Exploring Light Resonances in Two-Pion Photoproduction: A Regge Formalism Analysis

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With

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Joint Physics Analysis Center

Outline

- Motivation
- Double pion photoproduction:
  - Kinematics
  - CLAS6 Measurements
  - Model description:
    - Resonance Production
    - Non-resonant Production: Deck Mechanism
  - Results
- Conclusions and Further Work

Motivation for Double Pion Photoproduction Study

Understanding the Significance

- Access to Previous CLAS6 Measurements: Building upon prior research.
- Model Challenges: Addressing limitations of earlier models at high momentum transfer.
- Upcoming CLAS12 and GleuX Measurements: Anticipating new data for validation.
- Exploring $\pi^- \eta \Delta^{++}$ Exotic Channel: Expanding the scope of applicability.
- Relevance to Two-Pion Electroproduction: Implications for broader studies.

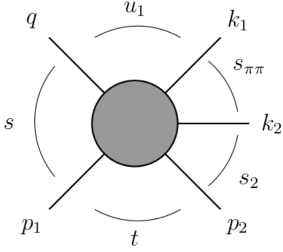
Two-Pion Photoproduction: Kinematics

Process:

$$\gamma(q, \lambda_\gamma) + p(p_1, \lambda_1) \rightarrow \pi^+(k_1) + \pi^-(k_2) + p(p_2, \lambda_2)$$

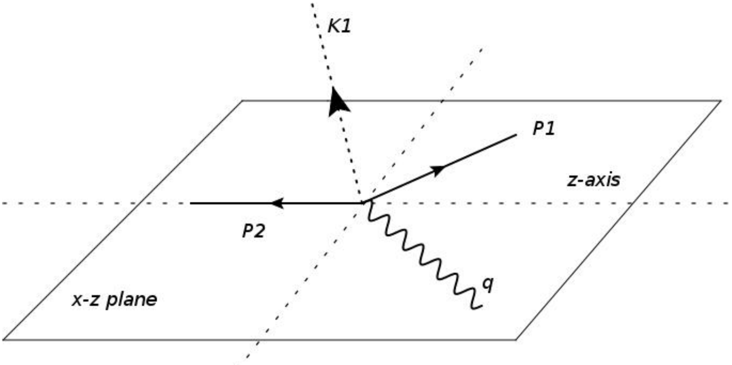
Kinematic Variables:

$$s = (p_1 + q)^2$$
$$s_i = (k_i + p_2)^2$$
$$t = (p_1 - p_2)^2$$
$$s_{12} = s_{\pi\pi} = (k_1 + k_2)^2 = m_{12}^2 = m_{\pi\pi}^2$$



Helicity Frame: $\Omega^H(\theta^H, \phi^H)$

$$\boldsymbol{p_1^H} = |\vec{p_1}|(\sin\theta_1, 0, \cos\theta_1)$$
$$\boldsymbol{p_2^H} = |\vec{p_2}|(0, 0, -1)$$
$$\boldsymbol{q^H} = |\vec{q}|(-\sin\theta_q, 0, \cos\theta_q)$$
$$\boldsymbol{k_1^H} = |\vec{k_1}|(\sin\theta^H\cos\phi^H, \sin\theta^H\sin\phi^H, \cos\theta^H) = -\boldsymbol{k_2^H}$$

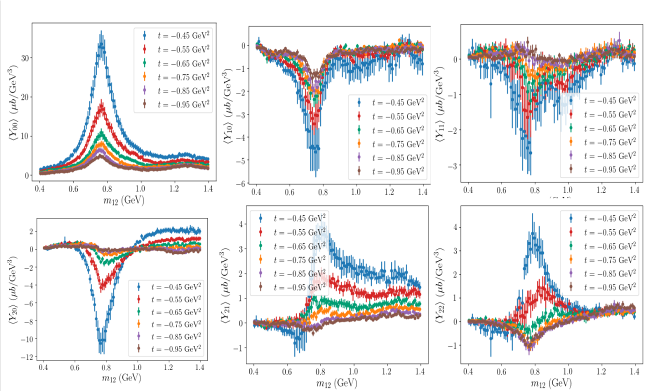


Two-Pion Photoproduction: CLAS6 Measurements

$E_\gamma = [3.0, 3.8]$ GeV, $t = [-0.45, -0.95]$ GeV^2 [Phys.Rev.D 80 (2009) 072005]

$$< Y_{LM} > = \sqrt{4\pi} \int d\Omega^H \frac{d\sigma}{dt dm_{12} d\Omega^H} \text{Re } Y_{LM}(\Omega^H)$$

$$< Y_{00} > = \frac{d\sigma}{dt dm_{12}}$$

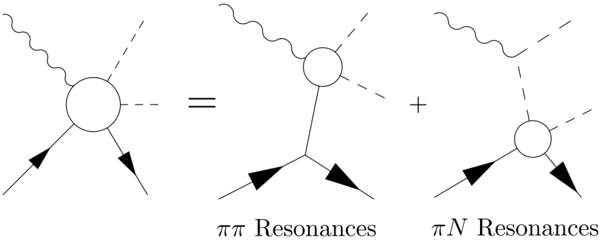


Model Description

For the process $\gamma(q, \lambda_\gamma) + p(p_1, \lambda_1) \to \pi^+(k_1) + \pi^-(k_2) + p(p_2, \lambda_2)$, we consider

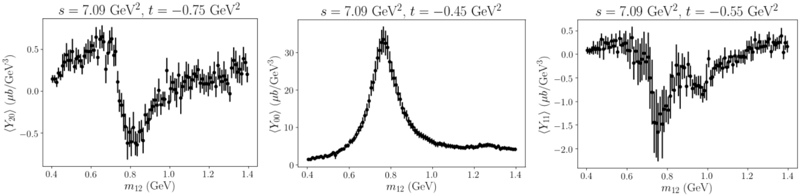
$2 \to 3$ Dynamics

Built from known dynamics in $2 \to 2$ subchannels:



- $\pi\pi$ resonances are directly implemented in our model.
- πN resonances are embedded in the Deck mechanism.

Meson Resonances Below 1 GeV



$f_0(500)$ $\qquad I^G(J^{PC}) = 0^+(0^{++})$

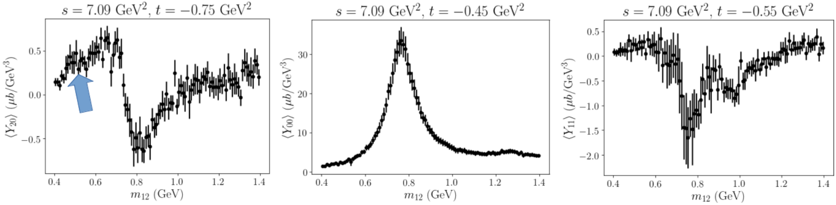
also known as σ; was $f_0(600)$

See the review on "Scalar Mesons below 1 GeV."

Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)−i(200–350) MeV

Mass (Breit-Wigner) = 400 to 800 MeV

Full width (Breit-Wigner) = 100 to 800 MeV



R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022)

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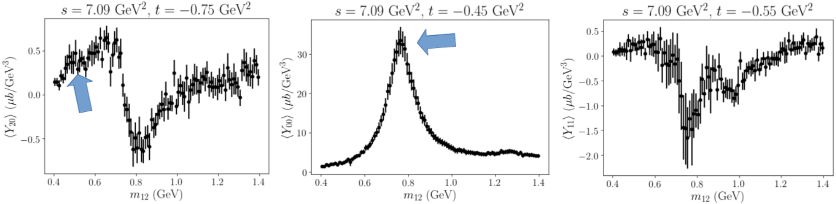

$\rho(770)$ $I^G(J^{PC}) = 1^+(1^{--})$

See the review on "Spectroscopy of Light Meson Resonances."

T-Matrix Pole $\sqrt{s}$ = (761–765) $- i$ (71–74) MeV

Mass (Breit-Wigner) = 775.26 $\pm$ 0.23 MeV

Full width (Breit-Wigner) = 149.1 $\pm$ 0.8 MeV

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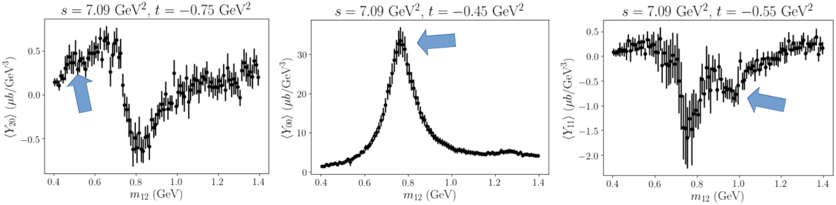
Mass (T-Matrix Pole $\sqrt{s}$) = (400–550)$-i$(200–350) MeV
Mass (Breit-Wigner) = 400 to 800 MeV
Full width (Breit-Wigner) = 100 to 800 MeV

$f_0(980)$ $I^G(J^{PC}) = 0^+(0^{++})$

See the review on "Scalar Mesons below 1 GeV."

T-matrix pole $\sqrt{s}$ = (980–1010) $- i$ (20–35) MeV [h]
Mass (Breit-Wigner) = 990 $\pm$ 20 MeV [h]
Full width (Breit-Wigner) = 10 to 100 MeV [h]



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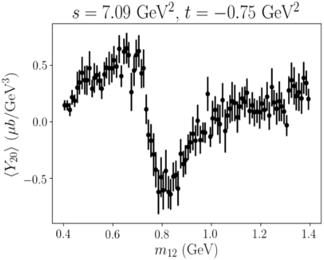
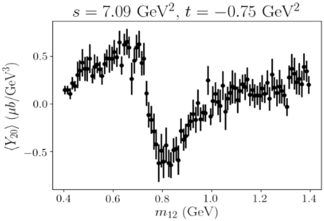




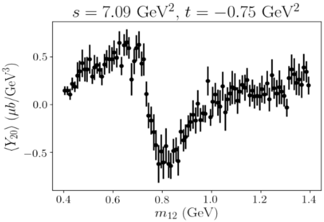
Meson Resonances Above 1 GeV

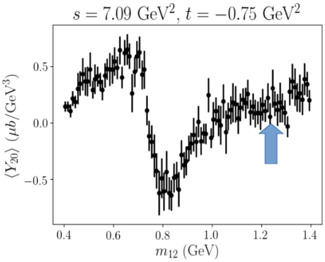
$f_2(1270)$ $\quad I^G(J^{PC}) = 0^+(2^{++})$

Mass (T-Matrix Pole$\sqrt{s}$) = (1260–1283) − i(90–110) MeV

Mass (Breit-Wigner) = 1275.4 ± 0.8 MeV

Full Width (Breit-Wigner) = 186.6 ± 2.3 MeV





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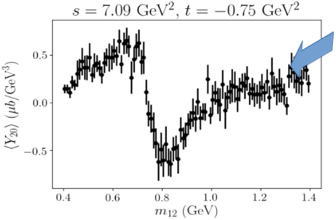
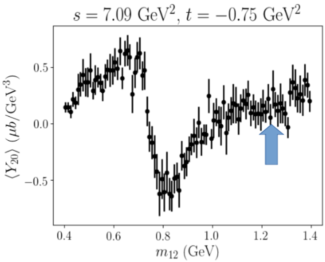




$f_0(1370)$ $I^G(J^{PC}) = 0^+(0^{++})$

See the review on "Spectroscopy of Light Meson Resonances" and a note on "Non-$q\bar{q}$ Candidates" in PDG 06, Journal of Physics **G33** 1 (2006).

Mass (T-Matrix Pole$\sqrt{s}$) = (1250–1440) − i(60–300) MeV

Mass (Breit-Wigner) = 1200 to 1500 MeV

Full Width (Breit-Wigner) = 200 to 500 MeV

R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022)

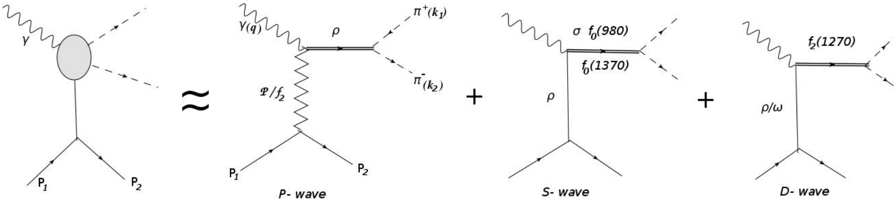
Resonant Production

- Effective Lagrangian: One particle exchange model
- “Reggeize”:

$$R_N(s,t) = \frac{1 + e^{i\pi\alpha_N(t)}}{\sin(\pi\alpha(t))}\left(\frac{s}{s_0}\right)^{\alpha_N(t)}$$

- “Breit-Wignerize”: [Phys. Rev. D 98, 030001 (2018)]

$$\mathrm{BW}^{\mathrm{dep}}(s,l) = \frac{n(s)}{m_{\mathrm{BW}}^2 - s - i m_{\mathrm{BW}}\Gamma_{\mathrm{tot}}(s)}\,, \text{ where } n(s) = \left(\frac{q}{q_0}\right)^l F_l(q,q_0)$$



Partial Waves Amplitudes

$$\mathcal{M}_{\lambda_1,\lambda_2,\lambda_q}(s,t,s_{\pi\pi},\Omega_H) = \sum_{lm} \mathcal{M}^{lm}_{\lambda_1,\lambda_2,\lambda_q}(s,t,s_{\pi\pi},\Omega_H) Y_{lm}(\Omega_H)$$

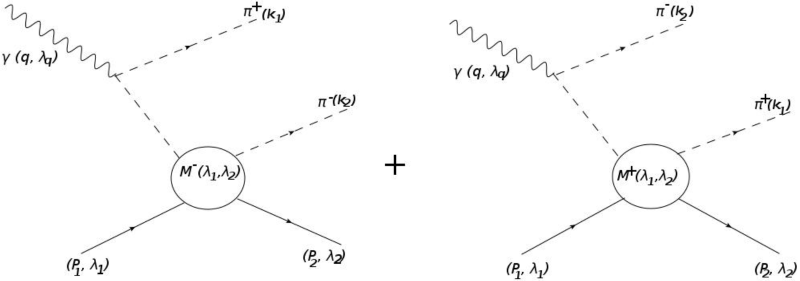
$$\mathcal{M}^{l-m}_{-\lambda_1,-\lambda_2,-\lambda_q} = (-1)^{m-\lambda_2-\lambda_q+\lambda_1} \mathcal{M}^{lm}_{\lambda_1,\lambda_2,\lambda_q}$$

- The model's rigidity hinders its ability to accurately describe the data.
- To address this, we introduce free parameters to redefine the t-dependence g^{lm} in the resonant components, while keeping Deck and ρ production via pomeron components fixed:

$$\tilde{\mathcal{M}}_{\lambda_1,\lambda_2,\lambda_q}(s,t,s_{\pi\pi},\Omega_H) = \sum_{lm} g^{lm} \mathcal{M}^{lm}_{\lambda_1,\lambda_2,\lambda_q}(s,t,s_{\pi\pi},\Omega_H) Y_{lm}(\Omega_H)$$

- In total, we have 30 free parameters: 2 for $f_0(500)$, $f_0(980)$, $f_0(1375)$, and background, 6 for ρ via f_2 and background, and 10 for $f_2(1270)$.

Non Resonant Production: Deck Mechanism

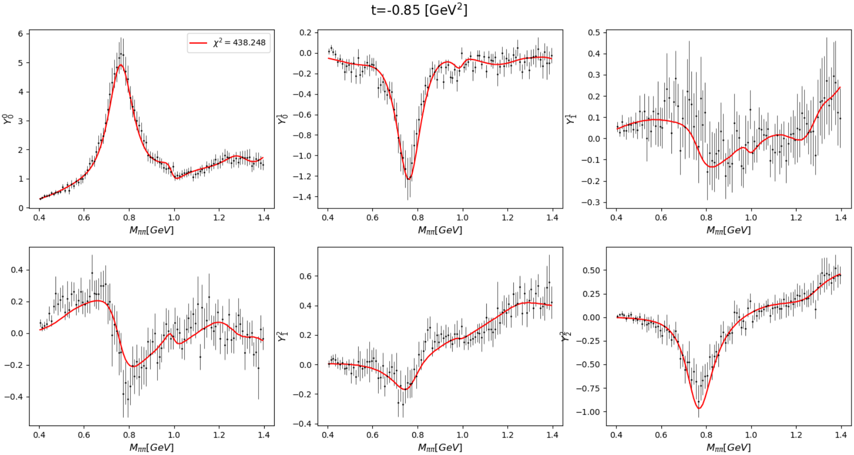


The Deck Mechanism describes non-resonant production with the following equation:

$$\mathcal{M}^{\text{Deck, GI}}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12},\Omega) = e\Big[-\frac{\epsilon(q,\lambda_q)\cdot k_2}{q\cdot k_2}\beta(t_2)\mathcal{M}^{+}_{\lambda_1\lambda_2}(s_1,t;t_2) + \frac{\epsilon(q,\lambda_q)\cdot k_1}{q\cdot k_1}\beta(t_1)\mathcal{M}^{-}_{\lambda_1\lambda_2}(s_2,t;t_1)\Big]$$

Results- Preliminary

E_γ=3.7 GeV:



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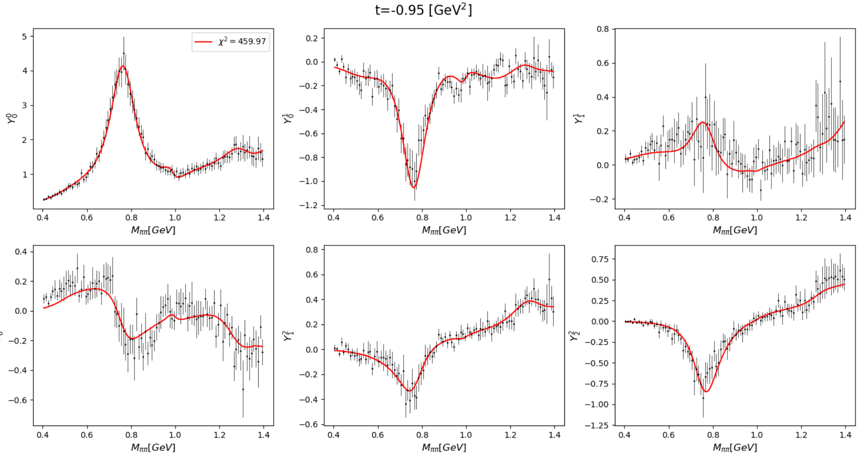


Conclusions and Further Work

- The presented global analysis has demonstrated strong performance.
  ➡ Can we streamline empirical parameterizations for couplings and prune redundant ones?
- Future work includes extending our analysis to predict angular moments at different E_γ values, particularly for the upcoming CLAS12 experiment.
- Investigating angular moments at $m_{\pi N}$ for further insights.
- Expanding the Deck mechanism to encompass other final states, such as $\pi^- \eta \Delta^{++}$, opens new opportunities for investigation.
- Exploring SDMEs for particles like ρ and Δ.
- Potential research in two-pion electroproduction for $F_\pi(Q^2)$ determination.


Thank you

Back-Up: Deck Model

The Deck Model amplitude can be written as:

$$\mathcal{M}^{\text{Deck}}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12},\Omega) = e\Big[-\frac{\epsilon(q,\lambda_q)\cdot k_2}{q\cdot k_2}\beta(t_2)\mathcal{M}^{+}_{\lambda_1\lambda_2}(s_1,t;t_2) + \frac{\epsilon(q,\lambda_q)\cdot k_1}{q\cdot k_1}\beta(t_1)\mathcal{M}^{-}_{\lambda_1\lambda_2}(s_2,t;t_1)\Big]. \quad (1)$$

Where $\beta(t_i) = \exp((t_\pi - t_i^{\text{min}})/\Lambda_\pi^2)$, $\Lambda_\pi = 0.9$ GeV, $t_\pi = (q-k_i)^2$ and

$$t_1^{\text{min}} = m_\pi^2 - \frac{1}{2s}\Big[(s-m_p^2)(s-s_2+m_\pi^2) - \lambda^{1/2}(s,0,m_p^2))\lambda^{1/2}(s,s_2,m_\pi^2)\Big], \quad (2)$$

$$t_2^{\text{min}} = m_\pi^2 - \frac{1}{2s}\Big[(s-m_p^2)(s-s_1+m_\pi^2) - \lambda^{1/2}(s,0,m_p^2)\lambda^{1/2}(s,s_1,m_\pi^2)\Big] \quad (3)$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$.

Pion-proton Scattering:

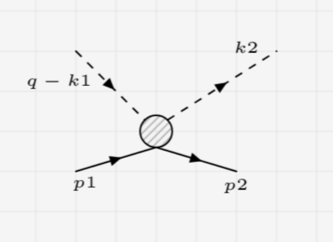


Figure: Feynman diagram for $\pi^- p \to \pi^- p$

Assuming that the intermediate pion is offshell, then the pion-proton scattering amplitude will read:

$$\mathcal{M}_\lambda^- = \bar{u}_\lambda(p_2)\left[A^-(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q-k_1+k_2)^\mu B^-(s,t,t_\pi)\right]u_\lambda(p_1), \quad (4)$$

where $t_\pi = (q-k_1)$

Pion-proton Scattering:

Similarly for the positive exchanged pion:

$$\mathcal{M}^{+}_{\lambda} = \bar{u}_{\lambda}(p_2)\left[A^{+}(s,t,t_{\pi}) + \frac{1}{2}\gamma_{\mu}(q-k_2+k_1)^{\mu}B^{+}(s,t,t_{\pi})\right]u_{\lambda}(p_1), \quad (5)$$

where $t_{\pi} = (q-k_2)$.
In the πN center of mass frame the t-channel A and B defined as follows:

$$\frac{1}{4\pi}A^{\pm} = \frac{\sqrt{s}+m_p}{Z_1^{+}Z_2^{+}}f_1^{\pm} - \frac{\sqrt{s}-m_p}{Z_1^{-}Z_2^{-}}f_2^{\pm}, \quad (6)$$

$$\frac{1}{4\pi}B^{\pm} = \frac{1}{Z_1^{+}Z_2^{+}}f_1^{\pm} - \frac{1}{Z_1^{-}Z_2^{-}}f_2^{\pm}. \quad (7)$$

Where f_1 and f_2 are called the reduced helicity amplitudes, $Z_i^{\pm} = \sqrt{E_i \pm m_p}$.

Pion-proton scattering

The partial wave decomposition:

$$f_1 = \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=0}^{\infty} f_{l+}(s) P'_{l+1}(\cos\theta) - \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=2}^{\infty} f_{l-}(s) P'_{l-1}(\cos\theta), \tag{8}$$

$$f_2 = \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P'_l(\cos\theta). \tag{9}$$

The scalar functions A and B in our case depends on the pion virtuality, in which it enters in Z_1 and hence P_1 since

$$E_1 = \frac{s_i - t_\pi + m_p^2}{2\sqrt{s_i}} \tag{10}$$

Also our scattering angle is proportional to the virtuality as follows:

$$\cos\theta = \frac{2s_i(t - 2m_p^2) + (s_i - t_\pi + m_p^2)(s_i - m_\pi^2 + m_p^2)}{\sqrt{\lambda(s_i, t_\pi, m_p^2)}\,\sqrt{\lambda(s_i, m_\pi^2, m_p^2)}}. \tag{11}$$

Differential Cross Section

The differential cross section is given as:

$$\frac{d^5\sigma}{dtd\sqrt{s_{12}}d\Omega d\Phi} = I(\Omega,\Phi) = I^0(\Omega) + \boldsymbol{I}(\Omega)\cdot P_\gamma(\Phi) \qquad (12)$$

where $\Omega = (\theta,\phi)$ and the intensity vector is defined as:

$$I^0(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2}\mathcal{M}_{\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \qquad (13)$$

$$I^1(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2}\mathcal{M}_{-\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \qquad (14)$$

$$I^2(\Omega) = i\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2}\lambda\mathcal{M}_{-\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \qquad (15)$$

where $0 < P_\gamma < 1$ is the degree of linear polarization of the photon beam.

Model Parameters

Model parameters not fit from data:

$$m_\sigma = 0.5 \text{ GeV}$$
$$\Gamma_\sigma = 0.5 \text{ GeV}$$
$$m_\rho = 0.775 \text{ GeV}$$
$$\Gamma_\rho = 0.149 \text{ GeV}$$
$$m_{f_0} = 0.99 \text{ GeV}$$
$$\Gamma_{f_0} = 0.2 \text{ GeV}$$

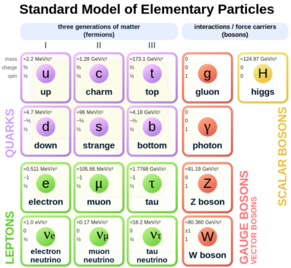
$$\alpha_0^{\mathbb{P}} = 1.08$$
$$\alpha_1^{\mathbb{P}} = 0.2 \text{ GeV}^{-2}$$
$$\alpha_0^{f_2} = 0.5$$
$$\alpha_1^{f_2} = 0.9 \text{ GeV}^{-2}$$
$$\alpha_0^{\rho} = 0.55$$
$$\alpha_1^{\rho} = 0.8 \text{ GeV}^{-2}$$

Angular Moments and Partial Waves

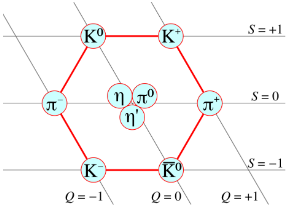
$$
\begin{aligned}
\langle Y_{00}\rangle &= |S|^2 + |P_-|^2 + |P_0|^2 + |P_+|^2 + |D_-|^2 + |D_0|^2 + |D_+|^2 + |F_-|^2 + |F_0|^2 + |F_+|^2 \\
\langle Y_{10}\rangle &= SP_0^* + P_0S^* + \sqrt{\frac{3}{5}}\left(P_-D_-^* + P_-^*D_- + P_+D_+^* + D_+P_+^*\right) + \sqrt{\frac{4}{5}}\left(P_0D_0^* + D_0P_0^*\right) \\
&+ \sqrt{\frac{24}{35}}\left(D_-F_-^* + F_-D_-^* + D_+F_+^* + F_+D_+^*\right) + \sqrt{\frac{216}{280}}\left(D_0F_0^* + F_0D_0^*\right) \\
\langle Y_{11}\rangle &= \left(-P_-S^* - SP_-^* + P_+S^* + SP_+^*\right) + \sqrt{\frac{1}{20}}\left(P_-D_0^* + D_0P_-^* - P_+D_0^* - D_0P_+^*\right) \\
&+ \sqrt{\frac{3}{20}}\left(-P_0D_-^* - D_-P_0^* + P_0D_+^* + D_+P_0^*\right) \\
&+ \sqrt{\frac{9}{140}}\left(D_-F_0^* + F_0D_-^* - D_+F_0^* - F_0D_+^*\right) \\
&+ \sqrt{\frac{9}{70}}\left(-D_0F_-^* - F_-D_0^* + D_0F_+^* + F_+D_0^*\right)
\end{aligned}
$$

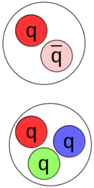
Angular Moments and Partial Waves

$$
\begin{aligned}
\langle Y_{20}\rangle &= SD_0^* + D_0S^* + \sqrt{\frac{1}{5}}\left(2|P_0|^2 - |P_-|^2 - |P_+|^2 + |F_-|^2 + |F_+|^2\right)\\
&+ \sqrt{\frac{18}{35}}\left(P_-F_-^* + F_-P_-^* + P_+F_+^* + F_+P_+^*\right)\\
&+ \sqrt{\frac{27}{35}}\left(P_0F_0^* + F_0P_0^*\right) + \sqrt{\frac{5}{49}}\left(|D_-|^2 + |D_+|^2\right) + \sqrt{\frac{20}{49}}|D_0|^2 + \sqrt{\frac{16}{45}}|F_0|^2\\
\langle Y_{21}\rangle &= \frac{1}{2}\left(SD_+^* + D_+S^* - SD_-^* - D_-S^*\right)\\
&+ \sqrt{\frac{3}{20}}\left(P_0P_+^* + P_+P_0^* - P_-P_0^* - P_0P_-^*\right)\\
&+ \sqrt{\frac{9}{140}}\left(P_-F_0^* + F_0P_-^* - P_+F_0^* - F_0P_+^*\right)\\
&+ \sqrt{\frac{6}{35}}\left(P_0F_+^* + F_+P_0^* - P_0F_-^* - F_-P_0^*\right)\\
&+ \sqrt{\frac{5}{196}}\left(D_0D_+^* + D_+D_0^* - D_0D_-^* - D_-D_0^*\right)\\
&+ \sqrt{\frac{1}{90}}\left(F_0F_+^* + F_+F_0^* - F_0F_-^* - F_-F_0^*\right)\\
\langle Y_{22}\rangle &= \sqrt{\frac{3}{10}}\left(P_-P_+^* + P_+P_-^*\right) + \sqrt{\frac{3}{140}}\left(P_-F_+^* + F_+P_-^* + P_+F_-^* + F_-P_+^*\right)\\
&+ \sqrt{\frac{4}{30}}\left(-F_+F_-^* - F_-F_+^*\right) + \sqrt{\frac{3}{196}}\left(-D_-D_+^* - D_+D_-^*\right)
\end{aligned}
$$

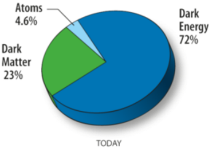


Standard (Quark) Model





Beyond the Standard Model



Beyond the Standard (Quark) Model