The light baryon resonance spectrum in a coupled-channel approach – recent results of the Jülich-Bonn model

MENU 2023 - The 16th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon

October 17, 2023 | Deborah Rönchen | Institute for Advanced Simulation, Forschungszentrum Jülich

In collaboration with: M. Döring, M. Mai, T. Mart, Ulf-G. Meißner, C.-W. Shen, Y.-F. Wang, R. Workman
(Jülich-Bonn and Jülich-Bonn-Washington collaborations)

Supported by DFG, NSFC, MKW NRW
HPC support by Jülich Supercomputing Centre

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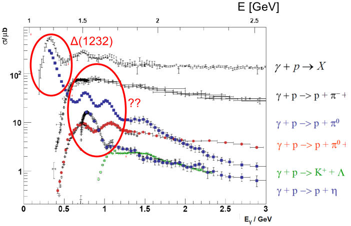
The excited baryon spectrum:

Connection between experiment and QCD in the non-perturbative regime

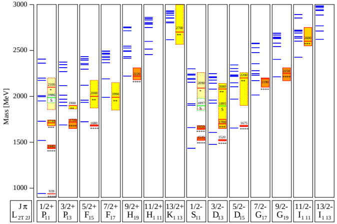
Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

$\Longleftrightarrow$

Theoretical predictions of excited hadrons
e.g. from relativistic quark models:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Major source of information:

In the past: **elastic or charge exchange πN scattering**

- “missing resonance problem”

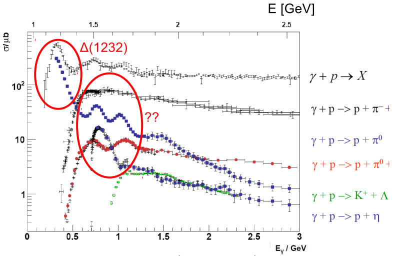
In recent years: **photoproduction reactions**

- large data base, high quality (double) polarization observables, towards a complete experiment
  Reviews: Prog.Part.Nucl.Phys. 125, 103949 (2022), Prog.Part.Nucl.Phys. 111 (2020) 103752

In the future: **electroproduction reactions**

- 10^5 data points for πN, ηN, KY, $\pi\pi N$ Review: e.g. Prog.Part.Nucl.Phys. 67 (2012)

From experimental data to the resonance spectrum



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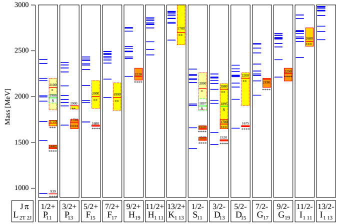

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Different modern analyses frameworks:

- **(multi-channel) K-matrix:** GWU/SAID, BnGa (phenomenological),
  Gießen (microscopic Bgd)
- **dynamical coupled-channel (DCC):** 3d scattering eq., off-shell intermediate states
  ANL-Osaka (EBAC), Dubna-Mainz-Taipeh, Jülich-Bonn
- **unitary isobar models:** unitary amplitudes + Breit-Wigner resonances
  MAID, Yerevan/JLab, KSU
- **other groups:** Mainz-Tuzla-Zagreb PWA (MAID + fixed-t dispersion relations, L+P), JPAC (amplitude analysis with Regge phenomenology), Ghent (Regge-plus-resonance), truncated PWA
- …

Detailed comparison of MAID, GWU/SAID, BnGa and JüBo: EPJ A 52, 284 (2016)

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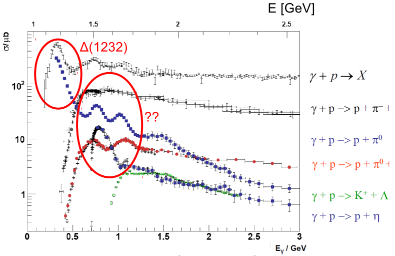

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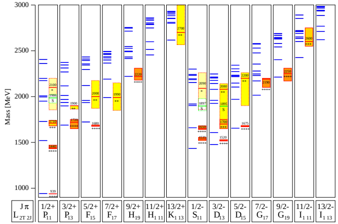

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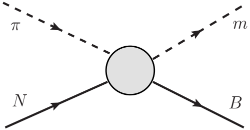
Jülich-Bonn DCC approach for hadronic reactions



The Jülich-Bonn DCC approach for N^* and Δ resonances

pion-induced reactions

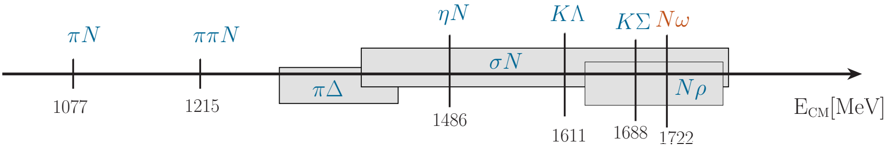
EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

- channels ν, μ, γ:



$\hookrightarrow \pi N \to \omega N$ included by Y.-F. Wang (PRD 106 (2022)), talk on Monday on compositeness of resonances (2307.06799 [nucl-th])

$\hookrightarrow$ hidden charm reactions: C.-W. Shen, Z.-L. Wang CP C 42 (2018), EPJ C 82 (2022)

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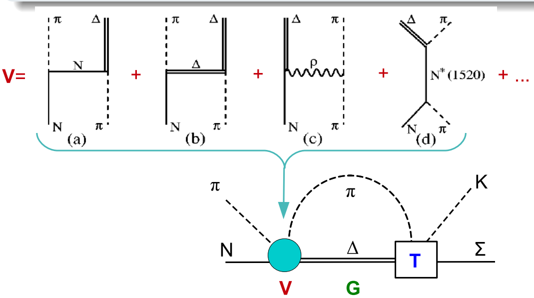

- potentials V constructed from effective $\mathcal{L}$
- t- and u-channel: T^{NP}

  dynamical generation of poles
- s-channel diagrams: T^{P}

  genuine resonance states
- contact terms

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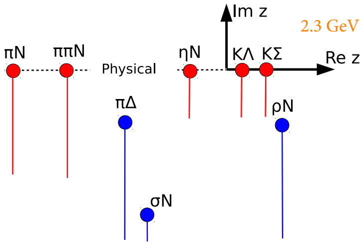
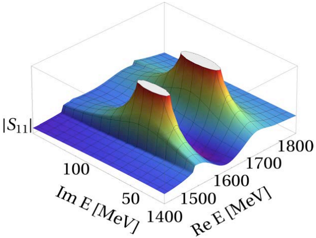
Resonance states

- (2 body) unitarity and analyticity respected (no on-shell factorization, dispersive parts included)
- opening of **inelastic channels** $\Rightarrow$ **branch point** and new **Riemann sheet**

Resonances: poles in the full T-matrix

- on the unphysical Riemann sheet
- Pole position E_0 is the same in all channels
- Re(E_0) = "mass", -2Im(E_0) = "width"
  residues $\longrightarrow$ branching ratios



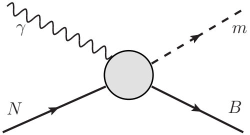


3-body $\pi\pi N$ channel:

- parameterized effectively as $\pi\Delta$, σN, ρN
- $\pi N/\pi\pi$ subsystems fit the respective phase shifts

↳ branch points move into complex plane

Photoproduction



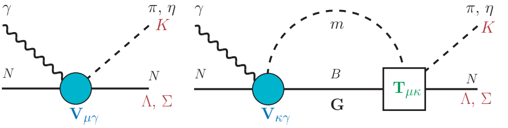
Photoproduction in a semi-phenomenological approach

EPJ A 50, 101 (2015)

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_{\kappa} V^{IJ}_{\kappa\gamma}$$

(partial wave basis)

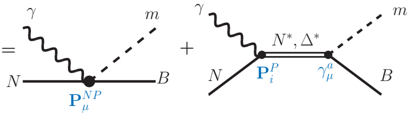


$m = \pi, \eta, K$, $B = N, \Delta, \Lambda$

$T_{\mu\kappa}$: full hadronic T-matrix as in pion-induced reactions

Photoproduction potential: approximated by energy-dependent polynomials (field-theoretical description numerically too expensive)

$\mathbf{V}_{\mu\gamma}(E,q) =$



$$= \frac{\tilde{\gamma}^a_{\mu}(q)}{m_N} P^{\mathrm{NP}}_{\mu}(E) + \sum_i \frac{\gamma^a_{\mu;i}(q) P^{\mathrm{P}}_i(E)}{E - m^b_i}$$

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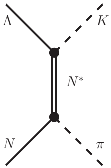
Simultaneous fit of pion- & photon-induced reactions

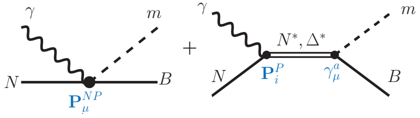
Free parameters

- $\pi N \to \ \pi N, \eta N, KY$:
  s-channel: resonances (T^P)



m_{bare} + f_{mBN^*}

- $\gamma p \to \pi N, \eta N, KY$: couplings of the polynomials and s-channel parameters



- couplings in contact terms
- t- & u-channel parameters: cut-offs, mostly fixed to values of previous JüBo studies (couplings fixed from SU(3))

$\Rightarrow \sim$ 900 fit parameters in total, $\sim$ 72,000 data points

↳ calculations on a supercomputer [JURECA, Jülich Supercomputing Centre, Journal of large-scale research facilities, 2, A**62** (2016)]

- large number of fit parameters, many from polynomials
- can be regarded as advantage: prevents the inclusion of superfluous s-channel states to improve fit

Extension to $K\Sigma$ photoproduction on the proton

JüBo2022 **Eur.Phys.J.A 58 (2022) 229**

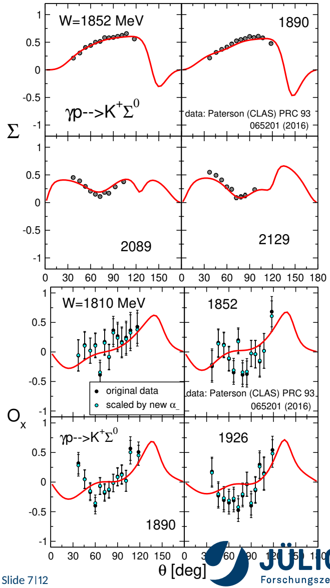
Simultaneous analysis of $\pi N \to \pi N, \eta N, K\Lambda, K\Sigma$ and
$\gamma p \to \pi N, \eta N, K\Lambda, K\Sigma$

- almost 72,000 data points in total, $W_{\text{max}} = 2.4$ GeV
  - $\gamma p \to K^+\Sigma^0$: $d\sigma/d\Omega$, P, Σ, T, $C_{x',z'}$, $O_{x,z}$ = 5,652
  - $\gamma p \to K^0\Sigma^+$: $d\sigma/d\Omega$, P = 448
- polarizations scaled by new Λ decay constant α_- (Ireland PRL 123 (2019), 182301), if applicable
- χ^2 minimization with MINUIT on JURECA [Jülich Supercomputing Centre, JURECA: JLSRF 2, A62 (2016)]

Resonance analysis:

- all 4-star N and Δ states up to $J = 9/2$ are seen (exception: $N(1895)1/2^-$) + some states rated less than 4 stars
- no additional s-channel diagram, but indications for new dyn. gen. poles

Selected fit results



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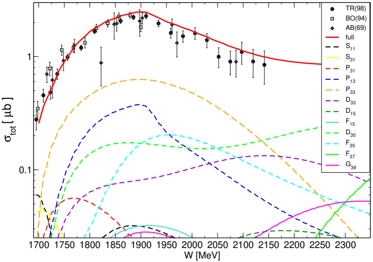
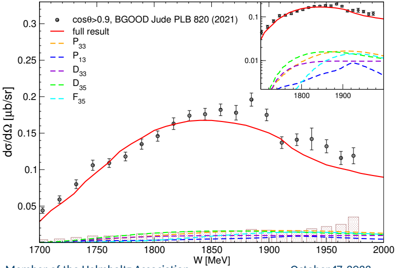
Resonance contributions to $K\Sigma$ photoproduction

$\gamma p \to K^+ \Sigma^0$

JüBo2022 Eur.Phys.J.A 58 (2022) 229



(Data not included in fit)



- dominant partial waves: $I = 3/2$

Exception: P_{13} partial wave ($I = 1/2$):

$N(1720)\ 3/2^+$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1726(8)	185(12)	5.9(1)	82(6)
2017	1689(4)	191(3)	0.6(0.4)	26(58)
PDG 2021	1675 ± 15	250^{+150}_{-100}	—	—

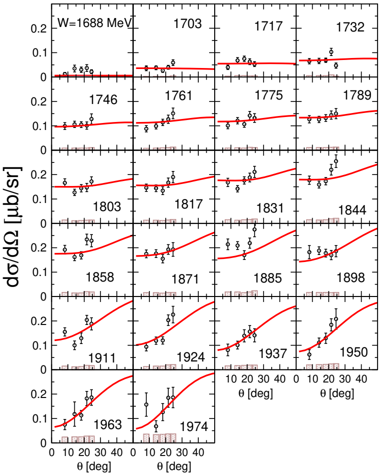
$N(1900)\ 3/2^+$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Sigma}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N\to K\Sigma}$ [deg]
2022	1905(3)	93(4)	1.3(0.3)	$-40(18)$
2017	1923(2)	217(23)	10(7)	$-34(74)$
PDG 2021	1920 ± 20	150 ± 50	4 ± 2	110 ± 30

- drop in cross section due to $N(1900)3/2^+$
- "cusp-like structure" only qualitatively explained

Resonance contributions to $K\Sigma$ photoproduction

JüBo2022 Eur.Phys.J.A 58 (2022) 229

$\gamma p \to K^+\Sigma^0$



Data: Jude et al. (BGOOD) PLB 820 (2021)

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Selected results $\gamma p \to K^0\Sigma^+$

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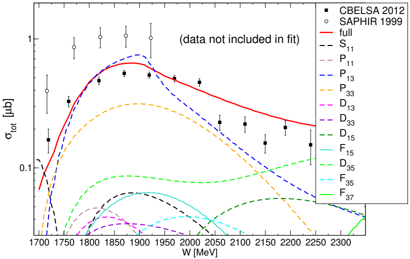
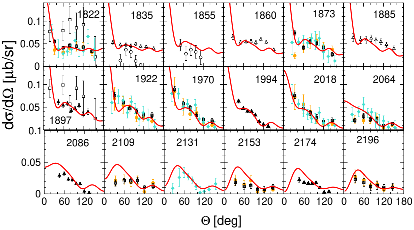


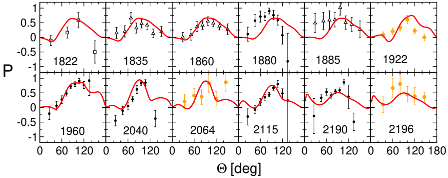
- much less data than for $K^+\Sigma^0$
  (448 vs 5,652 data points)
- in parts inconsistent data

  $\to$ difficult to achieve a good fit result
- cusp in σ_{tot} at $\sim$ 2 GeV not reproduced
  (data not included in fit)

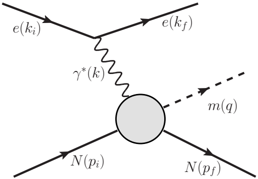
Selected fit results:



Data: open squares: SPAHIR 1999, cyan: SAPHIR 2005, orange: CBELSA/TAPS 2007, black squares: CBELSA/TAPS 2011, open circles: A2 2018, open triangles: A2 2013, black triangles: Hall B 2003, black circles: CLAS 2013



Electroproduction



Experimental studies of electroproduction:

major progress in recent years, e.g., from JLab, MAMI, ...

- 10^5 data points for πN, ηN, KY, $\pi\pi N$ electroproduction
- access the Q^2 dependence of the amplitude

  → expected to provide a link between perturbative QCD and the region where quark confinement sets in
- so far, no new N^* or Δ^* established from electroproduction: data not yet analyzed on the same level as photoproduction
  Reviews: Prog.Part.Nucl.Phys. 67 (2012); Few. Body Syst. 63 (2022) 3, 59

$\gamma^* p \to \pi^0 p$

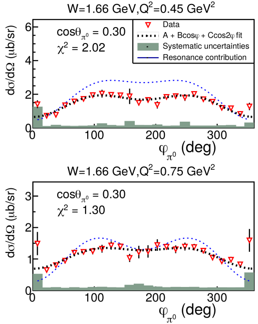


Figure and data from Markov et al. (CLAS) PRC 101 (2020), resonance contribution: JLab/YerPhI

Single-channels analyses, e.g.:

- MAID: π, η, kaon electroproduction (EPJA 34, 69 (2007), NPA 700, 429 (2002),)
- JLab: π electroproduction covering the resonance region (PRC 80 (2009) 055203)

Coupled-channels analyses:

- ANL-Osaka: extension of DCC analysis of pion electroproduction (PRC 80, 025207 (2009)) in progress (Few Body Syst. 59 (2018) 3, 24)
- Jülich-Bonn-Washington approach M. Mai *et al.* PRC 103 (2021): $\gamma^* p \to \pi^0 p$, $\pi^+ n$, ηp, $K\Lambda$

Jülich-Bonn-Washington parametrization

M. Mai *et al.* PRC 103 (2021), PRC 106 (2022), arXiv:2307.10051 [nucl-th]

$$\mathcal{M}_{\mu\gamma^*}(k,W,Q^2) = R_{\ell'}(\lambda, q/q_\gamma)\left(V_{\mu\gamma^*}(k,W,Q^2) + \sum_\kappa \int_0^\infty dp\, p^2\, T_{\mu\kappa}(k,p,W)G_\kappa(p,W)V_{\kappa\gamma^*}(p,W,Q^2)\right)$$

(Pseudo)-threshold behavior with meson/photon momenta

$$lim_{k\to 0} E_{\ell+} = k^\ell$$
$$lim_{q\to 0} L_{\ell+} = q^\ell$$
$$\dots$$

For $Q^2=0$ (real photons) identical to Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k,W,Q^2) = V^{\rm JUBO}_{\mu\gamma}(k,W)\cdot \tilde{F}_D(Q^2)\cdot e^{-\beta^0_\mu Q^2/m_p^2}\left(1 + Q^2/m_p^2\beta^1_\mu + (Q^2/m_p^2)^2\beta^2_\mu\right)$$

$\leftarrow$ Siegerts's theorem Siegert(1973) Amaldi et al.(1979) Tiator(2016)

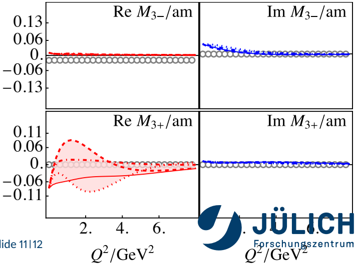
$$V^{L_{\ell\pm}} = (\text{const.})\cdot V^{E_{\ell\pm}}$$

...at pseudo-threshold

- simultaneous fit to πN, ηN, $K\Lambda$ electroproduction off proton ($W < 1.8$ GeV, $Q^2 < 8$ GeV^2)
- 533 fit parameters, 110.281 data points
- Input from JüBo: $V_{\mu\gamma}(k,W,Q^2=0)$, $T_{\mu\kappa}(k,p,W)$, $G_\kappa(p,W)$

  $\rightarrow$ universal pole positions and residues (fixed in this study)
- long-term goal: fit pion-, photo- and electron-induced reactions simultaneously

$\gamma^* p \rightarrow K\Lambda$ at $W = 1.7$ GeV



Summary and Outlook

Extraction of the N^* and Δ spectrum from experimental data: major progress in last decade

- new information from photoproduction data $\rightarrow$ new and upgraded states in PDG table
- however: less progress for Δ^* states
- wealth of high-quality electroproduction data, more at high Q^2 in the future (CLAS12)
  $\rightarrow$ to be included in modern coupled-channel analyses (in progress)

Jülich-Bonn DCC analysis:

- Extraction of the N^* and Δ spectrum in a simultaneous analysis of pion- and photon-induced reactions [Eur.Phys.J.A 58 (2022) 229]
- $\pi N \rightarrow \omega N$ channel included, prerequisite for ω photoproduction [Wang *et al.* PRD 106 (2022), 094031]
- Electroproduction: *Jülich-Bonn-Washington* approach [Mai *et al.* PRC 103 (2021), PRC 106 (2022), 2307.10051 [nucl-th]]

  - In progress: Baryon transition form factors

  - In progress: adaption of JüBo framework to $\bar{K}N$ reactions $\rightarrow \Lambda^*$, Σ^*

  - New interactive web interface: https://jbw.phys.gwu.edu (under construction)
    $\rightarrow$ multipoles, observables, data

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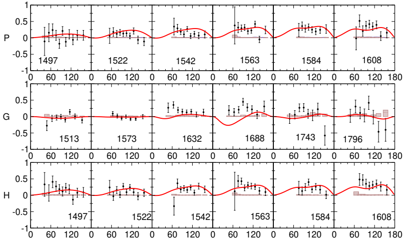
Thank you for your attention!

Appendix

New data for $\gamma p \to \eta p$ from CBELSA/TAPS

included in JüBo2022 Eur.Phys.J.A 58 (2022) 229

- T, P, H, G, E Müller PLB 803, 135323 (2020): very first data on H, G (and P) in this channel



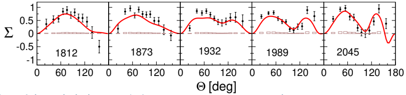
$N(1535)$ 1/2$^-$ $\ast\ast\ast\ast$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N \to K\Sigma}$ [deg]
2022	1504(0)	74 (1)	50(3)	118(3)
2017	1495(2)	112(1)	51(1)	105(3)
PDG 2022	1510 ± 10	130 ± 20	43 ± 3	-76 ± 5

$N(1650)$ 1/2$^-$ $\ast\ast\ast\ast$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N \to K\Sigma}$ [deg]
2022	1678(3)	127(3)	34(12)	71(45)
2017	1674(3)	130(9)	18(3)	28(5)
PDG 2022	1655 ± 15	135 ± 35	29 ± 3	134 ± 10

$\to \eta N$ residue $N(1650)1/2^-$ much larger (similarly observed by BnGa)

- Σ Afzal PRL 125, 152002 (2020): Backward peak in data

$\to$ Observation of $\eta' N$ cusp + importance of $N(1895)1/2^-$ (BnGa)



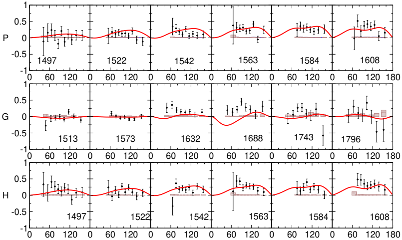
JüBo2022:

- no $\eta' N$ channel (or cusp), to be included in the future
- no $N(1895)1/2^-$ (not needed)
- backward peak from $N(1720)$ & $N(1900)3/2^+$

New data for $\gamma p \to \eta p$ from CBELSA/TAPS

included in JüBo2022 Eur.Phys.J.A 58 (2022) 229

- T, P, H, G, E Müller PLB 803, 135323 (2020): very first data on H, G (and P) in this channel



$N(1535)$ 1/2$^-$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N \to K\Sigma}$ [deg]
2022	1504(0)	74 (1)	50(3)	118(3)
2017	1495(2)	112(1)	51(1)	105(3)
PDG 2022	1510 ± 10	130 ± 20	43 ± 3	-76 ± 5

$N(1650)$ 1/2$^-$ ****	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{tot}}$ [%]	$\theta_{\pi N \to K\Sigma}$ [deg]
2022	1678(3)	127(3)	34(12)	71(45)
2017	1674(3)	130(9)	18(3)	28(5)
PDG 2022	1655 ± 15	135 ± 35	29 ± 3	134 ± 10

$\to \eta N$ residue $N(1650)1/2^-$ much larger (similarly observed by BnGa)

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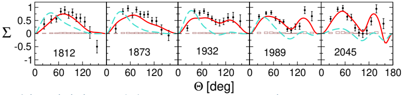


JüBo2022:

- no $\eta' N$ channel (or cusp), to be included in the future
- no $N(1895)1/2^-$ (not needed)
- backward peak from $N(1720)$ & $N(1900)3/2^+$ (turquoise lines: both states off)

Uncertainties of extracted resonance parameters

Challenges in determining resonance uncertainties, e.g.:

- **elastic πN channel**: not data but GWU SAID PWA are used by most groups

  $\rightarrow$ correlated χ^2 fit including the covariance matrix $\hat{\Sigma}$ [PRC 93, 065205 (2016)]

  $$\chi^2(A) = \chi^2(\hat{A}) + (A - \hat{A})^T \hat{\Sigma}^{-1} (A - \hat{A})$$

  $A \sim$ vector of fitted PWs, $\hat{A} \sim$ vector of SAID SE PWs

  $\rightarrow$ same χ^2 as fitting to data up to nonlinear and normalization corrections

- **error propagation** data $\rightarrow$ fit parameters $\rightarrow$ derived quantities:
  bootstrap method: generate pseudo data around actual data, repeat fit

  $\rightarrow$ numerically very challenging

- **model selection**, significance of resonance signals:

  determine minimal resonance content using Bayesian evidence [PRL 108, 182002; PRC 86, 015212 (2012)]
  or the LASSO method [J. R. Stat. Soc. B 58, 267 (1996), PRC 95, 015203 (2017)]:

  $$\chi_T^2 = \chi^2 + \lambda \sum_{i=1}^{i_{max}} |a_i| \,, \qquad \lambda \sim \text{penalty factor, } a_i \sim \text{fit parameter}$$

$\Rightarrow$ very challenging for coupled-channel analyses!

The SAID, MAID, BnGa and JüBo approaches

Detailed comparison: EPJ A 52, 284 (2016)

SAID PWA (gwdac.phys.gwu.edu)
based on Chew-Mandelstam K-matrix

- K-matrix elements parameterized as energy-dependent polynomials
- resonance poles are dynamically generated (except for the $\Delta(1232)$)
- masses, width and hadronic couplings from fits to pion-induced πN and ηN production
- photocouplings from photoproduction

Bonn-Gatchina (BnGa) PWA
(pwa.hiskp.uni-bonn.de)
Multi-channel PWA based on K-matrix (N/D)

- mostly phenomenological model
- resonances added by hand
- resonance parameters determined from large experimental data base:
  pion-, photon-induced reactions, 3-body final states
- PWA of $\bar{K}N$ scattering, hyperon spectrum EPJA 55,179 & 180 (2019)

MAID PWA (maid.kph.uni-mainz.de)
unitary isobar model

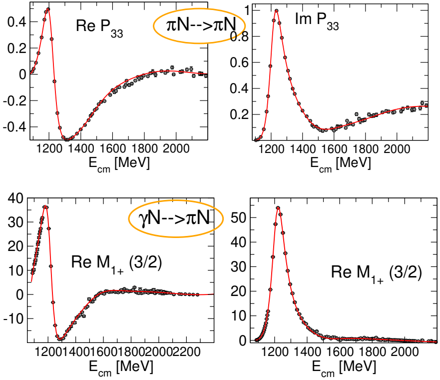
- resonances as multi-channel Breit-Wigner amplitudes
- background: Born terms + Regge exchanges
- photo- and electroproduction of pions, etas & kaons
- Mainz-Tuzla-Zagreb collaboration: MAID + fixed-t dispersion relations, L+P
  (pwatuzla.com/p/mtz-collab.html)

Jülich-Bonn (JüBo) DCC model
(collaborations.fz-juelich.de/ikp/meson-baryon/main)
Lippmann-Schwinger eq. formulated in TOPT

- hadronic potential from effective Lagrangians
- photoproduction as energy-dependent polynomials
- resonances as s-channel states (“by hand”), dynamical generation possible
- resonance parameters from pion- and photon-induced data
- Jülich-Bonn-Washington model: CC electroproduction analysis (jbw.phys.gwu.edu)

Excited states / Resonances

$J^P = 3/2^+, I = 3/2$



Points: SAID 2006 and CM12

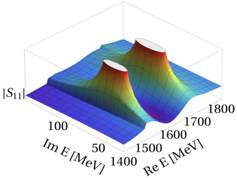
Breit-Wigner parameterization:

$$\mathcal{M}^{Res}_{ba} = -\frac{g_b g_a}{E^2 - M^2_{BW} + iE\Gamma_{BW}}$$

- M_{BW}, Γ_{BW} channel dependent
- background? overlapping resonances? thresholds?

Resonances: poles in the T-matrix

- Pole position E_0 is the same in all channels
- thresholds: branch points

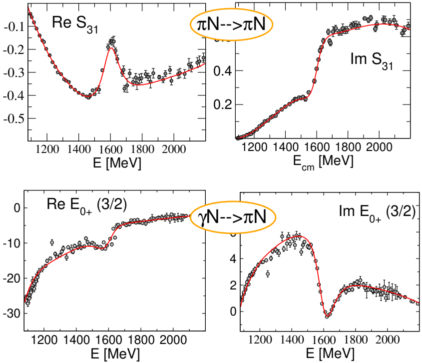


Re(E_0) = "mass"
-2Im(E_0) = "width"
residues→ branching ratios

JÜLICH Forschungszentrum

Excited states / Resonances

$J^P = 1/2^-, I = 3/2$



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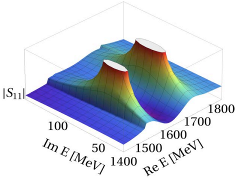

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residues$\to$ branching ratios

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Details of the formalism

Polynomials:

$$P_i^{\mathrm{P}}(E) = \sum_{j=1}^{n} g_{i,j}^{\mathrm{P}} \left(\frac{E - E_0}{m_N}\right)^j e^{-g_{i,n+1}^{P}(E-E_0)}$$

$$P_\mu^{\mathrm{NP}}(E) = \sum_{j=0}^{n} g_{\mu,j}^{\mathrm{NP}} \left(\frac{E - E_0}{m_N}\right)^j e^{-g_{\mu,n+1}^{\mathrm{NP}}(E-E_0)}$$

- $E_0 = 1077$ MeV
- $g_{i,j}^{\mathrm{P}}$, $g_{\mu,j}^{\mathrm{NP}}$: fit parameter
- $e^{-g(E-E_0)}$: appropriate high energy behavior
- $n = 3$

back

The scattering potential: s-channel resonances

$$V^{\mathrm{P}} = \sum_{i=0}^{n} \frac{\gamma^a_{\mu;i}\,\gamma^c_{\nu;i}}{z - m_i^b}$$

- i: resonance number per PW
- $\gamma^c_{\nu;i}$ ($\gamma^a_{\mu;i}$): creation (annihilation) vertex function with **bare coupling** f (free parameter)
- z: center-of-mass energy
- m_i^b: **bare mass** (free parameter)

- $J \leq 3/2$:

  $\gamma^c_{\nu;i}$ ($\gamma^a_{\mu;i}$) from effective $\mathcal{L}$

Vertex	$\mathcal{L}_{int}$
$N^*(S_{11})N\pi$	$\frac{f}{m_\pi}\,\bar\Psi_{N*}\,\gamma^\mu\,\vec\tau\,\partial_\mu\vec\pi\,\Psi$ + h.c.
$N^*(S_{11})N\eta$	$\frac{f}{m_\pi}\,\bar\Psi_{N*}\,\gamma^\mu\,\partial_\mu\eta\,\Psi$ + h.c.
$N^*(S_{11})N\rho$	$f\,\bar\Psi_{N*}\,\gamma^5\gamma^\mu\,\vec\tau\,\vec\rho_\mu\,\Psi$ + h.c.
$N^*(S_{11})\Delta\pi$	$\frac{f}{m_\pi}\,\bar\Psi_{N*}\,\gamma^5\vec S\partial_\mu\vec\pi\,\Delta^\mu$ + h.c.

- $5/2 \leq J \leq 9/2$:
  correct dependence on L (centrifugal barrier)

$$(\gamma^{a,c})_{\frac{5}{2}-} = \frac{k}{M}\,(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{5}{2}+} = \frac{k}{M}\,(\gamma^{a,c})_{\frac{3}{2}-}$$

$$(\gamma^{a,c})_{\frac{7}{2}-} = \frac{k^2}{M^2}\,(\gamma^{a,c})_{\frac{3}{2}-} \qquad (\gamma^{a,c})_{\frac{7}{2}+} = \frac{k^2}{M^2}\,(\gamma^{a,c})_{\frac{3}{2}+}$$

$$(\gamma^{a,c})_{\frac{9}{2}-} = \frac{k^3}{M^3}\,(\gamma^{a,c})_{\frac{3}{2}+} \qquad (\gamma^{a,c})_{\frac{9}{2}+} = \frac{k^3}{M^3}\,(\gamma^{a,c})_{\frac{3}{2}-}$$

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The scattering potential: t- and u-channel exchanges

	πN	ρN	ηN	πΔ	σN	KΛ	KΣ
πN	N,Δ,$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$	N, Δ, Ct., π, ω, a_1	N, a_0	N, Δ, ρ	N, π	Σ, Σ*, K*	Λ, Σ, Σ*, K*
ρN		N, Δ, Ct., ρ	-	N, π	-	-	-
ηN			N, f_0	-	-	K*, Λ	Σ, Σ*, K*
πΔ				N, Δ, ρ	π	-	-
σN					N, σ	-	-
KΛ						Ξ, Ξ*, f_0, ω, ϕ	Ξ, Ξ*, ρ
KΣ							Ξ, Ξ*, f_0, ω, ϕ, ρ

Free parameters: cutoffs Λ in the form factors: $F(q) = \left(\frac{\Lambda^2 - m_x^2}{\Lambda^2 + \vec{q}^{\,2}}\right)^n$, $n = 1,\ 2$

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Interaction potential from effective Lagrangian

J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); U.-G. Meißner, Phys. Rept. **161**, 213 (1988); B. Borasoy and U.-G. Meißner, Int. J. Mod. Phys. A **11**, 5183 (1996).

- consistent with the approximate (broken) chiral $SU(2) \times SU(2)$ symmetry of QCD

Vertex	$\mathcal{L}_{int}$	Vertex	$\mathcal{L}_{int}$
$NN\pi$	$-\frac{g_{NN\pi}}{m_\pi}\Psi\gamma^5\gamma^\mu\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi$	$NN\omega$	$-g_{NN\omega}\bar{\Psi}[\gamma^\mu - \frac{\kappa_\omega}{2m_N}\sigma^{\mu\nu}\partial_\nu]\omega_\mu\Psi$
$N\Delta\pi$	$\frac{g_{N\Delta\pi}}{m_\pi}\bar{\Delta}^\mu\vec{S}^\dagger\cdot\partial_\mu\vec{\pi}\Psi$ + h.c.	$\omega\pi\rho$	$\frac{g_{\omega\pi\rho}}{m_\omega}\epsilon_{\alpha\beta\mu\nu}\partial^\alpha\vec{\rho}^\beta\cdot\partial^\mu\vec{\pi}\omega^\nu$
$\rho\pi\pi$	$-g_{\rho\pi\pi}(\vec{\pi}\times\partial_\mu\vec{\pi})\cdot\vec{\rho}^\mu$	$N\Delta\rho$	$-i\frac{g_{N\Delta\rho}}{m_\rho}\bar{\Delta}^\mu\gamma^5\gamma^\mu\vec{S}^\dagger\cdot\vec{\rho}_{\mu\nu}\Psi$ + h.c.
$NN\rho$	$-g_{NN\rho}\Psi[\gamma^\mu - \frac{\kappa_\rho}{2m_N}\sigma^{\mu\nu}\partial_\nu]\vec{\tau}\cdot\vec{\rho}_\mu\Psi$	$\rho\rho\rho$	$g_{NN\rho}(\vec{\rho}_\mu\times\vec{\rho}_\nu)\cdot\vec{\rho}^{\mu\nu}$
$NN\sigma$	$-g_{NN\sigma}\bar{\Psi}\Psi\sigma$	$NN\rho\rho$	$\frac{\kappa_\rho g^2_{NN\rho}}{2m_N}\bar{\Psi}\sigma^{\mu\nu}\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\rho}_\nu)$
$\sigma\pi\pi$	$\frac{g_{\sigma\pi\pi}}{2m_\pi}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\sigma$	$\Delta\Delta\pi$	$\frac{g_{\Delta\Delta\pi}}{m_\pi}\bar{\Delta}_\mu\gamma^5\gamma^\nu\vec{T}\Delta^\mu\partial_\nu\vec{\pi}$
$\sigma\sigma\sigma$	$-g_{\sigma\sigma\sigma}m_\sigma\sigma\sigma\sigma$	$\Delta\Delta\rho$	$-g_{\Delta\Delta\rho}\bar{\Delta}_\tau(\gamma^\mu - i\frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta}\sigma^{\mu\nu}\partial_\nu)$
			$\cdot\vec{\rho}_\mu\cdot\vec{T}\Delta^\tau$
$NN\rho\pi$	$\frac{g_{NN\pi}}{m_\pi}2g_{NN\rho}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi(\vec{\rho}_\mu\times\vec{\pi})$	$NN\eta$	$-\frac{g_{NN\eta}}{m_\pi}\bar{\Psi}\gamma^5\gamma^\mu\partial_\mu\eta\Psi$
NNa_1	$-\frac{g_{NN\pi}}{m_\pi}m_{a_1}\bar{\Psi}\gamma^5\gamma^\mu\vec{\tau}\Psi\vec{a}_\mu$	NNa_0	$g_{NNa_0}m_\pi\bar{\Psi}\vec{\tau}\Psi\vec{a}_0$
$a_1\pi\rho$	$-\frac{2g_{\pi a_1\rho}}{m_{a_1}}[\partial_\mu\vec{\pi}\times\vec{a}_\nu - \partial_\nu\vec{\pi}\times\vec{a}_\mu]\cdot[\partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu]$	$\pi\eta a_0$	$g_{\pi\eta a_0}m_\pi\eta\vec{\pi}\cdot\vec{a}_0$
	$+\frac{2g_{\pi a_1\rho}}{2m_{a_1}}[\vec{\pi}\times(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)]\cdot[\partial^\mu\vec{a}^\nu - \partial^\nu\vec{a}^\mu]$		

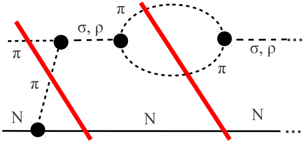
Theoretical constraints of the S-matrix

Unitarity: probability conservation

- 2-body unitarity
- 3-body unitarity:

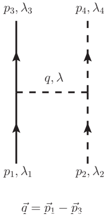
  discontinuities from t-channel exchanges

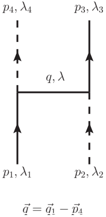
  → Meson exchange from requirements of the S-matrix [Aaron, Almado, Young, Phys. Rev. 174, 2022 (1968)]

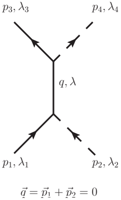


Analyticity: from unitarity and causality

- correct structure of branch point, right-hand cut (real, dispersive parts)
- to approximate left-hand cut → Baryon u-channel exchange







→ Resonances