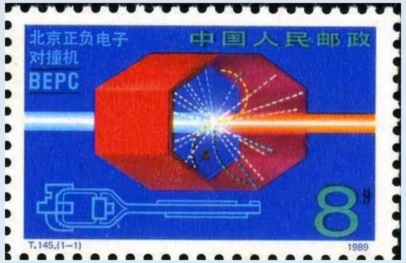
Three-body unitary coupled-channel analysis on $\eta(1405/1475)$



Jia-Jun Wu (UCAS)

Collaborators: Satoshi X. Nakamura, Qi Huang， Hai-Ping Peng，Yan Zhang, Yin-Chun Zhu

Phys.Rev.D 107 (2023) 9, L091505

MENU2023 2023.10.16 Mainz Germany

Outline

- The background of $\eta(1405/1475)$
- The data of $J/\psi \to \gamma\eta(1405/1475) \to \gamma K_s K_s \pi$
- Three-body unitary coupled channel model
- Dalitz fitting Result
- Theoretical analysis
- Summary and Outlook

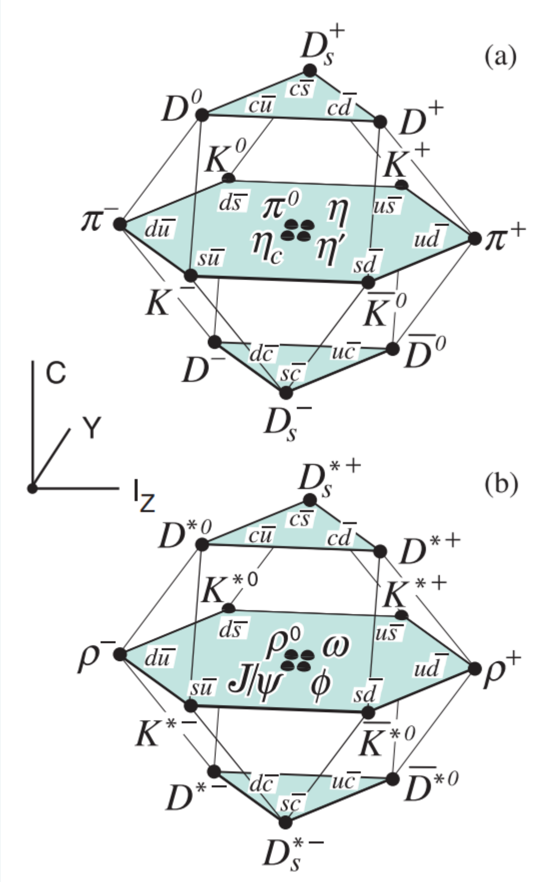
The background of $\eta(1405/1475)$ (PDG)

15. Quark Model

Revised August 2021 by C. Amsler (Stefan Meyer Inst.), T. DeGrand (Colorado U., Boulder) and B. Krusche (Basel U.).

$n^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I = 0 f'	I = 0 f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[d]	$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)$[a]		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$[c]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$[b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

[c] The $\eta(1475)$ and $\eta(1405)$ (not shown) may be manifestations of a single state [7].

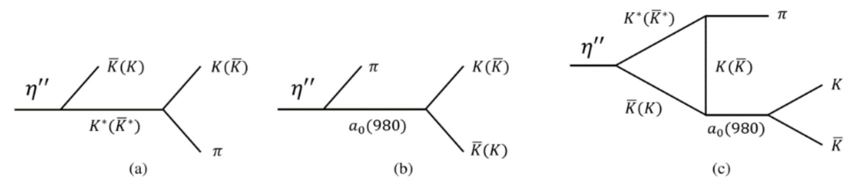


PHYSICAL REVIEW D **100**, 036005 (2019)

Internal particle width effects on the triangle singularity mechanism in the study of the $\eta(1405)$ and $\eta(1475)$ puzzle

Meng-Chuan Du[1,2,*] and Qiang Zhao[1,2,3,†]

claim that the TS mechanism plays a decisive role in the understanding of the $\eta(1405)$ and $\eta(1475)$ puzzle. Namely, the observed differences of η resonances within the mass region of 1.40–1.48 GeV are originated from the same state. For the isospin violated process $J/\psi \to \gamma\eta(1405/1475) \to f_0(980)\pi \to 3\pi$, we identify an additional contribution to the $a_0(980) - f_0(980)$ mixing via the TS mechanism.



PRL **108**, 081803 (2012) PHYSICAL REVIEW LETTERS week ending 24 FEBRUARY 2012

Puzzle of Anomalously Large Isospin Violations in $\eta(1405/1475) \to 3\pi$

Jia-Jun Wu,[1] Xiao-Hai Liu,[1] Qiang Zhao,[1,2,*] and Bing-Song Zou[1,2,†]
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[2]*Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China*
(Received 18 August 2011; published 22 February 2012)

The BES-III Collaboration recently reported the observation of anomalously large isospin violations in $J/\psi \to \gamma\eta(1405/1475) \to \gamma\pi^0 f_0(980) \to \gamma + 3\pi$, where the $f_0(980)$ in the $\pi\pi$ invariant mass spectrum appears to be much narrower (~ 10 MeV) than the peak width (~ 50 MeV) measured in other processes. We show that a mechanism, named as triangle singularity (TS), can produce a narrow enhancement between the charged and neutral $K\bar{K}$ thresholds, i.e., $2m_{K^\pm} \sim 2m_{K^0}$. It can also lead to different invariant mass spectra for $\eta(1405/1475) \to a_0(980)\pi$ and $K\bar{K}^* + \text{c.c.}$, which can possibly explain the long-standing puzzle about the need for two close states $\eta(1405)$ and $\eta(1475)$ in $\eta\pi\pi$ and $K\bar{K}\pi$, respectively. The TS could be a key to our understanding of the nature of $\eta(1405/1475)$ and advance our knowledge about the mixing between $a_0(980)$ and $f_0(980)$.

DOI: 10.1103/PhysRevLett.108.081803 PACS numbers: 13.20.Gd, 13.75.Lb, 14.40.Rt

The background of $\eta(1405/1475)$ (PDG)

15. Quark Model

Revised August 2021 by C. Amsler (Stefan Meyer Inst.), T. DeGrand (Colorado U., Boulder) and B. Krusche (Basel U.).

$n^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I = 0 f'	I = 0 f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
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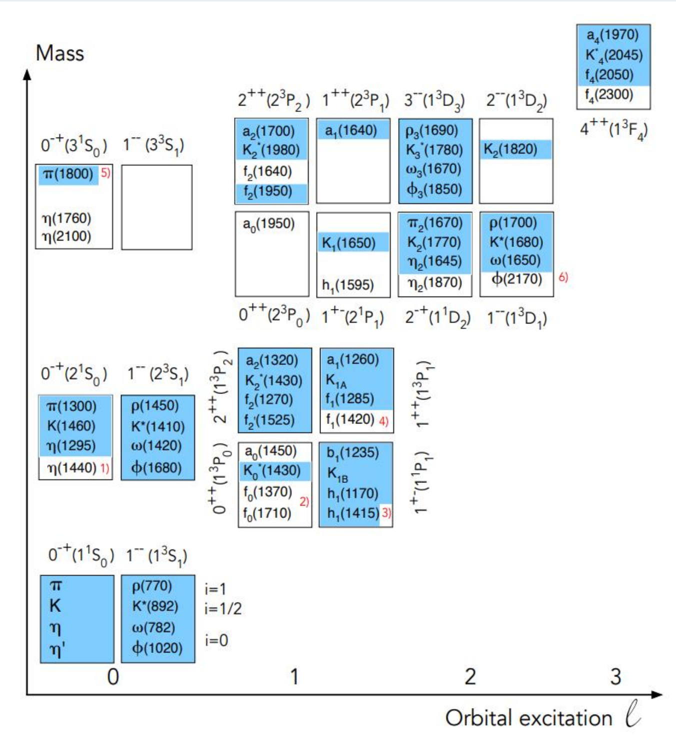
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

63. Spectroscopy of Light Meson Resonances

Written August 2021 by C. Amsler (Stefan Meyer Inst.), S. Eidelman (Budker Inst., Novosibirsk; Novosibirsk U.), A. Masoni (INFN, Cagliari) and G. Venanzoni (INFN, Pisa).



Hence, in radiative $J/\psi(1S)$ decay, π^-p and $\bar{p}p$ annihilation at rest two isoscalar signals are observed in the 1400 – 1500 MeV mass region, while the $\eta(1405)$ is not seen in $\gamma\gamma$ interactions nor in B decays. The $\eta(1475)$ could be the first radial excitation of the η', with the $\eta(1295)$ being the first radial excitation of the η. Ideal mixing, suggested by the $\eta(1295)$ and $\pi(1300)$ mass degeneracy,

that there is sufficient evidence to consider the 0^{-+} nonet with the $\eta(1440)$ in fig. 63.1 as established. Whether one or two different states – $\eta(1405)$ and $\eta(1475)$ – exist is an open question, in which case the $\eta(1405)$ would be supernumerary. There is a wide number of experimental results indicating the presence of two separate states but, as mentioned above, data are also consistent with one state only. Theoretical interpretations of the most recent data are not able to lift the ambiguity.

$\eta(1405/1475)$： Existed !
One Or Two ? What is it ?

Open Question

The background of $\eta(1405/1475)$ (EXP)

- $\pi^- p$ scattering: PRD40,693(1989)【$KsKs\pi$ 1&2】, E852 PLB516, 264(2001)【$K^+K^-\pi$ 2】
- $\bar{p}p$ annihilate exp: OBELX PLB361,187(1995);400,226(1997);462,453(1999); 545,261(2002)【$K^+K^-\pi$ 2】, EPJC33, 23 (2004)【$\pi\pi\eta$ 1】
- ψ decay :MARKIII, PRL 65, 2507 (1990))【$J/\psi \to \gamma K^+K^-\pi$ 2】,PRL69, 1328 (1992)【$J/\psi \to \gamma\pi\pi\eta$ 1】, DM2, PRD42, 10 (1990)【$J/\psi \to \gamma K^+K^-\pi$ 1】, PRD46, 1951 (1992)【$J/\psi \to \gamma K^+K^-\pi$ 2 $\gamma\pi\pi\eta$ 1】, BES, PRD77, 032005 (2008)【$J/\psi \to \omega K^+K^-\pi$ 1 $\phi K^+K^-\pi$ 0】, PRD87, 092006 (2013)【$\psi(3686) \to \omega K^+K^-\pi$ 1】PLB 446, 356【$J/\psi \to \gamma\pi\pi\eta$ 1】PRL 108 182001(2012)【$J/\psi \to \gamma\pi\pi\pi$ 1】 PLB 594 47(2004)【$J/\psi \to \gamma\gamma\rho$ 1】, PRD 97 051101(2018)【$J/\psi \to \gamma\gamma\phi$ 1(1475)】
- $\gamma\gamma$ collide: L3 PLB501 1(2001)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 1 $\pi\pi\eta$ 1】 JHEP 03, 018 (2007)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 1(1475)】 CLEO-II, PRD71, 072001(2005)【$\gamma\gamma \to \mathrm{KsK}^{\pm}\pi^{\mp}$ 0】
- B decay: BaBar, PRL101, 091801(2009)【$B^+ \to (\bar{\mathrm{K}}^*\mathrm{K}/\pi\pi\eta)\mathrm{K}$ 1 (1405)】

$\eta(1405)$: $a_0\pi, \bar{K}K^*$
$\eta(1475)$: $\bar{K}K^*$

The background of $\eta(1405/1475)$ (The)

- Quark Model：The radial excited states of（η, η'）are（$\eta(1295)$, $\eta(14??)$）
- Gluon Ball：The mass of pseudo-scalar gluon ball from the lattice calculation are all above 2.0 GeV【 UKQCD Collaboration, PLB 309, 378 (1993)/ C.J. Morningstar and M.J. Peardon, PRD 60, 034509 (1999)/ Y. Chen, A. Alexandru, S.J. Dong, T. Draper, I. Horva´th, F.X. Lee, K.F. Liu, N. Mathur et al., PRD 73, 014516 (2006)/ UKQCD Collaboration, PRD 82, 034501 (2010)/ F. Chen, X. Jiang, Y. Chen, K.-F. Liu, W. Sun, and Y.-B. Yang, arXiv:2111.11929 】， Some people recognize $\eta(1405)$ as a gluon ball 【 L. Faddeev, A. J. Niemi and U. Wiedner, PRD70, 114033 (2004) 】
- Hybrid：H. Y. Cheng, H. n. Li and K. F. Liu, PRD 79 014024(2009) ,
- Tetraquark state ： J. D. Weinstein and N. Isgur, PRD 27 588(1983) .
- Molecule: $K\bar{K}\pi$ Molecule state R. S. Longacre, PRD 42 (1990) 874.
- ……

The background of $\eta(1405/1475)$

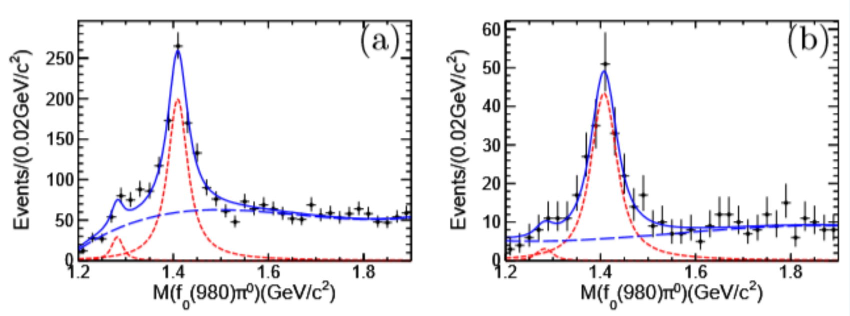
- The main two questions of $\eta(1405/1475)$

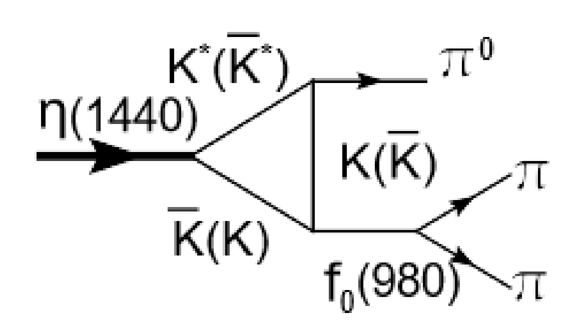
One or Two？

What is it？

Need more experimental data and a complete analysis are both necessary!

BESIII PRL 108 182001(2012) 【$J/\psi \to \gamma\pi\pi\pi$ 1】





The data of $J/\psi \to \gamma\eta(1405/1475) \to \gamma K_s K_s \pi$

BESIII JHEP 03 (2023) 121

JHEP

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Study of $\eta(1405)/\eta(1475)$ in $J/\psi \to \gamma K_S^0 K_S^0 \pi^0$ decay

BESIII

The BESIII collaboration

E-mail: besiii-publications@ihep.ac.cn

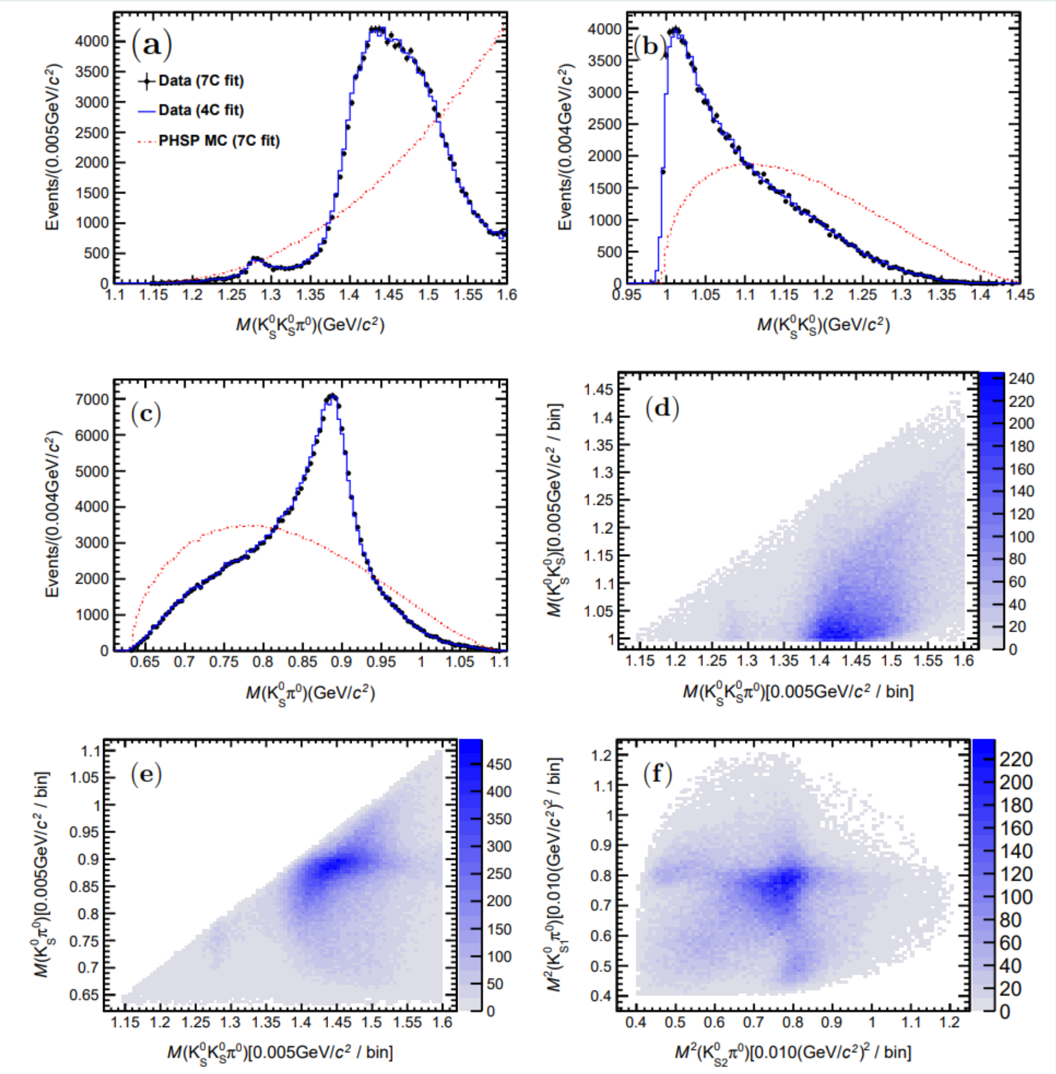
Abstract: Using a sample of $(10.09 \pm 0.04) \times 10^9$ J/ψ decays collected with the BESIII detector, partial wave analyses of the decay $J/\psi \to \gamma K_S^0 K_S^0 \pi^0$ are performed within the $K_S^0 K_S^0 \pi^0$ invariant mass region below 1.6 GeV/c^2. The covariant tensor amplitude method is used in both mass independent and mass dependent approaches. Both analysis approaches exhibit dominant pseudoscalar and axial vector components, and show good consistency for the other individual components. Furthermore, the mass dependent analysis reveals that the $K_S^0 K_S^0 \pi^0$ invariant mass spectrum for the pseudoscalar component can be well described with two isoscalar resonant states using relativistic Breit-Wigner model, i.e., the $\eta(1405)$ with a mass of $1391.7\pm0.7^{+11.3}_{-0.3}$ MeV/c^2 and a width of $60.8\pm1.2^{+5.5}_{-12.0}$ MeV, and the $\eta(1475)$ with a mass of $1507.6\pm1.6^{+15.5}_{-32.2}$ MeV/c^2 and a width of $115.8\pm2.4^{+14.8}_{-10.9}$ MeV. The first and second uncertainties are statistical and systematic, respectively. Alternate models for the pseudoscalar component are also tested, but the description of the $K_S^0 K_S^0 \pi^0$ invariant mass spectrum deteriorates significantly.

JHEP03(2023)121

BESIII
10^{10} J/ψ events

126436 events of $\gamma K_s K_s \pi$

For $K_s K_s \pi$ three-body "Bin-by-bin analysis"



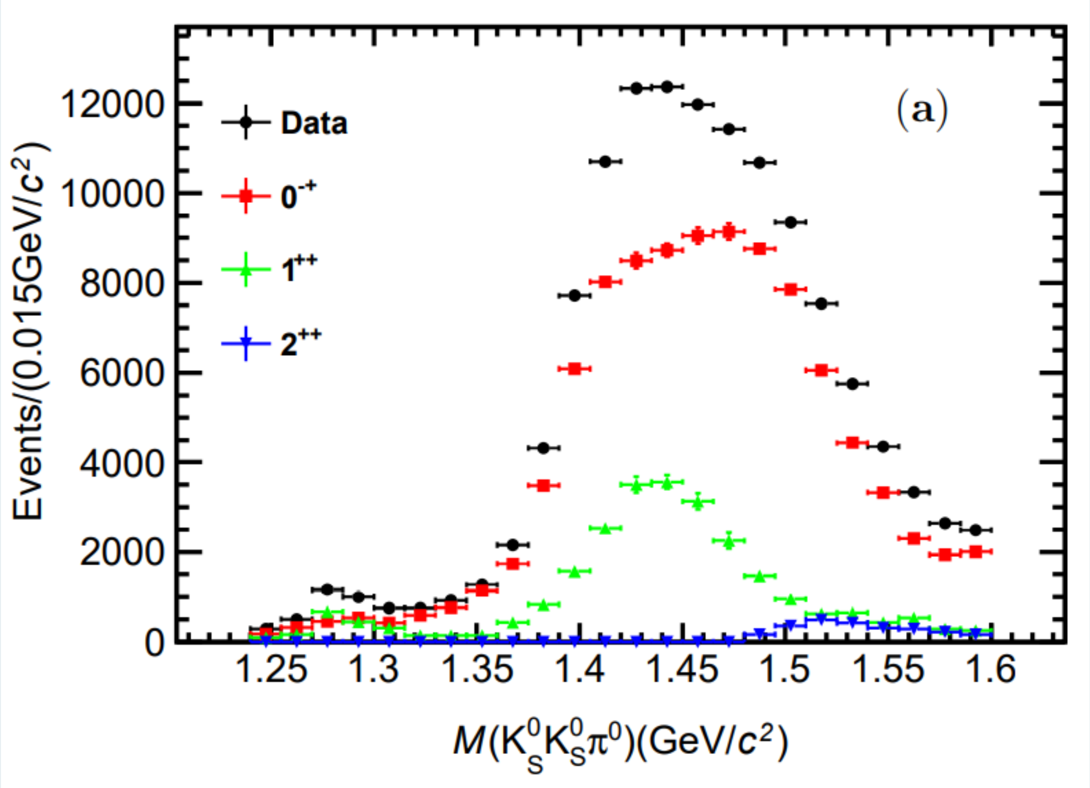
The data of $J/\psi \rightarrow \gamma\eta(1405/1475) \rightarrow \gamma K_s K_s \pi$

BESIII JHEP 03 (2023) 121

126436 events of $\gamma K_s K_s \pi$

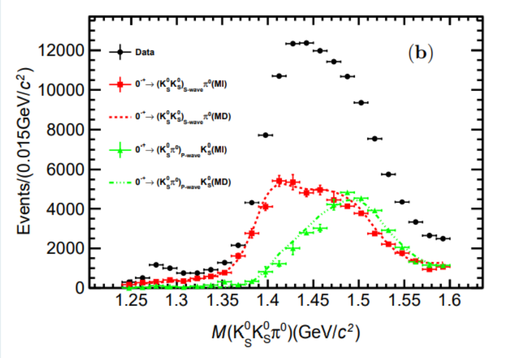
BESIII 10^{10} J/ψ events

For $K_s K_s \pi$ three body "Bin-by-bin analysis"



$$BW(s) = \frac{1}{M^2 - s - iM\Gamma(s)},$$

$$\Gamma(s) = \Gamma(M^2)\left(\frac{M}{\sqrt{s}}\right)\left(\frac{\rho(s)}{\rho(M^2)}\right)^{2l+1} B_l^2(\rho(s)),$$



To analysis the 0^{-+} $\bar{K}_s K_s \pi$, they direct use BW form the analysis the data to gain the each contributions

University of Chinese Academy of Sciences

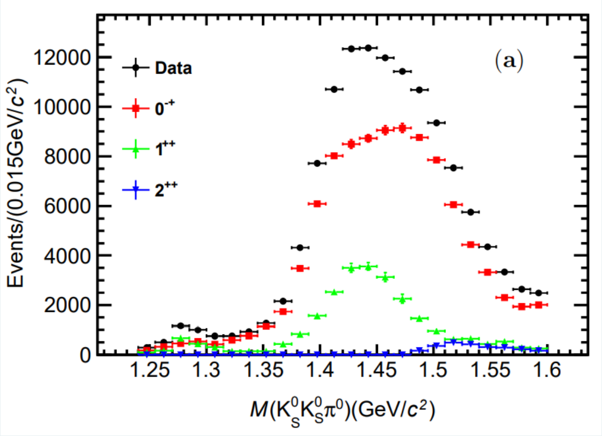
The data of $J/\psi \to \gamma\eta(1405/1475) \to \gamma K_s K_s \pi$

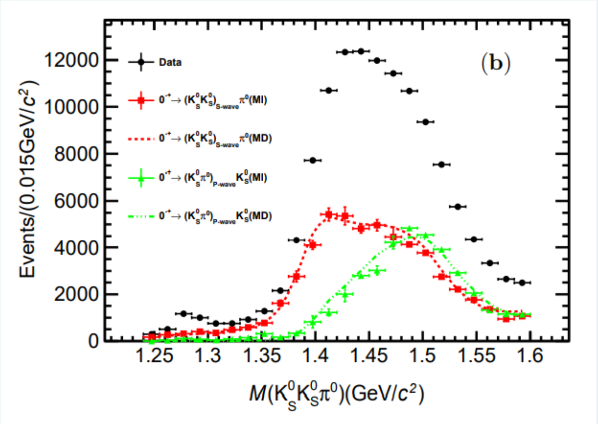
BESIII JHEP 03 (2023) 121

126436 events of $\gamma K_s K_s \pi$

BESIII 10^{10} J/ψ events

For $K_s K_s \pi$ three body "Bin-by-bin analysis" **+ BW model** ?



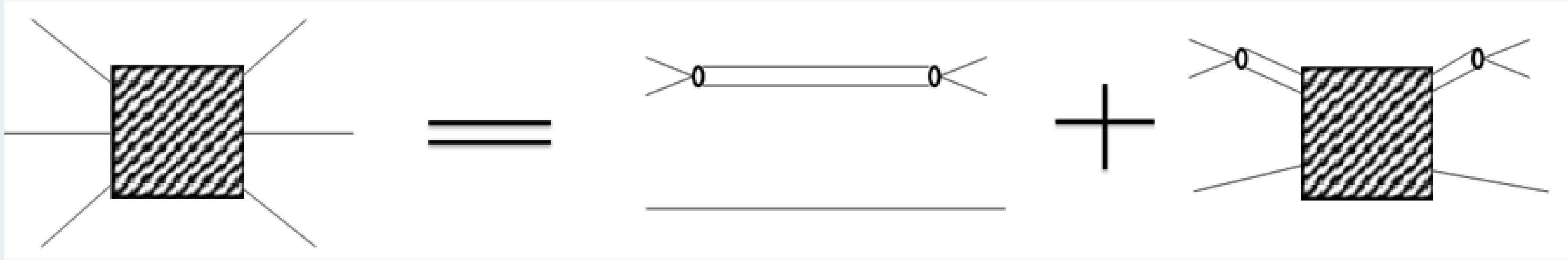


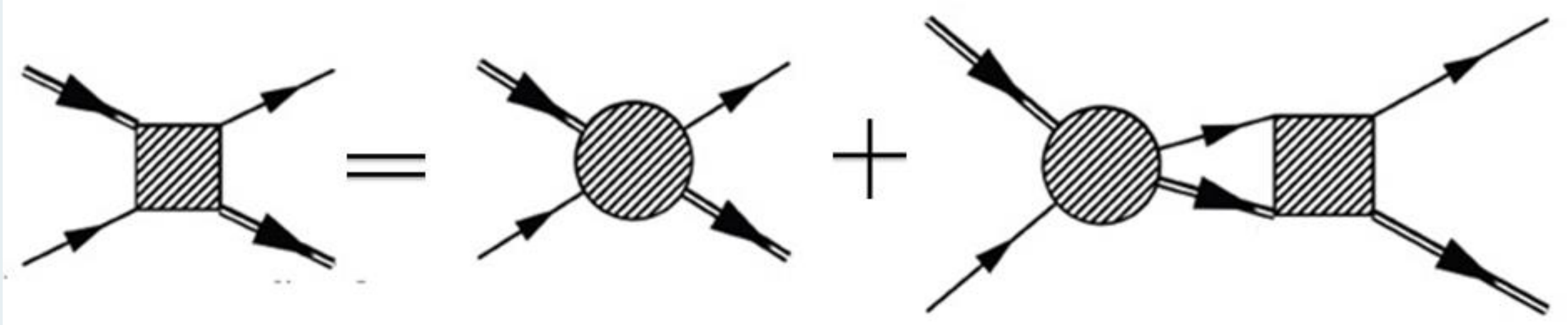
For obtaining 0^{-+} partial wave, they use Bin by Bin analysis, we trust it is almost model independent, while for the analysis of 0^{-+} partial wave, **BW form is not proper here.**

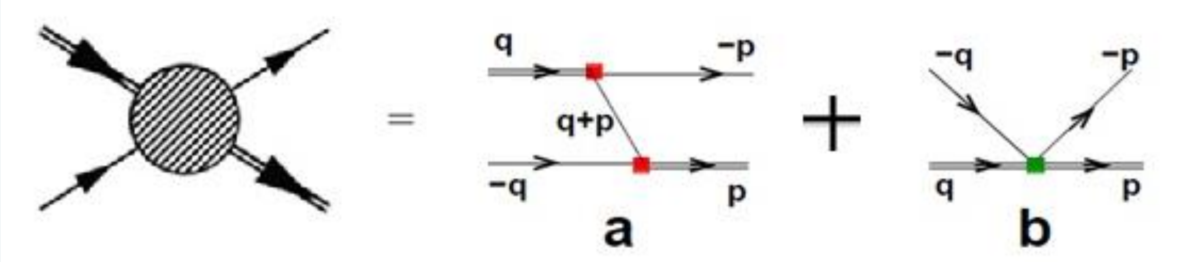
1. $\eta(1405/1475)$ refers to different resonances, which will destroy unitary
2. In the 1400-1500 Mev region, the threshold effect is important, such as $K^*\bar{K}$。
3. The complex kinematical effect, such as TS mechanism , how to estimate loop contribution.

For these reasons, it is necessary for a unitary couple channel for three-body system for extracting the pole positions of $\eta(1405/1475)$!

Three-body unitary coupled channel model







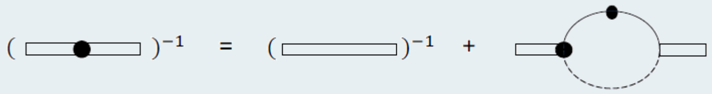
Three-body unitary coupled channel model

$$f_{ab,R}(q) = g_{ab,R}\frac{(1+q^2/c^2_{ab,R})^{-2-\frac{L}{2}}}{\sqrt{m_R E_a(q) E_b(q)}}\frac{q^L}{m_\pi^{L-1}}$$

$$F_{(cR)_l,\eta_i^*}(q) = C^i_{(cR)_l}\frac{[1+q^2/(\Lambda^i_{(cR)_l})^2]^{-2-\frac{l}{2}}}{\sqrt{8E_c(q)E_R(q)\,m_{\eta_i^*}}}\frac{q^l}{m_\pi^{l-1}}$$

$$\Gamma_{\gamma\eta_j^*,J/\psi} = \frac{g_{J/\psi\eta_j^*\gamma}\,(\boldsymbol{\epsilon}_{J/\psi}\times\boldsymbol{\epsilon}_\gamma)\cdot\boldsymbol{p}_\gamma}{\sqrt{8E_\gamma\, m_{J/\psi}\, m_{\eta_j^*}}}$$

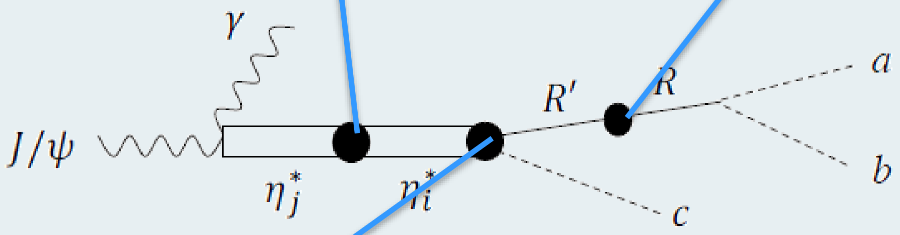
Bare vetex



Full propagator of η^*

Full propagator of R $\Gamma_{ab,R}(\boldsymbol{q}) = (t_a t_a^z t_b t_b^z | t_R t_R^z) Y_{s_R,s_R^z}(\hat{q}) f_{ab,R}(q)$

$$A_{\gamma abc,J/\psi} = \sum_{abc}^{\text{cyclic}}\sum_{RR's_R^z}\sum_{ij}\Gamma_{ab,R}(\boldsymbol{p}_a^*)\,\tau_{R,R'}(p_c,E-E_c)\,\bar{\Gamma}_{cR',\eta_i^*}(\boldsymbol{p}_c,E)\,\bar{G}_{ij}(E)\,\Gamma_{\gamma\eta_j^*,J/\psi}$$



$$[\tau^{-1}(p,E)]_{R,R'} = [E-E_R(p)]\delta_{R,R'} - \Sigma_{R,R'}(p,E)$$

Full propagator of R

$$[\bar{G}^{-1}(E)]_{ij} = (E-m_{\eta_i^*})\delta_{ij} - [\Sigma_{\eta^*}(E)]_{ij}$$

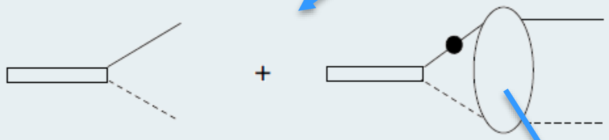
Full propagator of η^*

Self energy of R

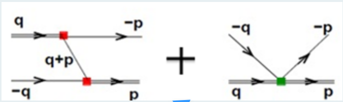
$$\Sigma_{R,R'}(p,E) = \sum_{ab}(t_a t_a^z t_b t_b^z | t_R, t_a^z + t_b^z)(t_a t_a^z t_b t_b^z | t_{R'}, t_a^z + t_b^z) \times\sqrt{\frac{m_R m_{R'}}{E_R(p)E_{R'}(p)}}\int q^2 dq\frac{M_{ab}(q)}{\sqrt{M^2_{ab}(q)+p^2}} \times\frac{\mathcal{B}_{ab}\, f_{R,ab}(q) f_{ab,R'}(q)}{E-\sqrt{M^2_{ab}(q)+p^2}+i\epsilon}$$

Self energy of η^*

$$[\Sigma_{\eta^*}(E)]_{ij} = \mathcal{B}_{Rc}\sum_{cRR'l}\int q^2 dq\, F_{(cR)_l,\eta_i^*}(q) \times\tau_{R,R'}(q,E-E_c(q))\bar{F}_{(cR')_l,\eta_j^*}(q,E)$$

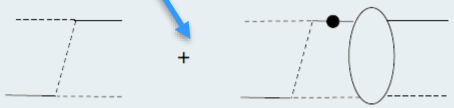


Full vertex of $\eta^* \to Rc$ including rescattering



$$\bar{\Gamma}_{cR,\eta_i^*}(\boldsymbol{p}_c,E) = \sum_{l,l^z}(ll^z s_R s_R^z | s_{\eta^*} s_{\eta^*}^z)(t_R t_R^z t_c t_c^z | t_{\eta^*} t_{\eta^*}^z) \times Y_{l,l^z}(-\hat{p}_c)\bar{F}_{(cR)_l,\eta_i^*}(p_c,E)$$

$$\bar{F}_{(cR)_l,\eta_i^*}(p_c,E) = F_{(cR)_l,\eta_i^*}(p_c) + \sum_{c'R'R''l'}\int q^2 dq \times X_{(cR)_l,(c'R'')_{l'}}(p_c,q;E) \times\tau_{R'',R'}(q,E-E_{c'})F_{(c'R')_{l'},\eta_i^*}(q)$$



The full amplitude of $Rc \to R'c'$ is solved from equation

$$X_{(c'R')_{l'},(cR)_l}(p',p;E) = V_{(c'R')_{l'},(cR)_l}(p',p;E) + \sum_{c'',R'',R''',l''}\int q^2 dq\, V_{(c'R')_{l'},(c''R''')_{l''}}(p',q;E) \times\tau_{R''',R''}(q,E)X_{(c''R'')_{l''},(cR)_l}(q,p;E)\ ,$$

Three-body unitary coupled channel model

Two body parameters are fitted by two-body scattering, we just fit the parameters in $\eta^* \rightarrow Rc$ and $Rc \rightarrow R'c'$ without Z diagrams

$$Rc = K^*(892)\overline{K}, \kappa\overline{K}, a0(980)\pi, a2(1320)\pi, f_0\eta, \rho\rho, f_0\pi$$

Additional experimental constrains：

$$R_{exp1} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\eta\pi+\pi-)]} \sim 6.8 - 11.9$$

$$R_{exp2} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\rho 0\gamma)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]} \sim 0.015 - 0.043$$

$$f_{ab,R}(q) = g_{ab,R}\frac{(1+q^2/c^2_{ab,R})^{-2-\frac{L}{2}}}{\sqrt{m_R E_a(q) E_b(q)}}\frac{q^L}{m_\pi^{L-1}}$$

$$F_{(cR)_l,\eta_i^*}(q) = C^i_{(cR)_l}\frac{[1+q^2/(\Lambda^i_{(cR)_l})^2]^{-2-\frac{l}{2}}}{\sqrt{8E_c(q)E_R(q)\, m_{\eta_i^*}}}\frac{q^l}{m_\pi^{l-1}}$$

$$\Gamma_{\gamma\eta_j^*,J/\psi} = \frac{g_{J/\psi\eta_j^*\gamma}\,(\boldsymbol{\epsilon}_{J/\psi}\times\boldsymbol{\epsilon}_\gamma)\cdot\boldsymbol{p}_\gamma}{\sqrt{8E_\gamma\, m_{J/\psi}\, m_{\eta_j^*}}}$$

Bare vetex

$$\Gamma_{ab,R}(\boldsymbol{q}) = (t_a t_a^z t_b t_b^z | t_R t_R^z) Y_{s_R,s_R^z}(\hat{q}) f_{ab,R}(q)$$

$$A_{\gamma abc,J/\psi} = \sum_{abc}^{\text{cyclic}}\sum_{RR's_R^z}\sum_{ij}\Gamma_{ab,R}(\boldsymbol{p}_a^*)\,\tau_{R,R'}(p_c, E-E_c)\,\bar{\Gamma}_{cR',\eta_i^*}(\boldsymbol{p}_c, E)\,\bar{G}_{ij}(E)\,\Gamma_{\gamma\eta_j^*,J/\psi}$$

$$[\tau^{-1}(p,E)]_{R,R'} = [E - E_R(p)]\delta_{R,R'} - \Sigma_{R,R'}(p,E)$$

Full propagator of R

$$[\bar{G}^{-1}(E)]_{ij} = (E - m_{\eta_i^*})\delta_{ij} - [\Sigma_{\eta^*}(E)]_{ij}$$

Full propagator of η^*

Self energy of R

$$\Sigma_{R,R'}(p,E) = \sum_{ab}(t_a t_a^z t_b t_b^z | t_R, t_a^z + t_b^z)(t_a t_a^z t_b t_b^z | t_{R'}, t_a^z + t_b^z) \times\sqrt{\frac{m_R m_{R'}}{E_R(p)E_{R'}(p)}}\int q^2 dq \frac{M_{ab}(q)}{\sqrt{M^2_{ab}(q)+p^2}} \times\frac{\mathcal{B}_{ab}\, f_{R,ab}(q) f_{ab,R'}(q)}{E - \sqrt{M^2_{ab}(q)+p^2} + i\epsilon}$$

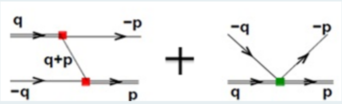
Self energy of η^*

$$\bar{\Gamma}_{cR,\eta_i^*}(\boldsymbol{p}_c, E) = \sum_{l,l^z}(ll^z s_R s_R^z | s_{\eta^*} s^z_{\eta^*})(t_R t_R^z t_c t_c^z | t_{\eta^*} t^z_{\eta^*}) \times Y_{l,l^z}(-\hat{p}_c)\bar{F}_{(cR)_l,\eta_i^*}(p_c, E)$$

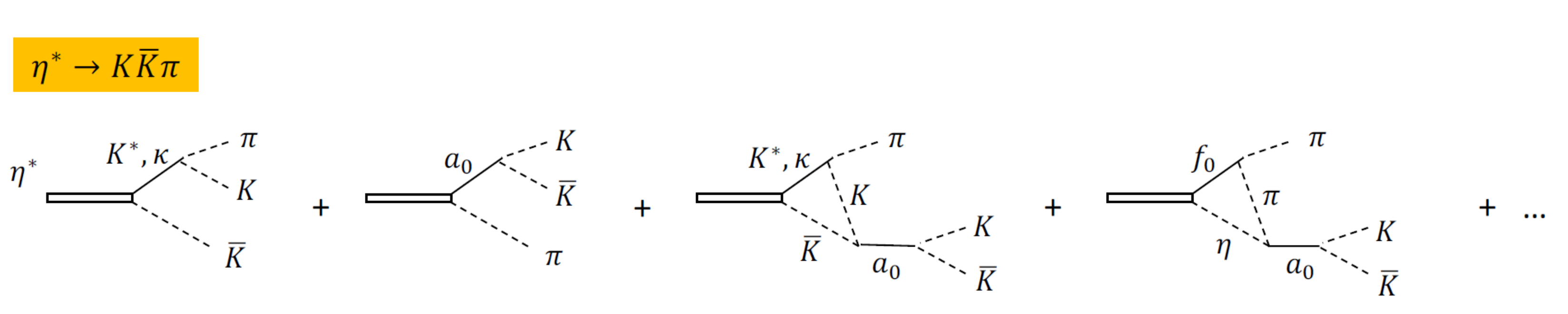
$$[\Sigma_{\eta^*}(E)]_{ij} = \mathcal{B}_{Rc}\sum_{cRR'l}\int q^2 dq\, F_{(cR)_l,\eta_i^*}(q) \times\tau_{R,R'}(q, E - E_c(q))\bar{F}_{(cR')_l,\eta_j^*}(q,E)$$

$$X_{(c'R')_{l'},(cR)_l}(p',p;E) = V_{(c'R')_{l'},(cR)_l}(p',p;E) + \sum_{c'',R'',R''',l''}\int q^2 dq\, V_{(c'R')_{l'},(c''R''')_{l''}}(p',q;E) \times\tau_{R''',R''}(q,E)X_{(c''R'')_{l''},(cR)_l}(q,p;E)\ ,$$

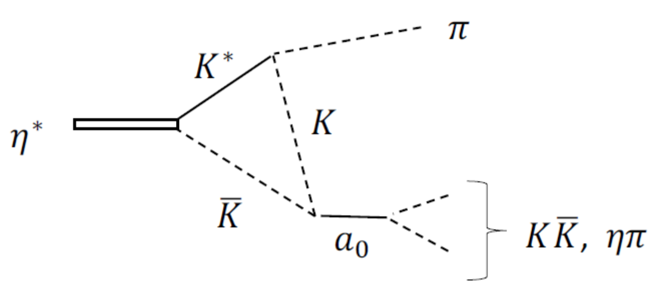
$$\bar{F}_{(cR)_l,\eta_i^*}(p_c,E) = F_{(cR)_l,\eta_i^*}(p_c) + \sum_{c'R'R''l'}\int q^2 dq \times X_{(cR)_l,(c'R'')_{l'}}(p_c,q;E) \times\tau_{R'',R'}(q,E-E_{c'})F_{(c'R')_{l'},\eta_i^*}(q)$$

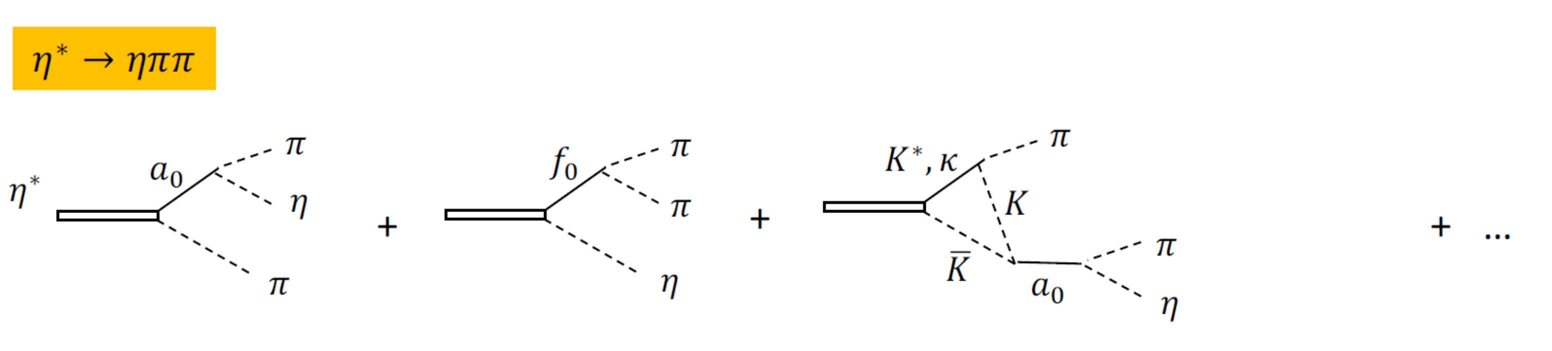


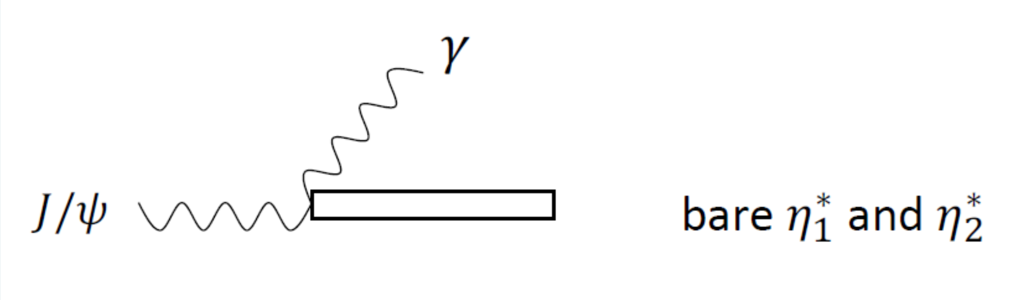
Three-body unitary coupled channel model



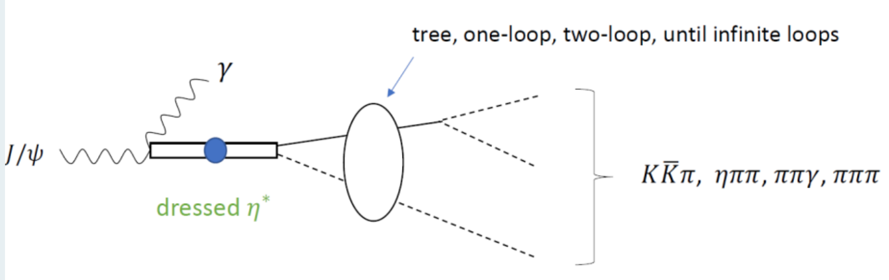
TS in single loop is automatically included



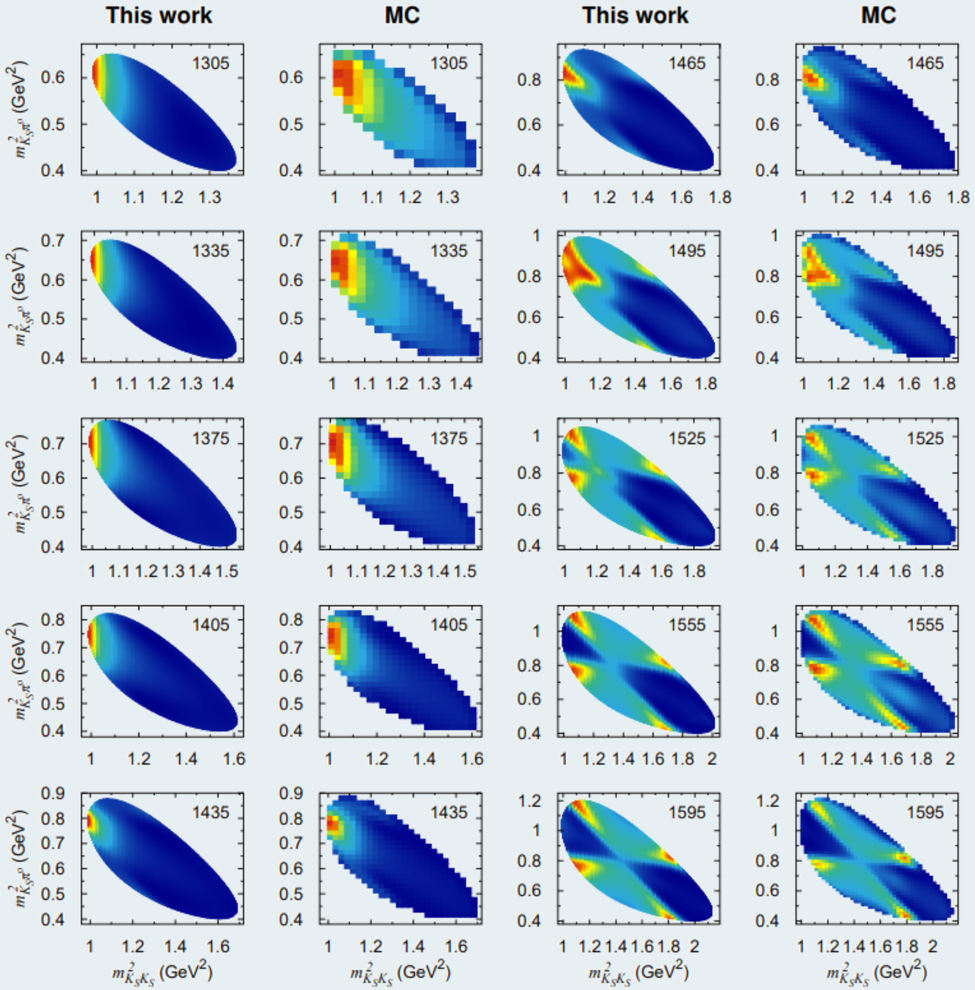


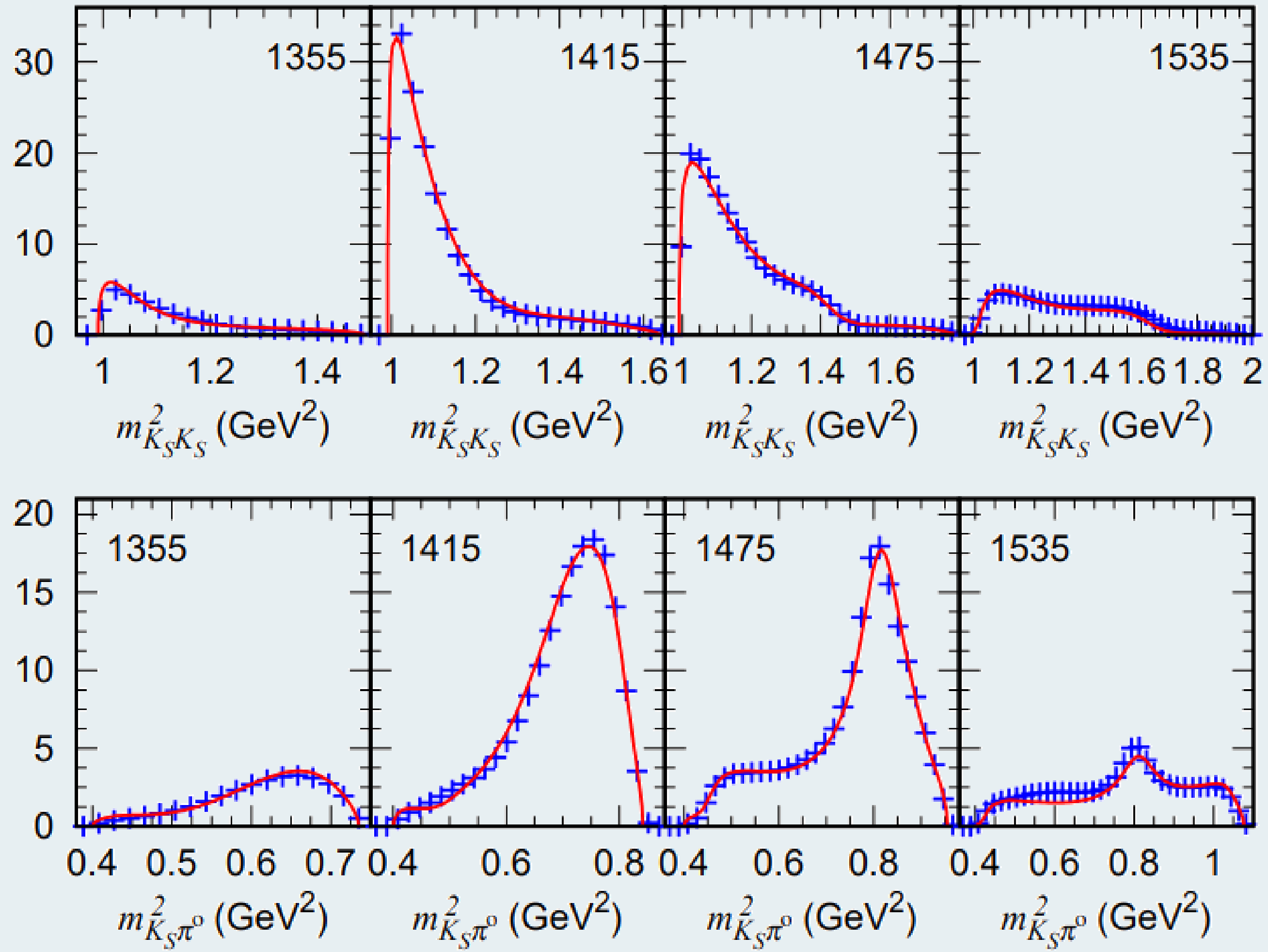


These two coupling are not constrained by coupled channel.

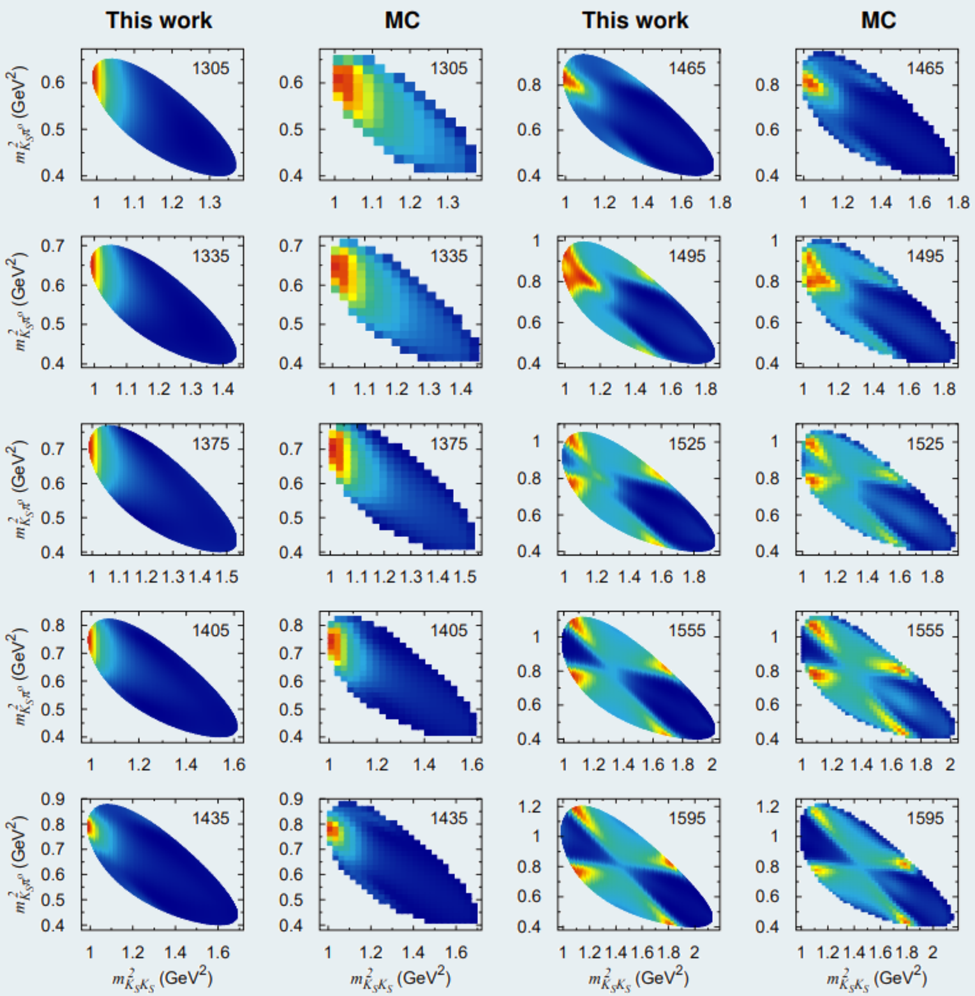


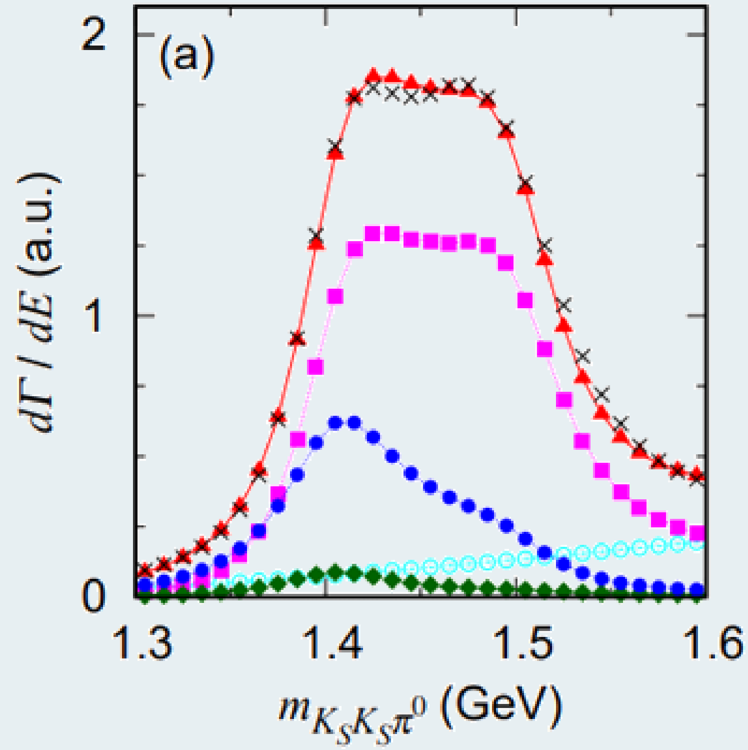
Dalitz fitting Result





Dalitz fitting Result



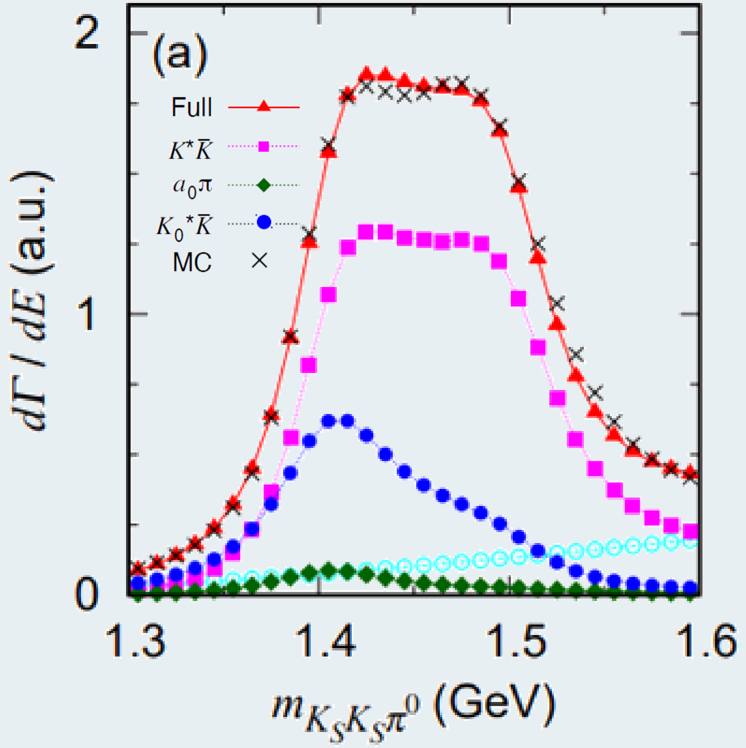


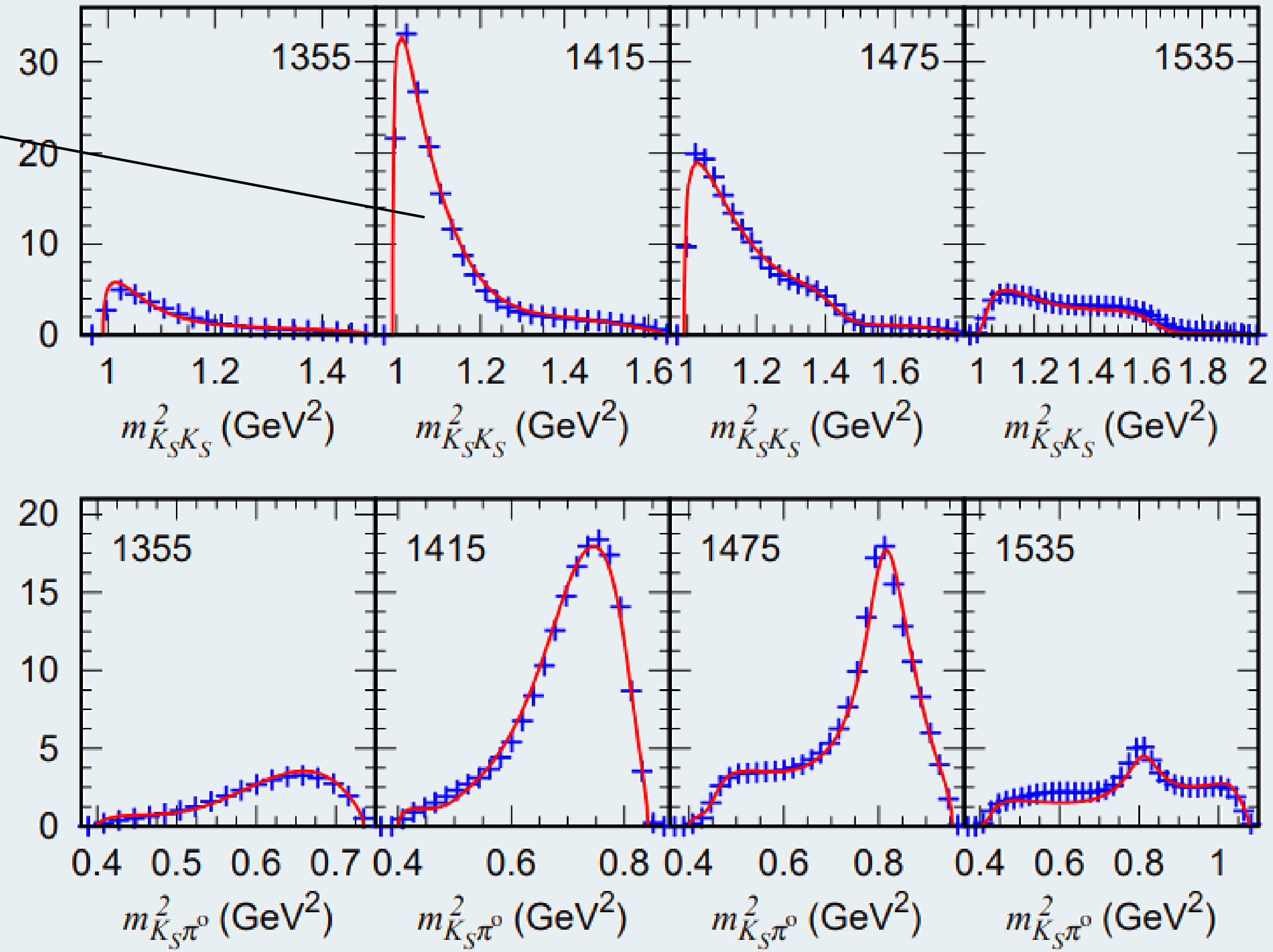
$$R_{exp1} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\eta\pi+\pi-)]} \sim 6.8 - 11.9 \qquad 6.9$$

$$R_{exp2} = \frac{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(\rho\ 0\gamma)]}{\Gamma[J/\psi \to \gamma\eta(1405/1475) \to \gamma(KK\pi^-)]} \sim 0.015 - 0.043 \qquad 0.026$$

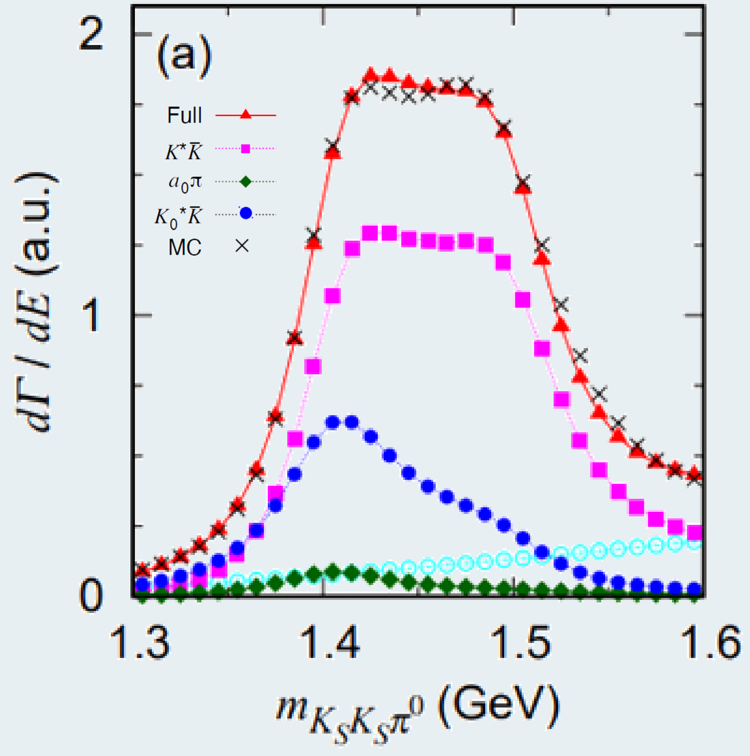
Theoretical analysis

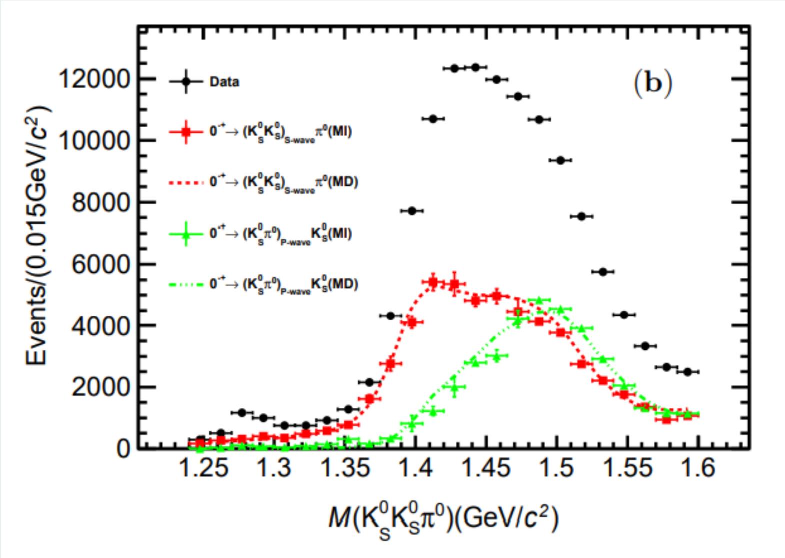
From the reflection of $\bar{K}K^*$,
rather than $a_0\pi$



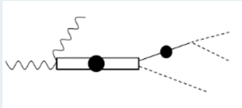


Theoretical analysis

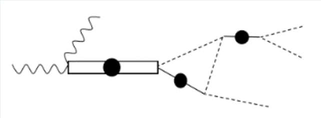




Our results is different from experimental partial wave analysis based on BW model for the three body in 0-+ sector. Although Dalitz distribution is similar, but the mechanisms are different.



1. We include $\kappa\bar{K}$
2. We consider the R_{exp1}， which suggest that the $a_0\pi$ should be not dominated, that's why we need $\kappa\bar{K}$
3. Including coupled channel contribution



We have four conclusion

1. The flat structure mainly from $\bar{K}K^*$
2. $\bar{K}K^*$ mainly from tree diagram
3. $a_0\pi$ and $\kappa\bar{K}$ mainly from the TS in the single loop
4. Nedd two bare η^*, around 1.6 GeV and above 2.0 GeV

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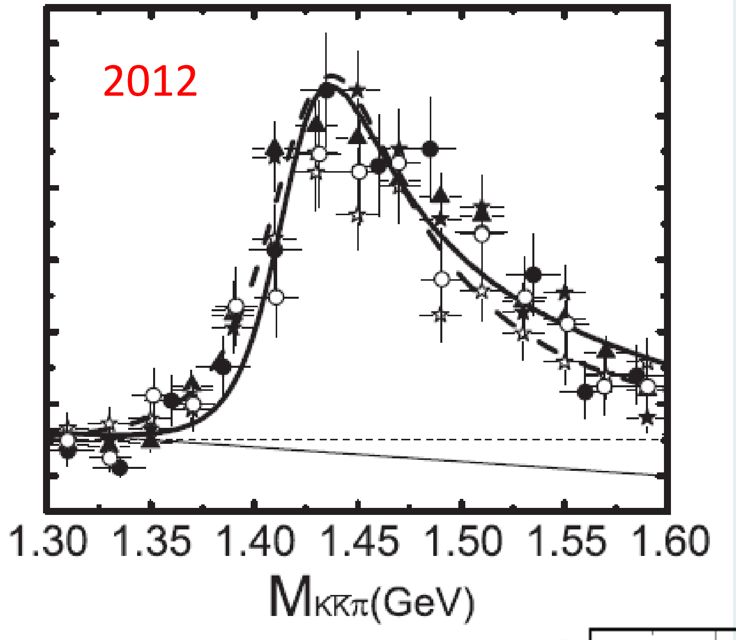
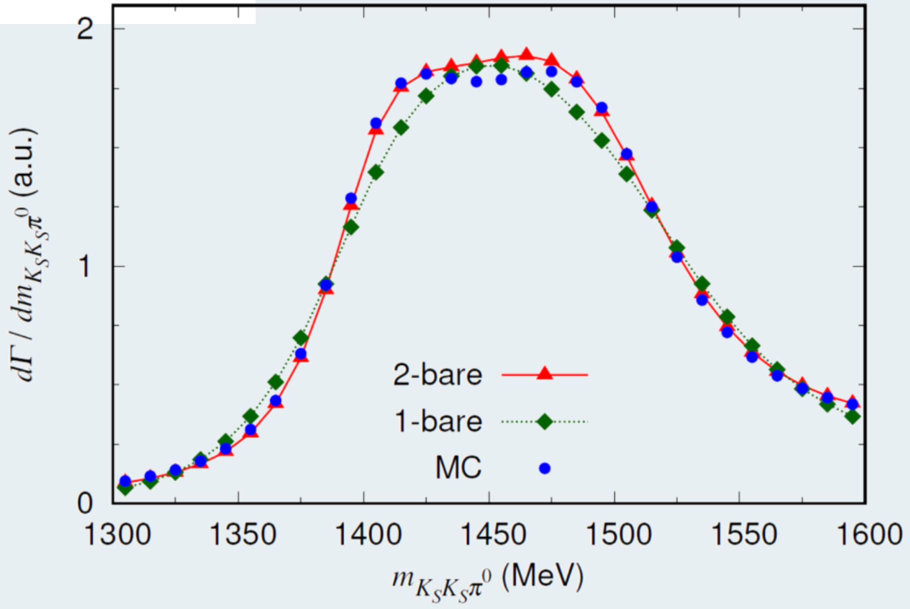
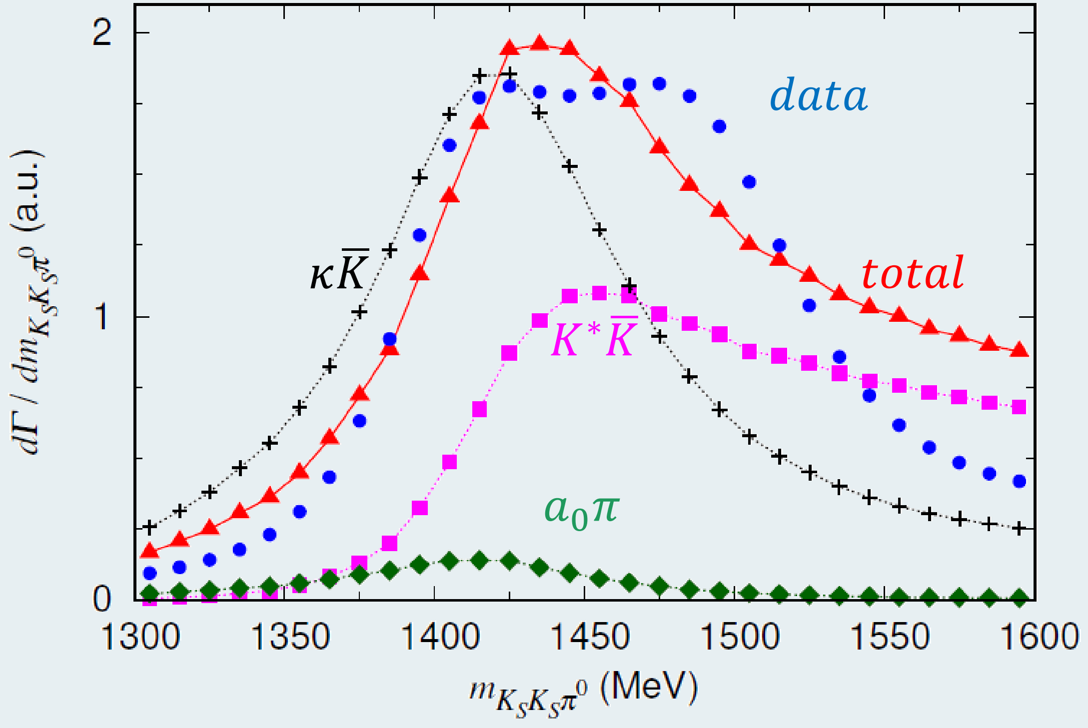


Theoretical analysis

1. The flat structure mainly from $\bar{K}K^*$
2. $\bar{K}K^*$ mainly from tree diagram
3. $a_0\pi$ and $\kappa\bar{K}$ mainly from the TS in the single loop
4. Need two bare $\eta^*\ s$

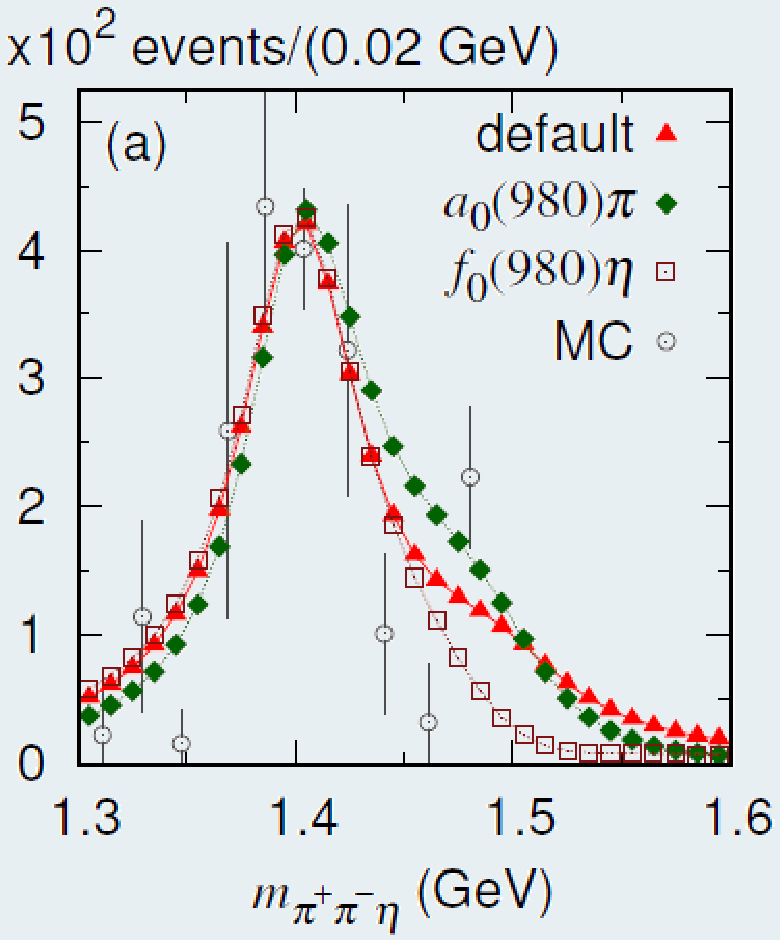
In our process, barely rely on the kinematic and P-wave vertex, it can not generate the flat structure by one bare state.

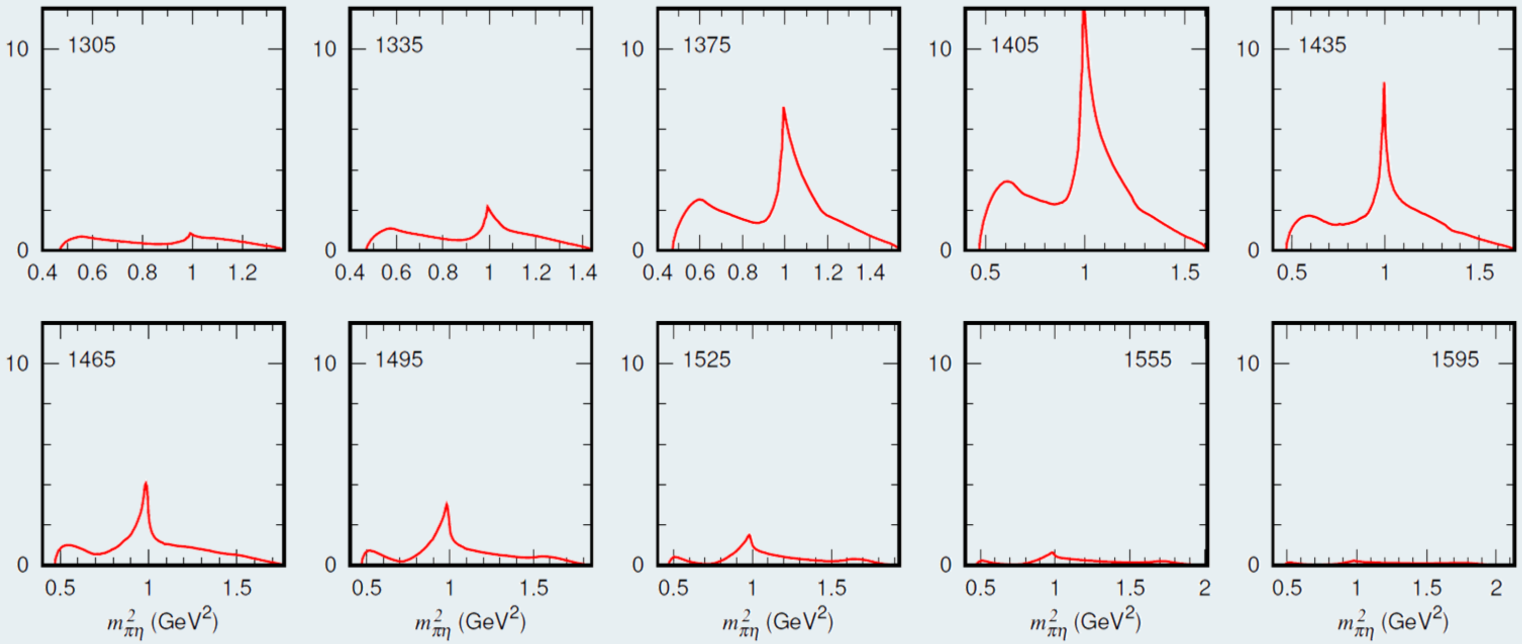




Theoretical analysis

Post predicted the invariant mass spectrum in $\eta(1405/1475) \rightarrow \pi\pi\eta$

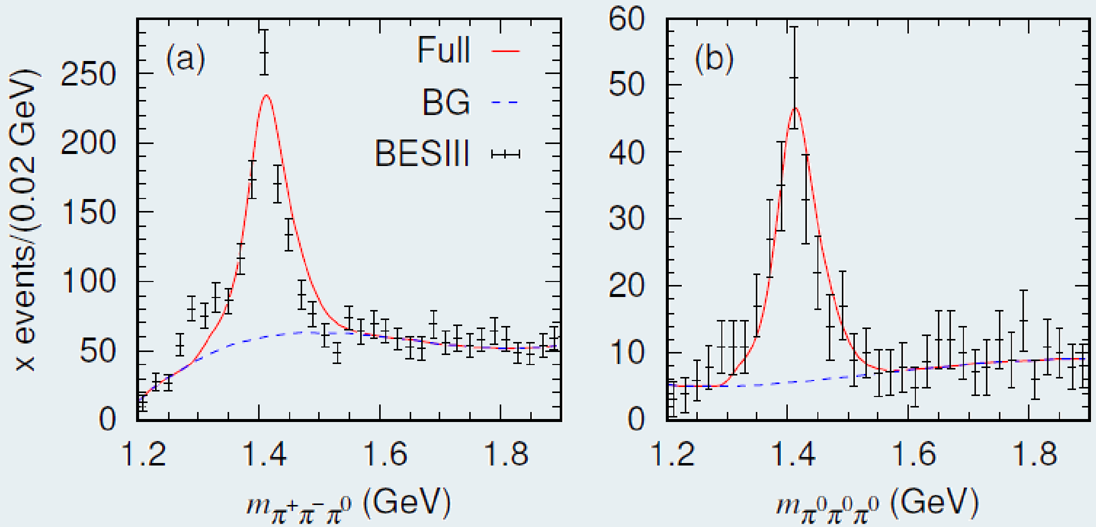


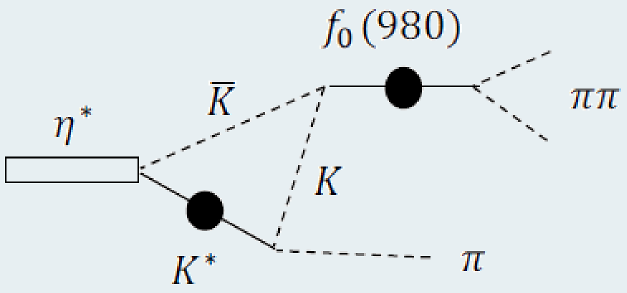


Mainly from $a_0\pi$ and $f_0\eta$ intermediate states

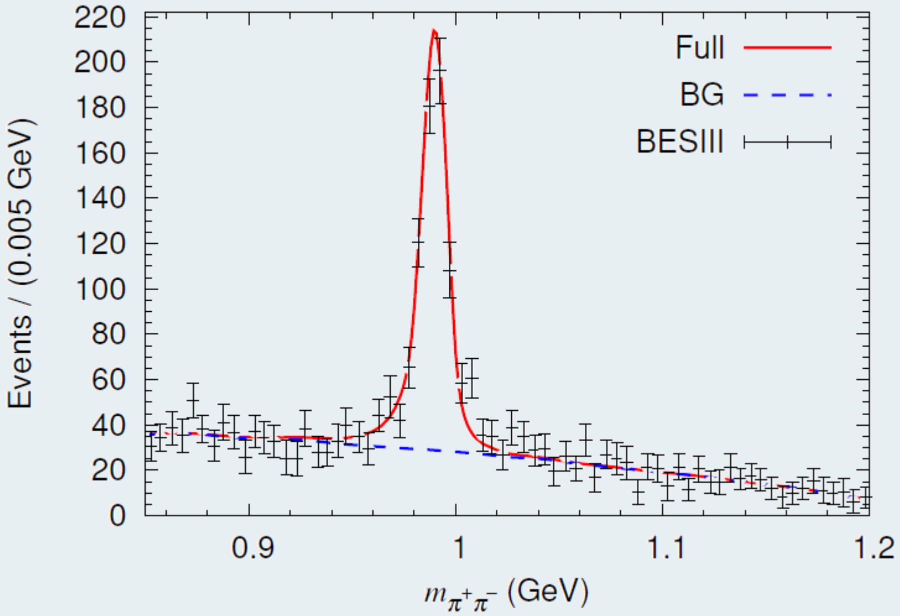
Theoretical analysis

Post predicted the invariant mass spectrum in $\eta(1405/1475) \to \pi\pi\pi$





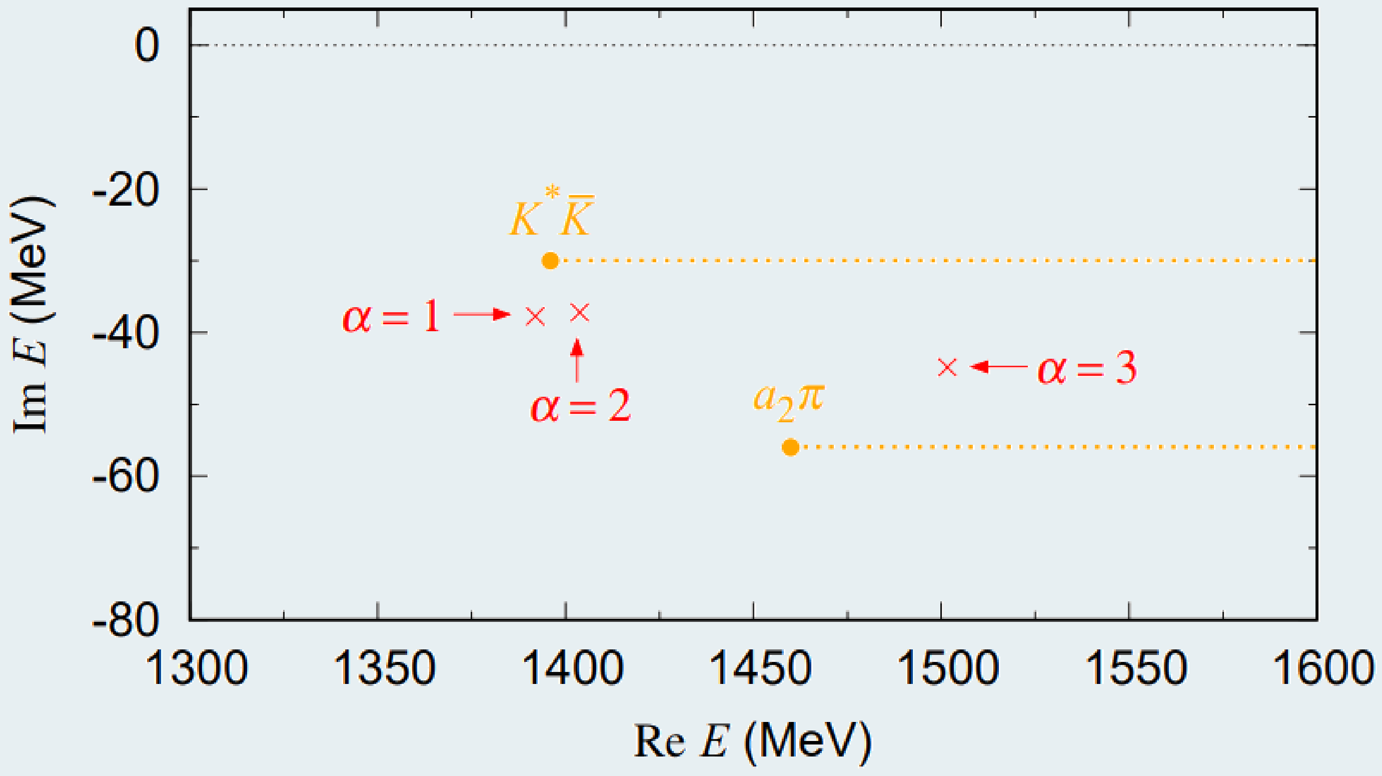
(a)



Mainly come from TS mechanism.

Theoretical analysis

The pole positions of $\eta(1405/1475)$



	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	BESIII [27]	
M	1392	1404	1502	1391.7±0.7	1507.6±1.6
Γ	76	74	90	60.8±1.2	115.8±2.4
RS	(pp)	(up)	(up)		

Bare states are not the physical states. They are just the bound states of quark and gluon, while the physical states are also effected the interaction from other hadronic channel. The masses of bare states will shifted through these interaction then form the corresponding physical states

1.6GeV - bare state here looks like the radial excited states in the $q\bar{q}$ model,

2.0 GeV - bare state would be various explanations, such as gluon ball, hybrid, higher excited states of $q\bar{q}$ state, $qq\overline{qq}$ states ….

Summary and Outlook

Based current experimental MC data and our three-body unitary model analysis

The main two questions of $\eta(1405/1475)$：

One or Two？ Two

What are they？ Until now we have confirmed result, we just find the higher bare state will dropped lower by involving the coupled channel interaction.

We need： More experimental data, such as high statistic data of $J/\psi \to \gamma\eta(1405/1475) \to \gamma\pi\pi\eta$;

And other data of $X \to Y\ \eta(1405/1475) \to Y(K\bar{K}\pi, \pi\pi\eta, \gamma\pi\pi, \pi\pi\pi)$

Thanks very much !