On the nature of the N^* and Δ resonances via coupled-channel dynamics

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Outline

Based on arXiv: 2307.06799 [nucl-th]
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Introduction

Criteria of Compositeness

Weinberg's criterion

- Hadrons $\to$
  quarks & gluons v.s. hadron exchanges?
- Weinberg's criterion (deuteron) [Weinberg, PR 137, B672 (1965)]

$$a = -\frac{2(1-Z)}{2-Z}R + \mathcal{O}(L)$$
$$r = -\frac{Z}{1-Z}R + \mathcal{O}(L)$$

  Z: "elementariness". $R \sim 4.3$ fm: deuteron radius.
  $L \sim m_\pi^{-1}$ interaction range.
  a: scattering length. r: effective range.

- Conditions:
  - S-wave
  - near-threshold
  - stable

Modern generalization

- Modern hadron spectroscopy & exotic states $\to$ unstable resonances, coupled-channel
- Deviation from naive quark models $\to$ likely hadronic molecules [Guo *et al.*, RMP 90, 015004 (2018)]
- Criteria
  - pole-counting rule [Morgan, NPA 543, 632 (1992)]: hadron exchanges $\to$ one near-threshold pole (S-wave only)
  - spectral density functions [Baru *et al.*, PLB 586, 53 (2004)]
  - complex elementariness of Gamow states
    [Hyodo, PRL 111, 132002 (2013)] [Guo and Oller, PRD 93, 096001 (2016)]
    [Sekihara, PRC 95, 025206 (2017)]
- Higher partial waves, broad resonances, not close to thresholds $\to$ extremely difficult
- Comprehensive data driven models
  $\to$ meaningful results

Introduction

Spectroscopy & Jülich-Bonn Model

(See also Deborah's talk tomorrow)

- Jülich-Bonn model $\rightarrow$ coupled-channel partial-wave analyses and resonance extraction
- Hadron exchange potentials + s-channel states + scattering equation
- Fit to a worldwide collection of data
- Unitarity, Analyticity $\rightarrow$ searching resonance poles on the second sheet [Döring *et. al.*, NPA 829, 170 (2009)]
- Applications
  - Hadronic part: πN induced reactions $\rightarrow$ N^* and Δ spectral

    [Schütz *et. al.*, PRC 51, 1374 (1995)] [Schütz *et. al.*, PRC 49, 2671 (1994)][Schütz *et. al.*, PRC 57, 1464 (1998)] [Krehl *et. al.*, PRC 62, 025207 (2000)]

    [Gasparyan *et. al.*, PRC 68, 045207 (2003)][Döring *et. al.*, NPA 851, 58 (2011)] [Rönchen *et. al.*, EPJA 49, 44 (2013)][Wang *et. al.*, PRD 106, 094031 (2022)]
  - Photoproduction $\rightarrow$ much more data

    [Rönchen *et. al.*, EPJA 50, 101 (2014)] [Rönchen *et. al.*, EPJA 51, 70 (2015)][Rönchen *et. al.*, EPJA 54, 110 (2018)] [Rönchen *et. al.*, EPJA 58, 229 (2022)]
  - Electroproduction (Jülich-Bonn-Washington) [Mai *et. al.*, PRC 103, 065204 (2021)] [Mai *et. al.*, PRC 106, 015201 (2022)]
  - Hidden charm sector and P_c states [Shen *et. al.*, CPC 42, 023106 (2018)] [Wang *et. al.*, EPJC 82, 497 (2022)]
- The N^* and Δ structures
  - meaningful for understanding QCD
  - spectral density functions & Gamow states $\rightarrow$ direct outputs of the model

Theoretical Framework

Toy model

Hilbert space

- Orthogonal and complete basis
  $\rightarrow$ bare state $|\psi_0\rangle$
  $+$ continuous two-body states $|\psi(\mathbf{k})\rangle$
- Hamiltonian (μ: reduced mass. $E_0 > 0$: bare state energy. g: coupling constant. $F(k,\Lambda) = \frac{\Lambda^2}{k^2+\Lambda^2}$: regulator)

$$\hat{H}_0 = E_0|\psi_0\rangle\langle\psi_0| + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{k^2}{2\mu}|\psi(\mathbf{k})\rangle\langle\psi(\mathbf{k})|$$

$$\hat{H}_I = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, gF(k,\Lambda)|\psi(\mathbf{k})\rangle\langle\psi_0| + h.c.$$

- Easy to be extended to coupled-channels

Solutions

- Eigenstates $|\Phi\rangle = c_0|\psi_0\rangle + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\chi(\mathbf{k})|\psi(\mathbf{k})\rangle$
  - bound states: $\hat{H}|\Phi(B)\rangle = -B|\Phi(B)\rangle$
  - scattering states: $\hat{H}|\Phi(\mathcal{E})\rangle = \mathcal{E}|\Phi(\mathcal{E})\rangle$
  - completeness: $|\Phi(B)\rangle\langle\Phi(B)| + \int_0^{+\infty} d\mathcal{E}|\Phi(\mathcal{E})\rangle\langle\Phi(\mathcal{E})| = \hat{I}$
  - elementariness & compositeness: $Z \sim |c_0|^2$, $X \sim \int \frac{d^3\mathbf{k}}{(2\pi)^3}|\chi(\mathbf{k})|^2$
- Scattering amplitude $\hat{T} = \hat{V} + \hat{V}\hat{G}\hat{T}$, $\hat{V} = \hat{H}_I\frac{|\psi_0\rangle\langle\psi_0|}{\mathcal{E}-E_0+i0^+}\hat{H}_I$
- Elementariness of a bound state: normalization $\langle\Phi(B)|\Phi(B)\rangle = 1$
  $Z = (1-\Sigma')^{-1}$, $\Sigma \equiv \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{g^2F^2(k,\Lambda)}{\mathcal{E}-k^2/(2\mu)}$

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Theoretical Framework

Spectral density functions

- Elementariness of a bound state: $Z_B \equiv |\langle\psi_0|\Phi(B)\rangle|^2$
- Spectral density function (Källén-Lehmann spectral representation): $w(\mathcal{E}) \equiv |\langle\psi_0|\Phi(\mathcal{E})\rangle|^2 = -\frac{1}{\pi}\mathrm{Im}D(\mathcal{E})$
- Bare state propagator: $D(\mathcal{E}) = (\mathcal{E} - E_0 - \Sigma)^{-1}$
- Quasi-bound states $\mathcal{E}_R = E_R - i\frac{\Gamma_R}{2}$ (narrow resonances close to and below a threshold):
  $w(\mathcal{E} \simeq E_R) = \frac{Z_B}{\pi}\frac{\Gamma_R/2}{(\mathcal{E}-E_R)^2+(\Gamma_R/2)^2}$
  Z_B: the elementariness of the bound state when the decay channel is switched off
- When $\Gamma_R \to 0$, $w(\mathcal{E} \simeq E_R) \to Z_B\delta(\mathcal{E} - E_R)$: compatible with Weinberg's criterion
- Estimation of the elementariness ($\Delta E \sim \Gamma_R$):

$$Z \simeq \frac{\int_{E_R-\Delta E}^{E_R+\Delta E} w(\mathcal{E})d\mathcal{E}}{\int_{E_R-\Delta E}^{E_R+\Delta E} BW(\mathcal{E})d\mathcal{E}}, BW(\mathcal{E}) \equiv \frac{1}{\pi}\frac{\Gamma_R/2}{(\mathcal{E}-E_R)^2+(\Gamma_R/2)^2}$$

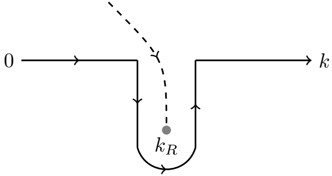
- Applications: [Baru *et al.*, PLB 586, 53 (2004)] [Baru *et al.*, EPJA 44, 93 (2010)][Hanhart *et al.*, PRD 81, 094028 (2010)] [Hanhart *et al.*, EPJA 47, 101 (2011)]...

Theoretical Framework

Gamow states

- Resonances $\to$ poles on the second Riemann sheet
- Analytical continuation $\to$ contour deformation (complex scaling): $\int_0^{+\infty} k^2 dk \to \int_C k^2 dk$
- Gamow states [Gamow, Zeitschrift für Physik 51, 204 (1928)] [Civitarese and Gadella, Physics Reports 396, 41(2004)]
  - with contour deformation when performing inner product: $|\Phi_R) = c_0 \left[|\psi_0\rangle - \int_C \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{gF(k,\Lambda)}{\frac{k^2}{2\mu} - \mathcal{E}_R} |\psi(\mathbf{k})\rangle \right]$
  - complex eigenvalue $\hat{H}|\Phi_R) = \mathcal{E}_R|\Phi_R) \to$ cannot be normalized $\to (\Phi_R^*|\Phi_R) = 1$
  - elementariness: $Z_R = (1 - \Sigma'|_{\text{2nd sheet}})^{-1} \to$ complex, not a probability
- Complex compositeness $\to$ off-shell residues $r(k)$ [Sekihara, PRC 95, 025206 (2017)]:

$$T^{\text{II}}(k, k', \mathcal{E}) = \frac{r(k)r(k')}{\mathcal{E} - \mathcal{E}_R} + \cdots, X_R = 1 - Z_R = \int_C \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{r^2(k)}{[\mathcal{E}_R - k^2/(2\mu)]^2}$$



Theoretical Framework

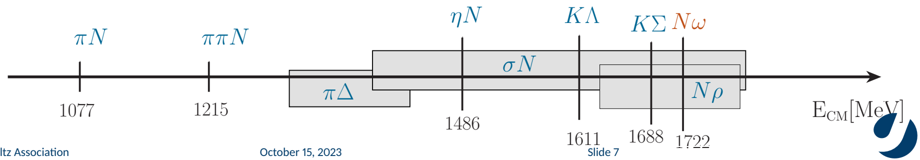
The Jülich-Bonn model

The model

- Scattering Eq. with CM energy z
  $T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$
- Effective three-body channels: ρN, $\pi\Delta$ and σN
- s-channel bare states $\rightarrow$ separation
  $T = T^P + T^{NP}$
- Potentials $\rightarrow$ hadron exchanges
- Technical details [Wang *et. al.*, PRD 106, 094031 (2022)]

Study of the structures

- Compared with the toy model
  - coupled-channel
  - more s-channel bare states
  - hadron-exchange potentials
- Spectral density functions $\rightarrow$ readily extracted from self-energies $w_i(z) = -\frac{1}{\pi}\mathrm{Im}D_i(z)$
- "Total elementariness" $\rightarrow Z = 1 - \prod_i (1 - Z_i)$
- Partial compositeness (Gamow states) $\rightarrow$ off-shell residues
  $X_\kappa = \int_C p^2 dp\, r_\kappa^2(p) G_\kappa^2(p, z_{\text{pole}})$



Numerical results

Numerical details

- Selection of the resonances
  - JüBo2022 solution ($K\Sigma$ photoproduction, no ωN) [Rönchen *et. al.*, EPJA 58, 229 (2022)]
  - For N^* $J \leq 5/2$, for Δ $J \leq 3/2$
  - Width $\Gamma_R < 300$ MeV
- Uncertainties $\to$ comparison of three results
  - spectral density functions directly from the model
  - locally constructed spectral density functions only by the pole position and residues (L_κ: loop functions)

$$w^{\mathrm{lc}}(z) = -\frac{1}{\pi}\mathrm{Im}\left[z - M_0 - \sum_\kappa g_\kappa^2 L_\kappa(z)\right]^{-1}$$

  - complex compositeness of the Gamow state, with naive measure [Sekihara, PRC 104, 035202 (2021)]

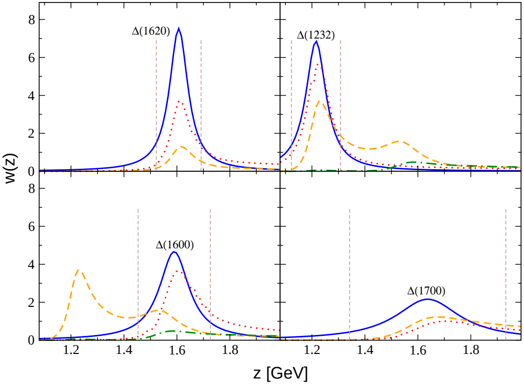
$$\tilde{X}_\kappa \equiv \frac{|X_\kappa|}{\sum_\alpha |X_\alpha| + |Z|}, \tilde{Z} \equiv \frac{|Z|}{\sum_\alpha |X_\alpha| + |Z|}$$

Numerical results

Spectral density functions – Δ states

Blue solid line: the Breit-Wigner denominator. Orange dashed (green dash-dotted) line: the 1st (2nd) spectral density function (model).

Red dotted line: the locally constructed function. Vertical lines: the integral region.



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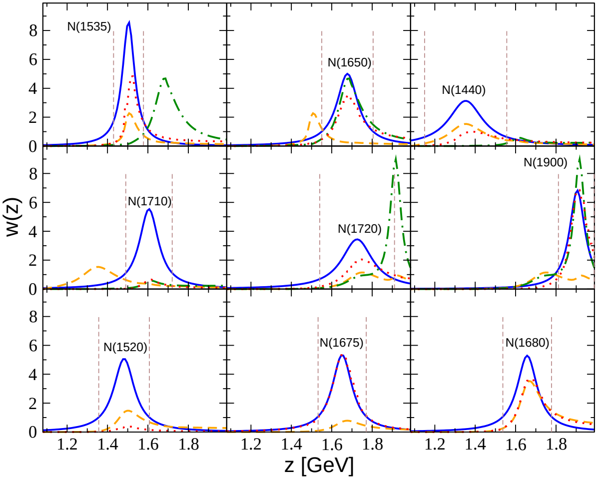
Numerical results

Spectral density functions – N^* states



Numerical results

Compositeness

State	Pole position (MeV)	$\mathcal{Z}_{\text{tot}}$	$\mathcal{Z}^{\text{lc}}$	$\tilde{Z}$
$N(1535)\,\frac{1}{2}^-$	$1504 - 37i$	29.0%	50.8%	39.4%
$N(1650)\,\frac{1}{2}^-$	$1678 - 64i$	92.8%	70.5%	8.5%
$N(1440)\,\frac{1}{2}^+$	$1353 - 102i$	49.5%	31.5%	36.9%
$N(1710)\,\frac{1}{2}^+$	$1605 - 58i$	20.6%	10.2%	40.3%
$N(1720)\,\frac{3}{2}^+$	$1726 - 93i$	79.3%	62.5%	41.4%
$N(1900)\,\frac{3}{2}^+$	$1905 - 47i$	100%	99.9%	38.5%
$N(1520)\,\frac{3}{2}^-$	$1482 - 63i$	29.4%	7.2%	40.4%
$N(1675)\,\frac{5}{2}^-$	$1652 - 60i$	16.6%	(F)	61.8%
$N(1680)\,\frac{5}{2}^+$	$1657 - 60i$	67.9%	69.9%	55.0%
$\Delta(1620)\,\frac{1}{2}^-$	$1607 - 42i$	18.9%	50.0%	69.4%
$\Delta(1232)\,\frac{3}{2}^+$	$1215 - 46i$	53.8%	(F)	30.5%
$\Delta(1600)\,\frac{3}{2}^+$	$1590 - 68i$	47.8%	77.5%	69.7%
$\Delta(1700)\,\frac{3}{2}^-$	$1637 - 148i$	59.7%	44.9%	47.8%

Numerical results

Results & Discussions

Results

- At least two results suggest **high compositeness**: $N(1535)\frac{1}{2}^-$, $N(1440)\frac{1}{2}^+$, $N(1710)\frac{1}{2}^+$, $N(1520)\frac{3}{2}^-$
- At least two results suggest **high elementariness**: $N(1650)\frac{1}{2}^-$, $N(1900)\frac{3}{2}^+$, $N(1680)\frac{5}{2}^+$, $\Delta(1600)\frac{3}{2}^+$
- Hints of compositions (Gamow states)
  - $N^*(1535)$: $X_{\eta N} = 35.8\%$
  - $N^*(1440)$: $X_{\pi N} = 59.0\%$
  - $N^*(1710)$: $X_{\eta N} = 44.9\%$
  - $N^*(1520)$: $X_{\pi\pi N} = 43.7\%$
- The compositions may be model-dependent: in this model all the σN bare couplings are switched off

On the states

- $N(1535)\frac{1}{2}^-$
  - might be dynamically generated [Kaiser *et. al.*, PLB 362, 23 (1995)] [Bruns *et. al.*, PLB 697, 254 (2011)]
  - ωN might be important [Wang *et. al.*, PRD 106, 094031 (2022)]
- $N(1440)\frac{1}{2}^+$
  - always dynamically generated in this model [Krehl *et. al.*, PRC 62, 025207 (2000)]
  - "radial excitation of the nucleon" is not favoured [Meißner and Durso, NPA 430, 670 (1984)]
  - a large σN component [Sekihara, PRC 104, 035202 (2021)]
- $\Delta(1232)\frac{3}{2}^+$: three Δ's (*s*-channel bare, initial/final $\pi\Delta$, *u*-channel exchange) $\to$ technical difficulties in this study

Conclusion and Outlook

- Jülich-Bonn model is a comprehensive coupled-channel model for partial-wave analyses and resonance extraction.
- In this work we let the model say something more: the information of the structures of the N^* and Δ states.
- The results for 8 states are relatively certain:
  - tend to be composite – $N^*(1535)$, $N^*(1440)$, $N^*(1710)$, $N^*(1520)$;
  - tend to be elementary – $N^*(1650)$, $N^*(1900)$, $N^*(1680)$, $\Delta(1600)$.
- Open questions:
  - understanding the relation between the spectral density functions and the Gamow states, e.g. the results for $N^*(1650)$;
  - understanding different descriptions, e.g. hadronic models and quark models.
- In the future:
  - study of the P_c states;
  - upgrade of the results when ωN photoproduction is included.


Thank
you

Backups

Details of the model

Dynamics I

Central part of this model: hadronic (πN induced) reactions

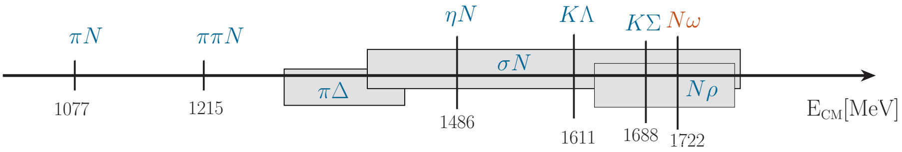
The Lippmann-Schwinger-like equation (CM frame)

$$T_{\mu\nu}(p'', p', z) = V_{\mu\nu}(p'', p', z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'', p, z) G_\kappa(p, z) T_{\kappa\nu}(p, p', z)$$

- Reaction channels $\nu \to \kappa \to \mu$ (after PW and isospin projection, JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)],$J \leq 9/2$)
- Propagator: G ($\pi\pi N$ channel: effective channels $\rho N, \sigma N, \pi\Delta$. E/ω - energy of the baryon/meson.)

$$G_\kappa(z,p) = \begin{cases} (z - E_\kappa - \omega_\kappa + i0^+)^{-1} & \text{(if } \kappa \text{ is a two-body channel)}\,, \\ \left[z - E_\kappa - \omega_\kappa - \Sigma_\kappa(z,p) + i0^+\right]^{-1} & \text{(if } \kappa \text{ is an effective channel)}\,. \end{cases}$$

- Second Riemann sheet $\to$ analytical continuation of G [Döring *et. al.*, NPA 829, 170 (2009)]
- Observables $\to$ dimensionless amplitude $\tau_{\mu\nu} = -\pi\sqrt{\rho_\mu \rho_\nu} T_{\mu\nu}$, $\rho \to$ kinematic factor
- Separating the amplitude $\to$ with/without s-channel poles $T = T^P + T^{NP}$

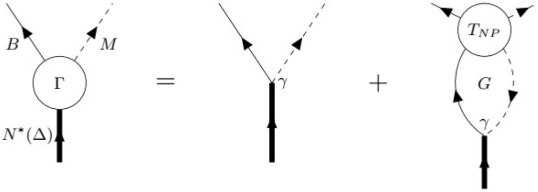


Details of the model

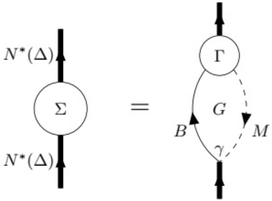
Dynamics II

- $T^{NP} = V^{NP} + \sum \int p^2 dp V^{NP} G T^{NP}$
- $T^{P}_{\mu\nu}(p'', p', z) = \sum_{i,j} \Gamma^{a}_{\mu,i}(p'') D_{ij}(z) \Gamma^{c}_{\nu,j}(p')$, $(D^{-1})_{ij} = \delta_{ij}(z - m_i^b) - \Sigma_{ij}(z)$
  - $\Gamma(\gamma)$: the dressed (bare) vertices (a - annihilation, c - creation)
  - Σ: self-energy functions (nucleon mass renormalization)
- $V^{NP}, \gamma \rightarrow$ constructed from effective Lagrangians $+$ regulators (cut-offs) (details: Supplemental material of [Wang *et. al.*, PRD 106, 094031 (2022)])
- s-channel contact terms: $D \sim (1 - \Sigma)^{-1}$

  [Rönchen *et. al.*, EPJA 51, 70 (2015)]



(a) The vertex.



(b) The self energy.

Local construction of the spectral density functions

- Local simulation of the amplitude

$$T^{\mathrm{lc}}_{\alpha\beta}(z) = \frac{c g_\alpha g_\beta f^a_\alpha(q_{\alpha z}) f^c_\beta(q_{\beta z})}{z - M_0 - \sum_\kappa g^2_\kappa L_\kappa(z)} + \cdots$$

- Loop functions (f: vertex function in this model)

$$L_\kappa(z) \equiv \int_0^\infty p^2 dp\, G_\kappa(p, z) f^a_\alpha(q_{\kappa z}) f^c_\alpha(q_{\kappa z})$$

- Parameters

$$h_\kappa \equiv \frac{g^2_\kappa}{g^2_1} = \left|\frac{r_\kappa f^a_1}{r_1 f^a_\kappa}\right|^2 ,$$

$$g^2_1 = -\frac{\Gamma_R}{2\sum_\kappa h_\kappa \mathrm{Im}(L^{II}_\kappa)} ,$$

$$M_0 = M_R - g^2_1 \sum_\kappa h_\kappa \mathrm{Re}(L^{II}_\kappa) ,$$

$$c = \frac{r^2_1}{g^2_1 f^a_1 f^c_1}\left(1 - g^2_1 \sum_\kappa h_\kappa \frac{d}{dz} L^{II}_\kappa \Big|_{z=M_R - i\Gamma_R/2}\right) .$$

- $g^2 < 0 \to$ construction fails. Estimation:

$$g^2_\kappa \to \left|\sqrt{\frac{2\pi\rho_\kappa}{\Gamma_R}} r_\kappa\right|^2 , M_0 \to M_0 - i\frac{\Gamma_0}{2}$$

Complex compositions of N^*'s – two-body channels

State	$X_{\pi N}$	$X_{\eta N}$	$X_{K\Lambda}$	$X_{K\Sigma}$	Z
$N(1535)\,\frac{1}{2}^-$	$0.12+0.14i$ (7.5%)	$0.67+0.57i$ (35.8%)	$0.03-0.01i$ (1.3%)	$0.01-0.03i$ (1.2%)	$0.32-0.92i$ (39.4%)
$N(1650)\,\frac{1}{2}^-$	$0.17+0.28i$ (22.2%)	$0.60-0.15i$ (41.4%)	$0.03+0.03i$ (3.0%)	$0.15-0.03i$ (10.2%)	$0.12+0.02i$ (8.5%)
$N(1440)\,\frac{1}{2}^+$	$0.69+0.37i$ (59.0%)	$0.00+0.00i$ (0.2%)	$0.00-0.00i$ (0.0%)	$0.00+0.00i$ (0.0%)	$0.34-0.36i$ (36.9%)
$N(1710)\,\frac{1}{2}^+$	$0.04-0.00i$ (1.4%)	$0.87+0.97i$ (44.9%)	$-0.10+0.18i$ (6.9%)	$-0.00+0.01i$ (0.3%)	$0.02-1.17i$ (40.3%)
$N(1720)\,\frac{3}{2}^+$	$-1.16-3.01i$ (32.6%)	$0.04-0.04i$ (0.5%)	$0.03+0.05i$ (0.6%)	$-0.08+0.13i$ (1.5%)	$2.71+3.09i$ (41.4%)
$N(1900)\,\frac{3}{2}^+$	$-0.00+0.00i$ (0.1%)	$-0.02+0.00i$ (0.2%)	$0.39+0.33i$ (6.0%)	$0.90-0.17i$ (10.7%)	$-3.04+1.22i$ (38.5%)
$N(1520)\,\frac{3}{2}^-$	$0.19+0.39i$ (15.9%)	$-0.00+0.00i$ (0.0%)	$0.00+0.00i$ (0.0%)	$0.00-0.00i$ (0.0%)	$0.87-0.66i$ (40.4%)
$N(1675)\,\frac{5}{2}^-$	$0.02+0.17i$ (9.4%)	$0.02-0.06i$ (3.3%)	$-0.00+0.00i$ (0.0%)	$0.00+0.00i$ (0.0%)	$1.01-0.49i$ (61.8%)
$N(1680)\,\frac{5}{2}^+$	$0.22+0.50i$ (36.3%)	$-0.00-0.00i$ (0.0%)	$0.00-0.00i$ (0.0%)	$0.00-0.00i$ (0.0%)	$0.68-0.48i$ (55.0%)

Complex compositions of N^*'s – effective channels

State	$X_{\rho N}(1)$	$X_{\rho N}(2)$	$X_{\rho N}(3)$	$X_{\pi\Delta}(1)$	$X_{\pi\Delta}(2)$
$N(1535)\ \frac{1}{2}^-$	$-0.17+0.27i$ (13.0%)	$\cdots$	$-0.00+0.00i$ (0.2%)	$\cdots$	$0.02-0.04i$ (1.6%)
$N(1650)\ \frac{1}{2}^-$	$-0.02-0.11i$ (7.4%)	$\cdots$	$-0.01-0.06i$ (4.3%)	$\cdots$	$-0.04+0.02i$ (3.0%)
$N(1440)\ \frac{1}{2}^+$	$-0.00+0.01i$ (0.6%)	$-0.01+0.00i$ (0.6%)	$\cdots$	$-0.02-0.03i$ (2.7%)	$\cdots$
$N(1710)\ \frac{1}{2}^+$	$0.00-0.00i$ (0.0%)	$-0.01+0.01i$ (0.3%)	$\cdots$	$0.17+0.01i$ (5.9%)	$\cdots$
$N(1720)\ \frac{3}{2}^+$	$0.19-0.07i$ (2.1%)	$0.58-0.07i$ (5.9%)	$0.10-0.07i$ (1.2%)	$-1.40+0.01i$ (14.1%)	$-0.01-0.01i$ (0.1%)
$N(1900)\ \frac{3}{2}^+$	$0.01-0.01i$ (0.1%)	$3.04-1.60i$ (40.3%)	$-0.00-0.00i$ (0.0%)	$-0.27+0.22i$ (4.1%)	$-0.00-0.00i$ (0.0%)
$N(1520)\ \frac{3}{2}^-$	$0.00+0.00i$ (0.1%)	$-0.01+0.01i$ (0.5%)	$-0.58+0.26i$ (23.5%)	$0.01-0.00i$ (0.3%)	$0.52-0.00i$ (19.3%)
$N(1675)\ \frac{5}{2}^-$	$-0.00-0.00i$ (0.1%)	$-0.14+0.26i$ (16.2%)	$0.00-0.01i$ (0.4%)	$0.10+0.12i$ (8.8%)	$-0.00+0.00i$ (0.0%)
$N(1680)\ \frac{5}{2}^+$	$0.00-0.01i$ (0.4%)	$0.00-0.00i$ (0.2%)	$-0.01-0.00i$ (0.7%)	$0.00+0.00i$ (0.0%)	$0.11-0.01i$ (7.4%)

Complex compositions of Δ's

State	$X_{\pi N}$	$X_{\rho N}(1)$	$X_{\rho N}(2)$	$X_{\rho N}(3)$	$X_{\pi\Delta}(1)$	$X_{\pi\Delta}(2)$	$X_{K\Sigma}$	Z
$\Delta(1620)\ \frac{1}{2}^-$	$0.10-0.03i$ (8.6%)	$-0.02-0.09i$ (8.0%)	$\cdots$	$-0.00-0.00i$ (0.3%)	$\cdots$	$0.13-0.09i$ (13.0%)	$0.01+0.00i$ (0.7%)	$0.79+0.20i$ (69.4%)
$\Delta(1232)\ \frac{3}{2}^+$	$0.63+1.16i$ (28.5%)	$0.02-0.01i$ (0.4%)	$-0.00+0.01i$ (0.2%)	$0.06-0.03i$ (1.4%)	$1.54-0.58i$ (35.4%)	$-0.01-0.01i$ (0.3%)	$-0.00+0.15i$ (3.3%)	$-1.24-0.70i$ (30.5%)
$\Delta(1600)\ \frac{3}{2}^+$	$-0.04+0.07i$ (3.5%)	$0.00-0.01i$ (0.3%)	$0.01-0.03i$ (1.3%)	$-0.00-0.00i$ (0.1%)	$-0.42+0.21i$ (21.0%)	$-0.00-0.00i$ (0.0%)	$-0.08+0.05i$ (4.1%)	$1.53-0.29i$ (69.7%)
$\Delta(1700)\ \frac{3}{2}^-$	$-0.03+0.05i$ (2.3%)	$-0.00+0.01i$ (0.4%)	$-0.03+0.02i$ (1.3%)	$-0.03+0.00i$ (1.3%)	$-0.01-0.03i$ (1.5%)	$0.45-0.95i$ (45.4%)	$0.00+0.00i$ (0.0%)	$0.64+0.91i$ (47.8%)

TABLE VII: The compositeness and elementariness of the selected Δ Gamow states. The percentages in the brackets are the naive measures from Eq. ($\boxed{87}$). The meaning of the channel indices are: $\rho N(1) \to |J-L| = \frac{1}{2}, S = \frac{1}{2}$; $\rho N(2) \to |J-L| = \frac{1}{2}, S = \frac{3}{2}$; $\rho N(3) \to |J-L| = \frac{3}{2}, S = \frac{3}{2}$; $\pi\Delta(1) \to |J-L| = \frac{1}{2}$; $\pi\Delta(2) \to |J-L| = \frac{3}{2}$.

Three $\Delta(1232)$'s in this model

- s-channel $\Delta(1232)$
  - generates the final $\Delta(1232)$ pole
  - spectral density functions $\rightarrow$ mostly elementary (53.8%)
  - complex compositeness $\rightarrow$ large πN and $\pi\Delta$ component (28.5%, 35.7%)
- u-channel $\Delta(1232)$: naive stable particle with mass $m = 1232$ MeV
- initial/final state $\Delta(1232)$
  - only simulating the πN system
  - spectral density functions $\rightarrow$ 69.6% elementariness
  - complex compositeness $\rightarrow$ $X_{\pi N} = 0.36 + 0.32i$, $Z \sim 59.5\%$