MENU 2023

International Conference on Meson-Nucleon Physics and the Structure of the Nucleon

Meson-baryon scattering and $\Lambda(1405)$ in chiral perturbation theory

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In collaboration with:

E. Epelbaum (RUB), J. Gegelia(RUB), and U.-G. Meißner (Bonn)

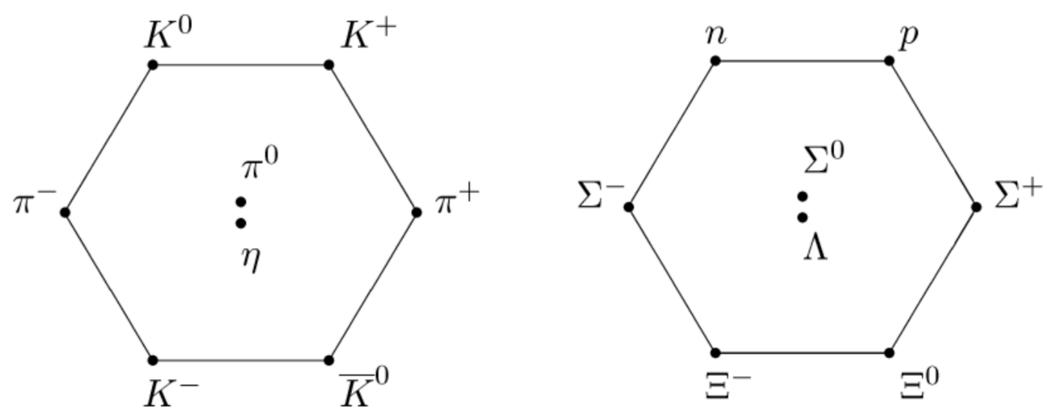
2023.10.16

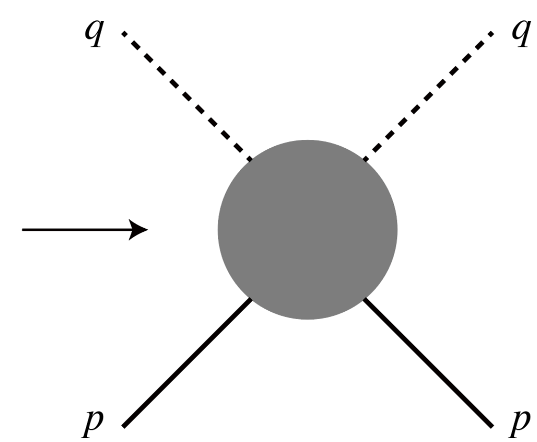
OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

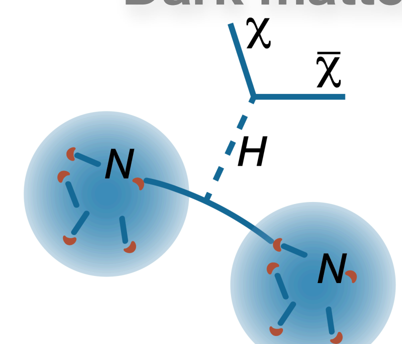
Meson-baryon scattering

Simple process: **lowest-lying** MB scattering





Interesting phenomena



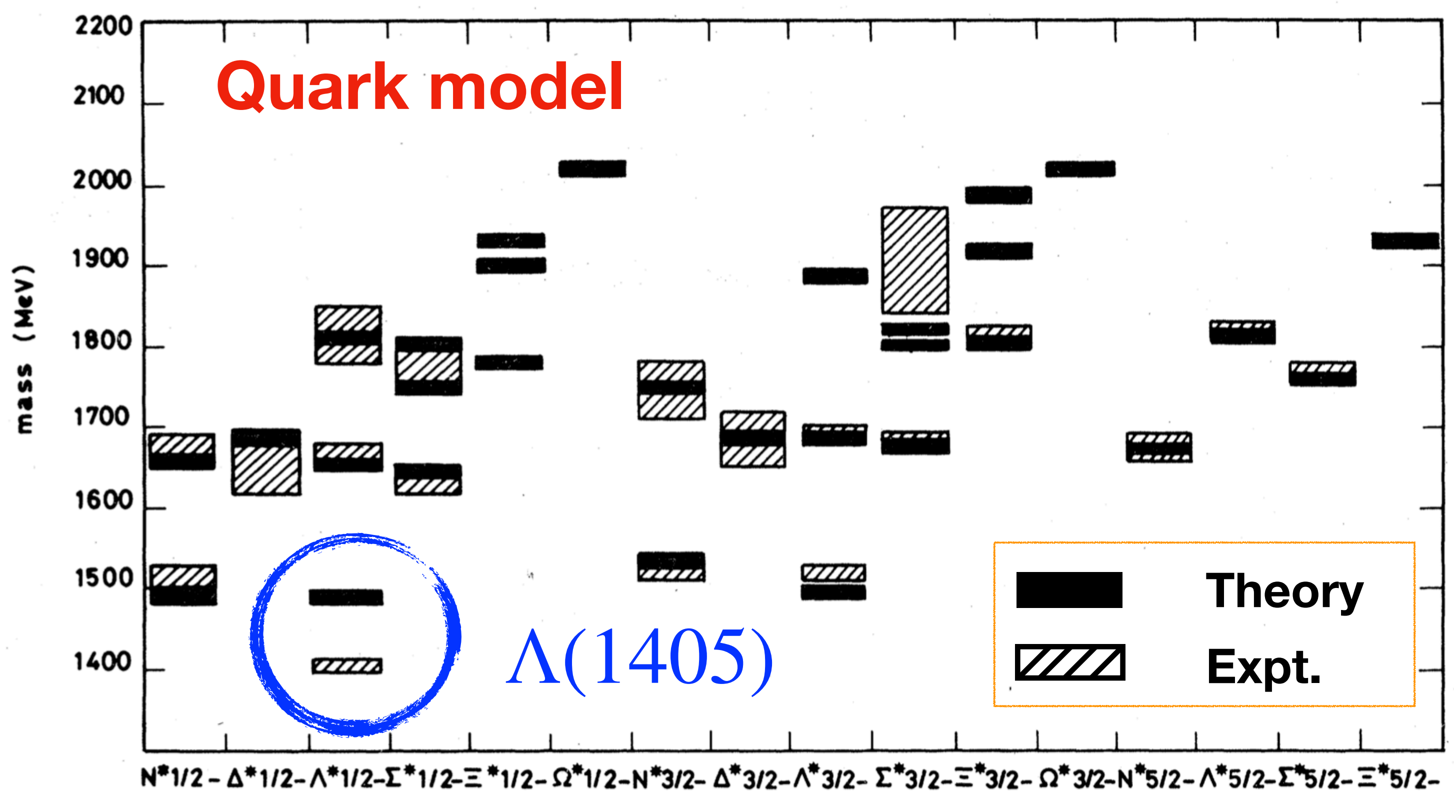
- πN scattering: 30K data points of GWU
  - ✓ Sigma term $\sigma_{\pi N}$, key input of neutralino-nucleon cross section

M. Hoferichter,et al.,PRL115,092301(2015) *J.R. Ellis, et al., PRD77(2008)065026*

- $\bar{K}N$ interaction is important in strangeness nuclear physics
  - ✓ Interaction is strongly attractive, generating $\Lambda(1405)$ resonance **CLAS, …**
  - ✓ $\bar{K}NN, \bar{K}NNN,$ multi-antikaonic nuclei **J-PARC, DAΦNE, …**
  - ✓ Kaon-condensate (?) in the interior of neutron star *S.Pal et al., NPA674(2000)553*
- S= -2 sector $\Xi(1620)$, $\Xi(1690)$; and other resonances **Belle, BES, ALICE,…**
- **Deepen understanding of SU(3) dynamics in nonperturbative QCD**

Λ(1405) resonance

- Λ(1405) state is an exotic candidate

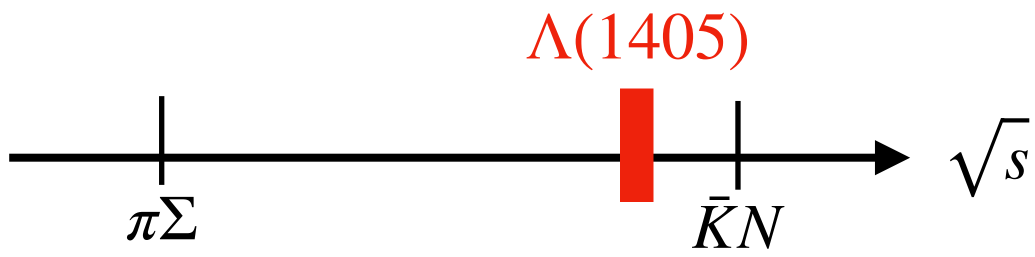


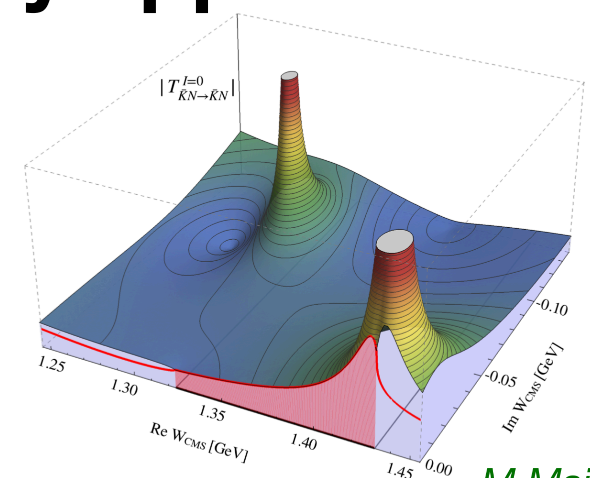
N. Isgur and G. Karl, Phys.Rev.D 18 (1978) 4187

- Variety of theoretical studies
  - QCD sum rules *L.S. Kisslinger,EPJA2011…*
  - Phenomenological potential model *A. Cieplý, NPA2015…*
  - Skyrme model *T. Ezoe,PRD2020…*
  - Hamiltonian effective field theory *Z.-W. Liu,PRD2017…*
  - Chiral unitary approach *N.Kaiser,NPA1995; E.Oset,NPA1998; J.A.Oller&U.-G.Meißner,PLB2001…*

Structure of Λ(1405) resonance

Double-pole predicted by **chiral unitary approach**





M.Mai, EPJ.ST(2021)

L.-S. Geng @12:30pm Tue.

- Pole positions

PDG(2022) Review 83

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. [14, 15], NLO	$1424^{+7}_{-23} - i\,26^{+3}_{-14}$	$1381^{+18}_{-6} - i\,81^{+19}_{-8}$
Ref. [17], Fit II	$1421^{+3}_{-2} - i\,19^{+8}_{-5}$	$1388^{+9}_{-9} - i\,114^{+24}_{-25}$
Ref. [18], solution #2	$1434^{+2}_{-2} - i\,10^{+2}_{-1}$	$1330^{+4}_{-5} - i\,56^{+17}_{-11}$
Ref. [18], solution #4	$1429^{+8}_{-7} - i\,12^{+2}_{-3}$	$1325^{+15}_{-15} - i\,90^{+12}_{-18}$

$\Lambda(1405)\ 1/2^-$ $\quad I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

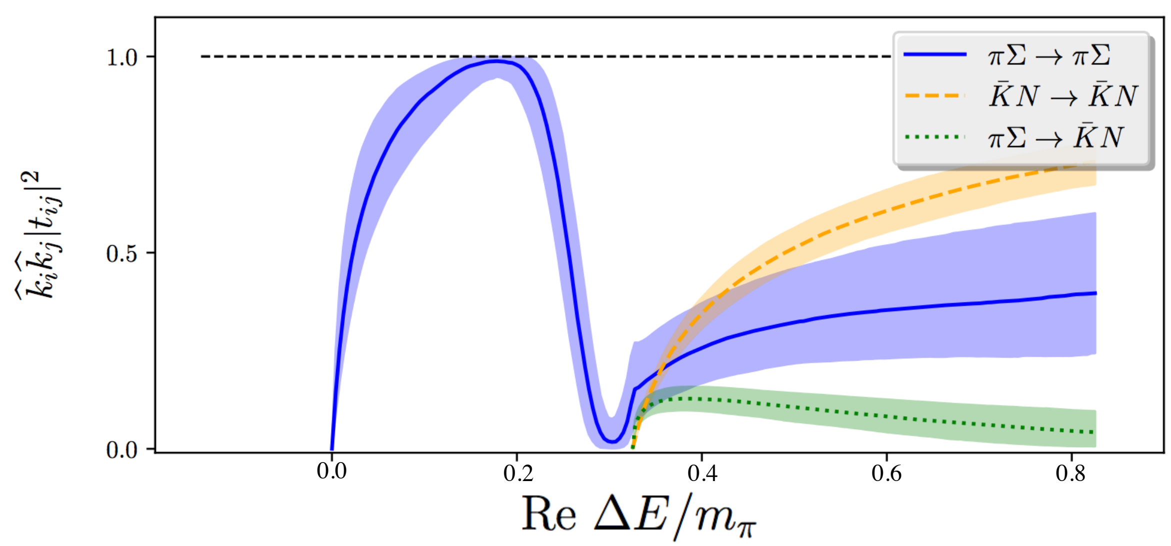
$\Lambda(1380)\ 1/2^-$ $\quad J^P = \frac{1}{2}^-$ Status: ∗∗

✓ NNLO study: $1425 \pm 1 - i(13 \pm 4)$ $\quad 1392 \pm 8 - i(102 \pm 15)$ J.-X. Lu, et al., PRL130(2023)071902

✓ pole 2: needs further studies to fix its position

Double-pole structure is verified by LQCD

J. Bulava @10am Mon.



$m_\pi \approx 200$ MeV, $m_K \approx 480$ MeV

Pole II	$1395(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}$ MeV
Pole I	$1456(14)_{\rm stat}(2)_{\rm model}(16)_{\rm a}$ MeV
	$-\,i \times 11.7(4.3)_{\rm stat}(4)_{\rm model}(0.1)_{\rm a}$ MeV

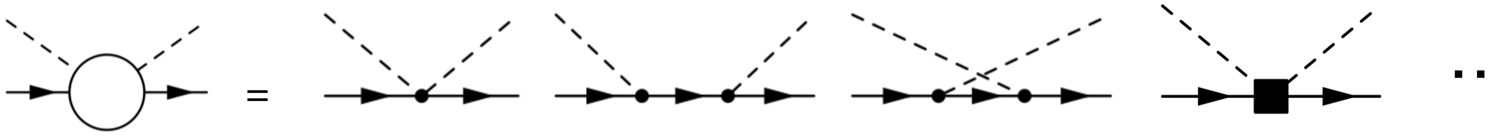
Baryon Scattering Coll., 2307.10413

Chiral Unitary approach

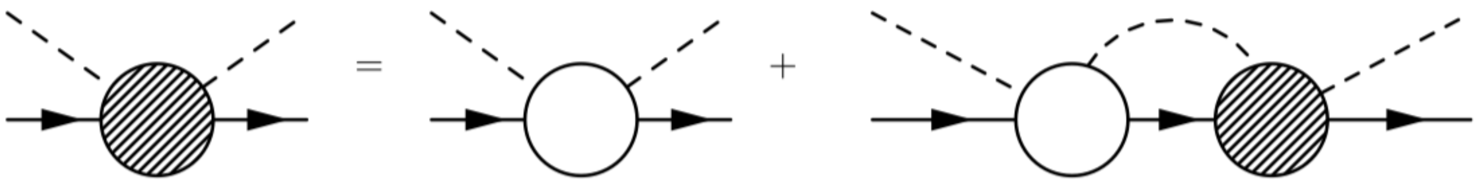
- **Chiral symmetry** of low-energy QCD + **Unitary Relation**

  *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP*120 (2021)103868 …

- Interaction kernel V: calculate in ChPT order by order
  - Leading, next-to-leading order, …



- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or (commonly-used) Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^4k}{(2\pi)^4} V(p',k)\, G(k)\, T(k,p)$$

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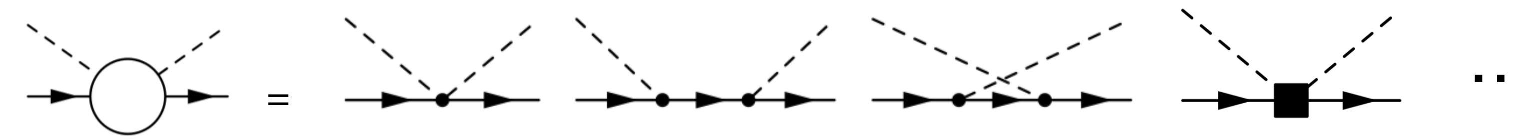

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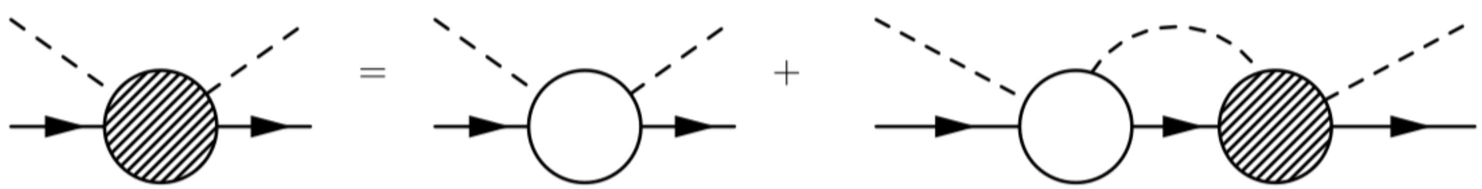

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  - On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k) \right) T(p',p)$

Neglecting off-shell effect

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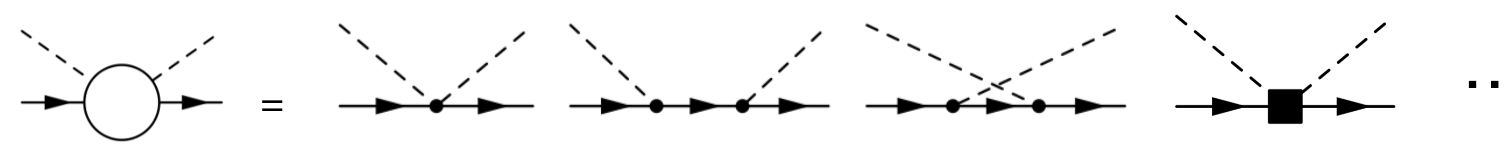

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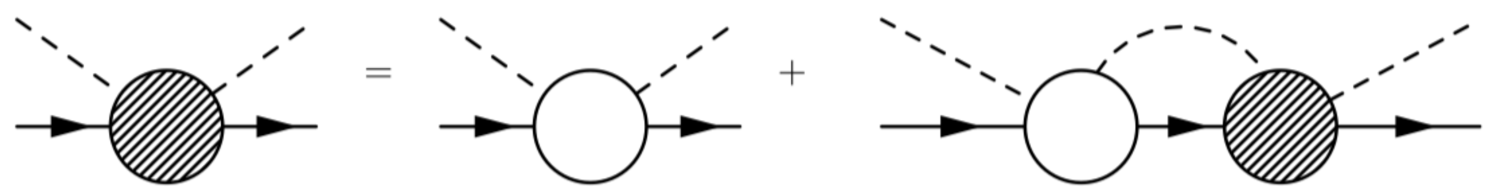

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  Neglecting off-shell effect

- Finite cutoff or subtraction constant
  to renormalize the loop integral

$$G^R(E,\Lambda) \text{ or } G^R(E,\alpha_i)$$

Cutoff/Model dependence

In this work

- Facing the rapid progress of precision experiments, **a model-independent formalism would be needed** ALICE, AMADEUS, J-PARC, STAR…
- We tentatively propose a renormalized framework for meson-baryon scattering using time-ordered perturbation theory with the covariant chiral Lagrangians
  - Obtain the potential and scattering equation on an equal footing
  - Include the **off-shell effects of potential** and utilize the **subtractive renormalization** to obtain the **renormalizable T-matrix**
  - **LO study:** pion-nucleon scattering and $S = -1$ sector
    - ✓ Investigate the $\Lambda(1405)$ state
  - **NLO study:** preliminary results of πN scattering

XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner,
Eur. Phys. J. C80 (2020) 406; Eur. Phys. J. C81 (2021) 582;
XLR, et al., work in progress

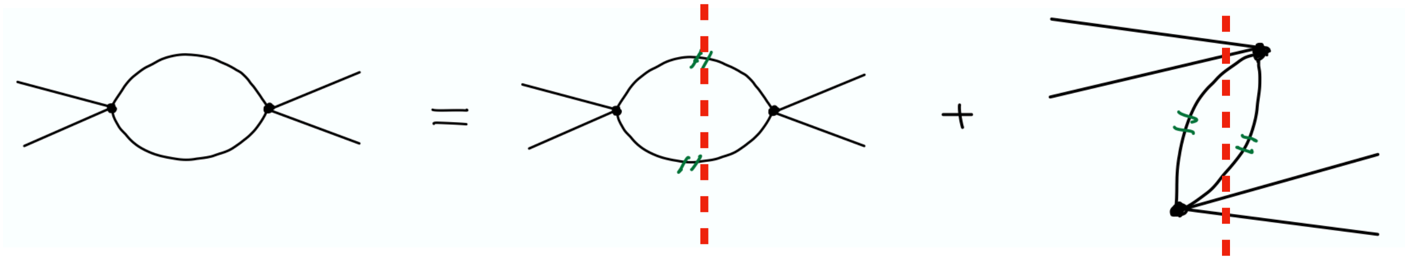
Theoretical framework

Time-ordered perturbation theory

- Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

  - Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
    - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
  - **TOPT or old-fashioned perturbation theory**



- Advantages
  - Explicitly show the unitarity
  - Easily to tell the contributions of a particular diagram

- Obtain the rules for time-ordered diagrams
  - Perform Feynman integrations over the zeroth components of the loop momenta
  - Decompose Feynman diagram into sums of time-ordered diagrams
  - Match to the rules of time-ordered diagrams

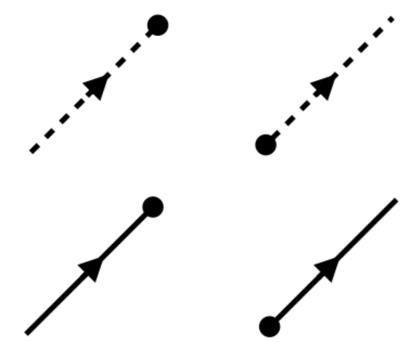
Diagrammatic rules in TOPT

- **External lines**

XLR, PoS(CD2021)007

Spin 0 boson (in, out)

Spin 1/2 fermion (in, out)



$$1$$

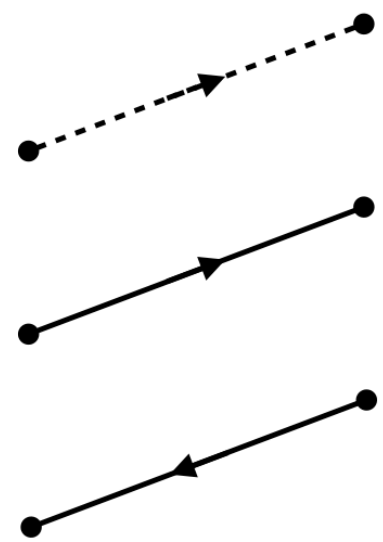
$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

- **Internal lines**

Spin 0 (anti-)boson

Spin 1/2 fermion

anti-fermion



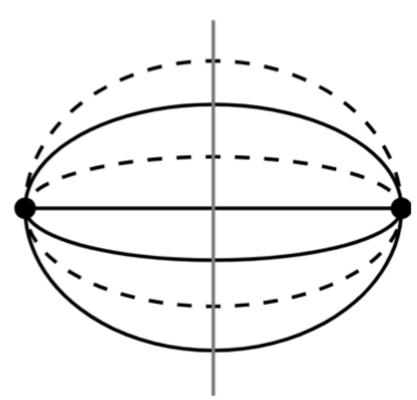
$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

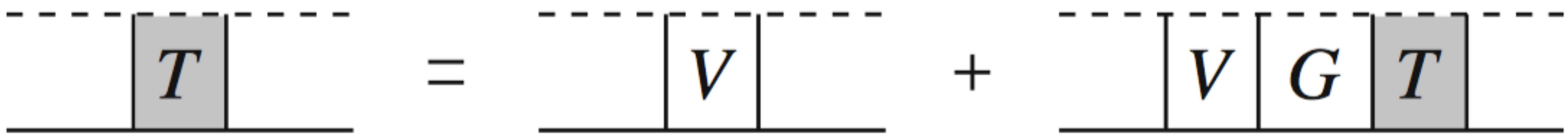
- Interaction vertices: **the standard Feynman rules**
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Meson−baryon scattering in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the one-meson and one-baryon **irreducible diagrams**
  - **Power counting:** Q/Λ_χ systematic ordering of all graphs
- Scattering equation



- Coupled-channel integral equation for T-matrix

$$T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) = V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) + \sum_{MB}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\,G_{MB}(E)\,T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)$$

- **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

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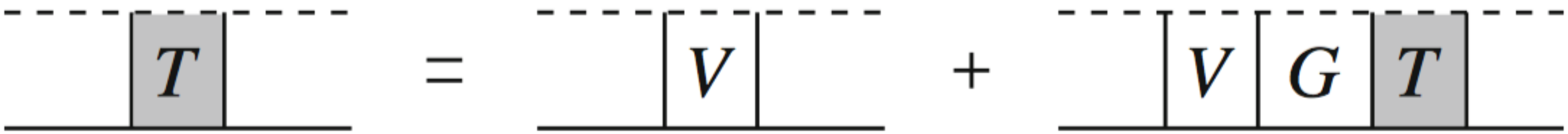

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  - **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)}\frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

Potential and scattering equation are obtained on an equal footing!

Results and discussion

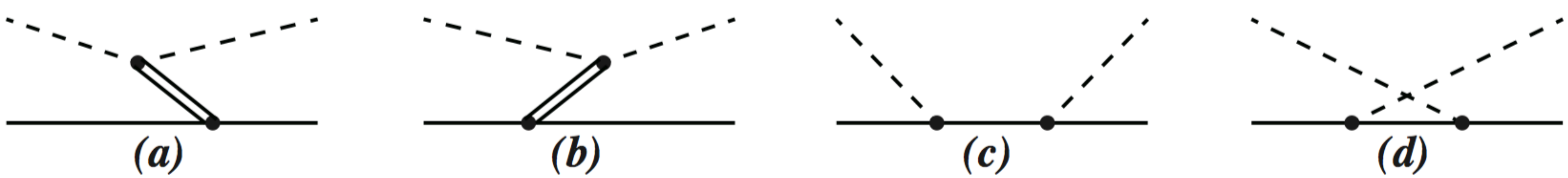
Leading order potential

Chiral effective Lagrangian

$$\mathcal{L}_{\rm LO} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle$$
$$-\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\,\langle \bar{B}\gamma_\mu[V^\mu, B]\rangle$$

- **Vector mesons included as explicit degrees of freedom**
  - ✓ One-vector meson exchange potential instead the Weinberg-Tomozawa term
  - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

Time ordered diagrams



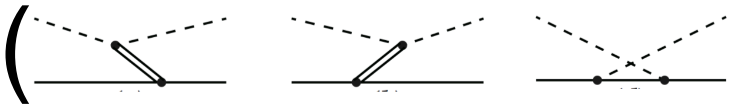
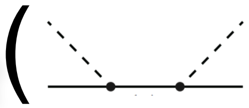
- LO potential in TOPT
  - ✓ Dirac spinor is decomposed as $u_B(p,s) = u_0 + [u(p) - u_0] \equiv (1,0)^\dagger \chi_s$+high order

$$V^{(a+b)}_{M_jB_j,M_iB_i} = -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right)$$
$$\times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)}+\frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]$$

$$V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.$$
$$V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.$$

Subtractive renormalization

- LO potential: **one-baryon irreducible and reducible parts**

$$V_{\rm LO} = V_I\Big($$  $$\Big) + V_R\Big($$  $$\Big)$$

- LO T-matrix

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO}\, G\, T_{\rm LO} \quad \Longrightarrow \quad \begin{cases} T_{\rm LO} = T_I + (1 + T_I\, G)\, T_R\, (1 + G\, T_I) \\ T_I = V_I + V_I\, G\, T_I \\ T_R = V_R + V_R\, G\, (1 + T_I\, G)\, T_R \end{cases}$$

- Irreducible part: $T_I \xrightarrow{\Lambda \sim \infty}$ Finite
- Reducible part: $T_R \xrightarrow{\Lambda \sim \infty}$ Divergent
  - ✓ **Potential can be rewritten as separable form**

$$V_R(p', p; E) = \xi^T(p')\, C(E)\, \xi(p) \qquad \text{C(E): constant} \qquad \xi^T(q) := (1, q)$$

  - ✓ T_R can be rewritten as $\quad T_R(p', p; E) = \xi^T(p')\, \chi(E)\, \xi(p) \qquad \chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

  - ✓ Using **subtractive renormalization,** replacing Green function $\quad G^{Rn} = G(E) - G(m_B)$

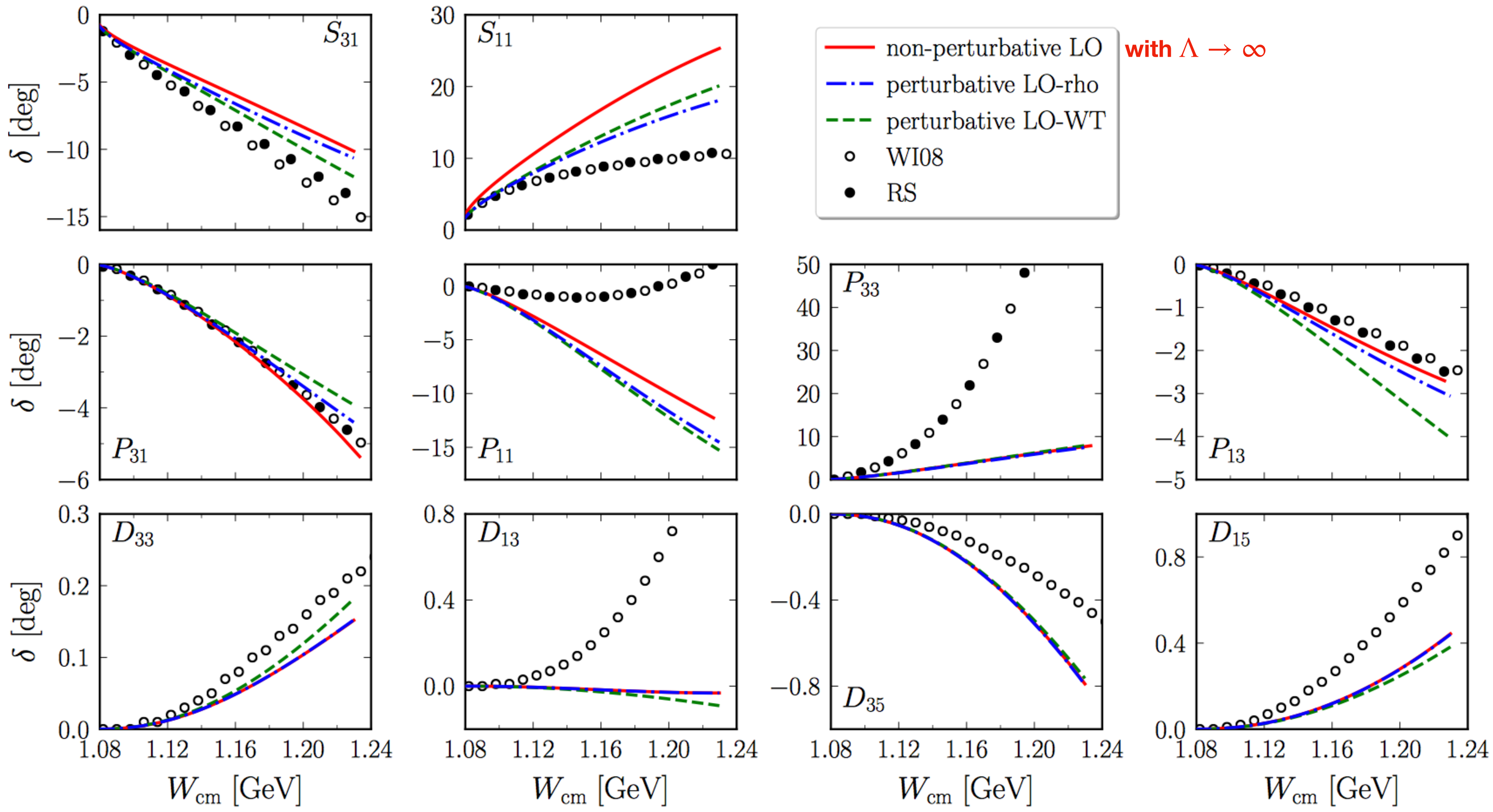
E. Epelbaum, et al., EPJA56(2020)152

Renormalized LO T-matrix

$$T_{\rm LO}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)$$

Pion–Nucleon scattering

- Description phase shifts of pion-nucleon scattering



- Rho-meson-exchange contribution is similar as WT term.
- Phase shifts from non-perturbative renormalized amplitude are only slightly different from the ones of the perturbative approach.

✓ Our non-perturbative treatment is valid, since ChPT has good convergence in SU(2) sector

XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

S=−1 meson-baryon scattering

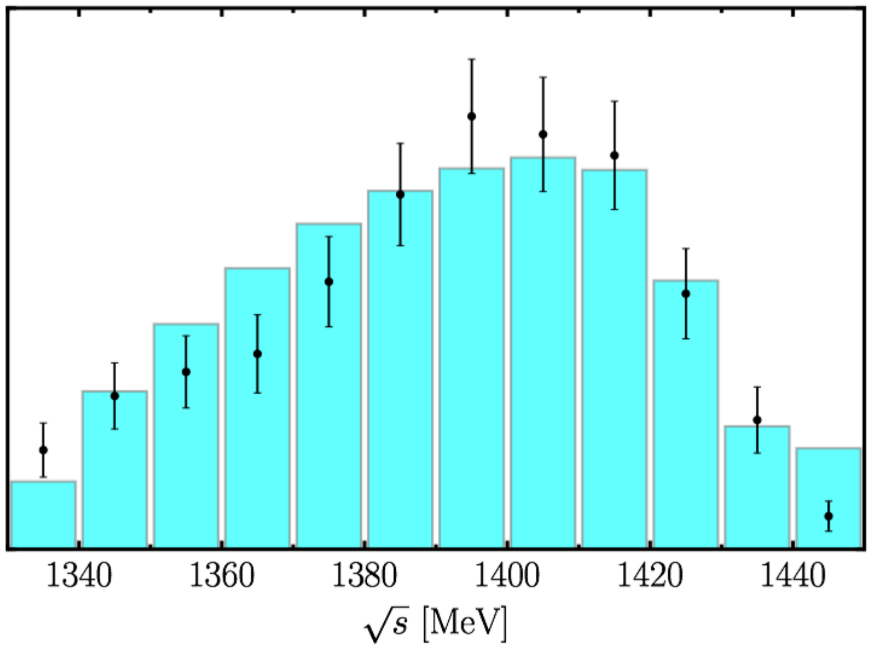
- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$
  - Solve the scattering equation in isospin basis by taking into account the off-shell effects of potential
  - **Use subtractive reormalization and take $\Lambda \to \infty$** to obtain the renormalized T-matrix

No free parameters needed to be fitted!

- Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work (LO)	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



- Consistent with M. Mai EPJA(2015), in particular for the lower pole

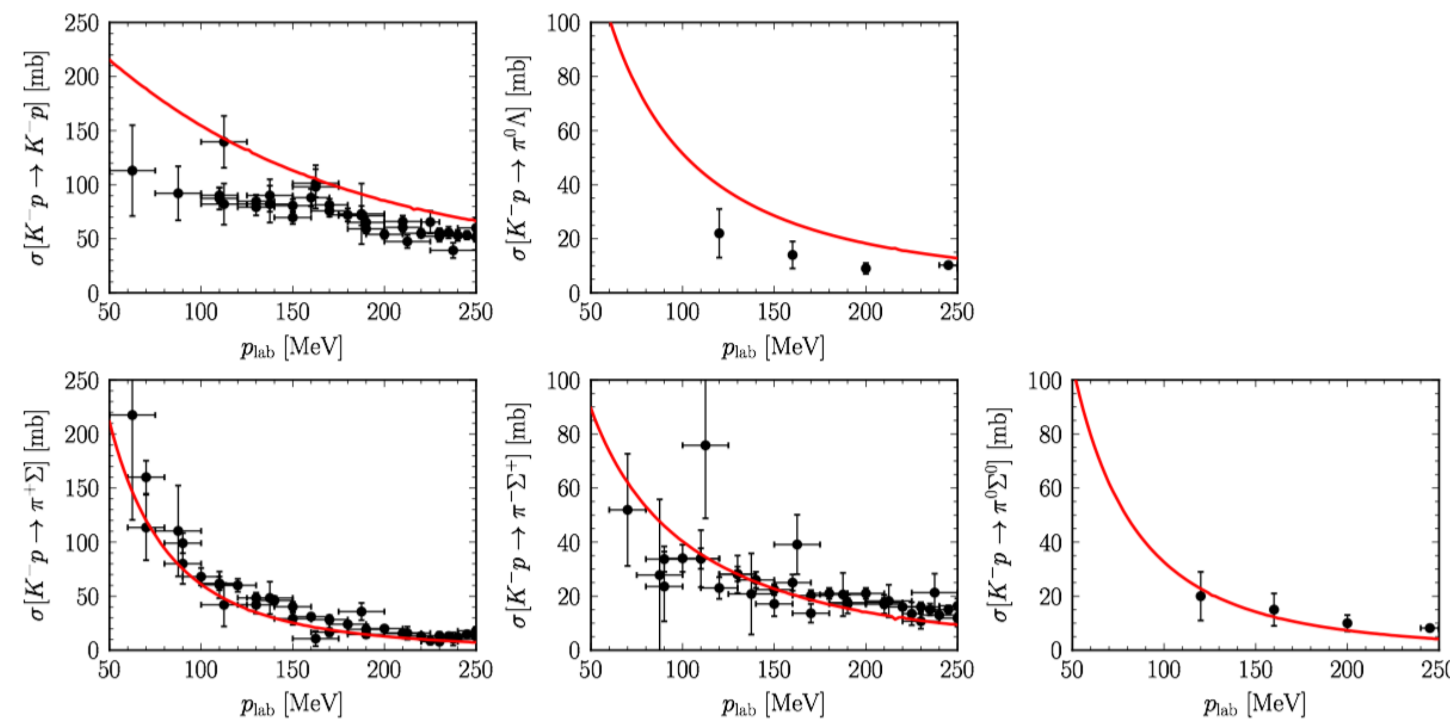
Couplings and scattering observables

- Coupling strength of the initial/final transition channel

	Lower pole		Higher pole	
	g_i	$\|g_i\|$	g_i	$\|g_i\|$
$\pi\Sigma$	$1.83 + i1.90$	2.64	$-0.38 + i0.84$	0.92
$\bar{K}N$	$-1.59 - i1.47$	2.17	$2.16 - i0.83$	2.31
$\eta\Lambda$	$-0.19 - i0.67$	0.69	$1.59 - i0.36$	1.63
$K\Xi$	$0.72 + i0.81$	1.08	$-0.10 + i0.34$	0.35

  - Two poles of $\Lambda(1405)$ have different coupling nature
    - ✓ Lower pole couples predominantly to the $\pi\Sigma$ channel
    - ✓ Higher pole couples strongly to the $\bar{K}N$ channel

- Total cross section of K^-p
  - Our LO prediction covers well $K^-p \to \pi^{\pm,0}\Sigma^{\pm,0}$ cross section
  - slightly larger than the data of $K^-p \to K^-p, \pi^0\Lambda$



XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582

Beyond leading order

- **Maintain the scattering T-matrix renormalizable**
  - Take LO potential non-perturbatively
  - Higher order corrections are perturbatively included

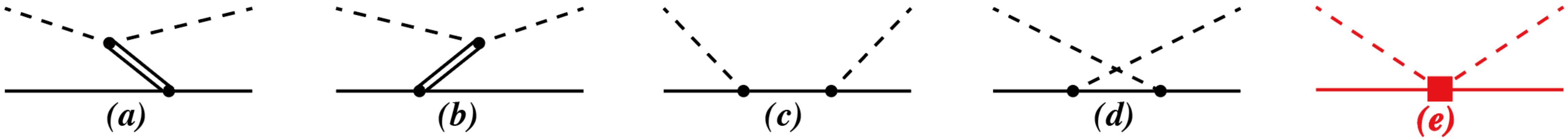
- E.g., up to NNLO
  - Potential: $V = V_{\rm LO} + V_{\rm NLO} + V_{\rm NNLO}$
  - T-matrix: $T = T_{\rm LO} + T_{\rm NLO} + T_{\rm NNLO}$

$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO} \quad \text{(non-perturbative)}$$

$$T_{\rm NLO} = V_{\rm NLO} + V_{\rm LO} G T_{\rm NLO} + V_{\rm NLO} G T_{\rm LO}$$

$$T_{\rm NNLO} = V_{\rm NNLO} + V_{\rm LO} G T_{\rm NNLO} + V_{\rm NLO} G T_{\rm NLO} + V_{\rm NNLO} G T_{\rm LO}$$

  - Use the subtractive renormalization scheme to remove divergent terms and power-counting breaking terms



- Chiral effective Lagrangian

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

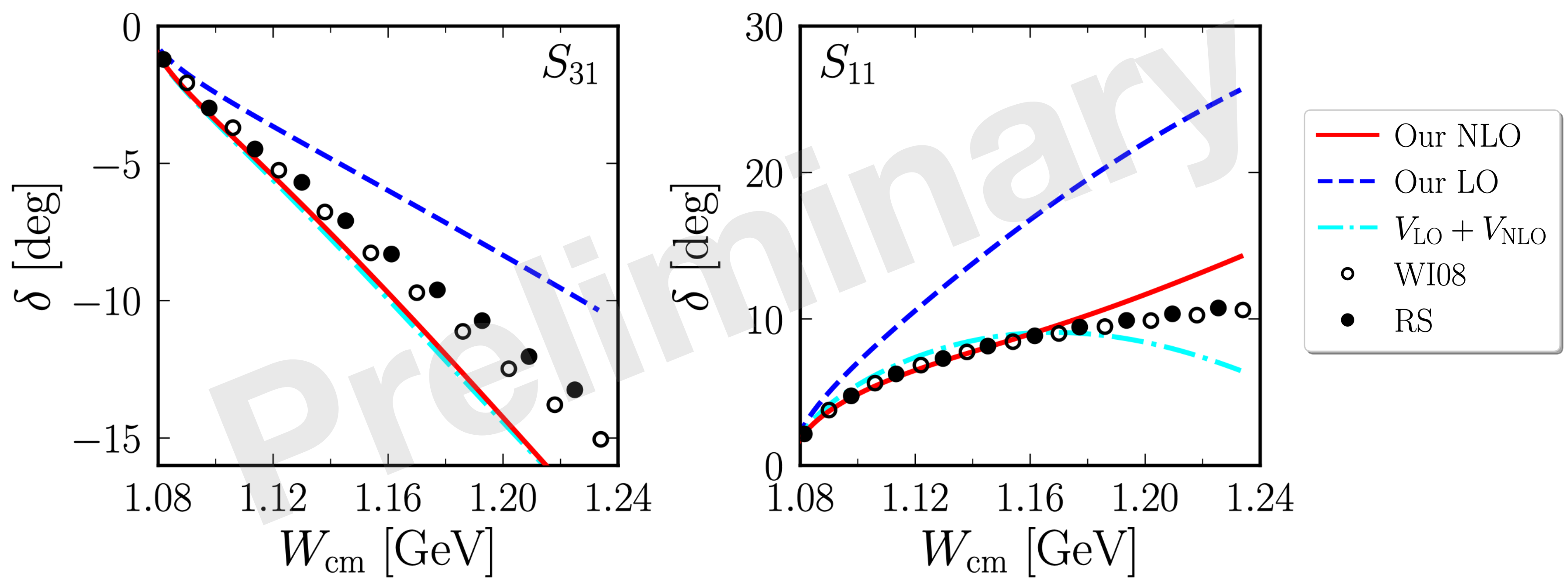
  - Fix $c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \mathrm{GeV}^{-1}$

D. Siemens, et al., PLB770 (2017) 27-34

- NLO potential

$$V = V_{\mathrm{LO}} + V_{\mathrm{NLO}}$$
$$= V^{(a+b+c+d)}\big|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c+d)}\big|_{u=u_1\sim\mathcal{O}(p)} + V^{(e)}\big|_{u=u_0\sim(1,0)^\dagger}$$

- Prediction for the πN phase shifts



Summary

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  - Time-ordered perturbation theory + Covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Use subtractive renormalization → **T-matrix is cutoff-independent**
  - **LO study:** πN scattering and the S=-1 sector
    - ✓ Obtain the two-pole structure of $\Lambda(1405)$
- Next-leading order study
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  - πN scattering: improve the description of phase shifts
  - Plan: extend to $\bar{K}N$ scattering, $\Lambda(1405)$, and other resonances

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Thank you for your attention!

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