Nucleon self-energy including two-loop contributions

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Contents

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Study the **pion mass dependence of the nucleon mass $m_N(M_\pi)$** by computing the self self-energy Σ

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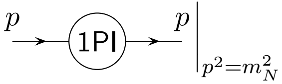

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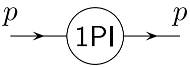

$$\Big|_{p^2=m_N^2}$$

Calculate $\Sigma(p)$ up to $\mathcal{O}(q^6) \rightarrow$ include two-loop diagrams

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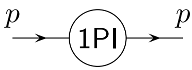

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Calculate $\Sigma(p)$ up to $\mathcal{O}(q^6)$ $\rightarrow$ include two-loop diagrams

1999 HBChPT [McGovern and Birse, Phys. Lett. B 446, 1999]
non-relativ., conserves PC, $\mathcal{O}(q^5)$ ($\mathcal{O}(q^6)$ from below)

$$m_N = m_0 + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln\frac{M}{\mu} + k_8 M^6 \ln^2\frac{M}{\mu} + k_9 M^6$$

with $k_i(m, g_A, F, l_3, l_4, c_{1-4}, d_{16}, d_{18}, e_i, \hat{g}_1)$

2008 IR [Schindler et al., Nuc.Phys.A 803.1, 2008] $\mathcal{O}(q^6)$

2024 **EOMS**: fully covariant, $\mathcal{O}(q^{6+})$ (∞-order $1/m$ for integrals) (based on [Fuchs et al., Phys.Rev.D 68, 056005, 2003])

Chiral perturbation theory

- Quantum chromodynamics (QCD) $\hat{=}$ strong interaction
- Chiral perturbation theory (ChPT) = EFT for low energies

Chiral Lagrangian up to chiral order $\mathcal{O}(q^4)$ in SU(2) [Fettes and Meißner., APhy 283.2, 2000]:

$$\mathcal{L} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

$$\mathcal{L}_\pi^{(2)} = -\frac{1}{2}M^2\vec{\pi}\cdot\vec{\pi} + \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + \frac{8\alpha-1}{8F^2}M^2\left(\vec{\pi}\cdot\vec{\pi}\right)^2 + \cdots + O(\pi^6)$$

$$\mathcal{L}_\pi^{(4)} = -\frac{(l_3+l_4)M^4}{F^2}\left(\vec{\pi}\cdot\vec{\pi}\right) + \cdots + O(\pi^3)$$

$$\mathcal{L}_{\pi N}^{(1)} = -\bar{\Psi}m\Psi + i\bar{\Psi}\partial\!\!\!/\Psi + \frac{g_A}{2F}\bar{\Psi}\gamma_5\vec{\tau}\cdot\partial\!\!\!/\vec{\pi}\Psi + \cdots + O(\pi^5)$$

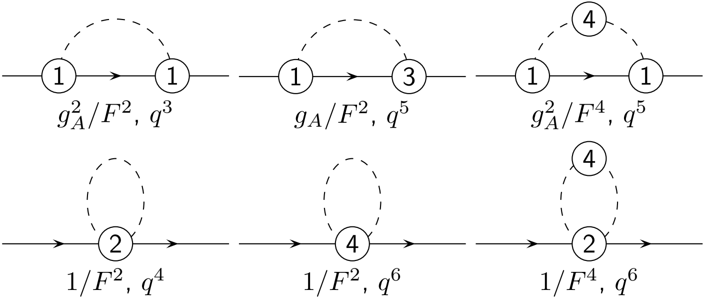
$$l_i \in \mathcal{L}_\pi^{(4)}, \quad c_i \in \mathcal{L}_{\pi N}^{(2)}, \quad d_i \in \mathcal{L}_{\pi N}^{(3)}, \quad e_i \in \mathcal{L}_{\pi N}^{(4)}$$

with $M/m < 1$ ($M = \mathcal{O}(q^1)$, $m = \mathcal{O}(\Lambda)$, $q/\Lambda < 1$)

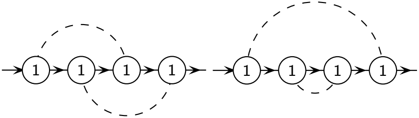
Diagrams – tree level and one-loop

Tree level

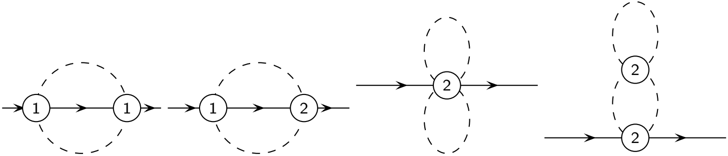
One-loop



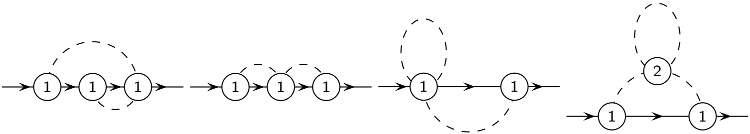
Diagrams – two-loop



$\frac{g_A^4}{F^4}$, q^5



$\frac{c_i^{0,1}}{F^4}$, $q^{5,6}$



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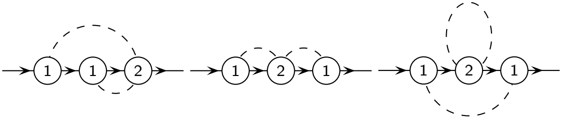

$c_i\,\frac{g_A^2}{F^4}$, q^6

Reduction to master integrals

[Tarasov, NPhyB 502.1, 1996]

Tensor integrals $\longrightarrow$ Scalar integrals $\longrightarrow$ Master integrals (MIs)

TARCER
[Mertig and Scharf, CPhyC 111.1, 1998]

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$\boxed{\text{Tensor integrals}} \longrightarrow \boxed{\text{Scalar integrals}} \xrightarrow[\text{TARCER}]{} \boxed{\text{Master integrals (MIs)}}$

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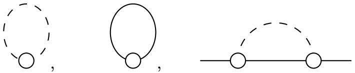
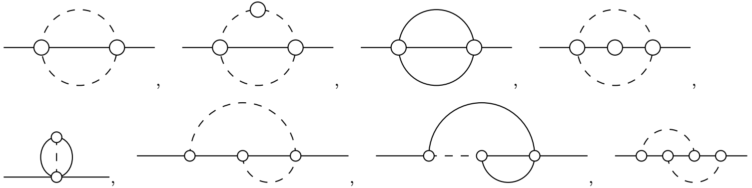
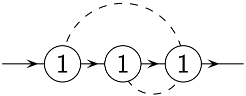
$T_\pi^{(1)}$, $T_N^{(1)}$, $T_{\pi N}^{(1)}$:



$T_{\mathrm{I}}^{(2)}$ - $T_{\mathrm{VIII}}^{(2)}$:



Reduced expression of a diagram



$$
\begin{aligned}
&= -i\Sigma_g^{(2)} = -\frac{ig_A^2\left((2d-3)m^2-(d-2)M^2\right)}{2(3d-4)F^4m}(T_\pi^{(1)})^2 - \frac{3ig_A^2}{8F^4m^3} \\
&\times \left(8m^4 - 3m^2(p^2-m^2) + 4m^3(\not{p}-m) - 3m(\not{p}-m)(p^2-m^2) + 3(p^2-m^2)^2\right)(T_N^{(1)})^2 \\
&+ \frac{ig_A^2\left(4(2d-3)m^2-(d-2)M^2\right)+(\dots)}{2(3d-4)F^4m}T_N^{(1)}T_\pi^{(1)} - \frac{3ig_A^2mM^2+(\dots)}{F^4}T_N^{(1)}T_{\pi N}^{(1)} \\
&- \frac{ig_A^2\left(\left(8d^2-32d+30\right)m^4+\left(-8d^2+33d-32\right)m^2M^2-\left(d^2-5d+6\right)M^4\right)+(\dots)}{(d-2)(3d-4)F^4m}T_{\mathrm{I}}^{(2)} \\
&+ \frac{4ig_A^2M^2\left(m^2-M^2\right)\left((4d-6)m^2+(d-2)M^2\right)+(\dots)}{(d-2)(3d-4)F^4m}T_{\mathrm{II}}^{(2)} \\
&- \frac{ig_A^2m(p^2-m^2)(\dots+\dots)}{2(d-2)(3d-4)F^4}T_{\mathrm{IV}}^{(2)} - \frac{3ig_A^2mM^2+(\dots)}{F^4}T_{\mathrm{V}}^{(2)} - \frac{3ig_A^2mM^4+(\dots)}{F^4}T_{\mathrm{VI}}^{(2)}
\end{aligned}
$$

Renormalization

Extended-on-mass-shell (EOMS) renormalization:
divergent + only power-counting breaking (PCB)

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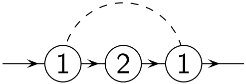
Dressed propagator (with contact terms Σ_c)
$\tilde{m} = m + \Sigma_c = m - 4c_1 M^2 - \hat{e}_1 M^4 + \hat{g}_1 M^6$



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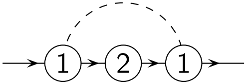
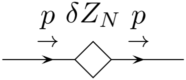


Renormalization corrections:

- One-loop: $\tilde{m}^{\mathsf{Bare}} = m_0 + \Sigma_c^{\mathsf{R}} - \Sigma^{(1),\mathsf{div+PCB}} + \mathcal{O}(\hbar^2)$
- Two-loop: δg_A, δd_i, δc_i, δe_i, δZ_N from [Siemens et al., PhyRevC 94, 014620, 2016] with main changes:
  - $m_N \to m_0 + \Sigma_c^{\mathsf{R}}$
  - chiral limit
- One-loop off-shell:  $\hat{=} \quad i\delta Z_N \left(\not{p} - \tilde{m}^{\mathsf{R}}\right)$
- No two-loop off-shell: $p^2 - \tilde{m}^2 = \mathcal{O}(M^3, \hbar)$

$1/m$ solution of the master integrals

Strategy of regions [Beneke and Smirnov, Nucl.Phys.B 522 321-344, 1998]:
Loop momenta $l_i \sim q$ or $l_i \gg q$ then Taylor series in small scale T_q

$$
\begin{aligned}
T_q\left[\int f(l_1,l_2,q)\mathrm{d}l_1\mathrm{d}l_2\right] &= \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2 \\
&+ \left(\int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2\right)
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$\rightarrow$ purely infrared singular, regular and mixed parts (IRS, IRR, M)

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&\quad + \left(\int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2\right)
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The infrared regular part

- is analytic in M^2 (no M^3 or $\ln(M/m)$)
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$\rightarrow m_N = m_0 + k_1 M^2 + \cdots + k_n M^6$ with k_i(LECs)

Numeric solution of the master integrals

Numerical calculation of the integrals via sector decomposition
SecDec [Binoth and Heinrich, Nucl. Phys. B 585, 741-759, 2000]
pySecDec [Borowka et al., Comput. Phys. Commun. 222, 313-326, 2018]

- Result computed in powers of ε with $d = 4 - 2\varepsilon$
  one-loop: $\frac{1}{\varepsilon}$, ε^0, ε
  two-loop: $\frac{1}{\varepsilon^2}$, $\frac{1}{\varepsilon}$, ε^0
- Double MIs: uncertainty/result $= 5 \times 10^{-5}$ ($\approx 1\,\mathrm{h}$)
- Significant differences to analytics for $T_{\mathrm{III}}^{(2)}$, $T_{\mathrm{VI}}^{(2)}$ and $T_{\mathrm{VIII}}^{(2)}$

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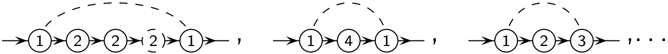
Renormalized result:

- Subtract the (finite) PCB terms
- Leading order removed $\rightarrow$ less relative accuracy

Comparison – $1/m$: EOMS vs. IR

Differences EOMS $\leftrightarrow$ IR:

- Renormalization corrections: different finite parts
- One-loop
  - IR $\hat{=}$ whole IRR part – EOMS $\hat{=}$ only the PCB
  - 

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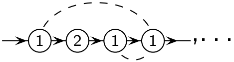
  - 

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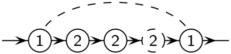
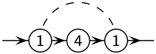
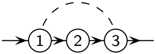
  - 

, 

, 

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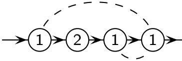
  - 

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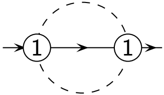
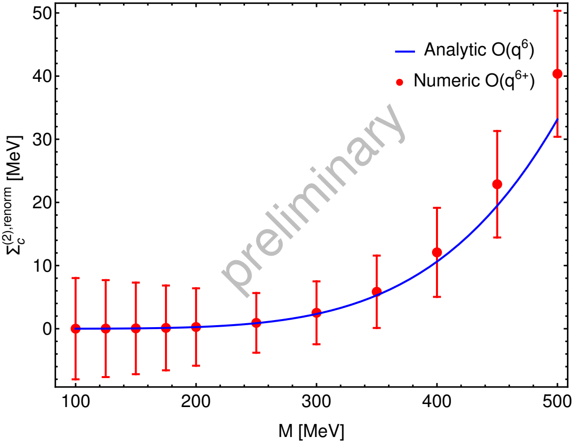
$m_N^{\mathsf{IR}} \to m_N^{\mathsf{EOMS}}$ by

$$x^{\mathsf{R,IR}} = x^{\mathsf{R,EOMS}} + \Delta x(m_0, \mathsf{LECs}), \quad x \in \{d_i, \hat{e}_1, \hat{e}_2, \hat{g}_1\}$$

Comparison – 1/m vs. numeric

$-i\Sigma_c^{(2)} =$ 

$$\Sigma_c^{(2),R} = \frac{5M^6}{2048\pi^4 F^4 m_0} - \frac{3M^6 \ln\left(\frac{M}{m_0}\right)}{512\pi^4 F^4 m_0}$$



Error/result:
single: 5×10^{-9},
double: 5×10^{-5}

Runtimes:
$T_\pi^{(1)}$: max 10 min
$T_I^{(2)}$: max 50 min
$T_{II}^{(2)}$: max 25 min

$\sqrt{p^2} = m_0 = 900$ MeV,
$F = 75$ MeV, $\mu = 1$ MeV

Summary and Outlook

Summary

- Expressed the self-energy in terms of MIs (including IRR)
- Applied strategy of regions for two-loop calculation
- Calculated $m_N(M_\pi)$ as **1/m result**
  - in EOMS
  - and confirmed HB and IR
- Numerical computation of all MIs
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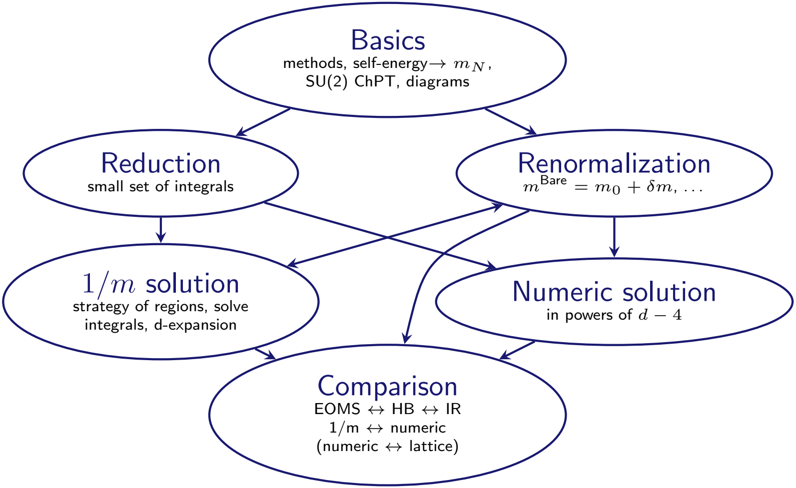
Outlook:

- Derive the full numerical result for m_N
- Comparison with lattice results

Appendix: Overview

$$m_N = m^{\text{Bare}} + \Sigma(p^2 = m_N^2)$$



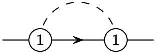
$$\to m_N(M_\pi, m_0, \text{LECs})$$

Appendix: LECs

c_1

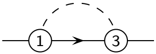
$\hat{e}_1 = 16e_{38}+2e_{115}+2e_{116}$

$\hat{g}_1$



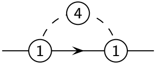
g_A^2/F^2, q^3

F, g_A



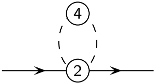
g_A/F^2, q^5

F, g_A, $2d_{16} - d_{18}$



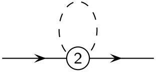
g_A^2/F^4, q^5

F, g_A, l_3, l_4



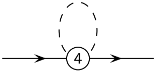
$1/F^4$, q^6

F, c_{1-3}, l_3, l_4



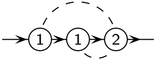
$1/F^2$, q^4

F, c_{1-3}



$1/F^2$, q^6

F, e_{16}, $\hat{e}_2 = 2e_{14} + 2e_{19} - e_{36} - 4e_{38}$,
$\hat{e}_3 = e_{15} + e_{20} + e_{35}$



F, g_A, c_{1-4}