Low-energy constants in the chiral Lagrangian with baryon fields from Lattice QCD data

Matthias F.M. Lutz

GSI Helmholtzzentrum für Schwerionenforschung GmbH

- **Large-N_c and chiral SU(3) expansions in QCD**
- **Chiral extrapolation for baryon masses**
- **Pion-nucleon sigma term from Lattice QCD data**
- **Summary and outlook**

arXiv:1111.0238, arXiv:1401.7805, arXiv:1801.06417, arXiv:1907.00714, arXiv:2301.06387

The chiral Lagrangian with baryon fields

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\,\eta \end{pmatrix}$$

Goldstone boson octet $(J^P = 0^-)$

baryon octet $(J^P = \frac{1}{2}^+)$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\,\Lambda \end{pmatrix}$$

✔ Leading order terms

covariant derivative $\partial_\mu = \partial_\mu + \cdots$

$$\begin{aligned} \mathcal{L} &= \operatorname{tr}\left\{\bar{B}(i\,\partial\cdot\gamma - M_{[8]})\,B\right\} + F\operatorname{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,[i\,U_\mu, B]\right\} + D\operatorname{tr}\left\{\bar{B}\,\gamma^\mu\gamma_5\,\{i\,U_\mu,\,B\}\right\} \\ &- \operatorname{tr}\left\{\bar{B}_\mu\cdot\left((i\,\partial\cdot\gamma - M_{[10]})\,g^{\mu\nu} - i\,(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu) + \gamma^\mu(i\,\partial\cdot\gamma + M_{[10]})\gamma^\nu\right)B_\nu\right\} \\ &+ C\left(\operatorname{tr}\left\{(\bar{B}_\mu\cdot i\,U^\mu)\,B\right\} + \text{h.c.}\right) + H\operatorname{tr}\left\{(\bar{B}^\mu\cdot\gamma_\nu\gamma_5\,B_\mu)\,i\,U^\nu\right\} \end{aligned}$$

- $U_\mu = \frac{1}{2}\,u^\dagger\left(\partial_\mu\,e^{i\,\frac{\Phi}{f}}\right)u^\dagger - \frac{i}{2}\,u^\dagger\,(v_\mu + a_\mu)\,u + \frac{i}{2}\,u\,(v_\mu - a_\mu)\,u^\dagger$ with $u = e^{i\,\frac{\Phi}{2\,f}}$
- from $B \to B' + e + \bar{\nu}_e$: $F \simeq 0.45$ and $D \simeq 0.80$
- from large-N_c: $H = 9\,F - 3\,D$ and $C = 2\,D$

Chiral symmetry breaking terms

$$\mathcal{L}_\chi^{(2)} = 2\,b_0\,\mathrm{tr}\left(\bar{B}\,B\right)\mathrm{tr}\left(\chi_+\right) + 2\,b_D\,\mathrm{tr}\left(\bar{B}\left\{\chi_+,\,B\right\}\right) + 2\,b_F\,\mathrm{tr}\left(\bar{B}\left[\chi_+,\,B\right]\right)$$
$$- \; 2\,d_0\,\mathrm{tr}\left(\bar{B}_\mu\cdot B^\mu\right)\mathrm{tr}\left(\chi_+\right) - 2\,d_D\,\mathrm{tr}\left(\left(\bar{B}_\mu\cdot B^\mu\right)\chi_+\right)$$

$$\chi_+ = \chi_0 - \frac{1}{8\,f^2}\left\{\Phi,\left\{\Phi,\,\chi_0\right\}\right\} + \mathcal{O}\left(\Phi^4\right)$$

$$\text{quark} - \text{mass matrix} \qquad \chi_0 \sim \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

✔ Relevance of low-energy parameters

- quark-mass dependence of the baryon masses $\leftrightarrow$ lattice QCD
- meson-baryon scattering $\leftrightarrow$ resonances in QCD
- nucleon sigma terms, $\langle N|\bar{u}\,u|N\rangle$, $\langle N|\bar{d}\,d|N\rangle$ and $\langle N|\bar{s}\,s|N\rangle$ relevant in WIMP scenarios – ATLAS

see e.g. arXiv:1805.09795

Quark-masses from Lattice QCD ensembles

$$
\begin{aligned}
m_\pi^2 &= 2\,B_0\,m - \frac{1}{18\,f^2}\Big\{-10\,m_\pi^2 + 4\,m_K^2 - 3\,m_\eta^2\Big\}\,\bar I_\pi - \frac{1}{6\,f^2}\,m_\pi^2\,\bar I_\eta \\
&\quad + \frac{8}{f^2}\,m_\pi^2\,(m_\pi^2 + 2\,m_K^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_\pi^4\,(2\,L_8 - L_5)\,, \\
m_K^2 &= B_0\,(m + m_s) - \frac{1}{6\,f^2}\Big\{m_\pi^2 - 4\,m_K^2 + 3\,m_\eta^2\Big\}\,\bar I_K + \frac{1}{3\,f^2}\,m_K^2\,\bar I_\eta \\
&\quad + \frac{12}{f^2}\,m_K^2\,(m_\pi^2 + m_\eta^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_K^4\,(2\,L_8 - L_5)\,, \\
m_\eta^2 &= \frac{2}{3}\,B_0\,(m + 2\,m_s) - \frac{1}{2\,f^2}\,m_\pi^2\,\bar I_\pi - \frac{1}{6\,f^2}\Big\{7\,m_\eta^2 - 4\,m_K^2\Big\}\bar I_\eta + \frac{4}{3\,f^2}\,m_K^2\,\bar I_K \\
&\quad + \frac{24}{f^2}\,m_\eta^2\,(2\,m_K^2 - m_\eta^2)\,(2\,L_6 - L_4) + \frac{8}{f^2}\,m_\eta^4\,(2\,L_8 - L_5) \\
&\quad + \frac{16}{5\,f^2}\,(3\,m_\pi^4 - 8\,m_K^4 - 8\,m_\eta^2\,m_K^2 + 13\,m_\eta^4)\,(3\,L_7 + L_8)\,,
\end{aligned}
$$

$$
\bar I_Q = \frac{m_Q^2}{(4\,\pi)^2}\,\log\left(\frac{m_Q^2}{\mu^2}\right) + \text{finite} - \text{box corrections}
$$

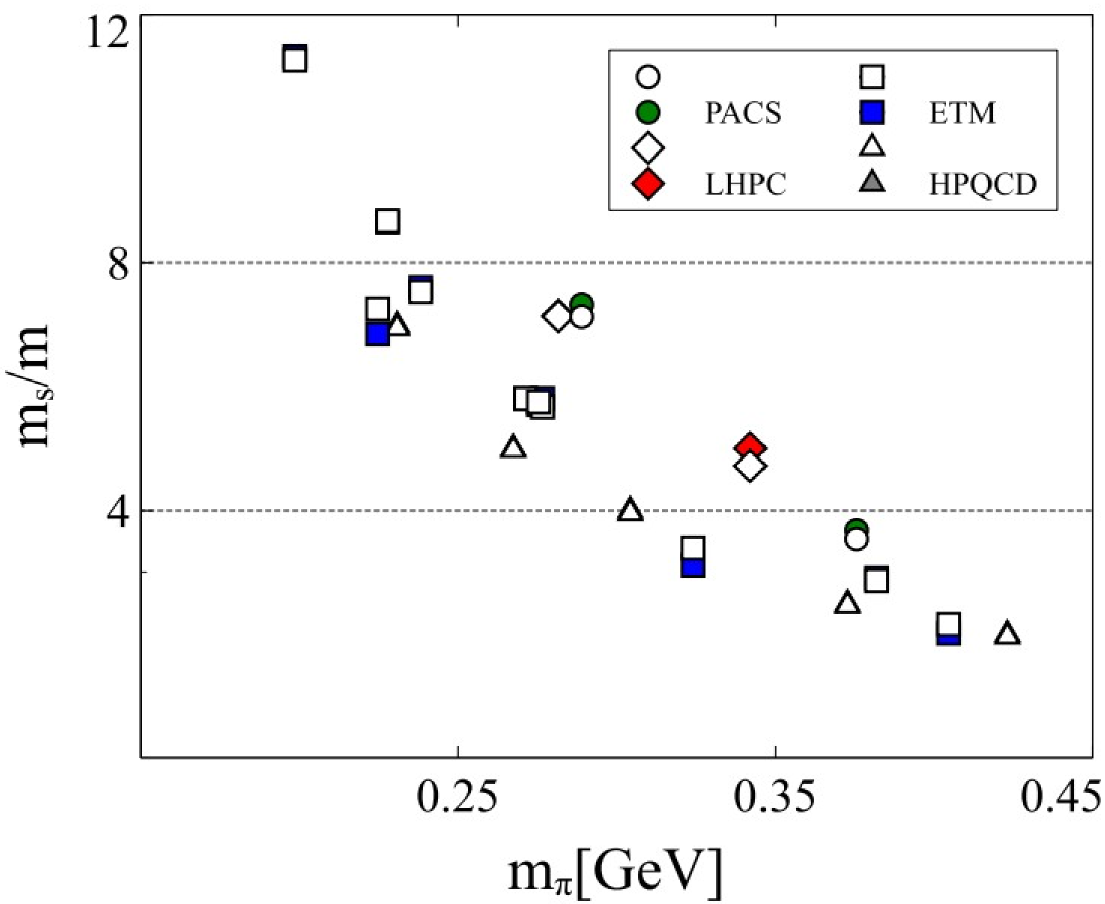
✔ Use on-shell meson masses

- for given pion and kaon mass determine $m = m_u = m_d$ and m_s
- such an analysis determines Gasser and Leutwyler LEC

Predictions for quark-mass ratios on lattice ensembles

✔ How to fit the lattice data?

- take pion and kaon mass of the ensemble → compute quark masses
- this requires the low-energy constants $L_4 - 2\,L_6, L_5 - 2\,L_8, L_8 + 3\,L_7$
- we do not fit to the quark-mass ratios given by the lattice groups!



✔ A fit to the D meson masses

- renormalization scale $\mu = 0.77$ GeV

$10^3\,(L_4 - 2\,L_6)$	-0.1575
$10^3\,(L_5 - 2\,L_8)$	-0.0370
$10^3\,(L_8 + 3\,L_7)$	-0.5207
m_s/m	26.600

- at physical quark masses our ratio compares well with lattice result

  $m_s/m = 26.66(32)$ from ETMC

  in Nucl. Phys. B887, 19 (2014)

arXiv:1801.10122

Predictions for quark-mass ratios on lattice ensembles

✔ How to fit the lattice data?

- take pion and kaon mass of the ensemble → compute quark masses
- this requires the low-energy constants $L_4 - 2\,L_6, L_5 - 2\,L_8, L_8 + 3\,L_7$
- we do not fit to the quark-mass ratios given by the lattice groups!

✔ A fit to baryon masses

- renormalization scale $\mu = 0.77$ GeV

	Fit (from CLS ensembles)	Fit (from arXiv:1907.00714)
$10^3\,(2\,L_6 - L_4)$	0.0411(3)	$0.0401^{(24)}_{(01)}$
$10^3\,(2\,L_8 - L_5)$	0.0826(12)	$0.1049^{(43)}_{(43)}$
$10^3\,(L_8 + 3\,L_7)$	-0.4768(4)	$-0.4818^{(09)}_{(13)}$
m_s/m	26.15(1)	$26.02^{(02)}_{(02)}$

- tiny statistical error from global fits
- systematic uncertainties drive the error

Quark-mass dependence of the baryon masses

✔ A challenge

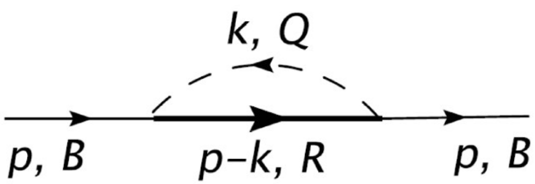
- 'poor' convergence in the heavy-baryon formulation of χPT

$$\text{e.g.} \qquad M_\Xi = (1018 + 1311 - 1007)\,\text{MeV} = 1322\,\text{MeV}$$

- conventional χPT inconsistent with three-flavor QCD lattice simulations?

✔ Multi-scale problem: how to powercount?

$$\frac{m_\pi}{M_N} \sim \frac{m_\pi}{M_\Delta} \sim \frac{m_\pi}{4\pi\, f} \sim Q$$



- insist on flavour SU(3) symmetric counting

$$\frac{m_Q}{M_B} \sim Q \quad \text{with} \quad Q \in [8] \qquad \text{and} \qquad B \in [8], [10]$$

- how to count mass differences?

$$M_R - M_B \sim Q \quad \text{with} \quad B \in [8] \quad \text{and} \quad R \in [10] \qquad \text{else} \qquad \sim Q^2$$

Quark-mass dependence of the baryon masses

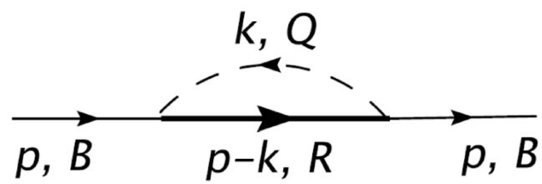
✔ A challenge

- ’poor’ convergence in the heavy-baryon formulation of χPT

$$\text{e.g.} \qquad M_\Xi = (1018 + 1311 - 1007)\,\text{MeV} = 1322\,\text{MeV}$$

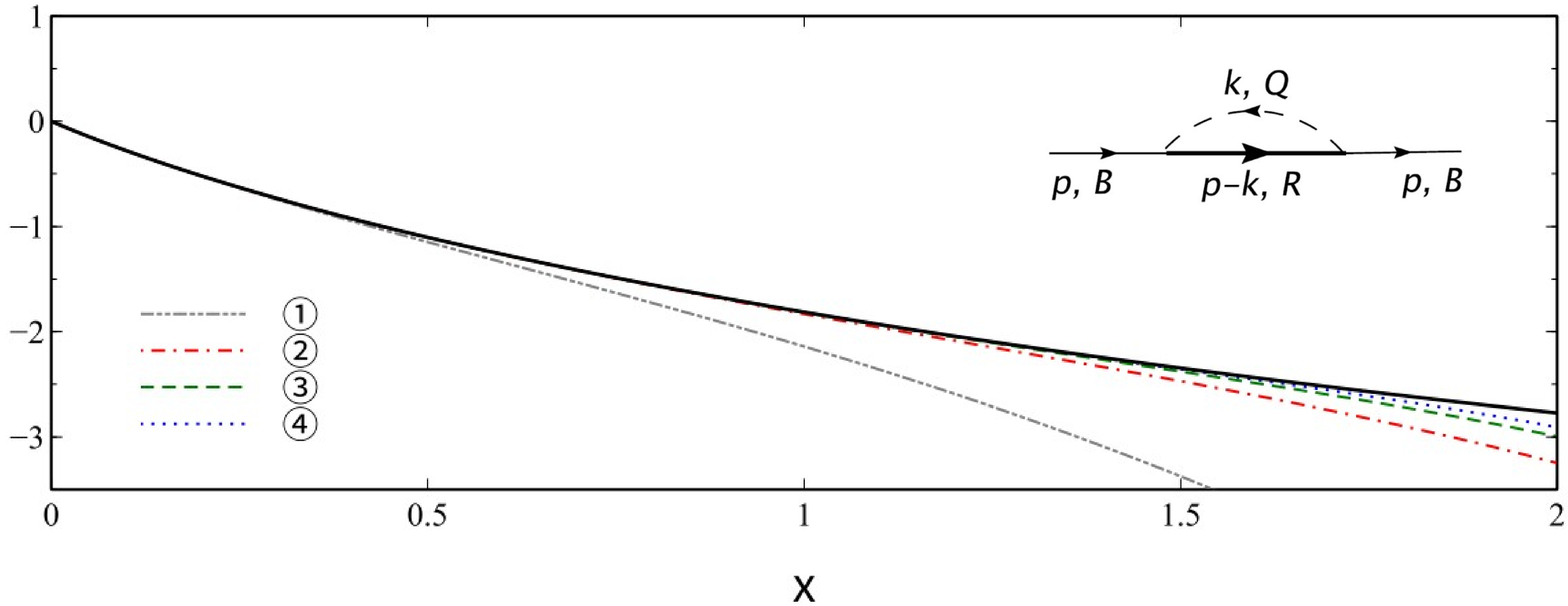
- conventional χPT inconsistent with three-flavor QCD lattice simulations?

✔ One-loop depends sensitively on internal masses

$$\Sigma_B(p) = \sum_{Q\in[8]} \sum_{R\in[8],[10]}$$  $$+\cdots$$

- chiral expansion in terms of physical meson and baryon masses
- reorganize conventional χPT keeping its model independence
- renormalization scale and reparametrization invariance

Chiral expansion of the scalar loop function $(4\pi)^2 \bar{I}_{QR}$



✔ **Convergence study for $M_R = M_B$ and $x = m_Q/M_R$**

- $(4\pi)^2 \, \bar{I}_{QR} = -\pi \sqrt{x^2} \, f_1(x^2) + x^2 \, f_2(x^2) - \frac{1}{2} \, x^2 \, f_3(x^2) \, \log x^2$
- the functions $f_n(x^2)$ are analytic in x^2 for $|x| < 2$

  $f_n(x^2) = 1 + \# x^2 + \# x^4 + \cdots$
- good convergence even for $m_K = M_N$ with $x \simeq 1$!

arXiv:1801.06417

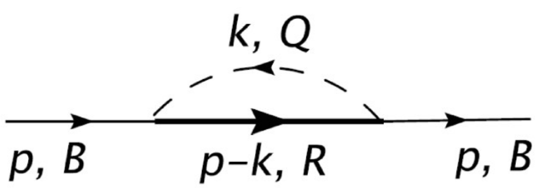
Quark-mass dependence of the baryon masses

✔ Good convergence of reordered chiral expansion

- use physical meson and baryon masses
- the full one-loop contributions can be decomposed into chiral moments
- taking empirical masses the N^4LO effects are less than 8 MeV

✔ Baryon masses determined by a non-linear system

$$M_B - \Sigma_B(M_B) = \begin{cases} M_{[8]} & \text{for} \quad B \in [8] \\ M_{[10]} & \text{for} \quad B \in [10] \end{cases}$$

$$\Sigma_B(p) = \sum_{Q\in[8]} \sum_{R\in[8],[10]}$$ 

$$+ \cdots$$

- numerical challenge

Lattice QCD for baryon octet and decuplet masses

✔ PACS-CS, HSC, LHPC, NPLQCD, QCDSF-UKQCD

- distinct Lattice actions, unphysical quark-masses, various lattice volumes
- despite that - global fits to such data were quite successful
- e.g. prediction of baryon masses on ETMC ensembles
- continum limit posses a challenge

✔ Results on CLS ensembles from Regensburg

arXiv:2211.03744

- large set of ensembles at different β values, quark masses and volumes
- ensembles at fixed m_s or $m_u = m_d = m_s$ or $m_u + m_d + m_s$ :: crucial for chiral SU(3)
- there are about 400 data points with $m_\pi, m_K < 550$ MeV
- a significant continuum limit extrapolation appears feasible

✔ Ensembles with physical pion masses: challenges

- excited state contamination of exponential signals?
- infinite volume extrapolation?
- from an EFT point of view unphysical quark-masses are more interesting!

Low-energy parameters from lattice QCD simulations

✔ A global fit to baryon masses on CLS/Regensburg ensembles

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- finite volume effects from chiral one-loop contributions are considered
- leading and subleading LEC have a quadratice lattice scale dependence

$$M_{[8]} \to M_{[8]} + a^2\,\gamma_{M_8}\,, \qquad b_0 \to b_0 + a^2\,\gamma_{b_0}\,, \quad b_D \to b_F + a^2\,\gamma_{b_D}\,, \quad b_F \to b_F + a^2\,\gamma_{b_F}\,,$$
$$M_{[10]} \to M_{[10]} + a^2\,\gamma_{M_{10}}\,, \qquad d_0 \to d_0 + a^2\,\gamma_{d_0}\,, \quad d_D \to d_D + a^2\,\gamma_{d_D}\,,$$

- global scale-setting with baryon octet and decplet masses
  physical baryon octet and decuplet masses are always reproduced
- acurracy level : self-consistent one-loop at $\mathrm{N^3LO}$
- assume an ad-hoc size for the systematic error of about 10-15 MeV

✔ Sum rules from QCD in the limit of large N_c

- significant parameter reduction
- e.g. only five symmetry conserving Q^2 parameters relevant at large-N_c
- all together we adjust 24 LEC to the lattice data set

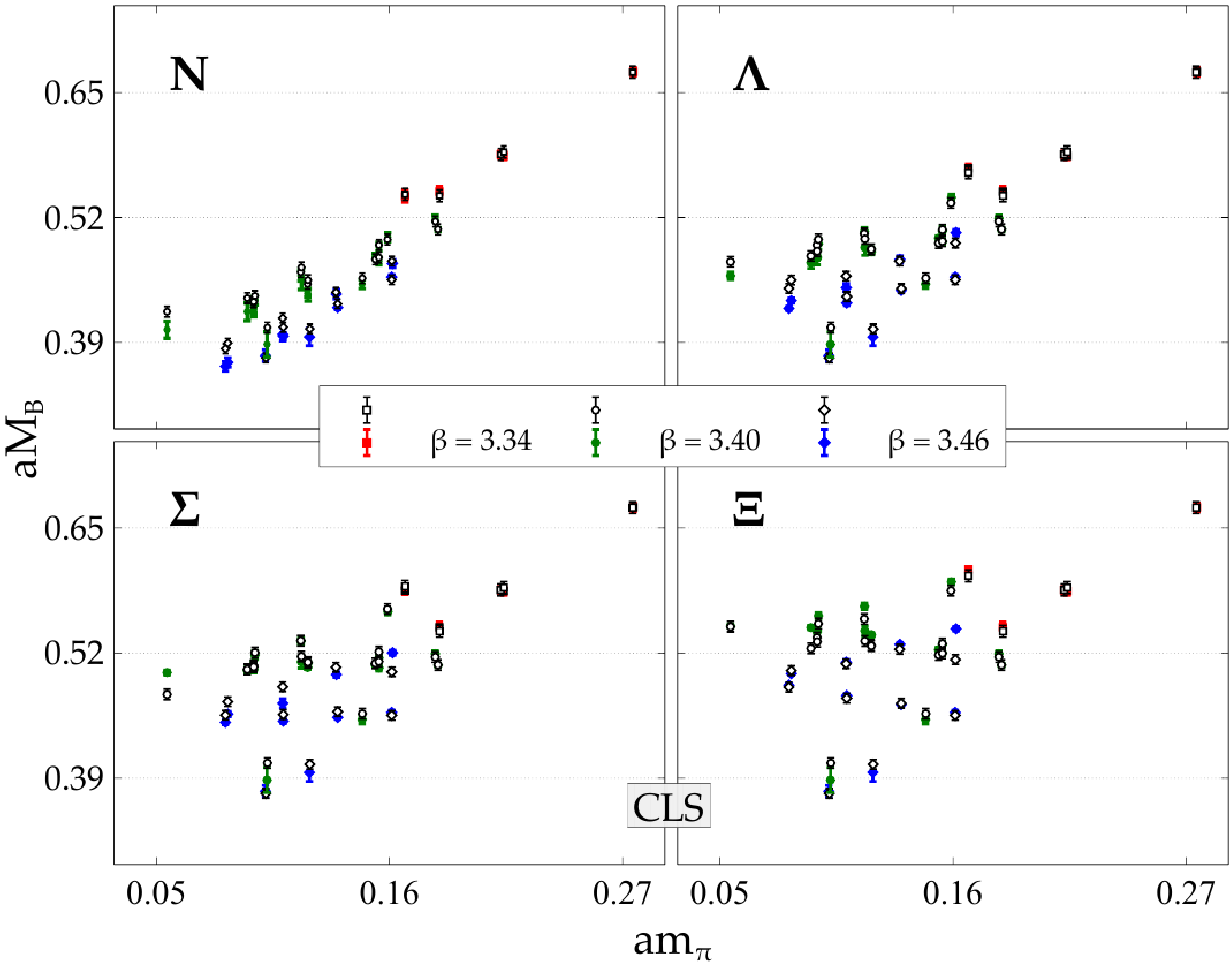
✔ A global fit to baryon masses on CLS/Regensburg ensembles

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- finite volume effects from chiral one-loop contributions are considered
- leading and subleading LEC have a quadratice lattice scale dependence
- global scale-setting with baryon octet and decplet masses

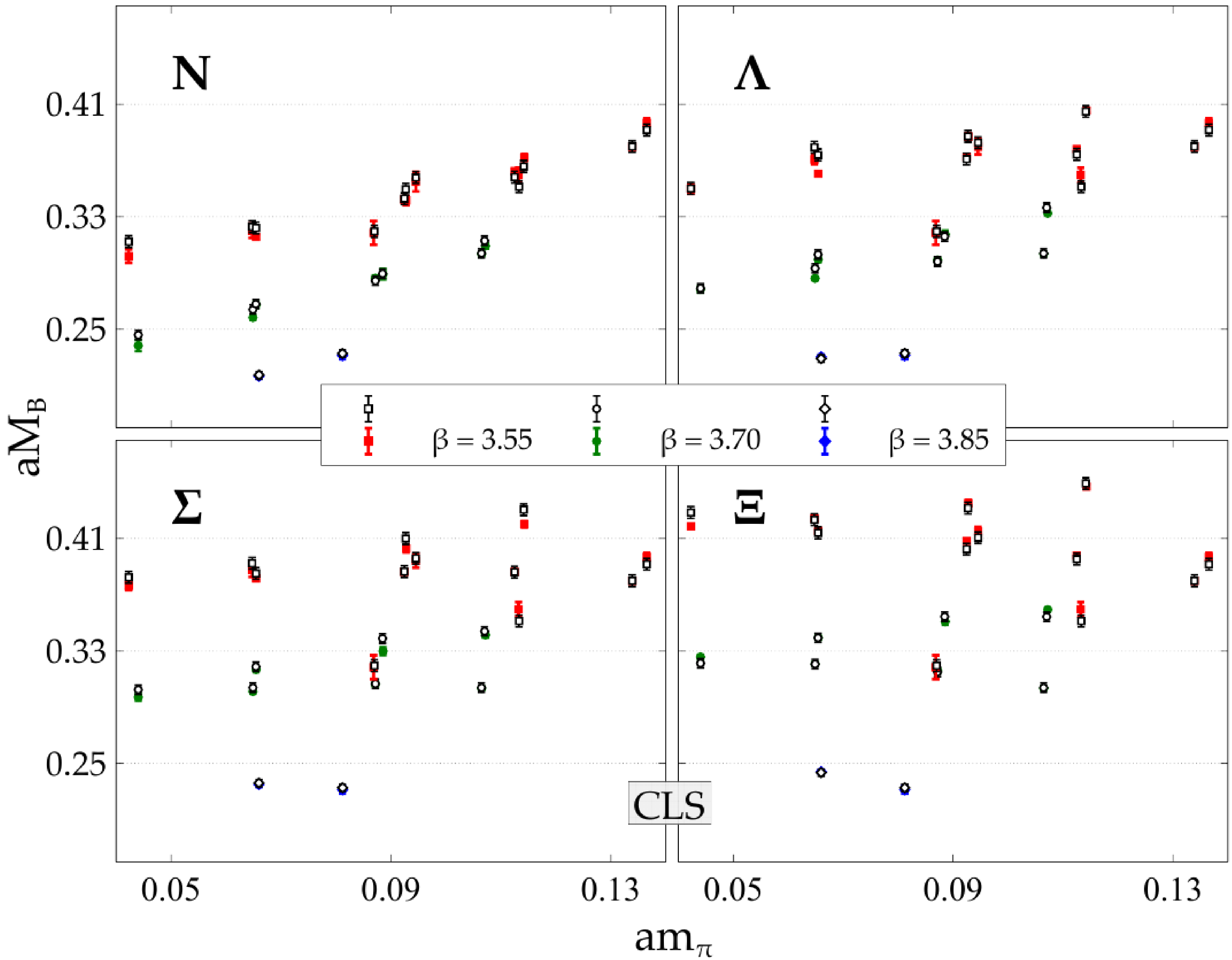
  physical baryon octet and decuplet masses are always reproduced
- acurracy level : self-consistent one-loop at N^{3}LO
- assume an ad-hoc size for the systematic error of about 10-15 MeV

	Fit	CLS [9]		Fit	CLS [9]
$a_{\rm CLS}^{\beta=3.34}$ [fm]	0.09337(22)	0.09757(56)	$a_{\rm CLS}^{\beta=3.55}$ [fm]	0.06314(12)	0.06379(37)
$a_{\rm CLS}^{\beta=3.40}$ [fm]	0.08251(7)	0.08524(49)	$a_{\rm CLS}^{\beta=3.70}$ [fm]	0.05003(16)	0.04934(28)
$a_{\rm CLS}^{\beta=3.46}$ [fm]	0.07478(8)	0.07545(44)	$a_{\rm CLS}^{\beta=3.85}$ [fm]	0.03845(11)	0.03873(22)
γ_{M_8} [GeV^3]	− 0.1322(10)		$\gamma_{M_{10}}$ [GeV^3]	− 0.0776(4)	
γ_{b_0} [GeV]	0.0619(8)		γ_{d_0} [GeV]	− 0.0115(9)	
γ_{b_D} [GeV]	− 0.1512(9)		γ_{d_D} [GeV]	0.0206(9)	
γ_{b_F} [GeV]	− 0.0071(4)		b_a	0.6305(8)	
$M_{[8]}$ [GeV]	0.8043(9)	$0.809^{(71)}_{(53)}$	$M_{[10]}$ [GeV]	1.1152(1)	$1.147^{(74)}_{(91)}$
b_0 [GeV^{-1}]	− 0.8144(9)	$-0.706^{(56)}_{(69)}$	d_0 [GeV^1]	− 0.4347(14)	$-0.84^{(44)}_{(34)}$
b_D [GeV^{-1}]	0.1235(2)	$0.083^{(33)}_{(35)}$	d_D [GeV^{-1}]	− 0.5169(13)	$-0.50^{(18)}_{(96)}$
b_F [GeV^{-1}]	− 0.2820(3)	$-0.384^{(28)}_{(44)}$			

Pion-mass dependence of the baryon octet masses



Pion-mass dependence of the baryon decuplet masses



Low-energy parameters from lattice QCD simulations

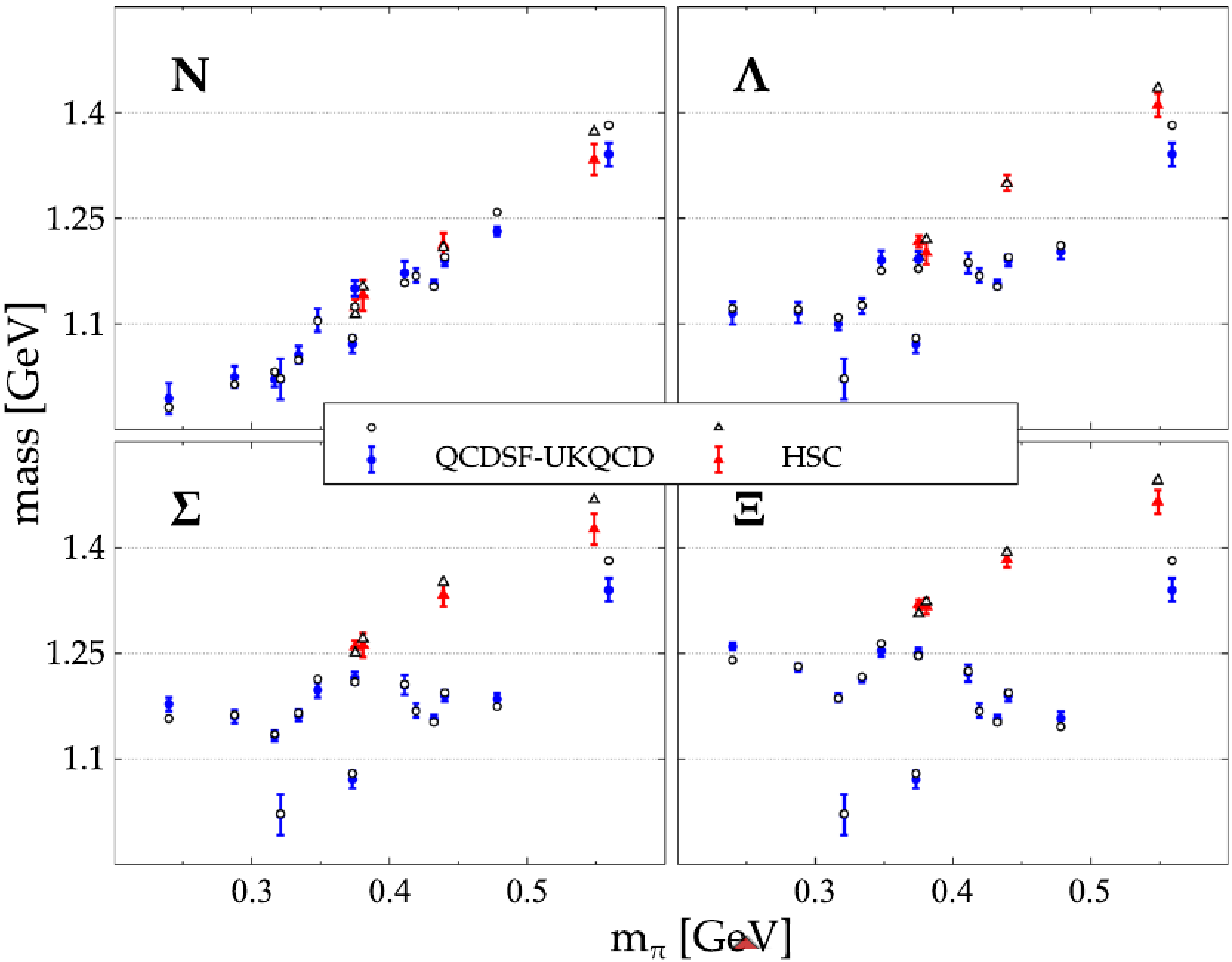
✔ Use the LEC from the fit to CLS/Regensburg ensembles

- consider all ensembles with $m_\pi < 550$ MeV and $m_K < 550$ MeV
- readjust lattice scales and the a^2 parts in the LEC
- fit to QCDSF-UKQCD, HSC, PACS-CS and ETMC data
- global scale-setting from all baryon octet and decplet masses
- assume an ad-hoc size for the systematic error of about 10-15 MeV

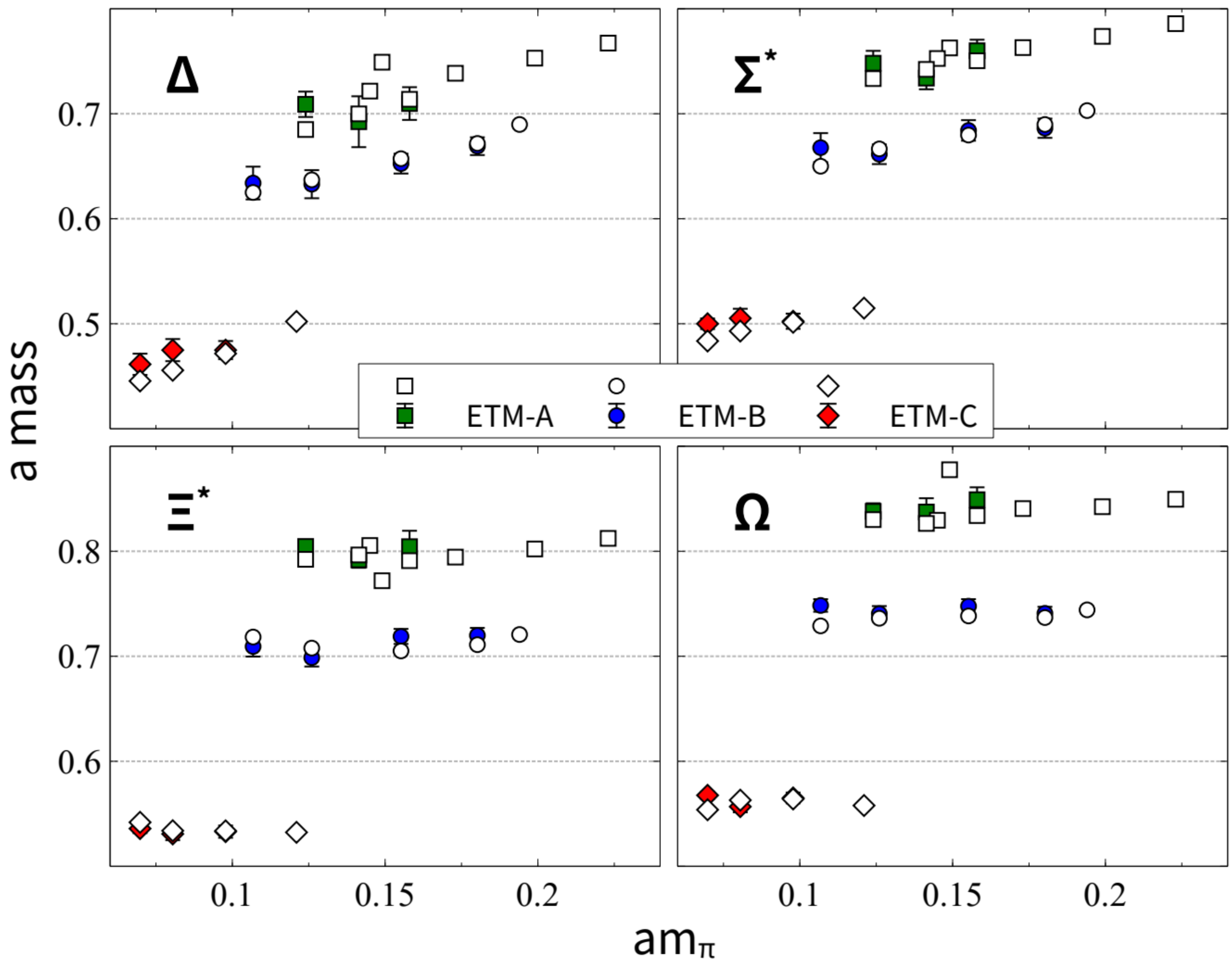
✔ Predict size of discretization effects for older Lattice Data sets

	ETMC	QCDSF	HSC	scale	Fit	Lattice
γ_{M_8} [GeV^3]	0.003(12)	− 0.284(2)	− 0.053(13)	$a_{\rm ETMC}^{\beta=1.90}$ [fm]	0.105(2)	0.0934(37)
γ_{b_0} [GeV]	0.201(10)	0.104(3)	0.117(25)	$a_{\rm ETMC}^{\beta=1.95}$ [fm]	0.094(1)	0.0820(37)
γ_{b_D} [GeV]	− 0.106(1)	− 0.001(5)	− 0.027(4)	$a_{\rm ETMC}^{\beta=2.10}$ [fm]	0.070(1)	0.0644(26)
γ_{b_F} [GeV]	− 0.112(14)	− 0.011(4)	0.019(4)	$a_{\rm QCDSF}$ [fm]	0.080(1)	0.0765(15)
$\gamma_{M_{10}}$ [GeV^3]	0.009(13)	− 0.128(11)	0.117(25)	$a_{\rm HSC}$ [fm]	0.125(2)	0.123(1)
γ_{d_0} [GeV]	0.238(21)	0.235(8)	0.173(12)			
γ_{d_D} [GeV]	− 0.147(13)	− 0.109(12)	− 0.038(2)			

Pion-mass dependence of the baryon octet masses



Pion-mass dependence of the baryon decuplet masses



Predictions for sigma terms

$$\sigma_{\pi N} = m \frac{\partial}{\partial m} m_N$$

✔ $\sigma_{\pi N}$ from pion-nucleon scattering and pionic atom data

- empirical value $\sigma_{\pi N} = 59.0(3.5)$ MeV (M. Hoferichter et al., arXiv:2305.07045)
- significant tension with QCD lattice results (only typical cases shown)

$\sigma_{\pi N} = 45.8(7.4)(2.8)$ MeV	Y. B. Yan et al., Phys. Rev. D 94 (2016) 054503
$\sigma_{\pi N} = 35(6)$ MeV	G. S. Bali et al., Phys. Rev. D 93 (2016) 094504
$\sigma_{\pi N} = 59.6(7.4)$ MeV	R. Gupta et al., arXiv:2105.12095

✔ From baryon masses on CLS ensembles

- $\sigma_{\pi N} = 43.9(4.7)$ MeV — G. S. Bali et al., arXiv:2211.03744
- $\sigma_{\pi N} = 43.6(3.8)$ MeV — A. Agadjanov et al., arXiv::2303.08741
- $\sigma_{\pi N} = 58.7(1.2)$ MeV — MFML et al., arXiv:2301.06387

  the inclusion of the isobar is cruical here

Summary & Outlook

✔ Chiral extrapolation of hadron masses

- resummed χPT : use physical masses in the loops
  - chiral expansion with up, down and strange quarks is useful
- so far we considered baryon masses at N^3LO
  - fits to masses of ground states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$
  - quantitative reproduction of the available lattice data set
- predict a large number of low-energy constants for the chiral Lagrangian of QCD
  - obtain a pion-nucleon sigma term comaptible with its empirical value
  - the decuplet baryons play an instrumental role

✔ QCD spectroscopy with coupled-channel dynamcis

- curent QCD lattice data provide many LEC relevant for scattering processes
- use as input in systematic coupled-channel computations
- analyze and predict the quark-mass dependence of hadron resonances in QCD

thanks to: Yonggoo Heo and Xiao-Yu Guo