Analysis of rescattering effects in 3π final states

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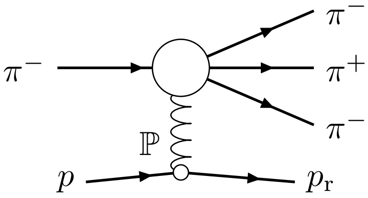
[DS, T. Isken, B. Kubis, M. Mikhasenko, M. Niehus
Eur. Phys. J. C 83, 510 (2023); arXiv:2212.11767]

COMPASS experiment



- large data set on diffractive $\pi^-\pi^-\pi^+$ production
- allows to study $I^G = 1^-$ processes
- $m_{3\pi}$ mass range from 0.5 to 2.5GeV
- set of 88 partial waves
- highly relevant question for COMPASS PWA framework:

[COMPASS; 2015]

to what extent and how are the $\pi^+\pi^-$ partial waves modified by the presence of the third spectator pion?

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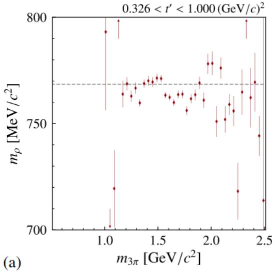

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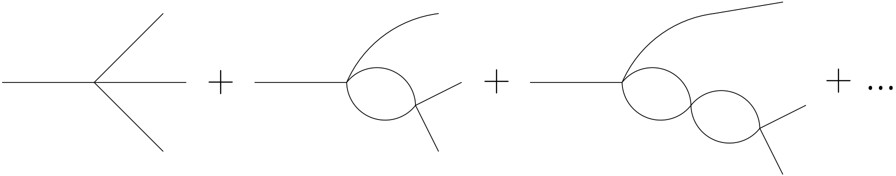
  to what extent and how are the $\pi^+\pi^-$ partial waves modified by the presence of the third spectator pion?
- ρ-mass dependence on $m_{3\pi}$

Motivation

- different decay processes that decay into $\rho\pi$ final states
- effects of the ρ described by $\pi\pi$ P-wave phase shift



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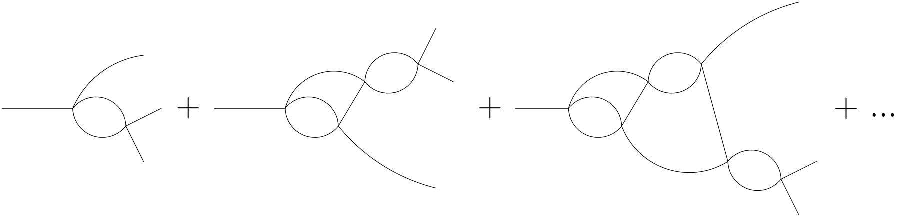
- include full rescattering effects
- Khuri–Treiman equations [Khuri, Treiman; 1960]



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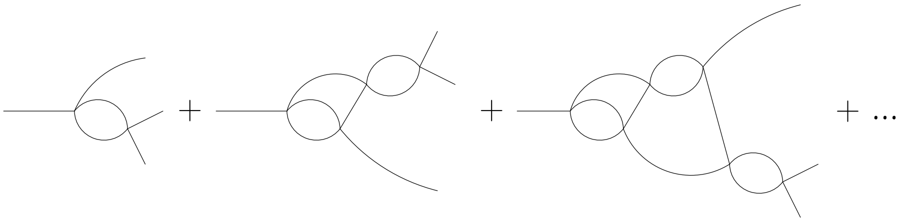

- which statistics to observe rescattering effects, depending on process and energy?

Khuri–Treiman equations

- amplitude for $\omega, \phi \to 3\pi$

$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

- unitarity condition

$$f_1(s) = \mathcal{F}(s) + \widehat{\mathcal{F}}(s)$$
$$\mathrm{disc}\mathcal{F}(s) = 2i\left(\mathcal{F}(s) + \cancel{\widehat{\mathcal{F}}(s)}\right)\sin\delta(s)e^{-i\delta(s)}\theta(s-4M_\pi^2)$$

- partial wave f_1 with angular momentum $\ell = 1$
- $\pi\pi$ P-wave phase shift δ
- homogeneous solution with $\widehat{\mathcal{F}}(s) = 0$ gives Omnès function

[Omnès; 1958]

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\mathrm{d}s'\frac{\delta(s')}{s'(s'-s)}\right)$$

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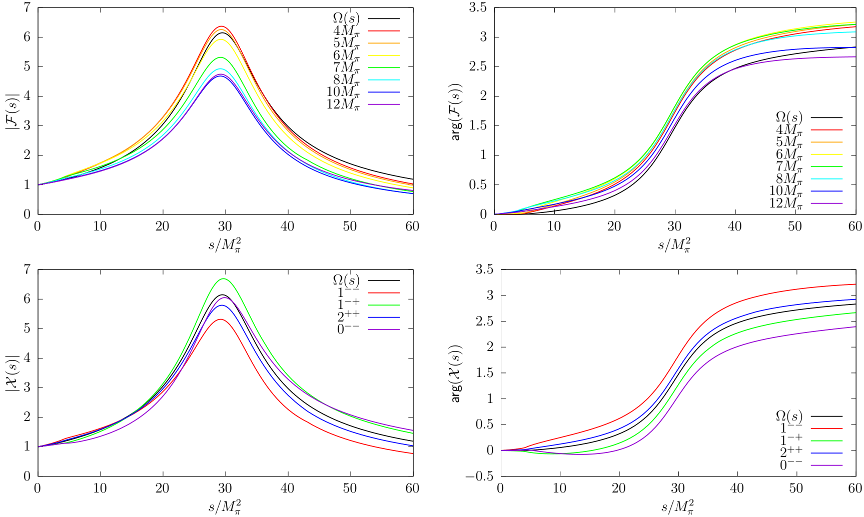
- partial wave f_1 with angular momentum $\ell = 1$
- $\pi\pi$ P-wave phase shift δ
- inhomogeneous solution leads to KT equations

$$\mathcal{F}(s) = \Omega(s)\left(P_n(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\sin\delta(s')\widehat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right)$$

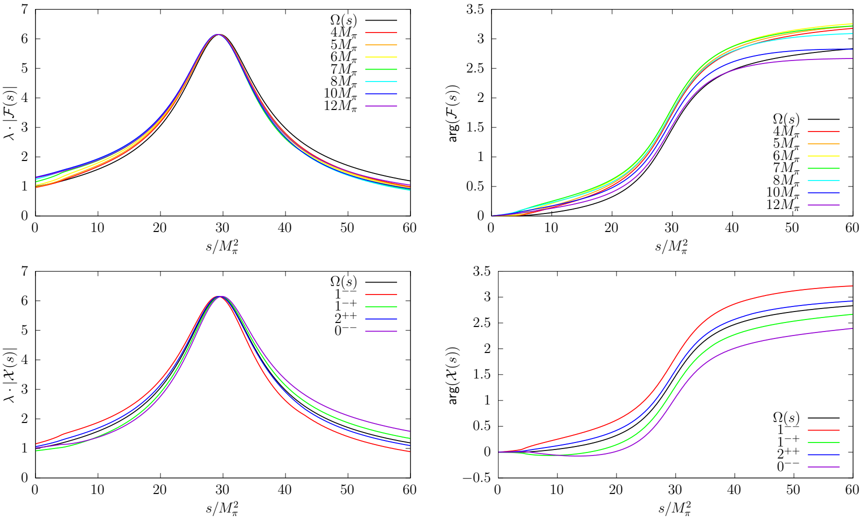
Decays

- four different processes
- angular-momentum quantum numbers of 3π system
- $(I^G)J^{PC} = (0^-)0^{--}$ (exotic), $(0^-)1^{--}$ (ω, ϕ),
  $(1^-)1^{-+}$ (π_1), $(1^-)2^{++}$ (a_2)
- restrictive analysis with only one free parameter: $\rightarrow$ normalization
- only consider P-waves
- not include processes that involve S-waves (1^{++}, a_1)
- only elastic unitarity

Basis functions (normalized $s = 0$)



Basis functions (normalized $s = M_\rho^2$)



Log-likelihood differences

- KT equations set as truth
- to which extent do Omnès functions reproduce these?
- probability density function

$$f(s,t) = \frac{|\mathcal{M}(s,t)|^2}{\int_D |\mathcal{M}(s,t)|^2 \mathrm{d}s\mathrm{d}t}$$

- likelihood function

$$L(\mathbb{D}, f) = \prod_{i=1}^{N} f(s_i, t_i)$$

- log-likelihood difference

$$\Delta\mathcal{L}(\mathbb{D}) = \ln(L(\mathbb{D}, f^{\text{Omnès}})) - \ln(L(\mathbb{D}, f^{\text{KT}}))$$

Log-likelihood differences

- use Kullback-Leibler divergence d_{KL} and variance ν_{KL}

  [Kullback, Leibler; 1951]

- probability of $\Delta\mathcal{L} > 0$ via cumulative distribution function

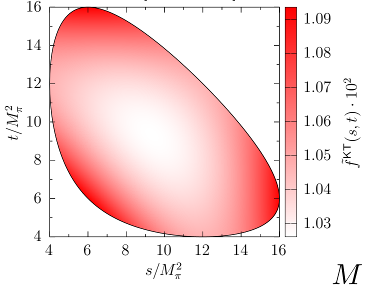
$$q(N) = 1 - \mathcal{N}(0, \mu(N), \sigma(N)) \quad \mu(N) = -N d_{\mathsf{KL}} \quad \sigma(N) = \sqrt{N \nu_{\mathsf{KL}}}$$

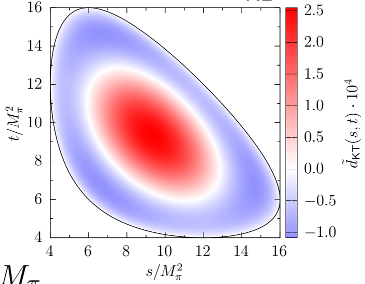
- number of events

$$N(q) = 2\nu_{\mathsf{KL}} \left(\frac{\mathrm{erf}^{-1}(1-2q)}{d_{\mathsf{KL}}} \right)^2$$

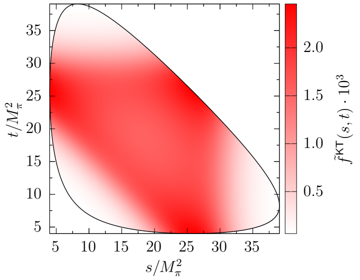
Dalitz plots for 1^{--}: ω, ϕ

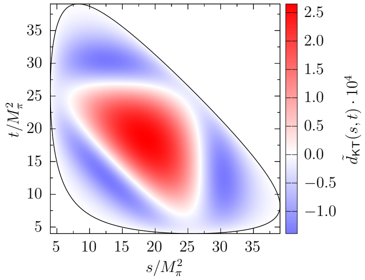
without phase space $\quad M = 5M_\pi \quad$ distribution of d_{KL}





$M = 7.25M_\pi$

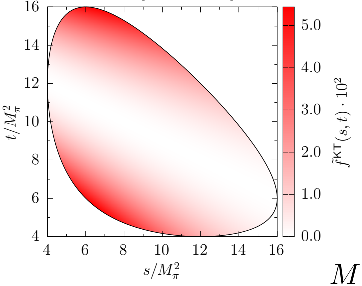


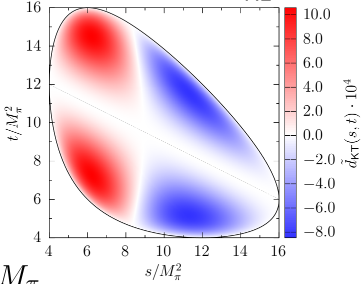


without phase space

$M = 5M_\pi$

distribution of d_{KL}





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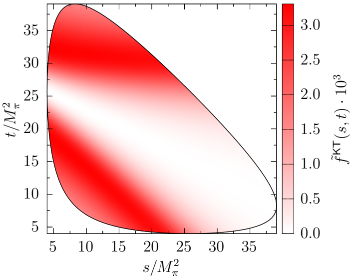
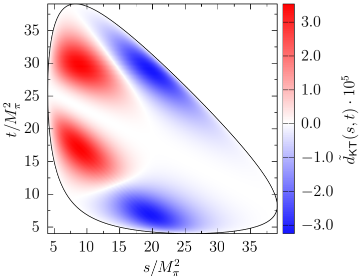
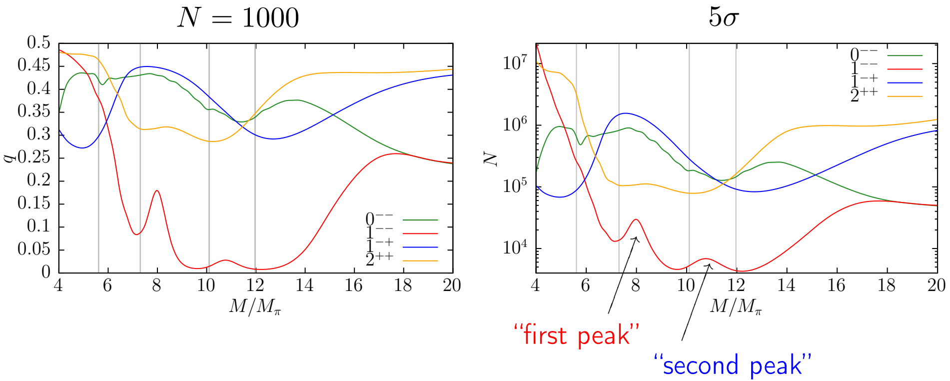




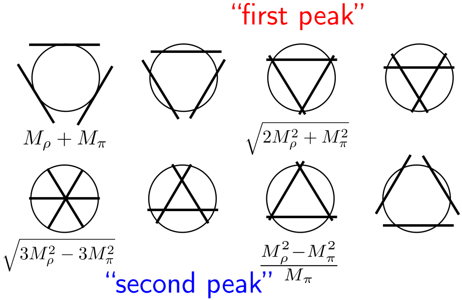
Results



- form dominated by angular momentum quantum number
- rise for limit $M \to \infty$ (but inelastic effects)
- two peaks in 1^{--}

Results

- schematic Dalitz plots for different decay masses



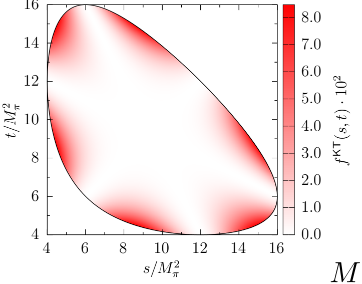
- difference decreases when ρ bands cross in Dalitz plot
- approx. $\sqrt{2M_\rho^2 + M_\pi^2}$
- second peak also due to kinematic effect

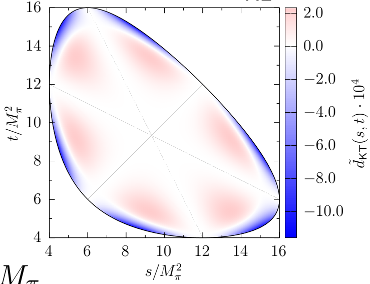
Conclusion

- compare Omnès to KT solutions:
  when are rescattering effects visible?
- in 0^{--}, 1^{-+} and 2^{++} many events ($\mathcal{O}(10^5) - \mathcal{O}(10^6)$) are needed for all decay masses
- for 1^{--} strong dependence on decay mass
- at ω mass: small difference [BESIII; 2018]
- at ϕ mass: rescattering effects easy to observe [KLOE; 2003]

Dalitz plots for 0^{--}

without phase space $M = 5M_\pi$ distribution of d_{KL}





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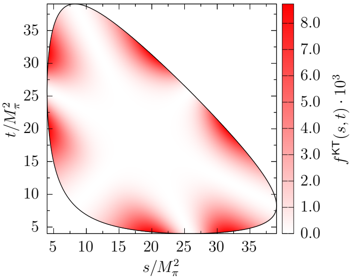

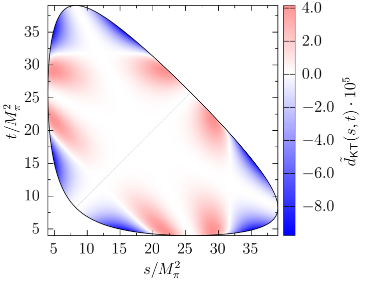

Dalitz plots for 2^{++}

without phase space $M = 5M_\pi$ distribution of d_{KL}