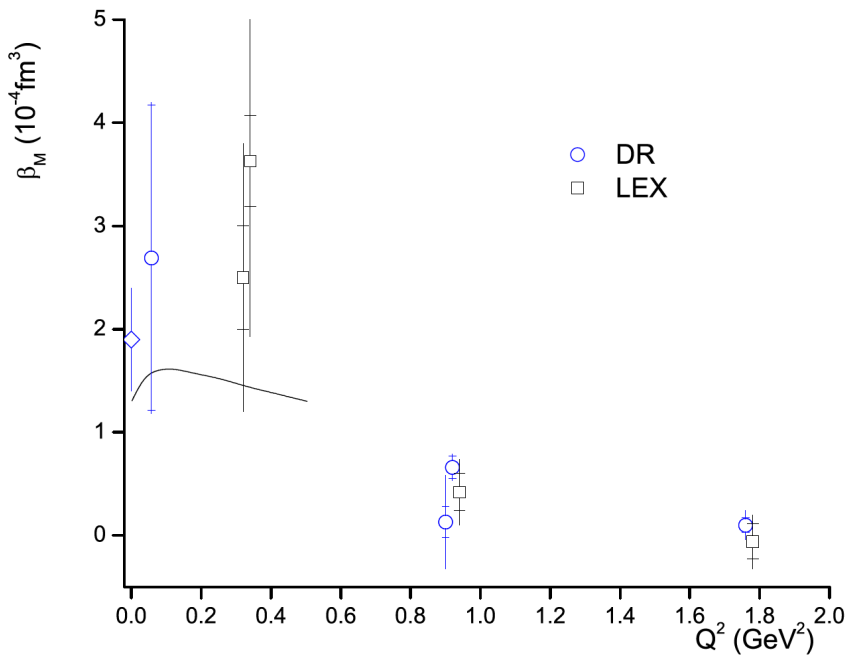


Early Experiments

Q^2 = 0.33 $(GeV/c)^2$ measured twice at MAMI:

- Phys. Rev. Lett 85, 708 (2000)
- Eur. Phys. J. A37, 1-8 (2008)





$a_E \approx 10^{-3}\ V_N$ (stiffness / relativistic character)

Data: non-trivial Q^2 dependence of a_E (?)

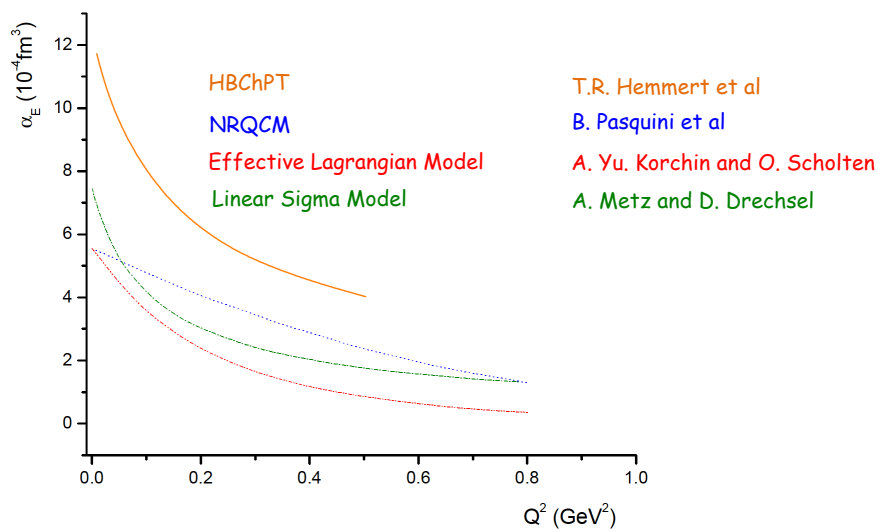
Theory: monotonic fall-off

β_M small ←→cancellation of competing mechanisms

Large uncertainties

Higher precision measurements needed

→ Quantify balance between dia/para-magnetism



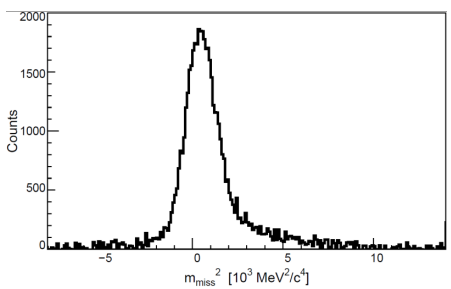
Recent Experiments

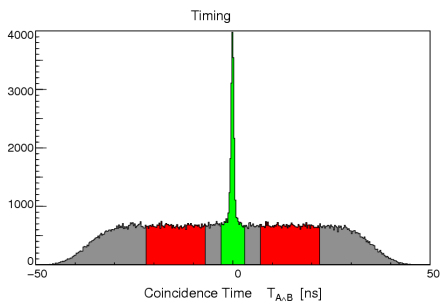
Recent Measurements: MAMI

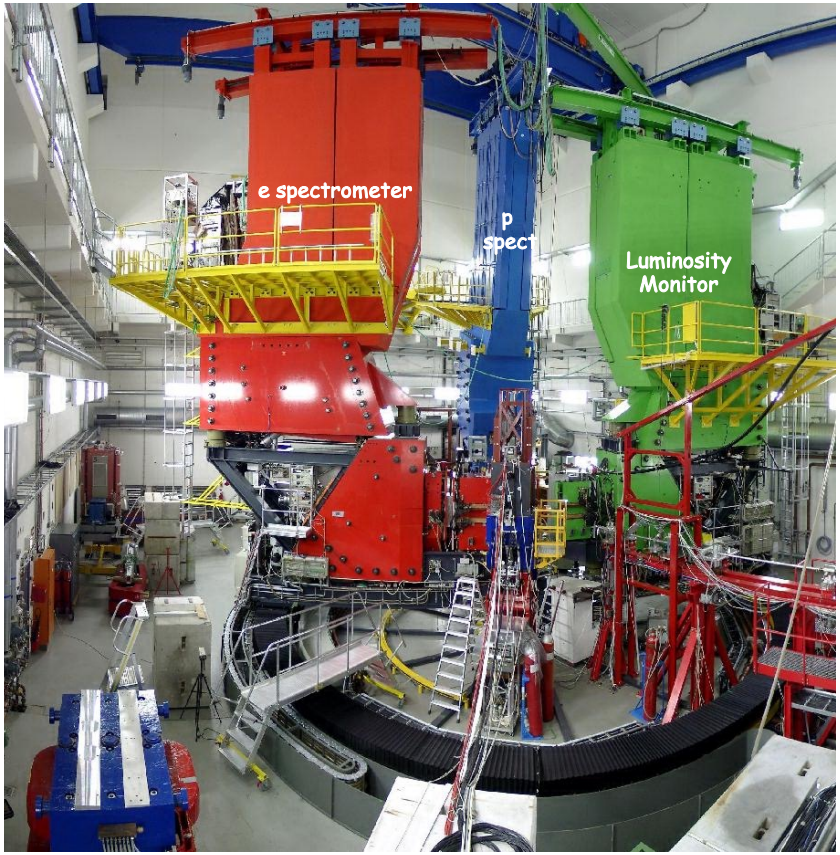
MAMI A1/1-09 (vcsq2) below threshold

MAMI A1/3-12 (vcsdelta) above threshold

Both experiments utilized the A1 setup at MAMI







A1/1-09 @ MAMI

For LEX the higher order terms have to be kept small / under control

$$d^5\sigma \;=\; d^5\sigma^{BH+Born} \;+\; q'_{cm}\cdot\phi\cdot\Psi_0 \;+\; \mathcal{O}(q'^2_{cm})$$

Refined analysis procedure / phase space masking to keep these terms smaller than ~ 2%-3% level

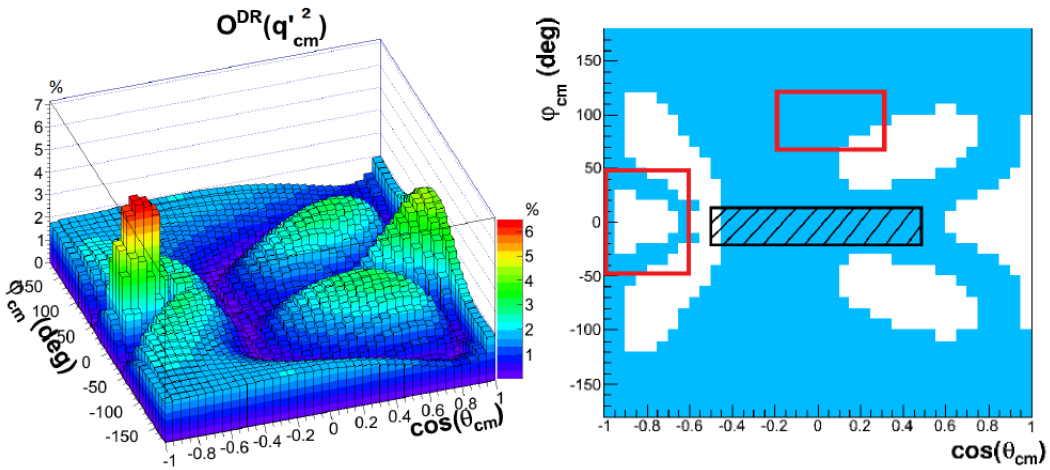
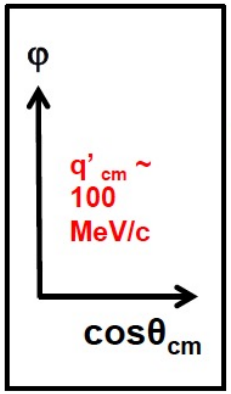


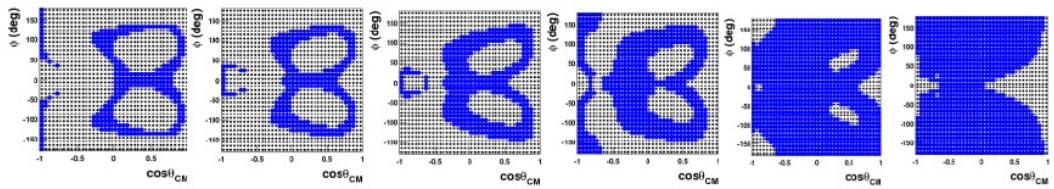
Figure 3.13: (Left) behavior of $\mathcal{O}^{DR}(q'^{\,2}_{cm})$ in the $(cos(\theta_{cm}),\varphi_{cm})$-plane at $q'_{cm} = 87.5\ MeV/c$ and (right) two-dimensional representation of the angular region where $\mathcal{O}^{DR}(q'^{\,2}_{cm}) < 2\%$ (blue), the red squares correspond to the two areas of interest to perform the GP extraction.

Figure from PhD thesis of L. Correa, Mainz / Cl. Ferrand

Blue bins = where the higher-order estimator is < 3%
(LEX truncation « valid »)



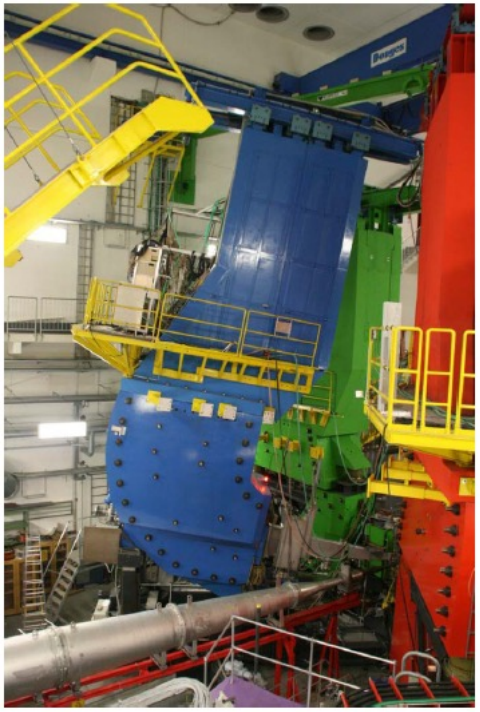
VCS expt :	Bates	MAMI	MAMI	MAMI	MAMI	JLab
Q2 (GeV2) =	0.06	0.10	0.20	0.33	0.45	0.92



New « vcsq2 » data:

- **OOP kinematics (to access the blue region)**
- **LEX Fit done with bin selection at Q^2 = 0.1 and 0.2 GeV^2.**
- **was found not necessary at Q^2 = 0.45 GeV^2.**

In-plane



8.5 deg OOP

(material from H. Fonvieille)

A1/1-09 @ MAMI

~ 1.0 GeV beam

Q^2 = 0.1 $(GeV/c)^2$, 0.2 $(GeV/c)^2$, and 0.45 $(GeV/c)^2$

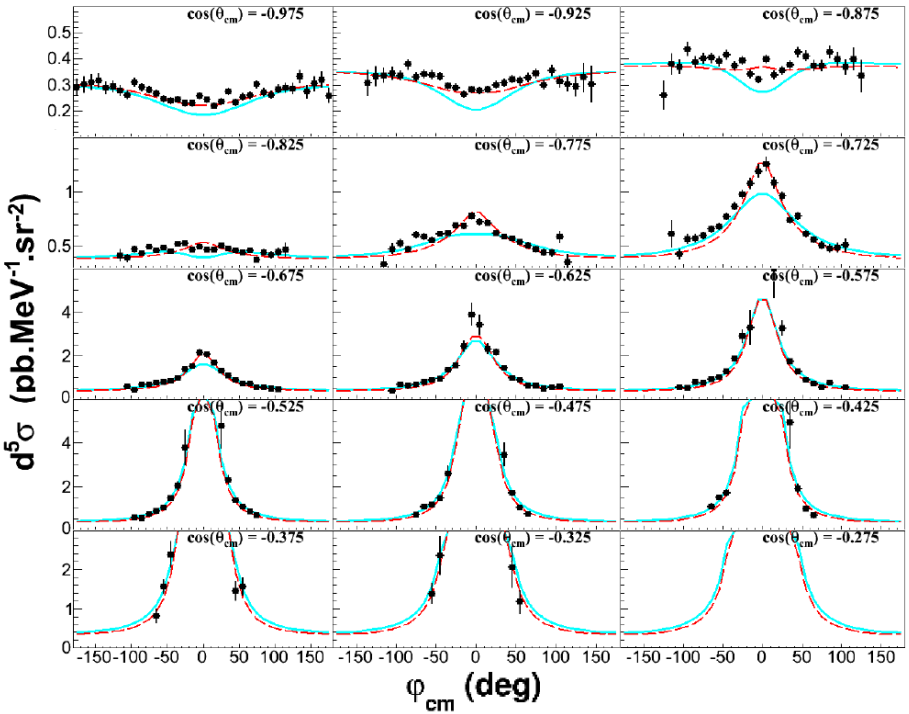


Figure 5.8: Setting INP: measured $ep \rightarrow ep\gamma$ cross section at fixed q'_{cm} = 112.5 MeV/c with respect to φ_{cm} for all the $cos(\theta_{cm})$-bins. The curves follow the convention of figure 5.6.

Figure from PhD thesis of L. Correa, Mainz / Cl. Ferrand

BH+B ---

Polarizability effect ---

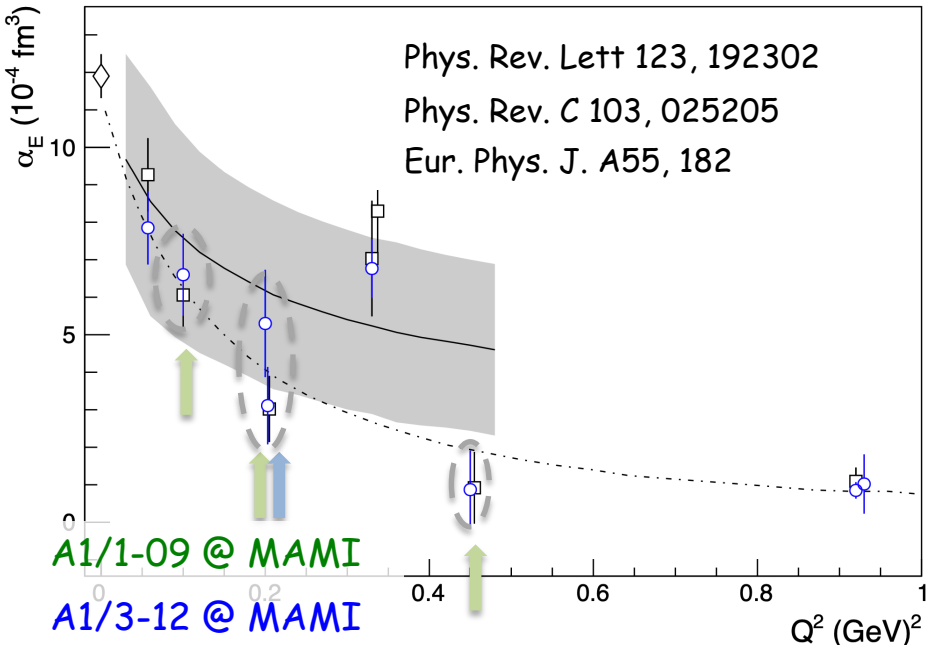
GP effect typically 5% – 15% of the cross section

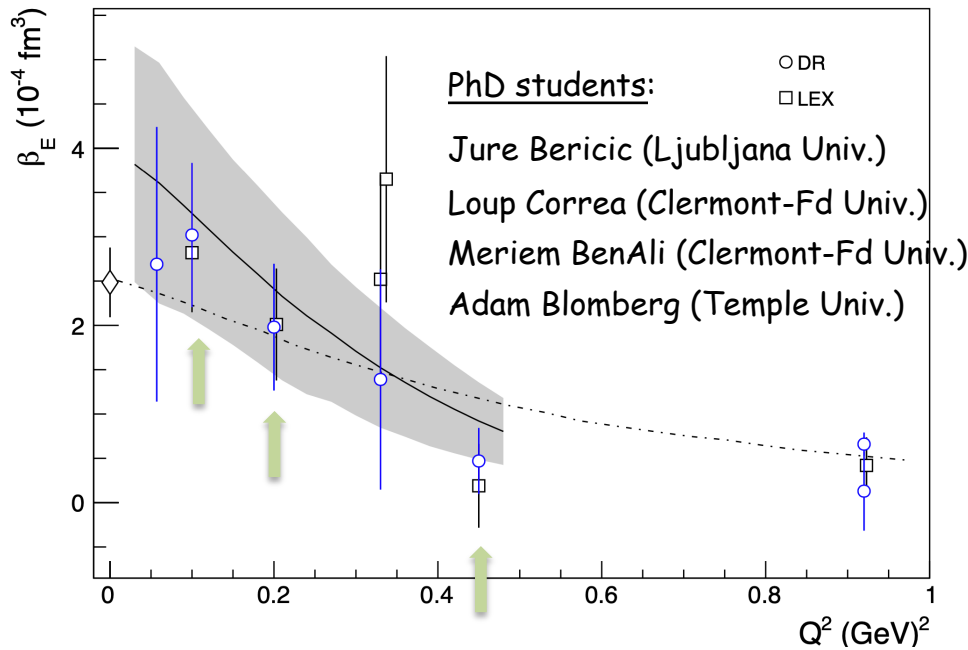
Polarizability fits:

DR fit:
DR calculation includes full dependency in q'_{cm}

LEX fit:
truncated in q'cm. Suppress contribution from higher order terms

MAMI Results

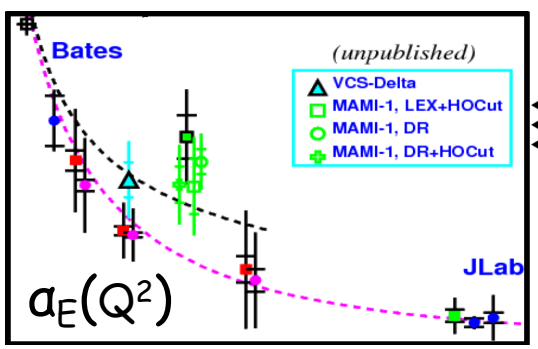




Revisiting the Q^2=0.33 GeV^2 data

Analysis revisited (unpublished):

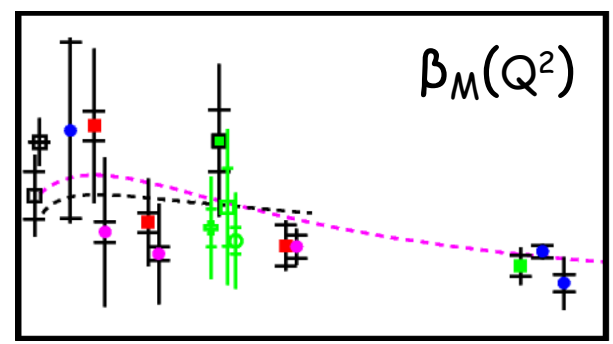
The α_E puzzle still holds



Re-fits at Q^2=0.33 GeV^2 (H.F.)

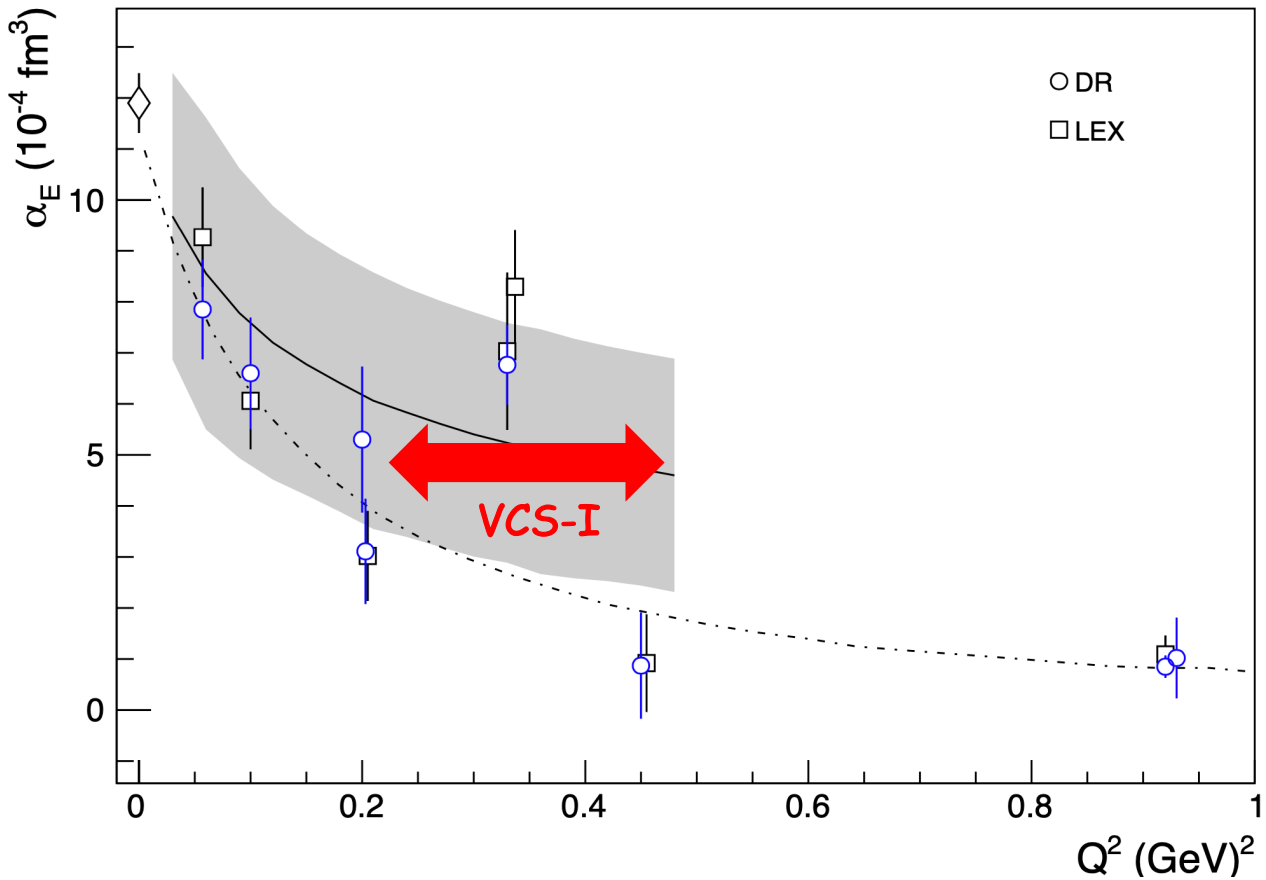
LEX and DR

Updated HO-cut

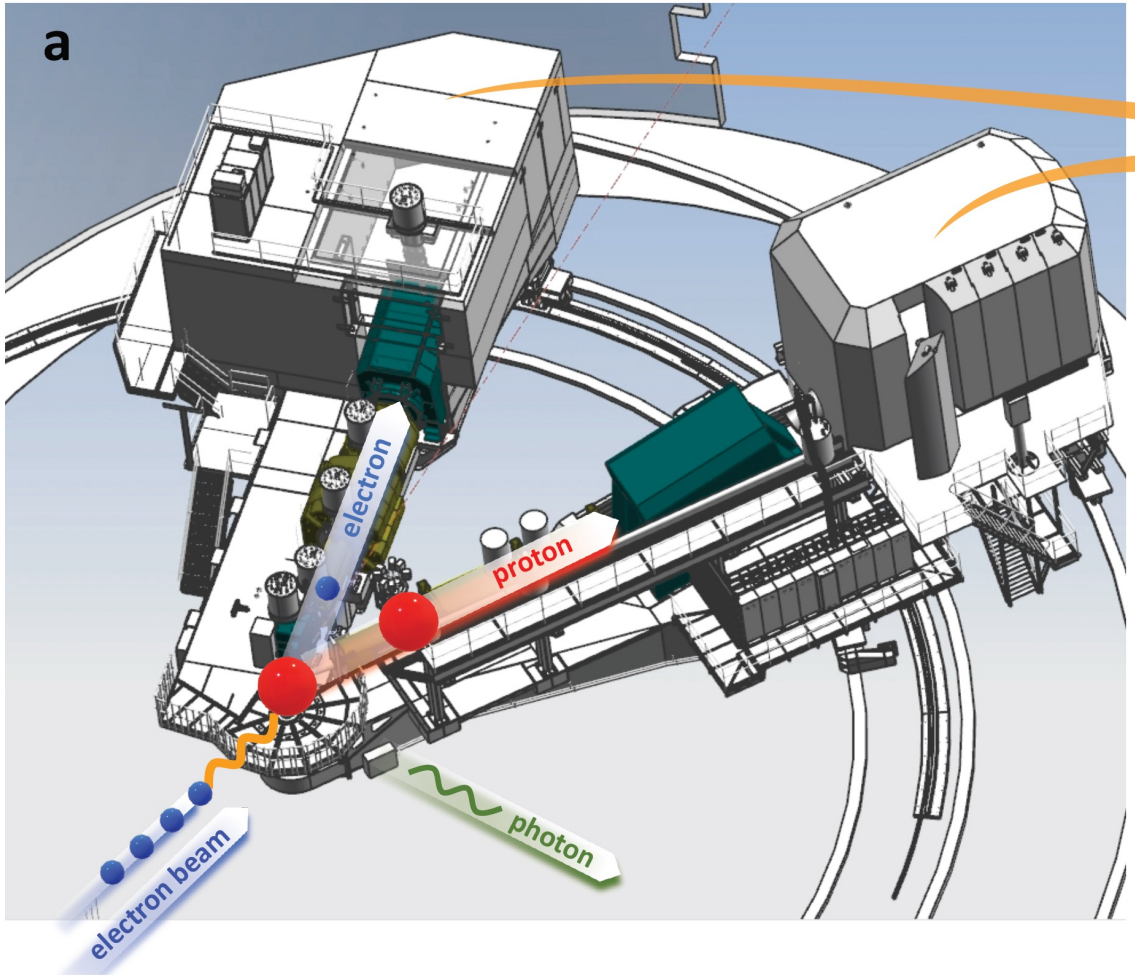


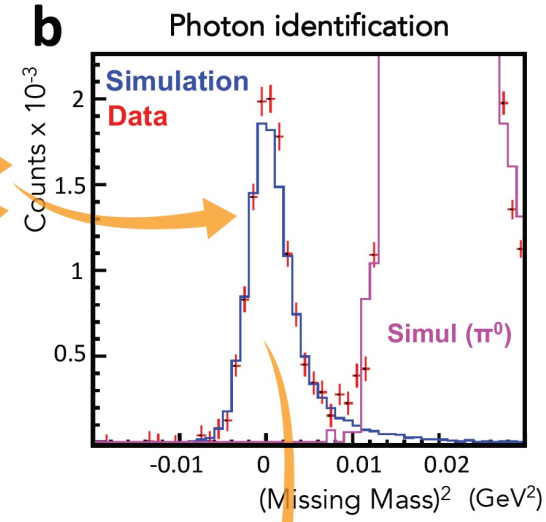
Jlab : VCS-I Experiment (E12-15-001) in Hall C

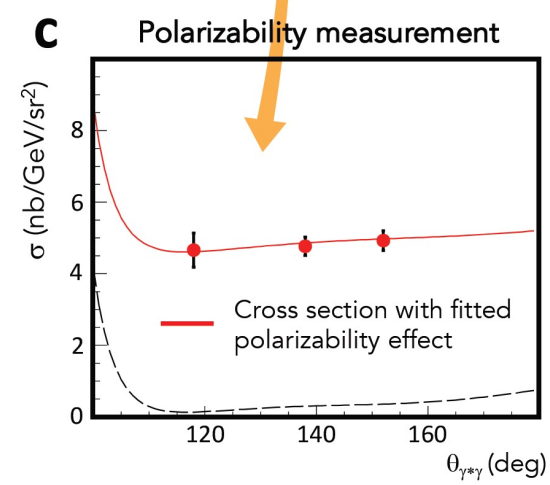
High precision measurements targeting
explicitly the kinematics of interest for α_E



The experiment







Hall C: SHMS, HMS

4.56 GeV

20 μA

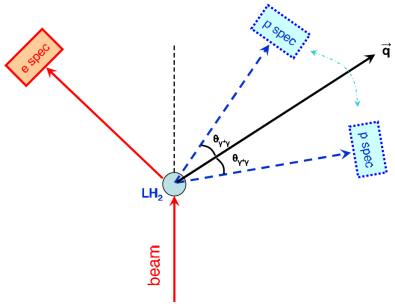
Liquid hydrogen 10 cm

cross sections & azimuthal asymmetries

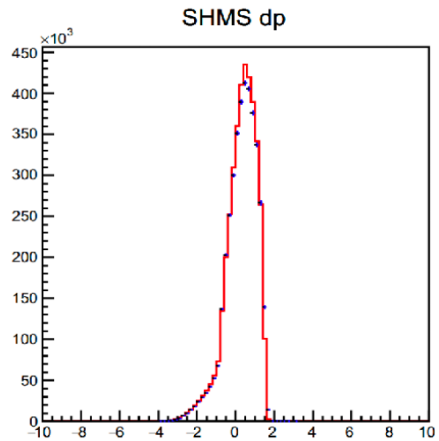
$$A_{(\phi_{\gamma^*\gamma}=0,\pi)} = \frac{\sigma_{\phi_{\gamma^*\gamma}=0} - \sigma_{\phi_{\gamma^*\gamma}=180}}{\sigma_{\phi_{\gamma^*\gamma}=0} + \sigma_{\phi_{\gamma^*\gamma}=180}}$$

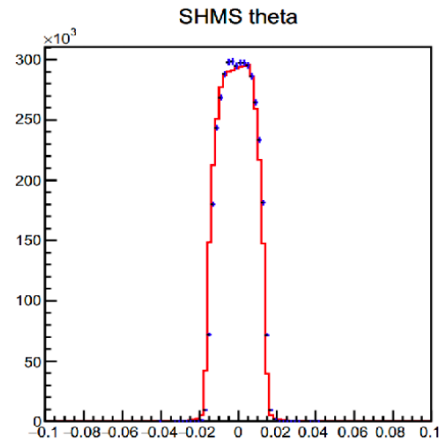
sensitivity to GPs

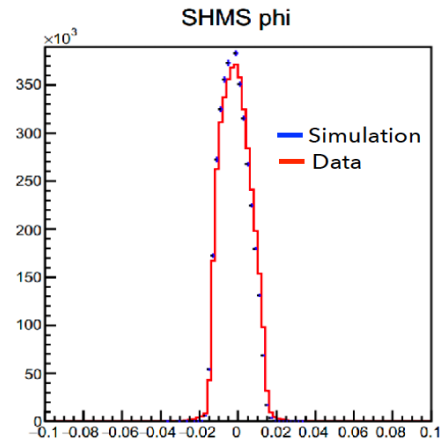
suppression of systematic asymmetries



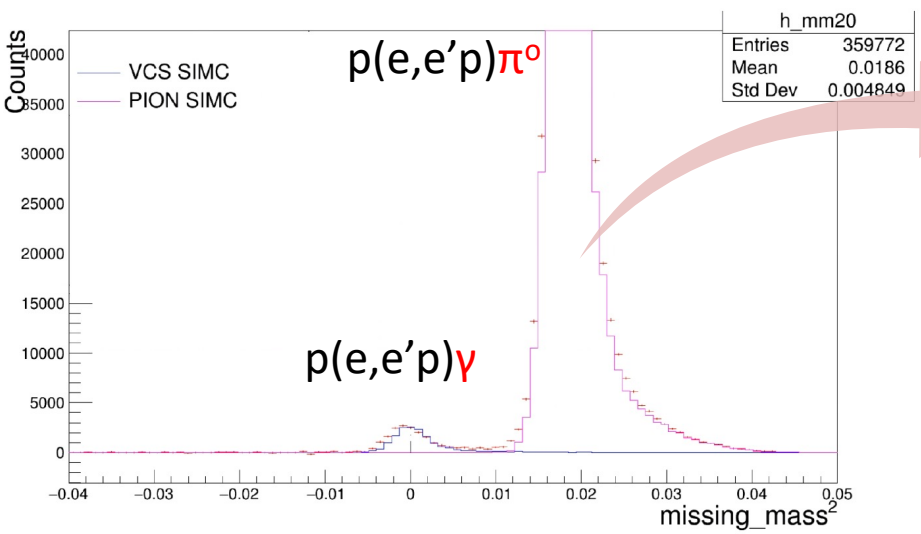
Elastic data

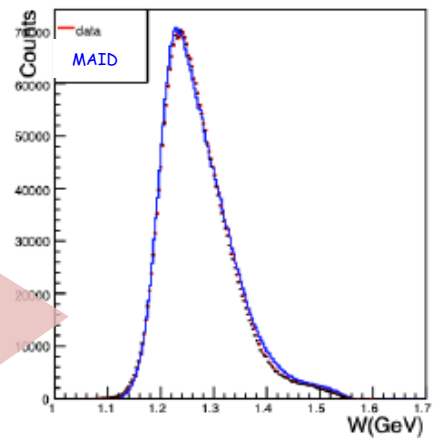


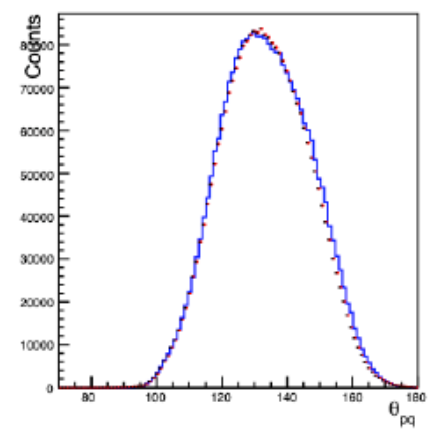


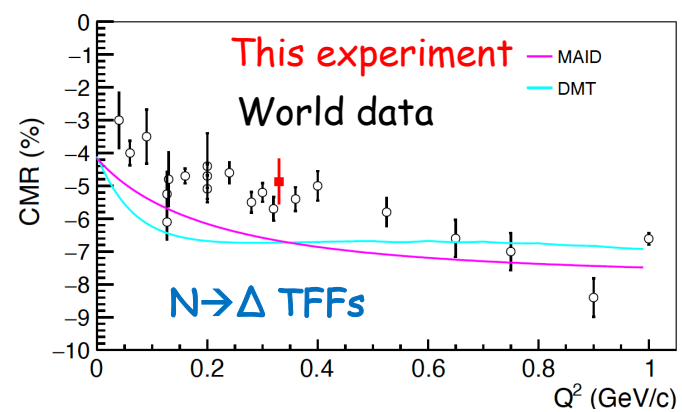


p(e,e'p)π⁰

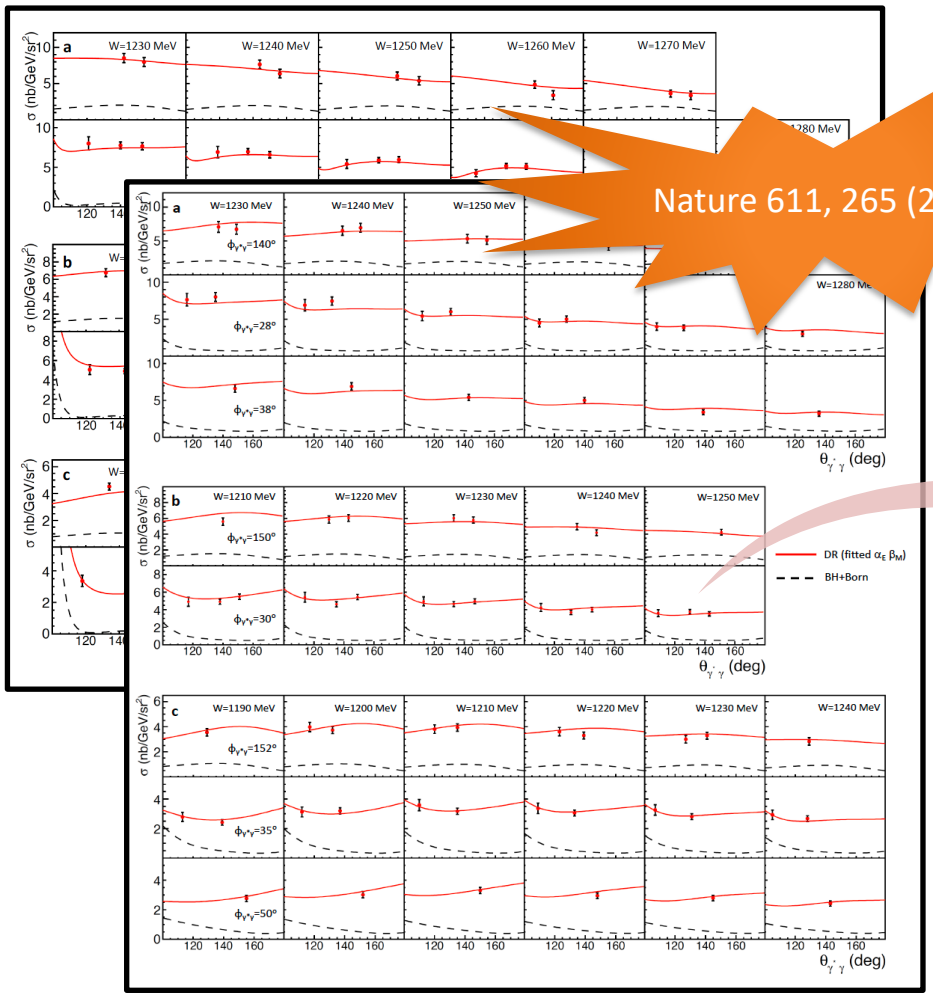




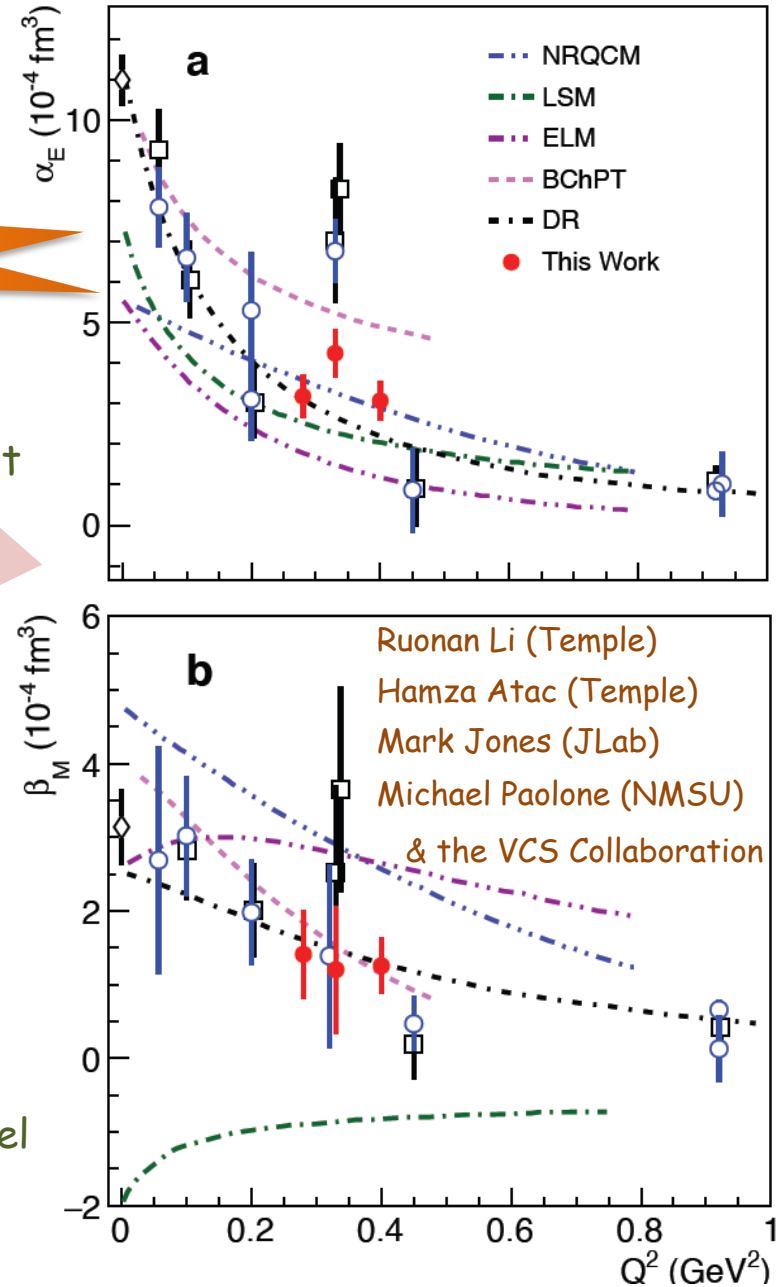




VCS-I results: GPs



DR fit



Ruonan Li (Temple)
Hamza Atac (Temple)
Mark Jones (JLab)
Michael Paolone (NMSU)
& the VCS Collaboration

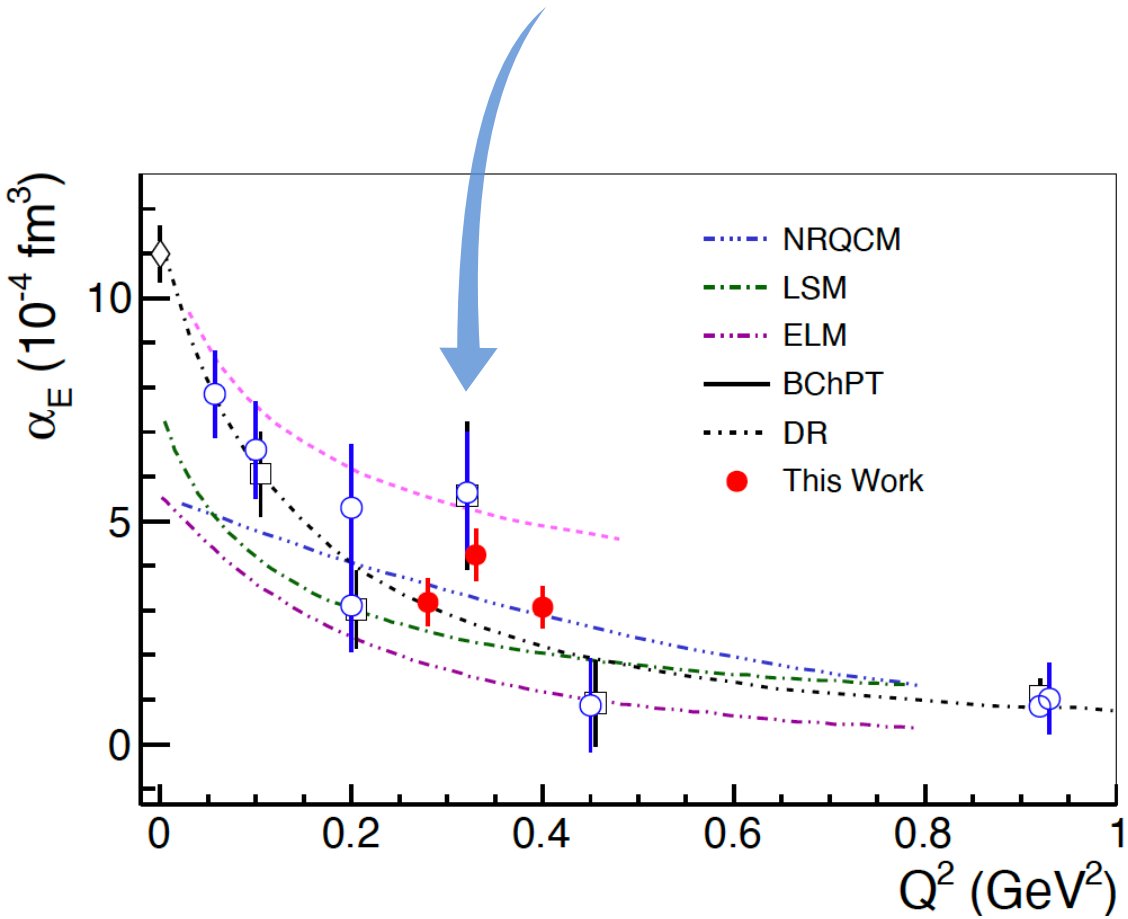
Experimental cross sections are compared to the DR model predictions for all possible values for the GPs

→ $\alpha_E(Q^2)$ and $\beta_M(Q^2)$ are fitted by a χ^2 minimization

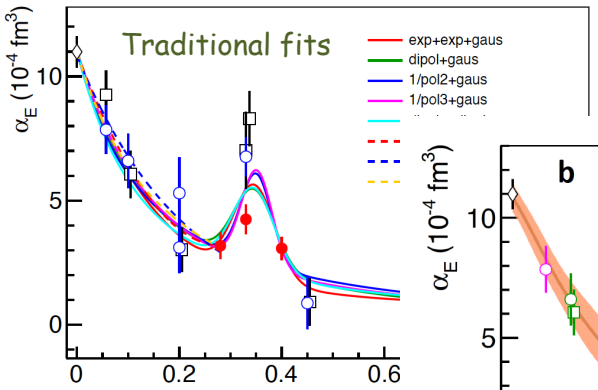
Electric GP (Q^2)

Is there a non-trivial structure?

MAMI-I re-analysis (unpublished)

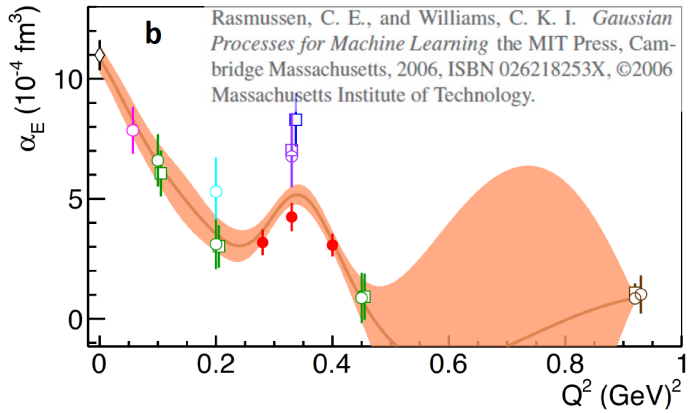


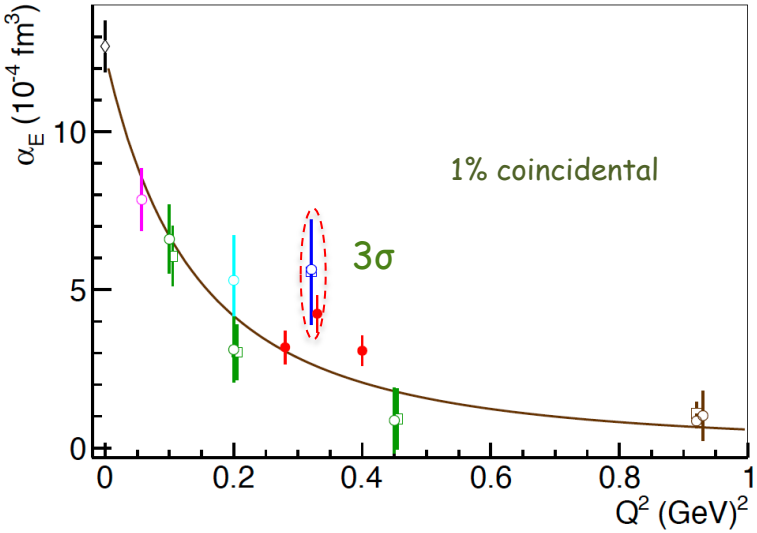
Electric GP



Data-driven techniques:

no underlying functional form is assumed





Is the observed a_E structure coincidental or not?

If true: Measure the shape precisely → input to theory

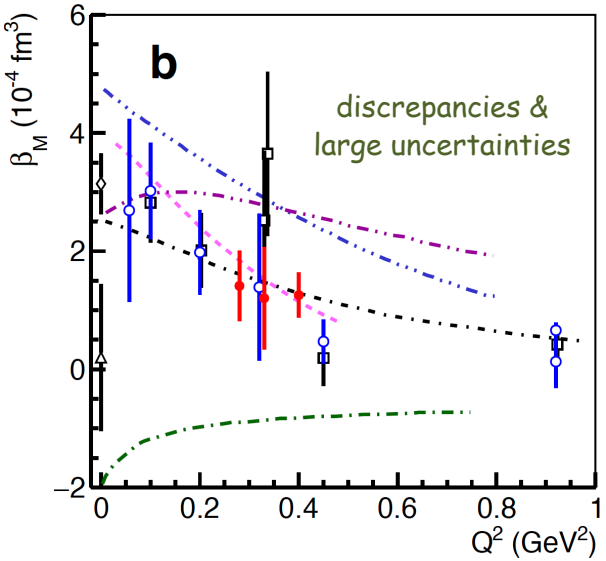
If not: We are able to show it with more measurements

Strong tension between world data (?)

Things we do not yet understand well?

Underestimated uncertainties? …

Magnetic GP



Magnetic GP: Large uncertainties & discrepancies

Needed to disentangle diamagnetism vs paramagnetism in the proton

Ability to measure a_E and β_M with superb precision and with consistent systematics across Q^2

Theory: BχPT

Eur. Phys. J. C (2017) 77:119
DOI 10.1140/epjc/s10052-017-4652-9

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

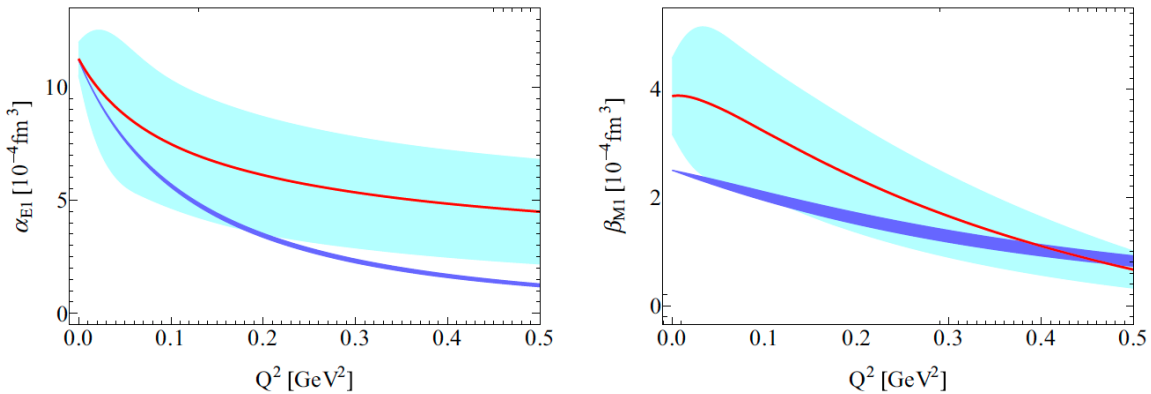
Generalized polarizabilities of the nucleon in baryon chiral perturbation theory

Vadim Lensky[1,2,3,a], Vladimir Pascalutsa[1], Marc Vanderhaeghen[1]

[1] Institut für Kernphysik, Cluster of Excellence PRISMA, Johannes Gutenberg Universität Mainz, 55128 Mainz, Germany
[2] Institute for Theoretical and Experimental Physics, Moscow 117218, Russia
[3] National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow 115409, Russia

BχPT calculation to NLO
in the δ-counting scheme

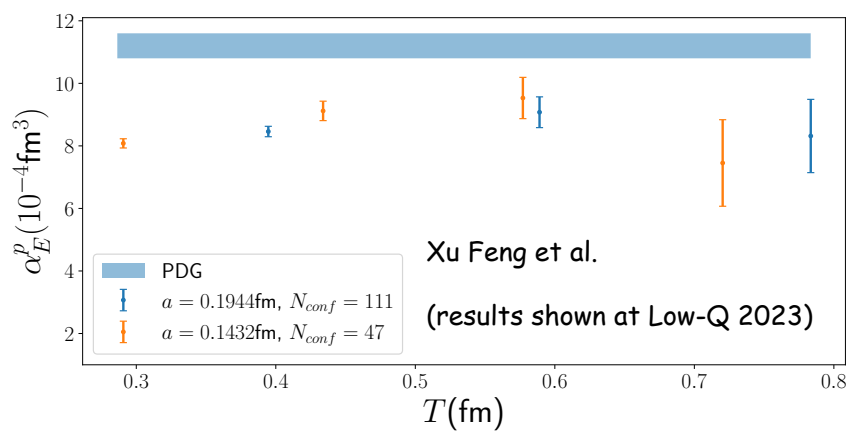
DR calculation
D. Drechsel, B. Pasquini, M. Vanderhaeghen,
Phys. Rep. 378,99 (2003)



Theory: Lattice QCD

Lattice QCD results for
the static polarizabilities

Next step: Lattice QCD
calculations for the GPs



Xu Feng et al.

(results shown at Low-Q 2023)

Spatial dependence of induced polarizations

Nucleon form factor data ➔ light-front quark charge densities

Formalism extended to the deformation of these quark densities when applying an external e.m. field:

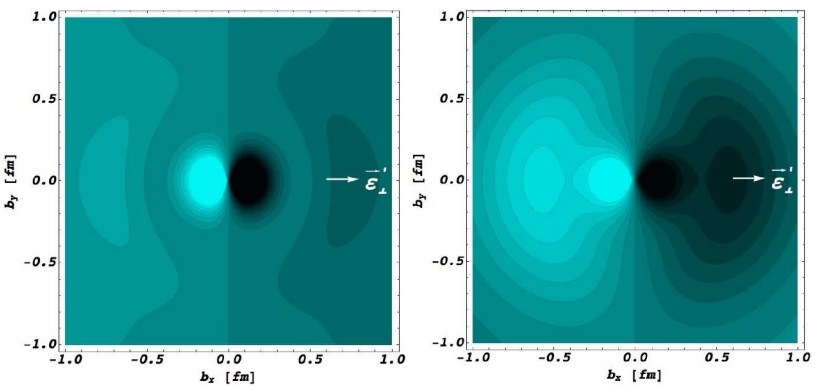
GPs ➔ spatial deformation of charge & magnetization densities under an applied e.m. field

Induced polarization in a proton when submitted to an e.m. field

Phys. Rev. Lett. 104, 112001 (2010)

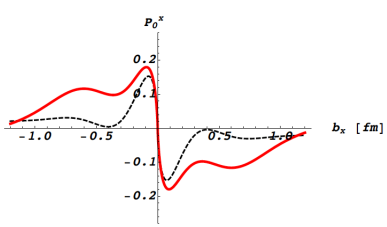
M. Gorchtein, C. Lorce, B. Pasquini, M. Vanderhaeghen

GP I GP II

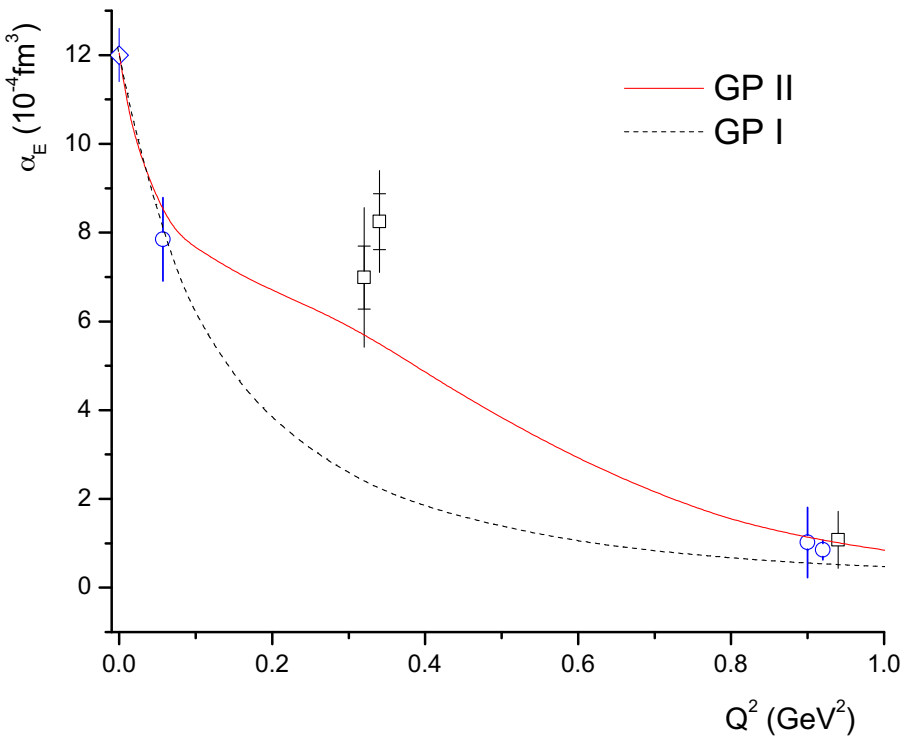


Light (**dark**) regions ➔ **largest** (**smaller**) values

(photon polarization along x-axis, as indicated)



Induced polarization along b_y=0



Spatial dependence of induced polarizations

Nucleon form factor data ➔ light-front quark charge densities

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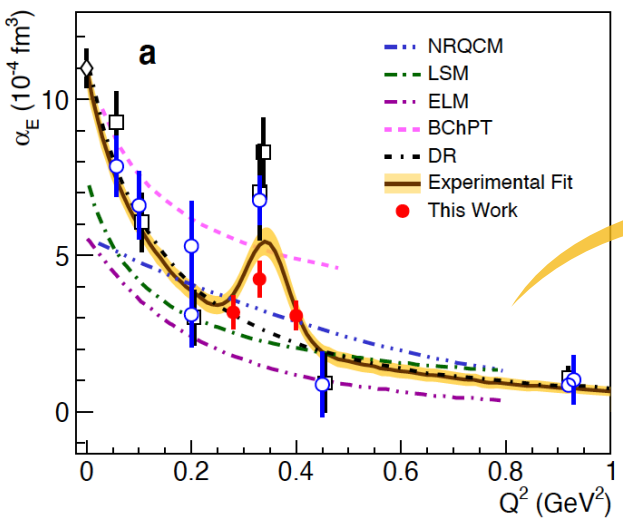
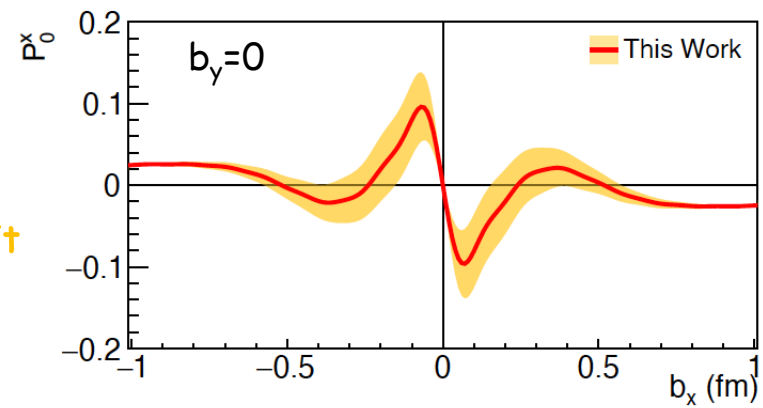


Experimental Fit

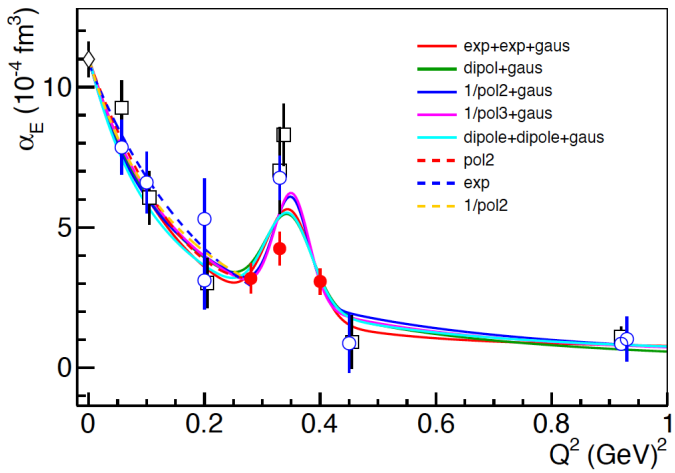


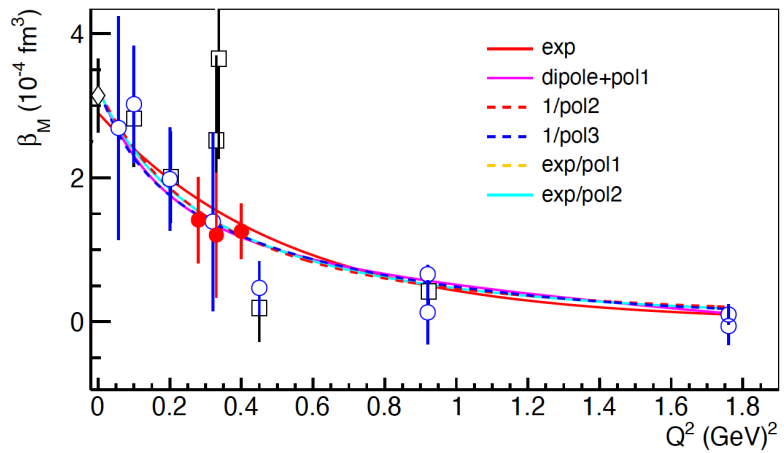
x-y defines the transverse plane with the z-axis being the direction of the fast-moving proton

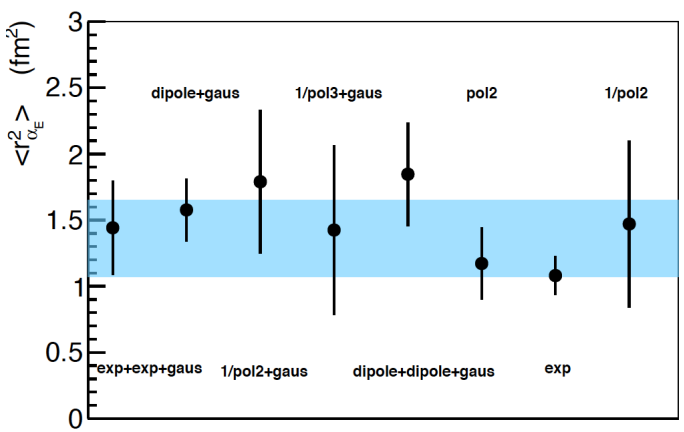
Polarizability radii

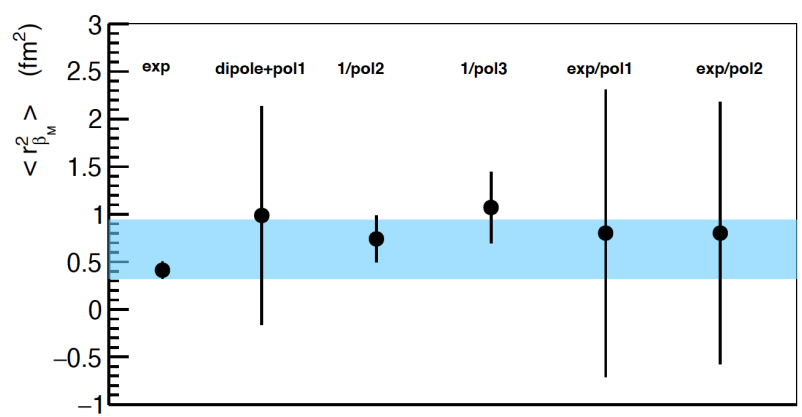
$$\langle r^2_{\alpha_E} \rangle = \frac{-6}{\alpha_E(0)} \cdot \frac{d}{dQ^2} \alpha_E(Q^2) \Big|_{Q^2=0}$$

$$\langle r^2_{\beta_M} \rangle = \frac{-6}{\beta_M(0)} \cdot \frac{d}{dQ^2} \beta_M(Q^2) \Big|_{Q^2=0}$$









$$\langle r^2_{\alpha_E} \rangle = 1.36 \pm 0.29 \text{ fm}^2$$

$$\langle r^2_{\beta_M} \rangle = 0.63 \pm 0.31 \text{ fm}^2$$

Polarizability radii

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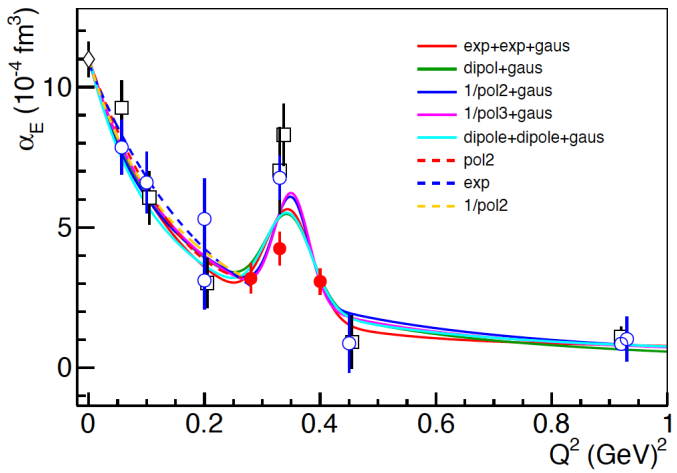

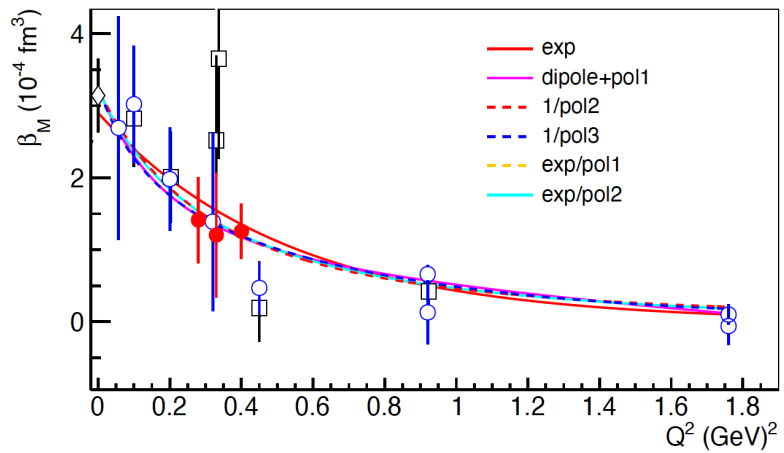

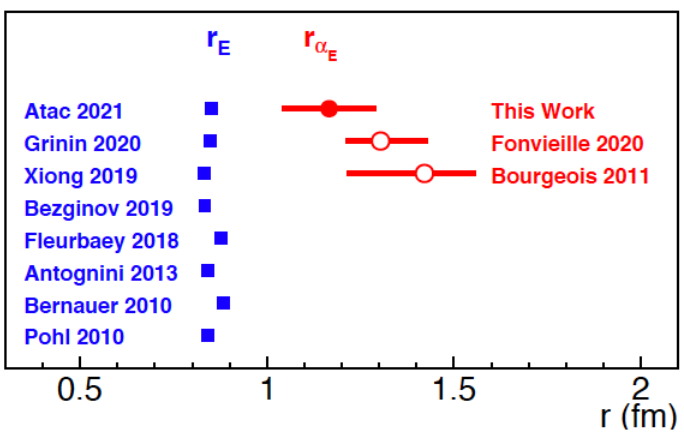

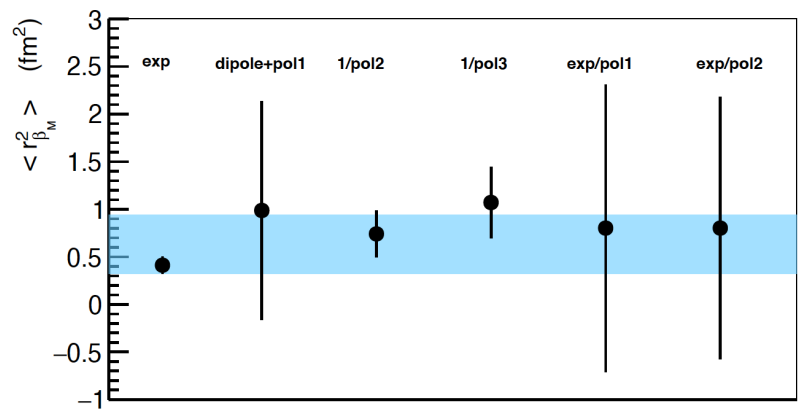

$$\langle r^2_{\alpha_E} \rangle = 1.36 \pm 0.29 \text{ fm}^2$$

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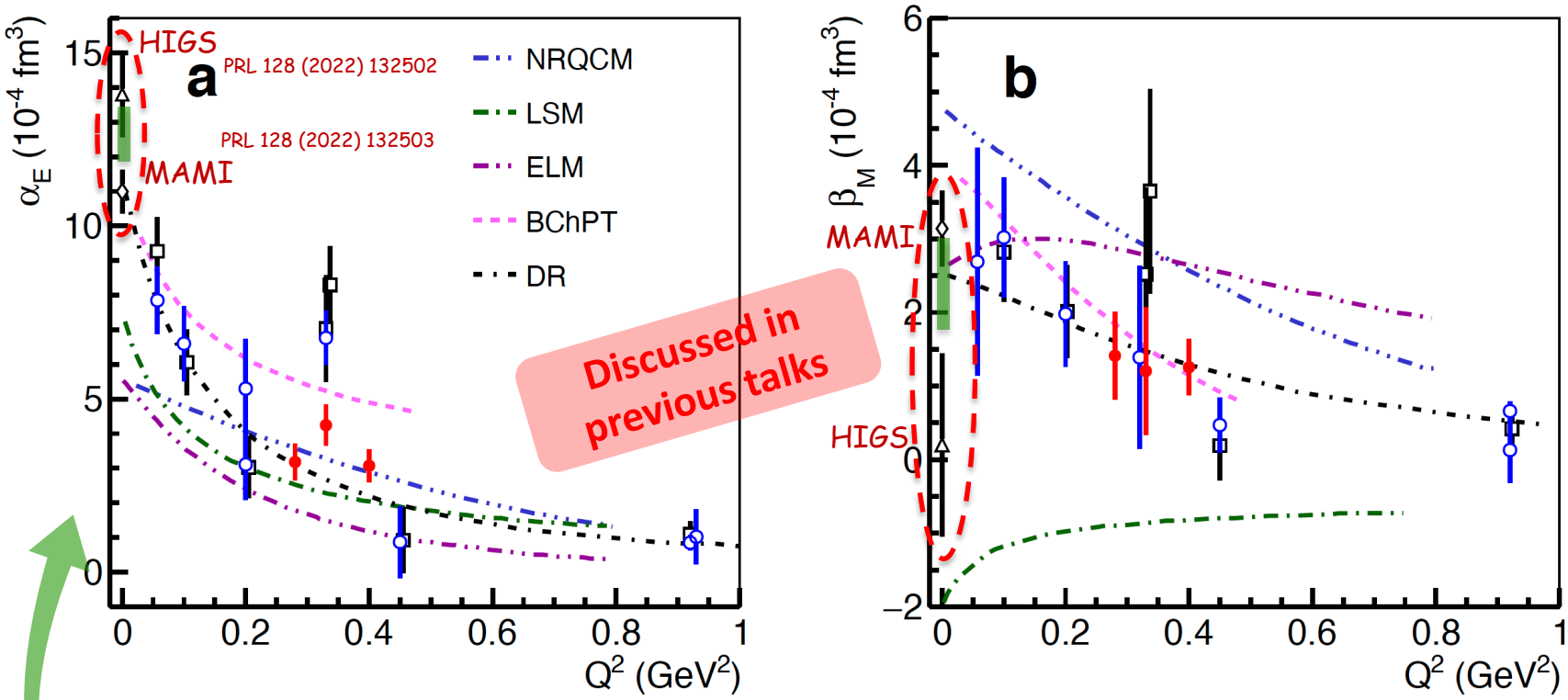
Static Polarizabilities



PHYSICAL REVIEW LETTERS **129**, 102501 (2022)

First Concurrent Extraction of the Leading-Order Scalar and Spin Proton Polarizabilities

E. Mornacchi,[1,*] S. Rodini,[2] B. Pasquini,[3,4] and P. Pedroni[4]
[1]*Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz,Germany*
[2]*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[3]*Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy*
[4]*INFN Sezione di Pavia, I-27100 Pavia, Italy*

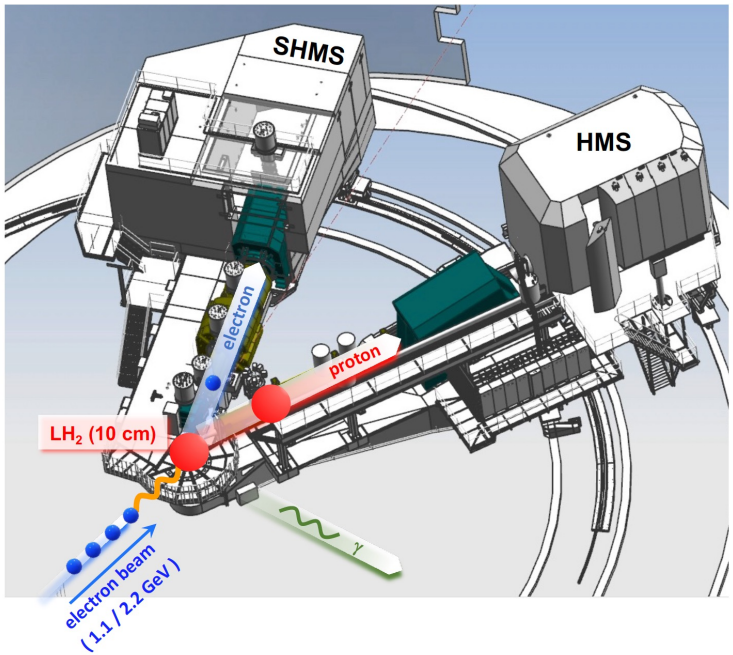
(Received 3 May 2022; revised 11 July 2022; accepted 2 August 2022; published 31 August 2022)

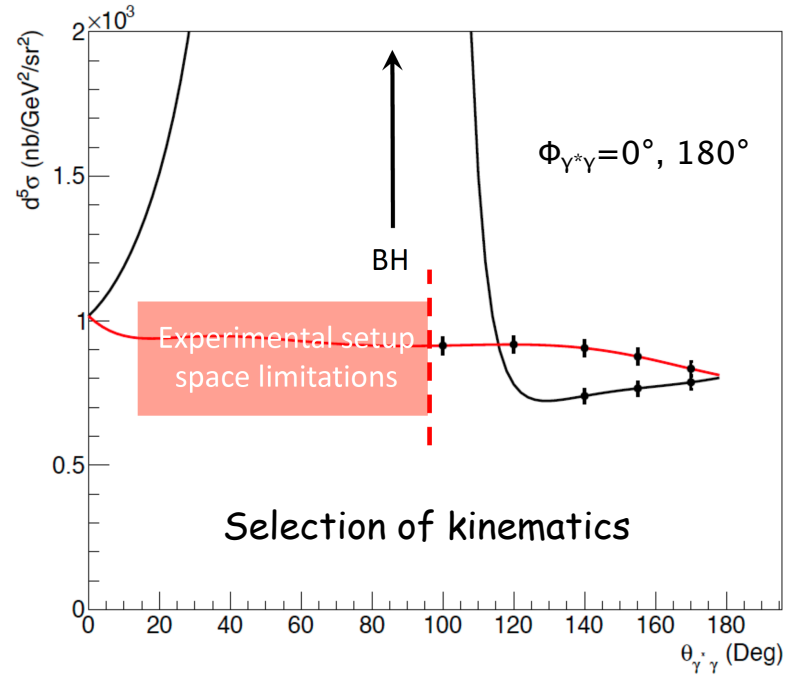
We performed the first simultaneous extraction of the six leading-order proton polarizabilities. We reached this milestone thanks to both new high-quality experimental data and an innovative bootstrap-based fitting method. These new results provide a self-consistent and fundamental benchmark for all future theoretical and experimental polarizability estimates.

$$\alpha_{E1} = [12.7 \pm 0.8(\text{fit}) \pm 0.1(\text{model})] \times 10^{-4}\ \text{fm}^3,$$
$$\beta_{M1} = [2.4 \pm 0.6(\text{fit}) \pm 0.1(\text{model})] \times 10^{-4}\ \text{fm}^3,$$
$$\gamma_{E1E1} = [-3.0 \pm 0.6(\text{fit}) \pm 0.4(\text{model})] \times 10^{-4}\ \text{fm}^4,$$
$$\gamma_{M1M1} = [3.7 \pm 0.5(\text{fit}) \pm 0.1(\text{model})] \times 10^{-4}\ \text{fm}^4,$$
$$\gamma_{E1M2} = [-1.2 \pm 1.0(\text{fit}) \pm 0.3(\text{model})] \times 10^{-4}\ \text{fm}^4,$$
$$\gamma_{M1E2} = [2.0 \pm 0.7(\text{fit}) \pm 0.4(\text{model})] \times 10^{-4}\ \text{fm}^4,$$

Moving Forward

VCS-II (E12-23-001) @ JLab



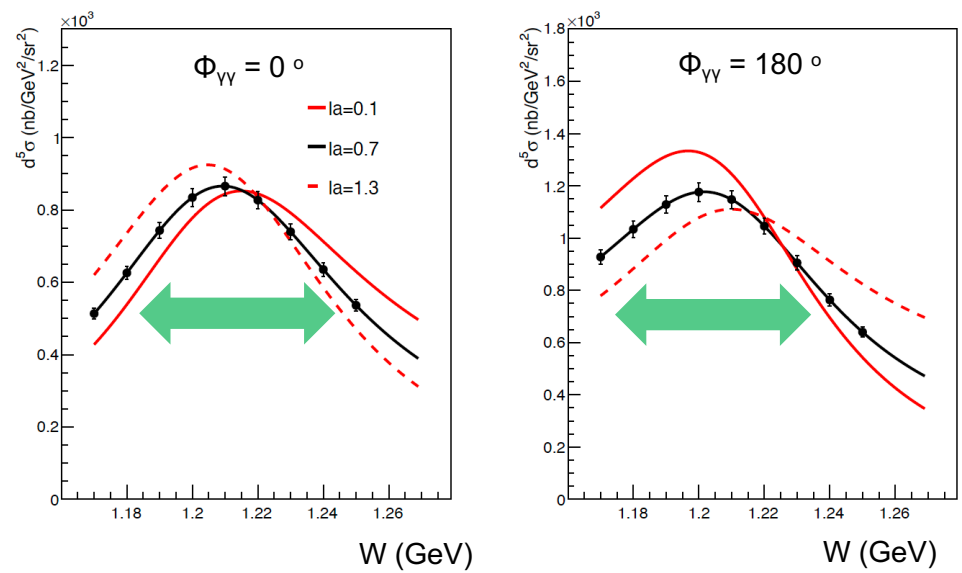


Extend Q^2 range & targeted measurements to fully exploit the sensitivity to the GPs

APPROVED PAC 51

Production (E_o = 1.1 GeV) :	6 days
Production (E_o = 2.2 GeV) :	53 days
Studies (optics/dummy/calibrations) :	3 days

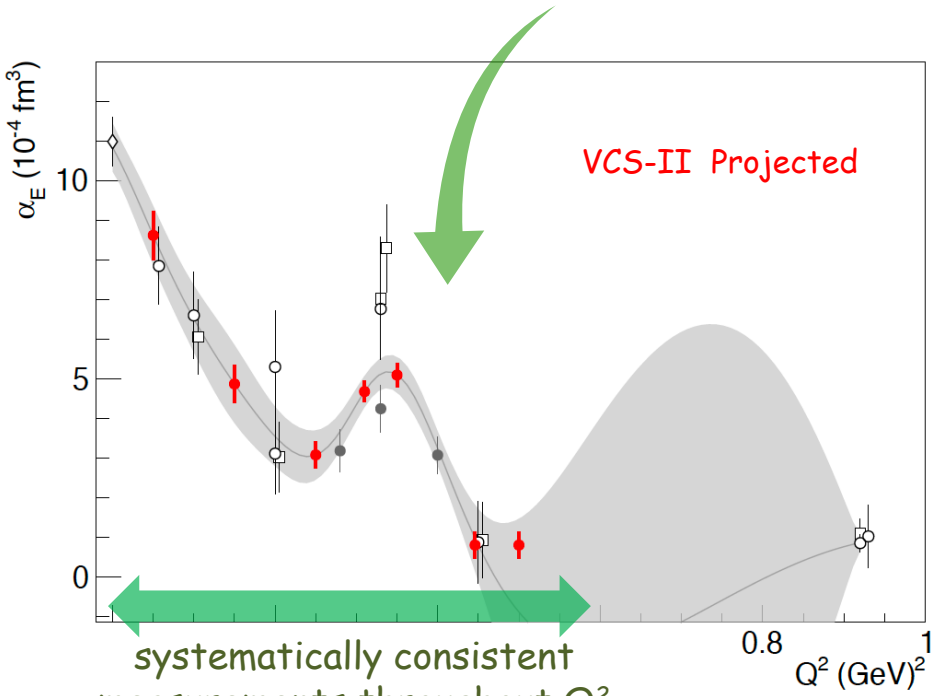
Total: 62 days



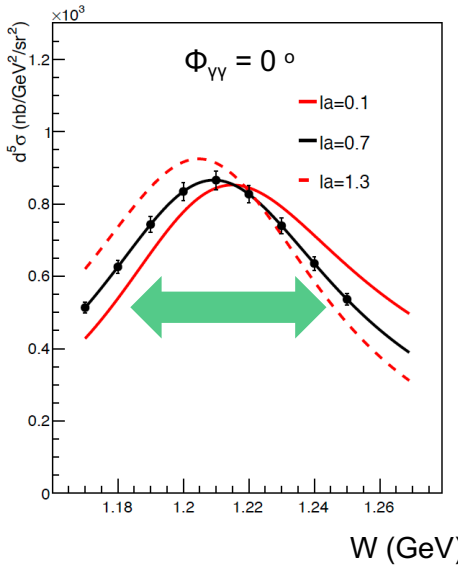
VCS-II Projected Measurements

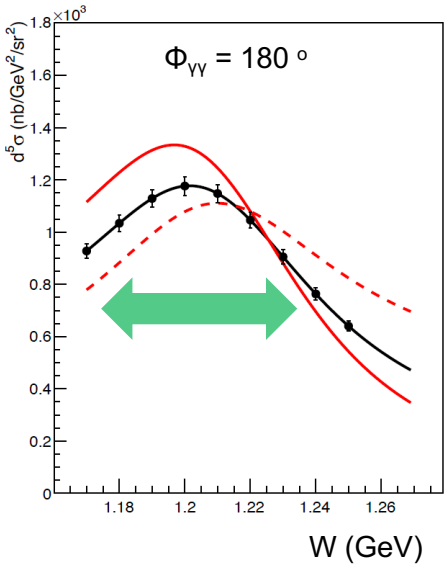
High precision measurements combined with a fine mapping in Q^2

Targeted measurements to fully exploit the sensitivity to the GPs



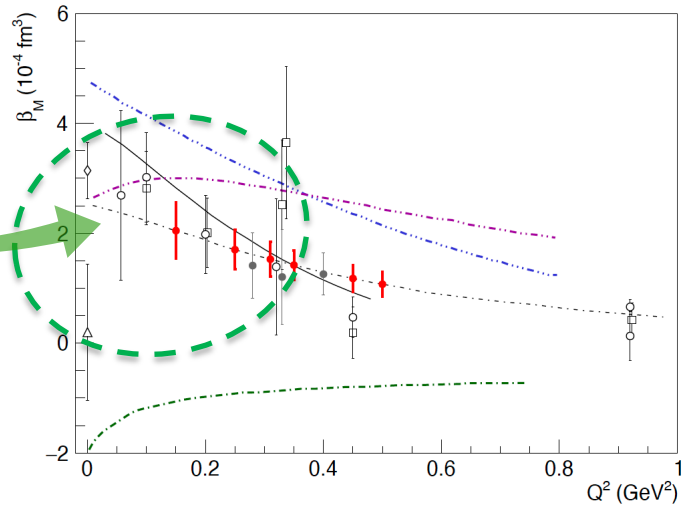
systematically consistent measurements throughout Q^2





Improve upon β_M :

Pin down the competing para/dia-magnetic contributions in the nucleon



Can we measure with a different method ?

Yes: positrons and/or beam spin asymmetries

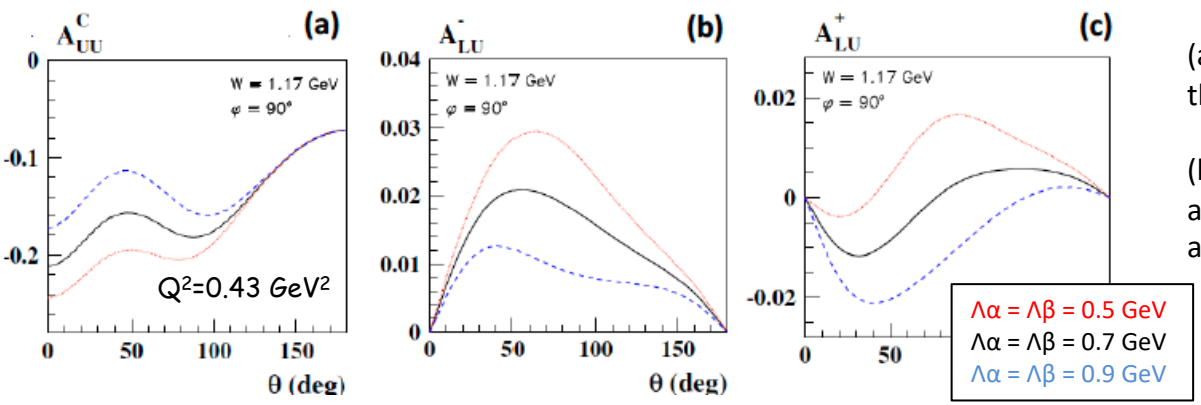
Positrons allow for an independent path to access experimentally the GPs

Eur. Phys. J. A 57 (2021) 11, 316

Virtual Compton scattering at low energies with a positron beam

Barbara Pasquini[a,1,2], Marc Vanderhaeghen[b,3]

[1]Dipartimento di Fisica, Università degli Studi di Pavia, 27100 Pavia, Italy
[2]Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, 27100 Pavia, Italy
[3]Institut für Kernphysik and PRISMA+ Cluster of Excellence, Johannes Gutenberg Universität, D-55099 Mainz, Germany



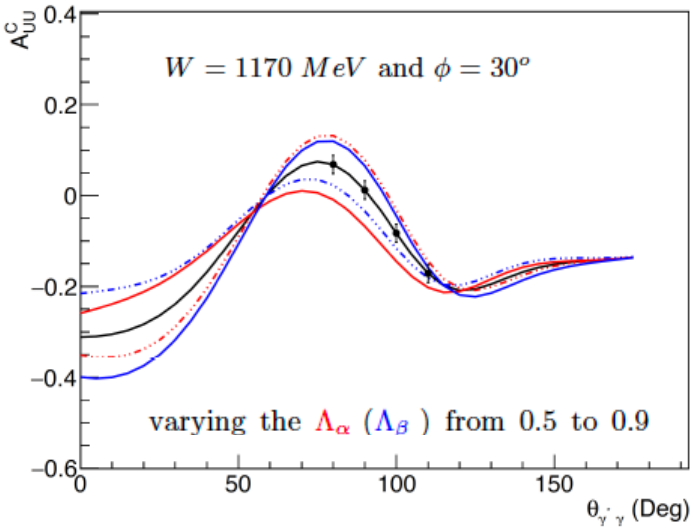
(a): The beam-charge asymmetry as a function of the photon scattering angle at Q2 = 0.43 GeV 2.

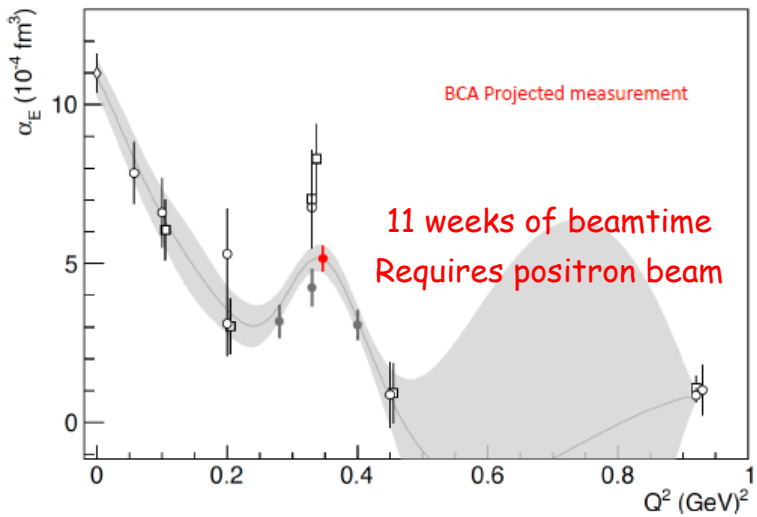
(b) & (c): The electron and positron beam-spin asymmetry as a function of the photon scattering angle for out-of-plane kinematics.

Unpolarized beam charge asymmetry (BCA) : $$A^C_{UU} = \frac{(d\sigma^+_+ + d\sigma^+_-) - (d\sigma^-_+ + d\sigma^-_-)}{d\sigma^+_+ + d\sigma^+_- + d\sigma^-_+ + d\sigma^-_-}$$

Lepton beam spin asymmetry (BSA) : $$A^e_{LU} = \frac{d\sigma^e_+ - d\sigma^e_-}{d\sigma^e_+ + d\sigma^e_-}$$

BCA (electrons & positrons)





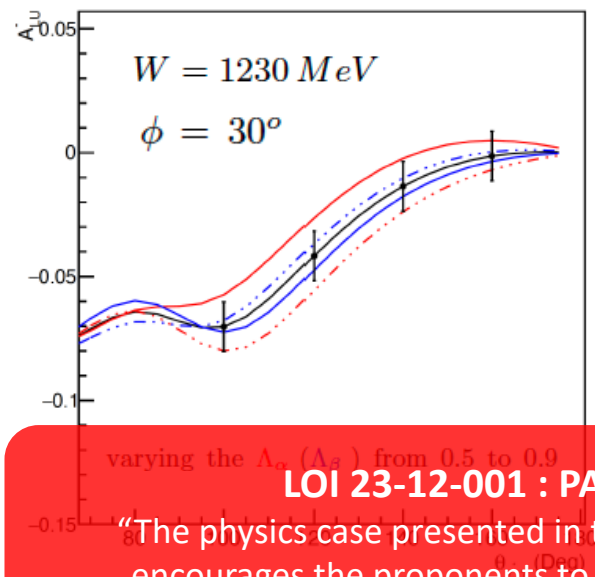
Hall C (SHMS / HMS)

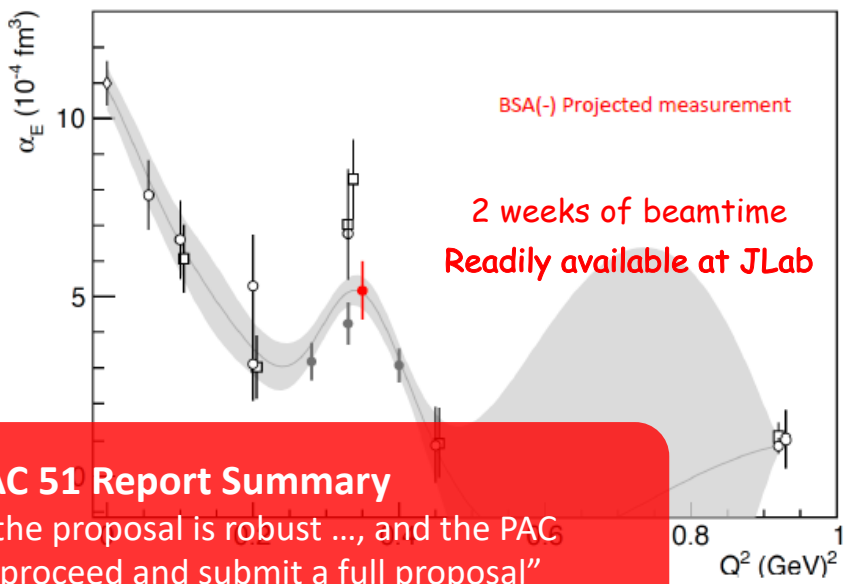
e^- : ~ 1 week @ 50 μA

and

e^+ : ~ 10 weeks @ 5 μA

BSA (electrons or positrons)





e^- (pol. 85% @ 70 μA)

~ 2 weeks of beamtime

or

e^+ (pol. 60% @ 50 nA)

~ 3 orders of magnitude more beamtime

LOI 23-12-001 : PAC 51 Report Summary

"The physics case presented in the proposal is robust …, and the PAC encourages the proponents to proceed and submit a full proposal"

Summary

Progress measuring proton's fundamental properties / response to an EM field

Insight to spatial deformation of the nucleon densities under an applied EM field, interplay of para/dia-magnetic mechanisms in the proton, polarizability radii, …

Electric GP:

- possibility for a non-trivial (non-monotonic) behavior in $a_E(Q^2)$ (albeit with a smaller magnitude than originally suggested)

  or

- at minimum: strong tension between world data

Experiment ahead of theory:

Stringent constraints to theoretical predictions / can improve further

High precision benchmark data for upcoming LQCD calculations

Future measurements:

Pin down precisely the shape of the a_E structure (if it exists) - important input for the theory

Independent cross-check
Measure via a different channel (BS asymmetries & positrons)

Thank you!