Light hybrid mesons and light glueballs

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Outline

- From QCD to conventional mesons: brief review
- A chiral model for mesons: eLSM
- Light hybrids: masses and decays
- Decay of the J/Psi into (isoscalar) hybrids
- Light glueballs: status and new results
- Conclusions

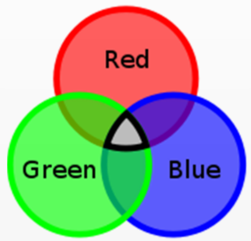
Symmetries of QCD

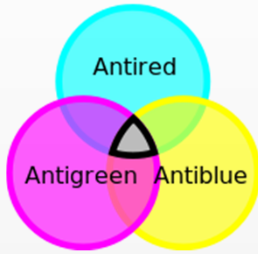
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



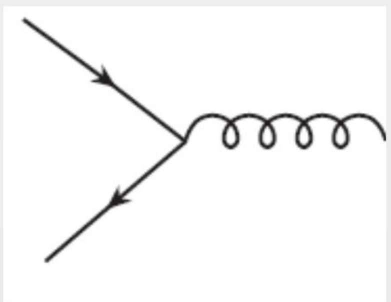


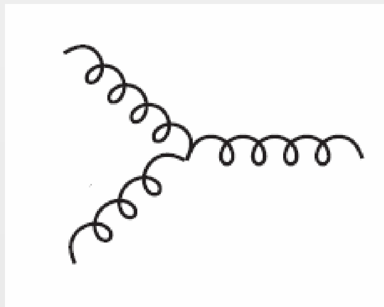
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \; i = u,d,s \,,\ldots$$

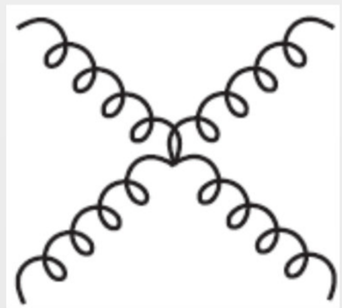
$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}}$$

8 type of gluons (R$\bar{G}$,B$\bar{G}$,…)

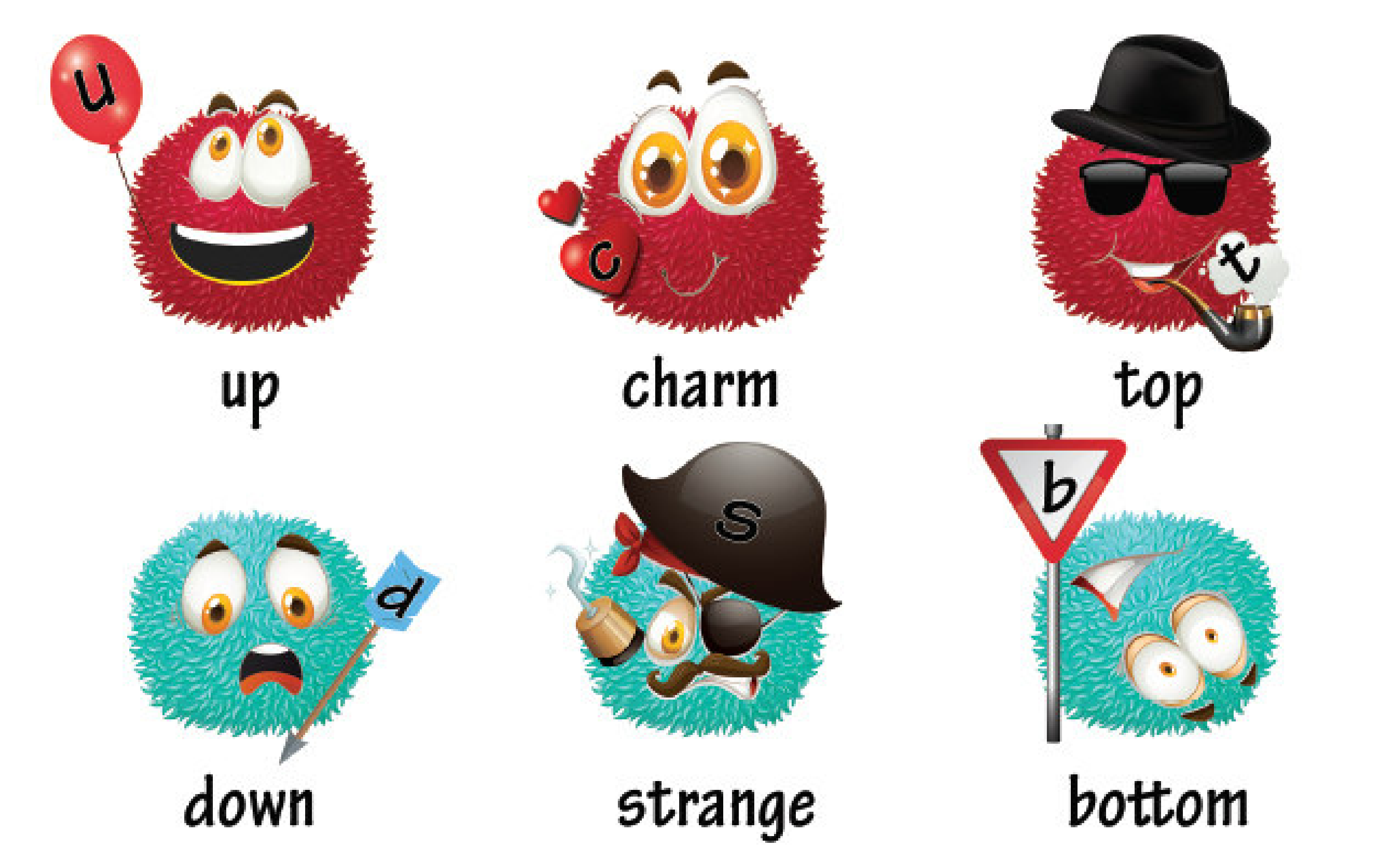
$$A_\mu^a \; ; \; a = 1,\ldots,8$$





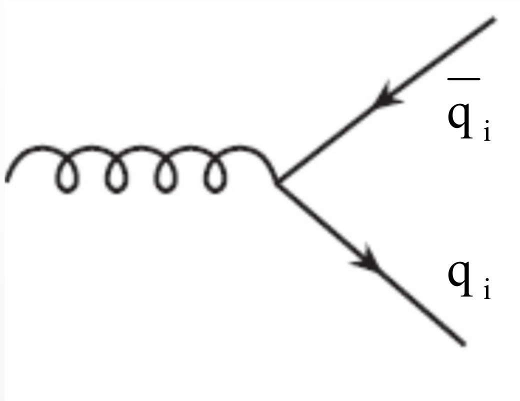


Confinement: quarks never ‘seen’ directly.
How they might look like ☺



Picture by Pawel Piotrowski

Flavor symmetry



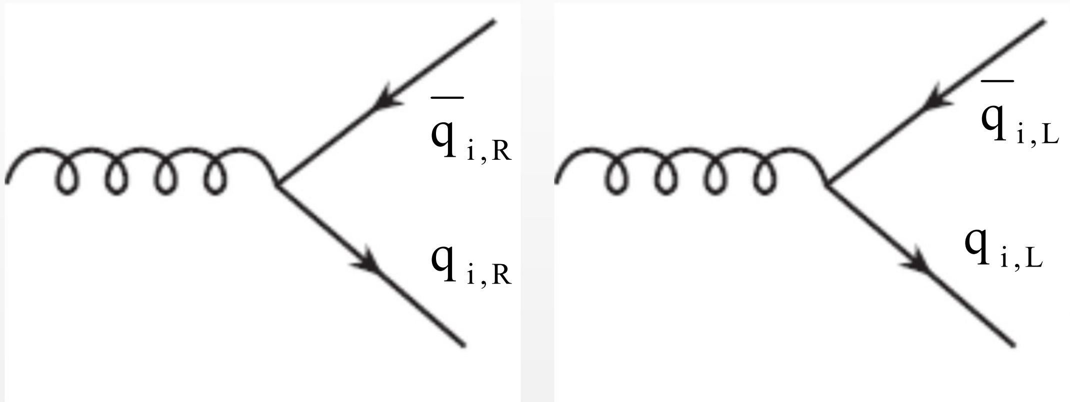
Gluon-quark-antiquark vertex

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$U \in U(3)_V \rightarrow U^+ U = 1$$

Chiral symmetry



$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U_{ij}^{R} q_{j,R} + U_{ij}^{L} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\boxed{\partial^{\mu}(\bar{q}^{i}\gamma_{\mu}\gamma_{5}q^{i}) = \frac{3g^2}{16\pi^2}\,\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})}$$

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

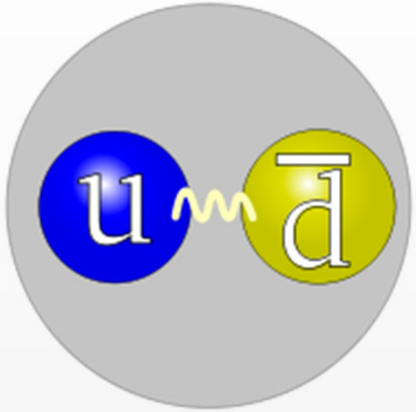
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + \ldots\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Rho-meson $m_{\rho^+} = 775 \text{ MeV}$

Pion $m_{\pi^+} = 139 \text{ MeV}$

$$m_u + m_d \approx 7 \text{ MeV}$$

Mass generation in QCD is a nonpert. penomenon based on SSB
(mentioned previusly).

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Motivation for the extended Linear Sigma Model (eLSM)

• Development of a a (chirally symmetric) linear sigma model for mesons and baryons including (axial-)vector, (axial-)tensor, hybrids, and –of course- glueballs

• Study of the model for T = μ =0 (spectroscopy in vacuum)

(decays, scattering lengths,…)

Interrelation between these two aspects!

• Second goal: properties at nonzero T and μ

(Condensates and masses in thermal/matter medium,…)

Fields of the eLSM

- Quark-antiquark mesons: scalar, pseudoscalar, vector and axial-vector quarkonia.

- Additional mesons: The scalar and the pseudoscalar glueballs

- Baryons: nucleon doublet and its partner

(in the so-called mirror assignment)

We construct the Lagrangian of the so-called Extended Linear Sigma Model (ELSM) according to

chiral invariance and dilatation symmetry and their explicit breakings.

Fields of the model

Field in eLSM	Assignment (predom.) [18]	Flavor content	I	J^{PC}
$\boldsymbol{a_0}$	$a_0(1450)$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K_0^{*[\pm,0]}}$	$K_0^*(1430)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{++}
$\sigma_N,\ \sigma_S$	$f_0(1370),\ f_0(1500)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	
$\boldsymbol{\pi}$	$\{\pi^0,\pi^\pm\}$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K^{[\pm,0]}}$	$K^{[0,\pm]}$, $K(1460)$, $K(1630)$, $K(1830)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{-+}
$\eta_N,\ \eta_S$	$\eta(547),\ \eta'(958)$, $\eta(1295)$, $\eta(1405)$, $\eta(1475)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	
$\boldsymbol{\rho^\mu}$	$\rho(770)$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K^{*\mu}},\ \boldsymbol{\bar{K}^{*\mu}}$	$K^*(892)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{-+}
$\omega_N^\mu,\ \omega_S^\mu$ (small mixing angle)	$\omega(782),\ \phi(1020)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	
$\boldsymbol{a_1^\mu}$	$a_1(1260)$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K_1^\mu},\ \boldsymbol{\bar{K}_1^\mu}$	$K_{1,A}\equiv K_1(1270)$, $K_1(1400)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{-+}
$f_{1N}^\mu,\ f_{1S}^\mu$ (small mixing angle)	$f_1(1285),\ f_1(1420)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	

and, in addition, the scalar/dilaton glueball G (plus evt other glueballs)

Model of QCD – eLSM

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right) + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi)\right] \\ &-m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\Phi^\dagger\Phi\right] - \lambda_1(\mathrm{Tr}\left[\Phi^\dagger\Phi\right])^2 - \lambda_2\mathrm{Tr}\left[(\Phi^\dagger\Phi)^2\right] \\ &+\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)\left((L^\mu)^2+(R^\mu)^2\right)\right] \\ &-\frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2+(R^{\mu\nu})^2\right] + \mathrm{Tr}\left[H\left(\Phi^\dagger+\Phi\right)\right] \\ &+c_1[\det(\Phi)-\det(\Phi^\dagger)]^2 + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\ &+h_2\mathrm{Tr}[\Phi^\dagger L_\mu L^\mu\Phi + \Phi R_\mu R^\mu\Phi^\dagger] + 2h_3\mathrm{Tr}[\Phi R_\mu\Phi^\dagger L^\mu]\end{aligned}$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N+a_0^0)+i(\eta_N+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N-a_0^0)+i(\eta_N-\pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$L^\mu, R^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N}\pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{\star +} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N}\mp a_1^0}{\sqrt{2}} & K^{\star 0} \pm K_1^0 \\ K^{\star -} \pm K_1^- & \bar{K}^{\star 0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011)**
D. Parganlija, P. Kovacs, G. Wolf , F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

Meson phenomenology - literature

1) N_f = 2 (with frozen glueball): Parganlija FG DHR **PRD82 (2010) 054024**

2) N_f = 2 (with glueball): Janowski Parganlija FG DHR **PRD84 (2011) 054007**

3) N_f = 3 (with frozen glueball): Parganlija Kovacs Wolf FG DHR **PRD87 (2013) 014011**

4) N_f = 3 (with glueball): Janowski FG DHR **PRD90 (2014) 114005**

5) Pseudoscalar glueball: Eshraim Janowski FG DHR **PRD87 (2013) 054036** Eshraim Schramm **PRD95 (2017) 014028**

6) N_f =4: Eshraim FG DHR **EPJ.A51 (2015) no.9,** Eshraim Fischer **112 EPJ A54 (2018) 139**

7) Vector glueball: Sammet Janowski FG **PRD95 (2017) no.11, 114004**

8) Excited (pseudo)scalar mesons: Parganlija FG **Eur.Phys.J. C77 (2017) 450**

9) Consistency with ChPT: Divotgey Kovacs FG DHR **Eur.Phys.J. A54 (2018) 5**

10) $f_0(500)$ as a four-quark in the vacuum: Lakaschus Mauldin FG DHR **arxiv: 1807.03735**

11) Hybrid mesons Eshraim et al., **EPJ+135 (2020) no.12, 945 arXiv:2001.06106**

12) Tensor mesons and tensor glueballs Vereijken et al, **PRD108 (2023) no.1, 014023 arXiv:2304.05225**
SJafarzade et al. **PRD106 (2022) no.3, 036008 [arXiv:2203.16585**

Baryon phenomenology - literature

1) Baryonic eLSM for $N_f = 2$: Gallas FG DHR **PRD82 (2010) 014004,** Gallas FG **IJMP.A29 (2014) 1450098**

2) Nucleon-nucleon scattering: Teilab Deinet FG DHR **Phys.Rev. C94 (2016) 044001**

3) $N_f = 3$ (with four multiplets): Olbrich Zetenyi FG DHR **Phys.Rev. D93 (2016) 034021**

4) $N_f = 3$ and axial-anomaly for baryons: Olbrich Zetenyi FG DHR **Phys.Rev. D97 (2018) no.1, 014007**

5) Nuclear matter: Gallas Pagliara FG **Nucl.Phys. A872 (2011) 13-24**

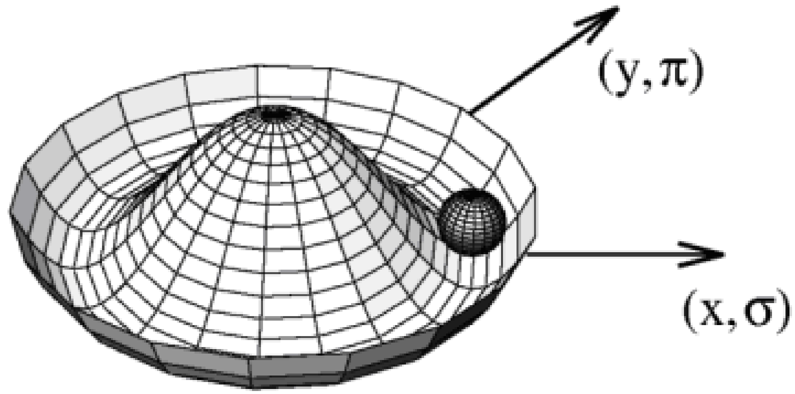
6) Inhomogenous condensation in nuclear matter: Heinz FG DHR **Nucl.Phys. A933 (2015) 34-42**

7) $N_f = 3$ Olbrich et al, **PRD93 (2016) no.3, 034021 ;PRD97 (2018) no.1, 014007**

Nonzero temperature and density (and critical endpoint) - literature

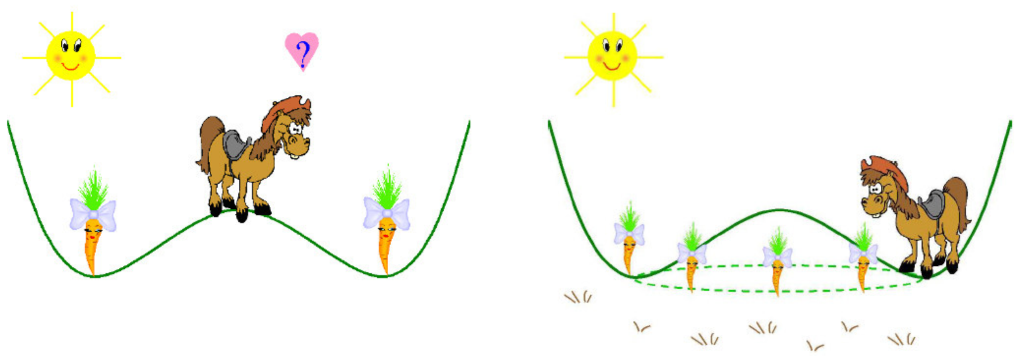
Kovacs et al., **PRD93 (2016) no.11, 114014 ; PRD106 (2022) no.11, 116016**

SSB and the donkey of Buridan



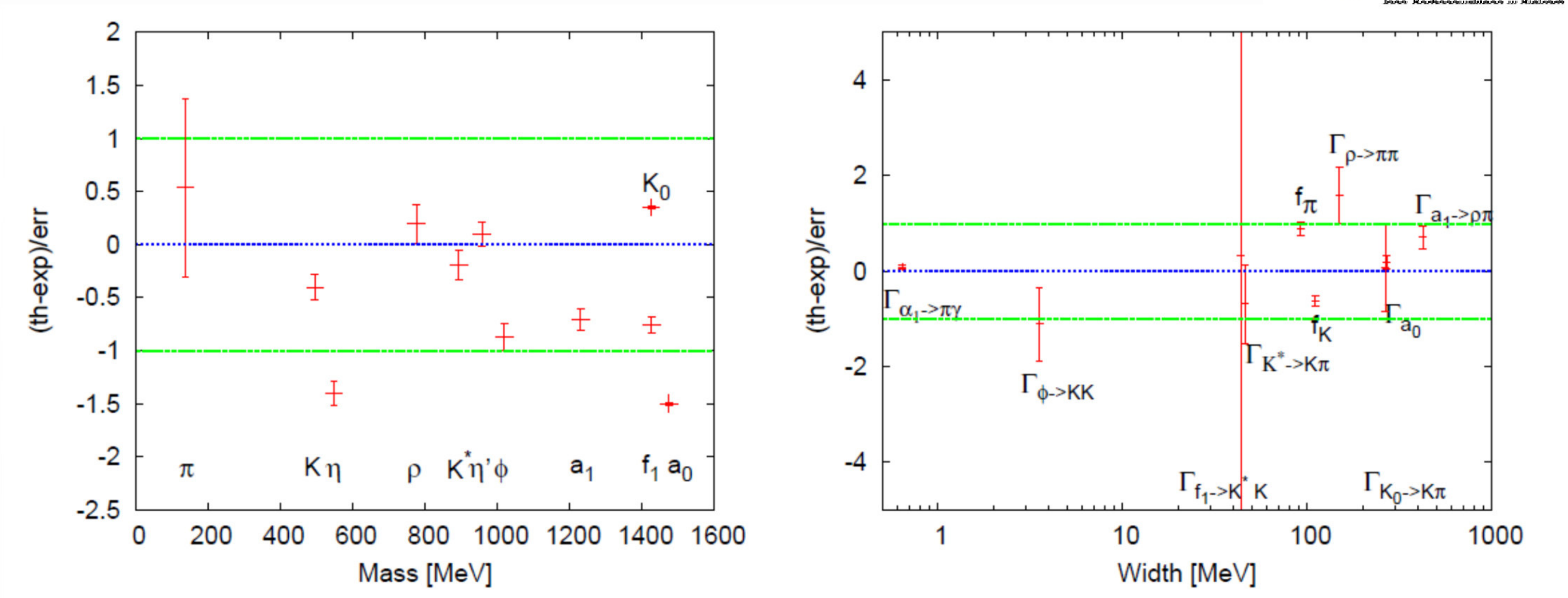
$$\boxed{\sigma_N \to \sigma_N + \phi}$$

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

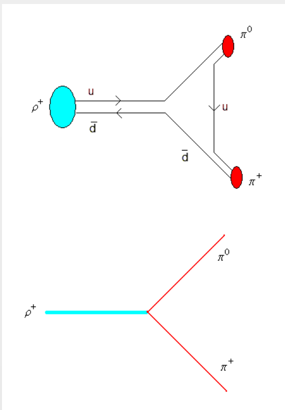
Results of the fit/2



arXiv:1208.0585

Microscpic

eLSM



Overall phenomenology is good. Further quantities calculated afterwards.

Scalar mesons $a_0(1450)$ and $K_0(1430)$ above 1 GeV and are quark-antiquark states. The chiral partner of the pion (the σ) is $f_0(1370)$.

Importance of the (axial-)vector mesons

Recent developments: tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d},\ d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s},\ d\bar{s}$ $s\bar{d},\ s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

PHYSICAL REVIEW D **106,** 036008 (2022)

From well-known tensor mesons to yet unknown axial-tensor mesons

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TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

J^{PC}, $^{2S+1}L_J$	$I = 1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$; $I = 1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$; $I = 0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)$**	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_L \times SU(3)_R \times \times U(1)_A$
0^{-+}, $^1S_0$	π; K; $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ $(\Phi^{ij} = \bar{q}^j_R q^i_L)$	$\Phi \to e^{-2i\alpha} U_L \Phi U_R^\dagger$
0^{++}, $^3P_0$	$a_0(1450)$; $K_0^*(1430)$; $f_0(1370), f_0(1710)$*	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, $^1S_1$	$\rho(770)$; $K^*(892)$; $\omega(782), \phi(1020)$	$V_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L_\mu^{ij} = \bar{q}^j_L \gamma_\mu q^i_L)$	$L_\mu \to U_L L_\mu U_L^\dagger$
1^{++}, $^3P_1$	$a_1(1260)$; $K_{1,A}$; $f_1(1285), f_1(1420)$	$A_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R_\mu^{ij} = \bar{q}^j_R \gamma_\mu q^i_R)$	$R_\mu \to U_R R_\mu U_R^\dagger$
1^{+-}, $^1P_1$	$b_1(1235)$; $K_{1,B}$; $h_1(1170), h_1(1380)$	$P_\mu^{ij} = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + iP_\mu$ $(\Phi_\mu^{ij} = \bar{q}^j_R i\overleftrightarrow{D}_\mu q^i_L)$	$\Phi_\mu \to e^{-2i\alpha} U_L \Phi_\mu U_R^\dagger$
1^{--}, $^3D_1$	$\rho(1700)$; $K^*(1680)$; $\omega(1650), \phi(?)$	$S_\mu^{ij} = \frac{1}{2}\bar{q}^j i\overleftrightarrow{D}_\mu q^i$		
2^{++}, $^3P_2$	$a_2(1320)$; $K_2^*(1430)$; $f_2(1270), f_2'(1525)$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}^j_L(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i_L)$	$L_{\mu\nu} \to U_L L_{\mu\nu} U_L^\dagger$
2^{--}, $^3D_2$	$\rho_2(?)$; $K_2(1820)$; $\omega_2(?), \phi_2(?)$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}^j_R(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_R)$	$R_{\mu\nu} \to U_R R_{\mu\nu} U_R^\dagger$
2^{-+}, $^1D_2$	$\pi_2(1670)$; $K_2(1770)$; $\eta_2(1645), \eta_2(1870)$	$P_{\mu\nu}^{ij} = -\frac{1}{2}\bar{q}^j(i\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$ $(\Phi_{\mu\nu}^{ij} = \bar{q}^j_R(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_L)$	$\Phi_{\mu\nu} \to e^{-2i\alpha} U_L \Phi_{\mu\nu} U_R^\dagger$
2^{++}, $^3F_2$	$a_2(?)$; $K_2^*(?)$; $f_2(?), f_2'(?)$	$S_{\mu\nu}^{ij} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$		
3^{--}, $^3D_3$	$\rho_3(1690)$; $K_3^*(1780)$; $\omega_3(1670), \phi_3(1850)$	$\vdots$	$\vdots$	$\vdots$

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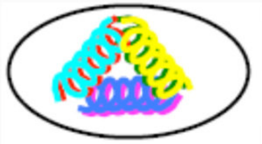
Table from:

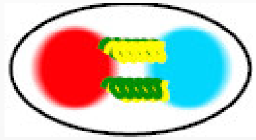
F.G., R. Pisarski, A. Koenigstein Phys.Rev.D 97 (2018) 9, 091901 e-Print: 1709.07454

Non-conventional mesons: beyond qq

1) Glueballs



2) Hybrids



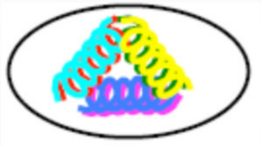
3) Four-quark states

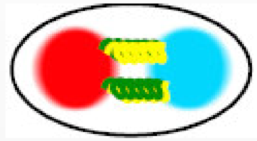
Compact diquark-antidiquark states

Molecular states (a type of dynamical generation)

Companion poles (another type of dynamical generation)

Non-conventional mesons: beyond qq

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

Toward a nonet of hybrid state/PDG

$\boxed{\pi_1(1600)}$ $I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Spectroscopy of Light Meson Resonances" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

π_1(1600) T-Matrix Pole $\sqrt{s}$

π_1(1600) MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1661^{+15}_{-11} OUR AVERAGE		Error includes scale factor of 1.2.		

π_1(1600) WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
240± 50 OUR AVERAGE		Error includes scale factor of 1.7. See the ideogram below.		

π_1(1600) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$\eta\pi$	
Γ_7	$f_1(1285)\pi$	seen

$\boxed{\pi_1(1400)}$ $I^G(J^{PC}) = 1^-(1^{-+})$

π_1(1400) MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1354 ±25 OUR AVERAGE		Error includes scale factor of 1.8. See the ideogram below.			

π_1(1400) WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
330 ±35 OUR AVERAGE					

π_1(1400) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen

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A unique I=1 hybrid state π1

PHYSICAL REVIEW LETTERS **122,** 042002 (2019)

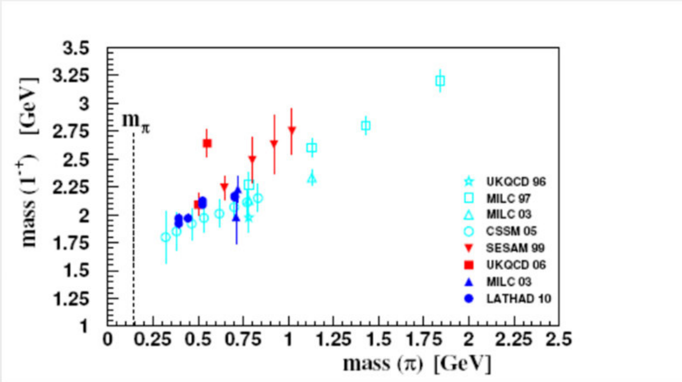
Determination of the Pole Position of the Lightest Hybrid Meson Candidate

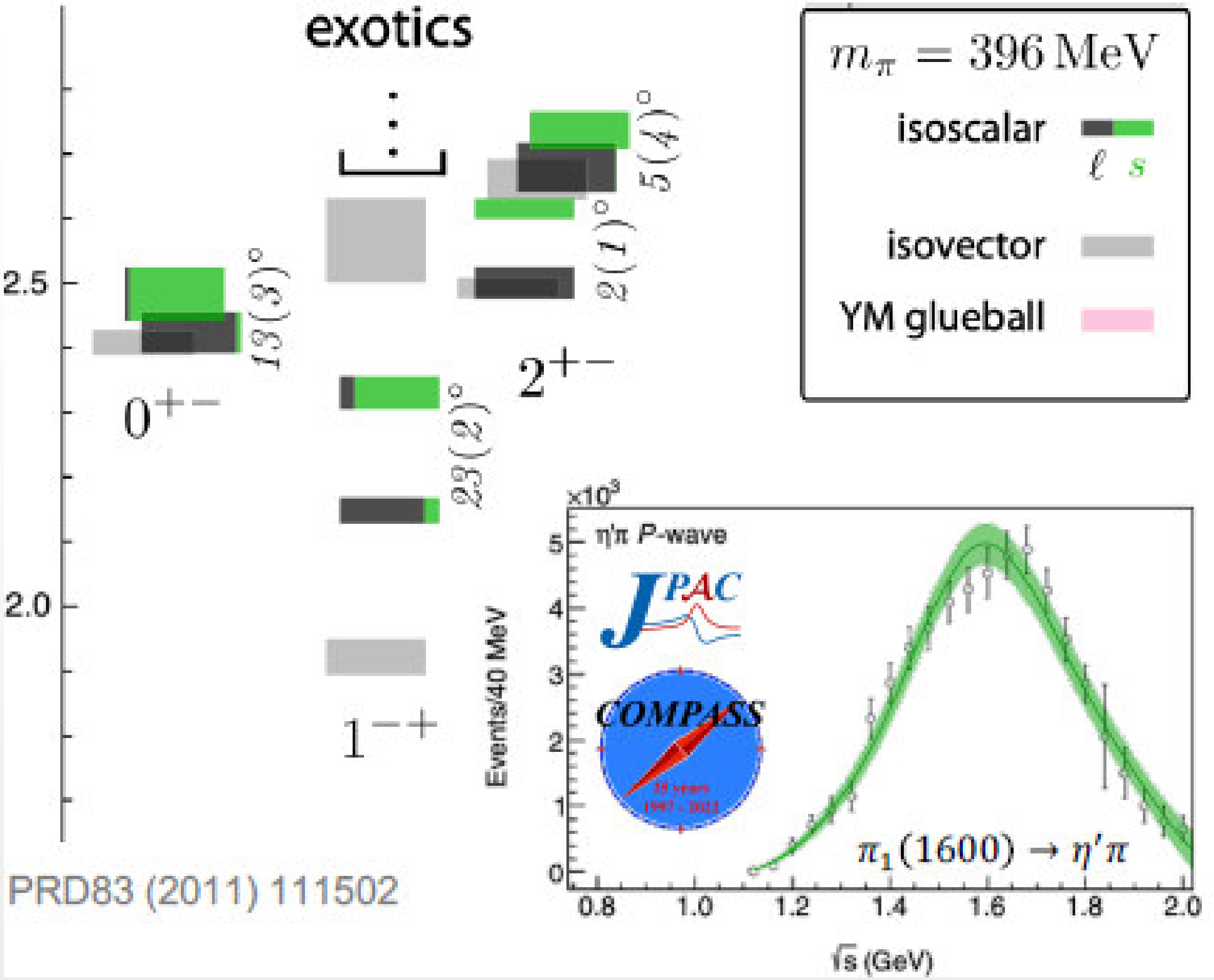
A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2] M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

Mapping states with explicit gluonic degrees of freedom in the light sector is a challenge, and has led to controversies in the past. In particular, the experiments have reported two different hybrid candidates with spin-exotic signature, $\pi_1(1400)$ and $\pi_1(1600)$, which couple separately to $\eta\pi$ and $\eta'\pi$. This picture is not compatible with recent Lattice QCD estimates for hybrid states, nor with most phenomenological models. We consider the recent partial wave analysis of the $\eta^{(\prime)}\pi$ system by the COMPASS Collaboration. We fit the extracted intensities and phases with a coupled-channel amplitude that enforces the unitarity and analyticity of the S matrix. We provide a robust extraction of a single exotic π_1 resonant pole, with mass and width $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, which couples to both $\eta^{(\prime)}\pi$ channels. We find no evidence for a second exotic state. We also provide the resonance parameters of the $a_2(1320)$ and $a_2'(1700)$.

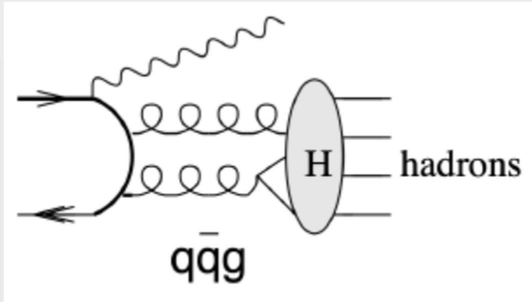
π1(1600) and π1(1400) are the same state
(in agreement with various models and lattice QCD)

C. Meyer and E. Swanson,
Hybrid Mesons,
Prog. Part. Nucl. Phys. 82 (2015) 21
[arXiv:1502.07276 [hep-ph]].





PRD83 (2011) 111502



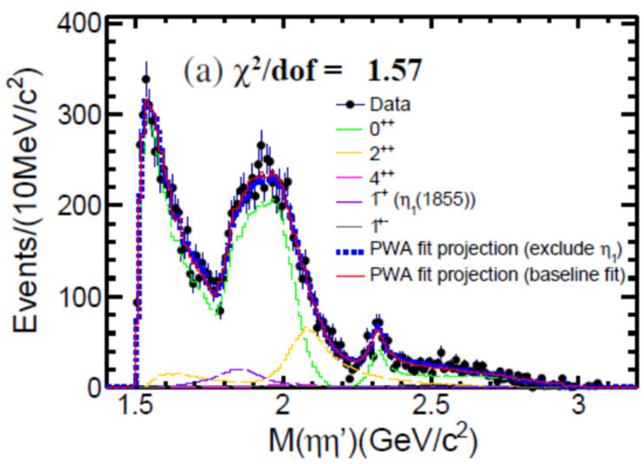
New experimental finding: η1(1855)

Observation of an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers in $J/\psi \to \gamma\eta\eta'$

M. Ablikim[1], M. N. Achasov[10,b], P. Adlarson[68], S. Ahmed[14], M. Albrecht[4], R. Aliberti[28], A. Amoroso[67A,67C], M. R. An[32],

Using a sample of $(10.09\pm0.04)\times10^9$ J/ψ events collected with the BESIII detector operating at the BEPCII storage ring, a partial wave analysis of the decay $J/\psi \to \gamma\eta\eta'$ is performed. The first observation of an isoscalar state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is reported in the process $J/\psi \to \gamma\eta_1(1855)$ with $\eta_1(1855) \to \eta\eta'$. Its mass and width are measured to be $(1855\pm9^{+6}_{-1})$ MeV/c^2 and $(188\pm18^{+3}_{-8})$ MeV, respectively, where the first uncertainties are statistical and the second are systematic, and its statistical significance is estimated to be larger than 19σ.

Phys.Rev.Lett. 129 (2022) 19, 192002 2202.00621 [hep-ex]



Hybrid mesons and their chiral partners

Eur. Phys. J. Plus (2020) 135:945
https://doi.org/10.1140/epjp/s13360-020-00900-z

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Check for updates

Hybrid phenomenology in a chiral approach

Walaa I. Eshraim[1,2], Christian S. Fischer[1,3], Francesco Giacosa[2,4,a], Denis Parganlija[5,6]

Resonance	Mass [MeV]
π_1^{hyb}	1660 [**input** using $\pi_1(1600)$ [9]]
$\eta_{1,N}^{hyb}$	1660
$\eta_{1,S}^{hyb}$	1751
K_1^{hyb}	1707
b_1^{hyb}	2000 [**input** set as an estimate]
$h_{1N,B}^{hyb}$	2000
$K_{1,B}^{hyb}$	2063
$h_{1S,B}^{hyb}$	2126

$\Gamma_{b_1^{hyb}\to\pi\omega(1650)}/\Gamma_{\pi_1^{hyb}\to\pi b_1}$	0.065
$\Gamma_{K_{1B}^{hyb}\to\pi K^*(1680)}/\Gamma_{\pi_1^{hyb}\to\pi b_1}$	0.19
$\Gamma_{h_{1,N}^{hyb}\to\pi\rho(1700)}/\Gamma_{\pi_1^{hyb}\to\pi b_1}$	0.16

Only ratios possible.
πb1 mode dominates!

A nonet of hybrid states?

Physics Letters B 834 (2022) 137478

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry [a,*], Christian S. Fischer [b,c], Francesco Giacosa [a,d]

arXiv:2203.04327

Besides π1(1600) and η1(1855), we expect also: K1(1750) and η1(1660). The last two not yet seen.

	M (MeV)
K_1^{hyb}	1761
η_1^L	1661
η_1^H	1855

Combined fit for the π1(1660) mesons (PDG values +latt)

$$m_{\pi_1} = 1661^{+15}_{-11}\ \text{MeV}$$

$$\Gamma_{\text{tot}} = 240 \pm 50\ \text{MeV}$$

$$D\text{-wave and } S\text{-wave: } \frac{BR(\pi_1 \to b_1\pi)_D}{BR(\pi_1 \to b_1\pi)_S} = 0.3 \pm 0.1$$

$$\frac{\Gamma_{f_1\pi}}{\Gamma_{\eta'\pi}} = 3.8 \pm 0.78$$

The following are the lattice estimates

1. $\Gamma_{b_1\pi} = 139\text{-}529$ MeV,
2. $\Gamma_{\rho\pi} = 0\text{-}20$ MeV,
3. $\Gamma_{K^*K} = 0\text{-}2$ MeV,
4. $\Gamma_{f_1\pi} = 0\text{-}24$ MeV,
5. $\Gamma_{f_1'\pi} = 0\text{-}2$ MeV,
6. $\Gamma_{\rho\omega} \leq 0.15$ MeV,
7. $\Gamma_{\eta\pi} = 0\text{-}1$ MeV,
8. $\Gamma_{\eta'\pi} = 0\text{-}12$ MeV.

A.J. Woss, et al., Hadron Spectrum, Decays of an exotic 1−+ hybrid meson resonance in QCD, Phys. Rev. D 103 (5) (2021) 054502, https://doi.org/10.1103/PhysRevD.103.054502, arXiv:2009.10034 [hep-lat].

Lagrangian part for π1

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$$\begin{aligned}\mathcal{L}^{\pi}_{hyb} &= g^{c}_{b_1\pi}\langle \pi_{1,\mu} b_1^{\mu}\pi\rangle + g^{d}_{b_1\pi}\langle \pi_{1,\mu\nu} b_1^{\mu\nu}\pi\rangle \\ &+ g_{f_1\pi}\langle \pi_{1,\mu} f_{1,N}^{\mu\nu}\partial_\nu\pi + \pi_{1,\mu} f_{1,S}^{\mu\nu}\partial_\nu\pi\rangle \\ &+ g_{\eta\pi}\langle \pi_{1,\mu}(\eta_N\partial^\mu\pi + \eta_S\partial^\mu\pi)\rangle + g_{\rho\pi}\langle \tilde{\pi}_{1,\mu\nu}\rho^{\mu\nu}\pi\rangle \\ &+ g_{\rho\omega}\langle \pi_{1,\mu}(\rho^{\mu\nu}\omega_\nu + \omega^{\mu\nu}\rho_\nu)\rangle.\end{aligned}$$

$$\begin{aligned}\Gamma_{b_1\pi} &= \frac{1}{2}\frac{k_{b_1}}{24\pi m^2_{\pi_1}}\left(\frac{1}{m^2_{b_1}}\left(E_{b_1} g^c_{b_1\pi} + 2g^d_{b_1\pi} m^2_{b_1} m_{\pi_1}\right)^2\right. \\ &\left. +2(2E_{b_1} g^2_{b_1\pi} m_{\pi_1} + g^c_{b_1\pi})^2\right)\end{aligned}$$

$$\frac{G_2}{G_0} = \sqrt{2}\frac{g^c_{b_1\pi}(-E_{b_1} + m_{b_1}) + 2g^d_{b_1\pi} m_{\pi_1}(-m^2_{b_1} + E_{b_1} m_{b_1})}{g^c_{b_1\pi}(E_{b_1} + 2m_{b_1}) + 2g^d_{b_1\pi} m_{\pi_1}(m^2_{b_1} + 2E_{b_1} m_{b_1})}$$

$$\Gamma_{\rho\pi} = g^2_{\rho\pi}\frac{k^3_\rho}{6\pi}$$

$$\Gamma_{K^*K} = g^2_{\rho\pi}\frac{k^3_{K^*}}{12\pi}$$

….and other decay terms…

Fit outcomes

$$\pi_1(1600)$$

Channel	Width (MeV)	Channel	Width (MeV)
$\Gamma_{b_1\pi}$	220 ± 34	$\Gamma_{f_1\pi}$	16.2 ± 3.1
$\Gamma_{\rho\pi}$	7.1 ± 1.8	$\Gamma_{f_1'\pi}$	0.83 ± 0.16
Γ_{K^*K}	1.2 ± 0.3	$\Gamma_{\eta\pi}$	0.37 ± 0.08
$\Gamma_{\rho\omega}$	0.08 ± 0.03	$\Gamma_{\eta'\pi}$	4.6 ± 1.0
		Γ_{tot}	250 ± 34

Predictions for other hybrids

$\eta_1^{hyb}(1660)$

Channel	Width (MeV)
	Set-1
$\Gamma_{a_1\pi}$	80 ± 15
Γ_{K^*K}	0.29 ± 0.075
$\Gamma_{\eta'\eta}$	0.41 ± 0.09
$\Gamma_{K_1(1270)K}$	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028
$\Gamma_{K^*K^*}$	0
$\Gamma_{\omega\phi}$	0
$\Gamma_{f_1\eta}$	0
Γ_{tot}	81 ± 15

$\eta_1(1855)$

Channel	Width (MeV)
	Set-1
$\Gamma_{K_1(1270)K}$	253 ± 92
Γ_{K^*K}	1.45 ± 0.37
$\Gamma_{\eta'\eta}$	2.28 ± 0.51
$\Gamma_{a_1\pi}$	0
$\Gamma_{\rho\rho}$	0
$\Gamma_{K^*K^*}$	0.075 ± 0.027
$\Gamma_{\omega\phi}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	2.15 ± 0.56
Γ_{tot}	259 ± 92

$K_1^{hyb}(1750)$

Channel	Width (MeV)
	Set-1
$\Gamma_{K_1(1270)\pi}$	125 ± 42
$\Gamma_{K_1(1400)\pi}$	103 ± 45
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28
$\Gamma_{\eta K}$	0.29 ± 0.07
$\Gamma_{\eta' K}$	2.77 ± 0.62
$\Gamma_{\rho K^*}$	0.045 ± 0.016
$\Gamma_{a_1 K}$	11.0 ± 2.32
$\Gamma_{\rho K}$	2.18 ± 0.56
$\Gamma_{\omega K}$	0.82 ± 0.21
$\Gamma_{\phi K}$	0.49 ± 0.12
$\Gamma_{K^*\pi}$	0.67 ± 0.17
$\Gamma_{K^*\eta}$	0.30 ± 0.08
$\Gamma_{\omega K^*}$	0.011 ± 0.004
$\Gamma_{b_1 K}$	64 ± 14
Γ_{tot}	312 ± 97

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Predictions for other hybrids

$\eta_1^{hyb}(1660)$

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$\Gamma_{\rho\rho}$	0.081 ± 0.028
$\Gamma_{K^*K^*}$	0
$\Gamma_{\omega\phi}$	0
$\Gamma_{f_1\eta}$	0
Γ_{tot}	81 ± 15

$\eta_1(1855)$

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$K_1^{hyb}(1750)$

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Nuclear Physics A 1037 (2023) 122683

NUCLEAR PHYSICS A

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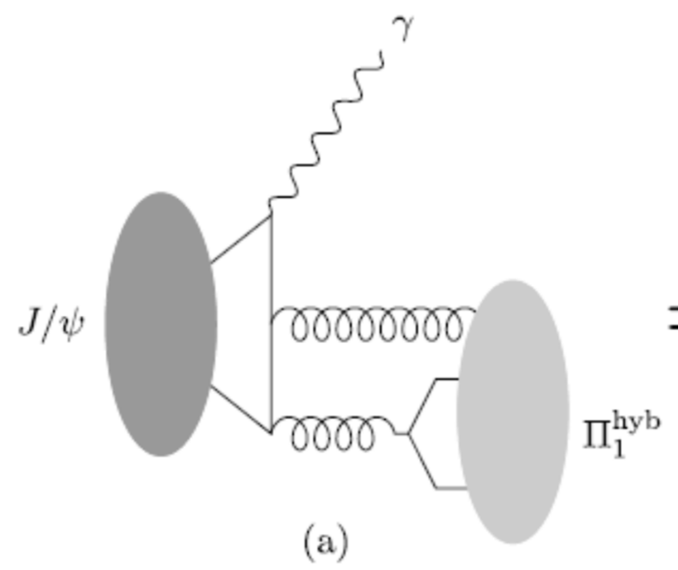
Radiative production and decays of the exotic $\eta_1'(1855)$ and its siblings

Vanamali Shastry [a,*], Francesco Giacosa [a,b]



J/Psi decay (via the so-called 'Sill' distribution)

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	Production Channel $(\phi_1\phi_2)$	Branching ratio (10^{-4}) Set-1 $\theta_h = 0^\circ$
$\eta_1^{hyb}(1660)$	$a_1\pi$	4.8 ± 1.4
	K^*K	$(1.73 \pm 0.49) \times 10^{-2}$
	$\eta'\eta$	$(2.28 \pm 0.65) \times 10^{-2}$
	$\rho\rho$	$(4.4 \pm 1.3) \times 10^{-3}$
$\eta_1(1855)$	$K_1(1270)K$	2.45 ± 0.70
	K^*K	$(1.86 \pm 0.53) \times 10^{-2}$
	K^*K^*	$(7.2 \pm 2.1) \times 10^{-4}$
	$f_1(1285)\eta$	$(27.6 \pm 7.9) \times 10^{-3}$
	$\eta\eta'$	$(2.70 \pm 0.76) \times 10^{-2}$ [10]

The branching ratios of the $J/\psi \to \gamma\eta_1^{hyb}(1660) \to \gamma\phi_1\phi_2$ and $J/\psi \to \gamma\eta_1'(1855) \to \gamma\phi_1\phi_2$

$$\Gamma_{A\to BC_1C_2} = \int_{s_{\rm th}}^{(\Delta M_{AC_2})^2} ds\ \Gamma_{A\to\mathcal{R}^*C_2}(s)\,d_s^i(s)$$

'Sill' implemented in all decays above

Light glueballs

Gluebals have along history

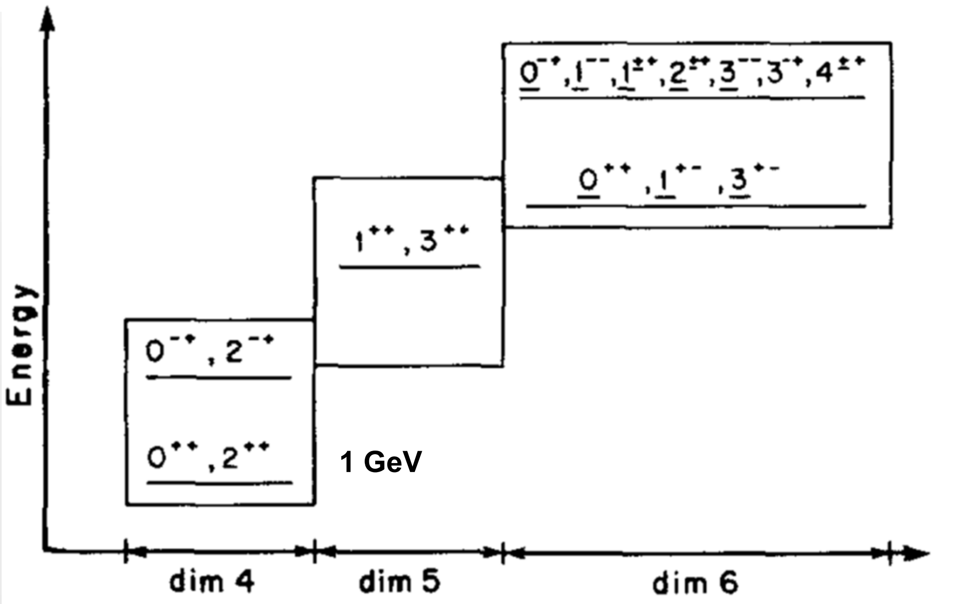
A. Chodos, et al., Phys. Rev. D 9 (1974) 3471.
R.L. Jaffe, K. Johnson, Phys. Lett. B60 (1976) 201

ANNALS OF PHYSICS **168**, 344–367 (1986)

Qualitative Features of the Glueball Spectrum*

R. L. JAFFE, K. JOHNSON, AND Z. RYZAK†

Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139



Lattice 2006

PHYSICAL REVIEW D 73, 014516 (2006)

Glueball spectrum and matrix elements on anisotropic lattices

Y. Chen,[1,2] A. Alexandru,[2] S.J. Dong,[2] T. Draper,[2] I. Horváth,[2] F.X. Lee,[3,4] K.F. Liu,[2] N. Mathur,[2,4] C. Morni
M. Peardon,[6] S. Tamhankar,[2] B.L. Young,[7] and J.B. Zhang[8]
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[2]Department of Physics & Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA
[3]Center for Nuclear Studies, Department of Physics, George Washington University, Washington, D.C. 20052 USA
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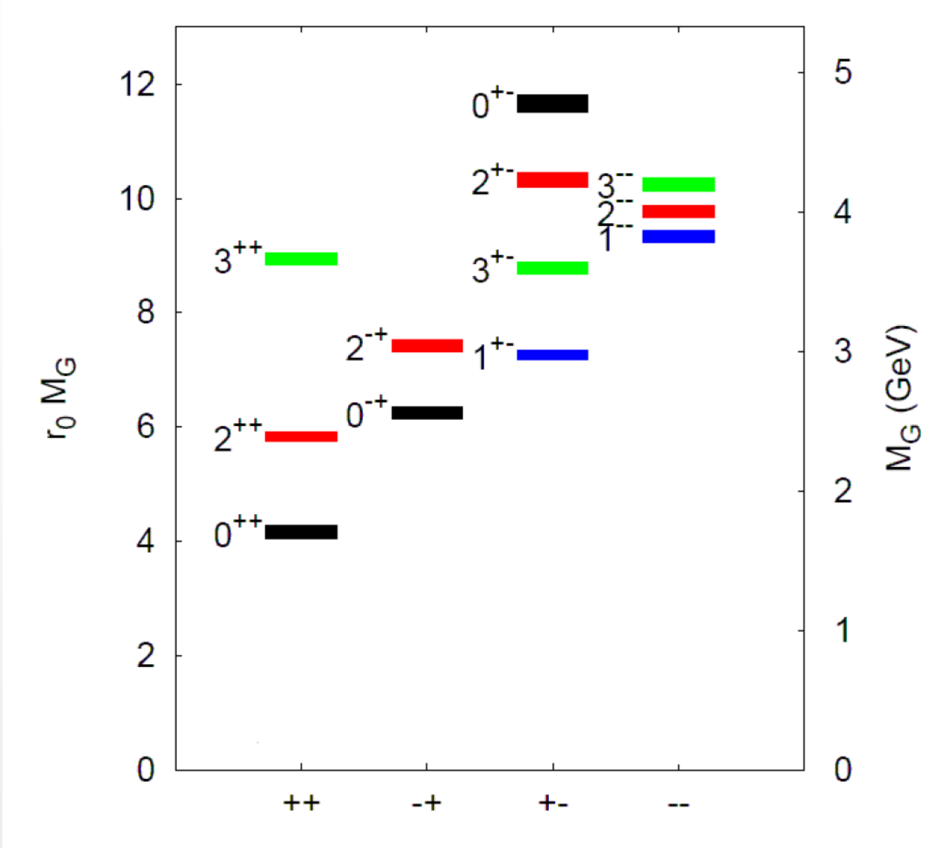
[5]Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
[6]School of Mathematics, Trinity College, Dublin, Dublin 2, Ireland
[7]Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA
[8]CSSM and Department of Physics, University of Adelaide, Adelaide, SA 5005, Australia
(Received 13 October 2005; published 26 January 2006)

The glueball-to-vacuum matrix elements of local gluonic operators in scalar, tensor, and pseudoscalar channels are investigated numerically on several anisotropic lattices with the spatial lattice spacing ranging from 0.1–0.2 fm. These matrix elements are needed to predict the glueball branching ratios in J/ψ radiative decays which will help identify the glueball states in experiments. Two types of improved local gluonic operators are constructed for a self-consistent check and the finite-volume effects are studied. We find that lattice spacing dependence of our results is very weak and the continuum limits are reliably extrapolated, as a result of improvement of the lattice gauge action and local operators. We also give updated glueball masses with various quantum numbers.



Quoted by the PDG in the 'Quark Model' review.

See also: Gregory et al, **JHEP 1210 (2012) 170**

Towards the glueball spectrum from unquenched lattice QCD

Conclusions and future prospects The most conservative interpretation of our results is that the masses in terms of lattice representations are broadly consistent with results from quenched QCD. We do not see any evidence for large unquenching effects, however a definitive calculation requires a continuum extrapolation, and the inclusion of fermionic operators.

Lattice: comparison

Table 2 The values of the glueball masses as given in three lattice works

$n\ J^{PC}$	M[MeV] Chen et al. [51]	Meyer [52]	A & T [53]	$n\ J^{PC}$	M[MeV] Chen et al. [51]	Meyer [52]	A & T [53]
$1\,0^{++}$	1710(50)(80)	1475(30)(65)	1653(26)	$1\,1^{--}$	3830(40)(190)	3240(330)(150)	4030(70)
$2\,0^{++}$		2755(30)(120)	2842(40)	$1\,2^{--}$	4010(45)(200)	3660(130)(170)	3920(90)
$3\,0^{++}$		3370(100)(150)		$2\,2^{--}$		3740(200)(170)	
$4\,0^{++}$		3990(210)(180)		$1\,3^{--}$	4200(45)(200)	4330(260)(200)	
$1\,2^{++}$	2390(30)(120)	2150(30)(100)	2376(32)	$1\,0^{+-}$	4780(60)(230)		
$2\,2^{++}$		2880(100)(130)	3300(50)	$1\,1^{+-}$	2980(30)(140)	2670(65)(120)	2944(42)
$1\,3^{++}$	3670(50)(180)	3385(90)(150)	3740(70)	$2\,1^{+-}$			3800(60)
$1\,4^{++}$		3640(90)(160)	3690(80)	$1\,2^{+-}$	4230(50)(200)		4240(80)
$1\,6^{++}$		4360(260)(200)		$1\,3^{+-}$	3600(40)(170)	3270(90)(150)	3530(80)
$1\,0^{-+}$	2560(35)(120)	2250(60)(100)	2561(40)	$2\,3^{+-}$		3630(140)(160)	
$2\,0^{-+}$		3370(150)(150)	3540(80)	$1\,4^{+-}$			4380(80)
$1\,2^{-+}$	3040(40)(150)	2780(50)(130)	3070(60)	$1\,5^{+-}$		4110(170)(190)	
$2\,2^{-+}$		3480(140)(160)	3970(70)				
$1\,5^{-+}$		3942(160)(180)					
$1\,1^{-+}$			4120(80)				
$2\,1^{-+}$			4160(80)				
$3\,1^{-+}$			4200(90)				

51. Y. Chen et al., Glueball spectrum and matrix elements on anisotropic lattices. Phys. Rev. D **73**, 014516 (2006). https://doi.org/10.1103/PhysRevD.73.014516. arXiv:hep-lat/0510074
52. H.B. Meyer, Glueball regge trajectories. arXiv:hep-lat/0508002
53. A. Athenodorou, M. Teper, The glueball spectrum of SU(3) gauge theory in 3 + 1 dimensions. JHEP **11**, 172 (2020). https://doi.org/10.1007/JHEP11(2020)172. arXiv:2007.06422 [hep-lat]

Table from 2212.03272

See also Bethe-Slapeter results: Huber, Fischer, Sanchis-Alepuz: Eur.Phys.J.C 81 (2021) 12, 1083 and *Eur.Phys.J.C* 80 (2020) 11, 1077

Which masses fit better?
According to Thermodynamics, the 'AT' ones

Uniwersytet

Eur. Phys. J. C (2023) 83:390
https://doi.org/10.1140/epjc/s10052-023-11557-0

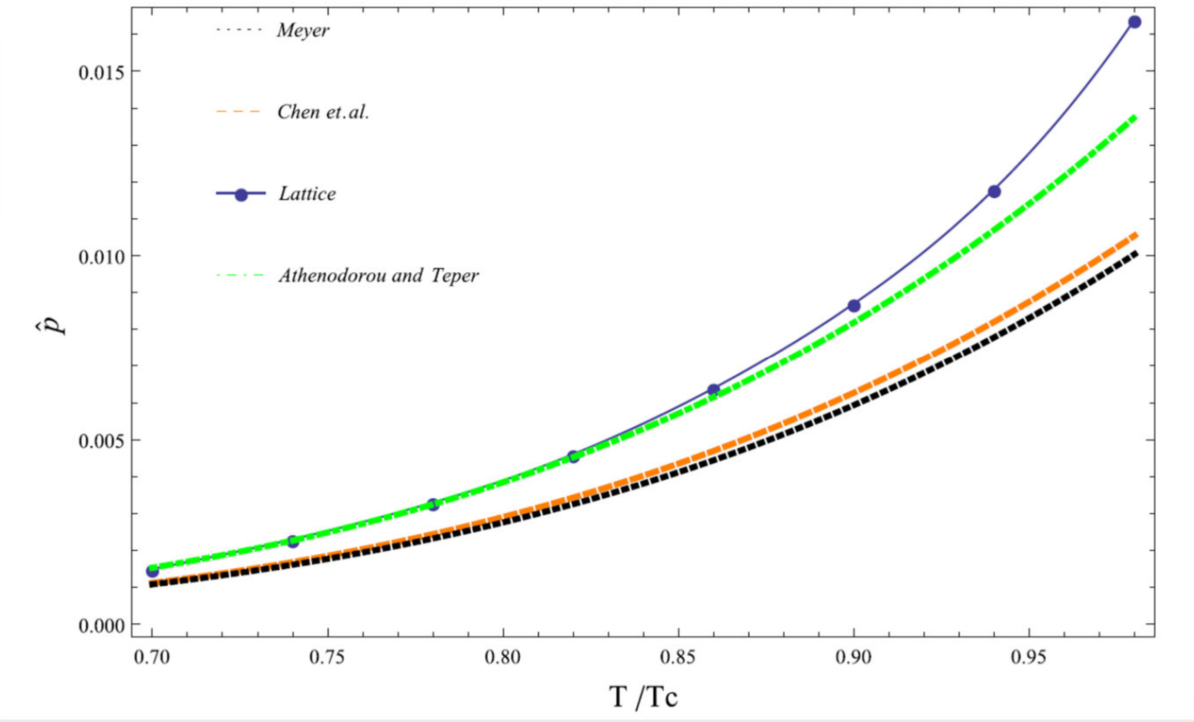
THE EUROPEAN PHYSICAL JOURNAL C

Check for updates

Regular Article - Theoretical Physics

Thermodynamics of the glueball resonance gas

Enrico Trotti[1,a], Shahriyar Jafarzade[1,b], Francesco Giacosa[1,2,c]



Scalar glueball

Uniwersytet Jana Kochanowskiego w Kielcach

PHYSICAL REVIEW D **90**, 114005 (2014)

Is $f_0(1710)$ a glueball?

Stanislaus Janowski,[1] Francesco Giacosa,[1,2] and Dirk H. Rischke[1]
[1]*Institute for Theoretical Physics, Goethe University, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*
[2]*Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland*
(Received 26 August 2014; published 2 December 2014)

eLSM result long ago!

PRL **110**, 021601 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2

Scalar Glueball in Radiative J/ψ Decay on the Lattice

Long-Cheng Gui,[1,2] Ying Chen,[1,2,*] Gang Li,[3] Chuan Liu,[4] Yu-Bin Liu,[5] Jian-Ping Ma,[6] Yi-Bo Yang,[1,2] and Jian-Bo Zhang[7]

(CLQCD Collaboration)

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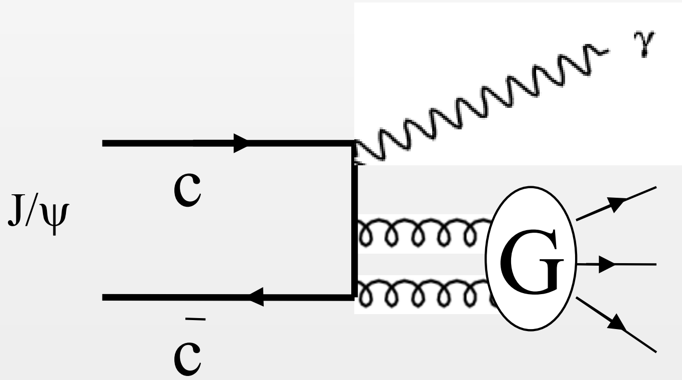
[3]*Department of Physics, Qufu Normal University, Qufu 273165, People's Republic of China*
[4]*School of Physics and Center for High Energy Physics, Peking University, Beijing 100871, People's Republic of China*
[5]*School of Physics, Nankai University, Tianjin 300071, People's Republic of China*
[6]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*
[7]*Department of Physics, Zhejiang University, Zhejiang 310027, People's Republic of China*
(Received 5 June 2012; published 10 January 2013)

The form factors in the radiative decay of J/ψ to a scalar glueball are studied within quenched lattice QCD on anisotropic lattices. The continuum extrapolation is carried out by using two different lattice spacings. With the results of these form factors, the partial width of J/ψ radiatively decaying into the pure gauge scalar glueball is predicted to be 0.35(8) keV, which corresponds to a branching ratio of $3.8(9) \times 10^{-3}$. By comparing with experiments, out results indicate that $f_0(1710)$ has a larger overlap with the pure gauge glueball than other related scalar mesons.



$\gamma f_0(1710) \rightarrow \gamma K \overline{K}$	$(8.5\ {}^{+1.2}_{-0.9}) \times 10^{-4}$
$\gamma f_0(1710) \rightarrow \gamma \pi \pi$	$(4.0\ \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \rightarrow \gamma \omega \omega$	$(3.1\ \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \rightarrow \gamma \eta \eta$	$(2.4\ {}^{+1.2}_{-0.7}) \times 10^{-4}$

$\gamma f_0(1500) \rightarrow \gamma \pi \pi$	$(1.01\ \pm 0.32) \times 10^{-4}$
$\gamma f_0(1500) \rightarrow \gamma \eta \eta$	$(1.7\ {}^{+0.6}_{-1.4}) \times 10^{-5}$

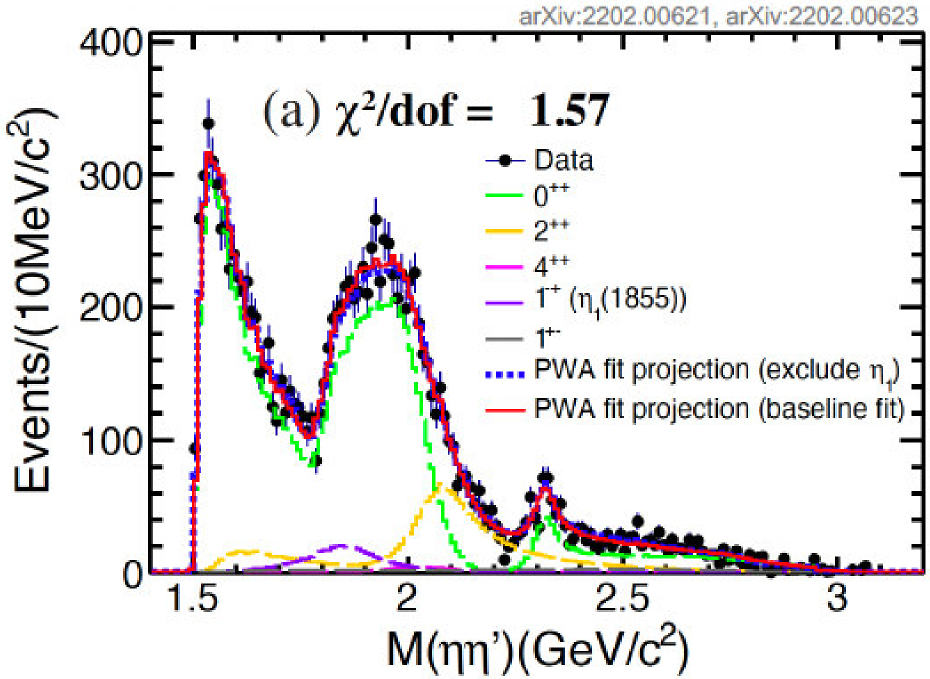
Recent BES results

Radiative J/ψ decays

- scalar glueball decays to $\eta\eta'$ expected to be suppressed $\frac{B(G\to\eta\eta')}{B(G\to\pi\pi)} < 0.04$

  PRD 92, 121902 (2015)

- significant $f_0(1500)$ contribution, but no $f_0(1710)$ (there is a small $f_0(1810)$ in the fit)
- $\frac{B(f_0(1500)\to\eta\eta')}{B(f_0(1500)\to\pi\pi)} = (8.96^{+2.95}_{-2.87})\times10^{-2}$,
- $\frac{B(f_0(1710)\to\eta\eta')}{B(f_0(1710)\to\pi\pi)} < 1.61\times10^{-3}$ (90% CL)
- $\frac{B(f_0(1810)\to\eta\eta')}{B(f_0(1710)\to\pi\pi)} = (1.39^{+0.62}_{-0.52})\times10^{-2}$



Nils Hüsken
on behalf of the BESIII collaboration

Workshop: Recent results and perspectives in hadron physics
Orsay, October 17th, 2022

$J/\psi \to \gamma\eta'\eta$

- PWA of $J/\psi \to \gamma\eta\eta'$ using 10 Billion J/ψ events
- Veto ϕ in $\gamma\eta$ system
- 15000 signal events and $\sim$ 8-13% background events remaining
- All kinematically allowed resonances as listed in the PDG considered
  - $J^{PC} = 0^{++}$, 2^{++} and 4^{++} ($\eta'\eta$ system)

Decay mode	Resonance	M (MeV/c^2)	Γ (MeV)	M_{PDG} (MeV/c^2)	Γ_{PDG} (MeV)	B.F. ($\times 10^{-5}$)	Sig.
$J/\psi \to \gamma X \to \gamma\eta\eta'$	$f_0(1500)$	1506	112	1506	112	3.05 ± 0.07	$\gg 30\sigma$
	$f_0(1810)$	1795	95	1795	95	0.07 ± 0.01	7.6σ
	$f_0(2020)$	1935 ± 5	266 ± 9	1992	442	1.67 ± 0.07	11.0σ
	$f_0(2100)$	2109 ± 11	253 ± 21	2086	284	0.33 ± 0.03	5.2σ
	$f_0(2330)$	2327 ± 4	44 ± 5	2314	144	0.07 ± 0.01	8.5σ
	$f_2(1565)$	1542	122	1542	122	0.20 ± 0.03	6.2σ
	$f_2(1810)$	1815	197	1815	197	0.37 ± 0.03	7.0σ
	$f_2(2010)$	2022 ± 6	212 ± 8	2011	202	1.36 ± 0.10	8.8σ
	$f_2(2340)$	2345	322	2345	322	0.25 ± 0.04	6.5σ
	$f_4(2050)$	2018	234	2018	234	0.11 ± 0.02	5.6σ

New development: glueball-glueball scattering

Eur. Phys. J. C (2022) 82:487
https://doi.org/10.1140/epjc/s10052-022-10403-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

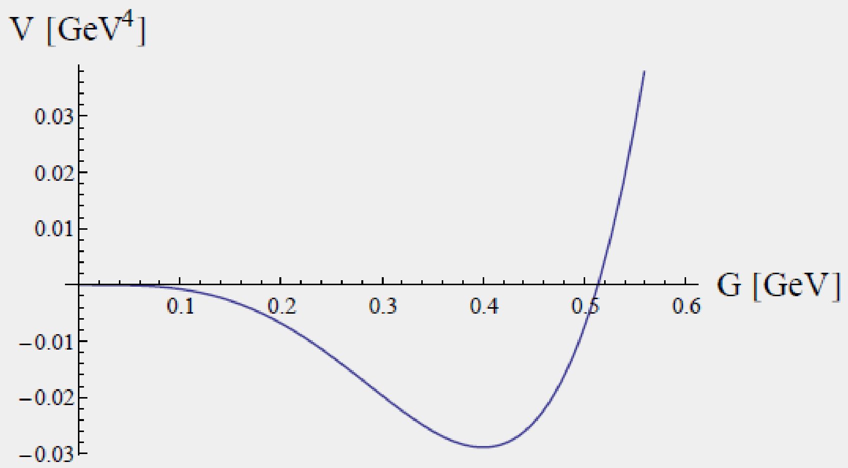
Glueball–glueball scattering and the glueballonium

Francesco Giacosa[1,2], Alessandro Pilloni[3,4], Enrico Trotti[1,a]

$$\mathcal{L}_{\rm dil} = \frac{1}{2}(\partial_\mu G)^2 - V(G),$$

with

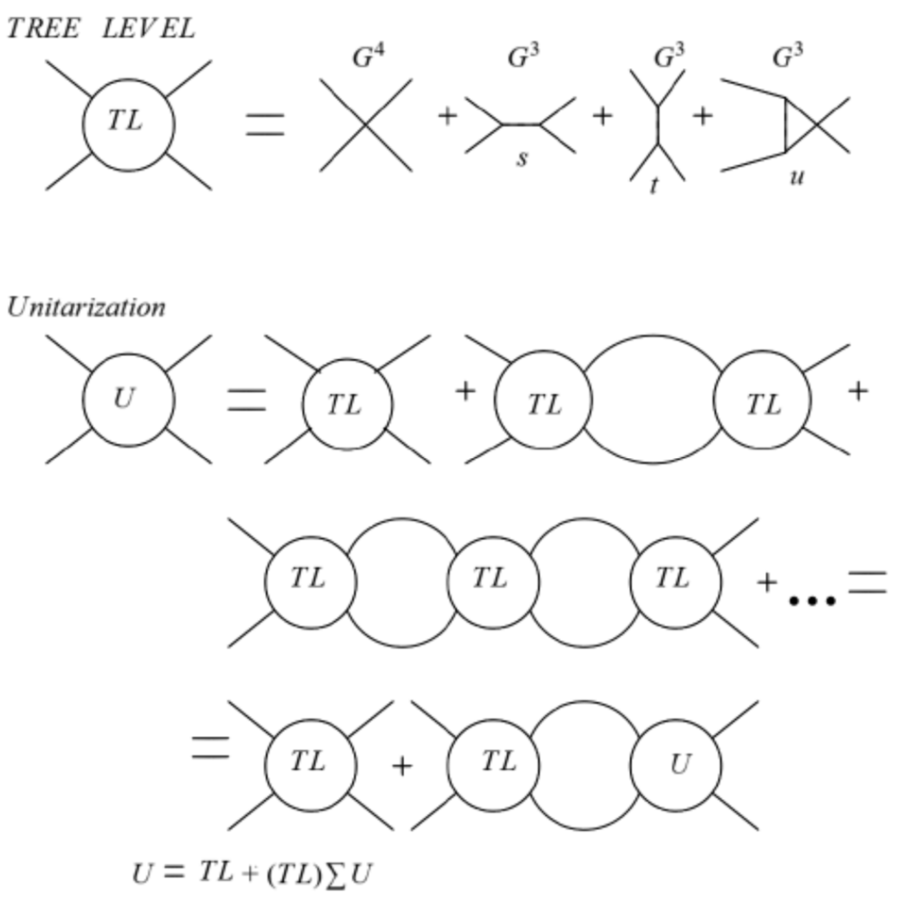
$$V(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right).$$

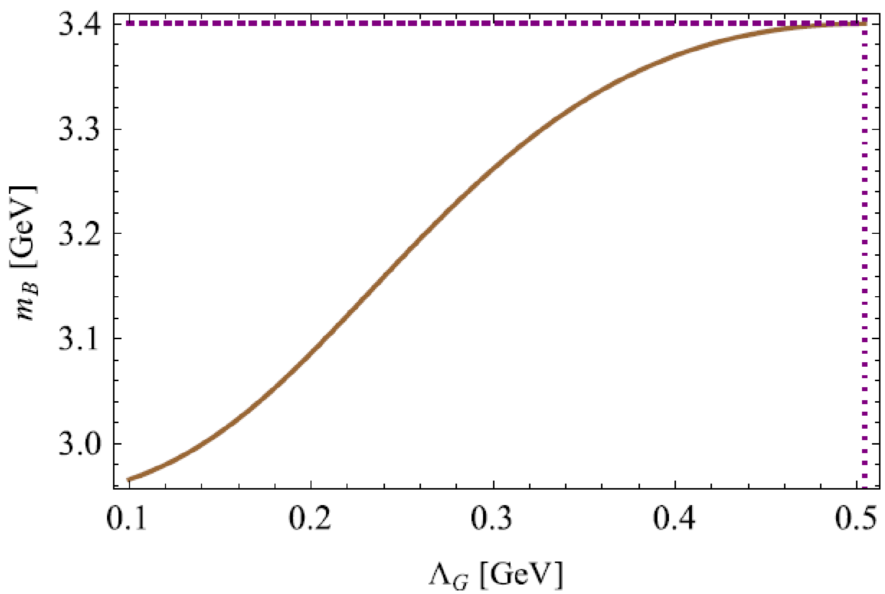


A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

Glueballonium mass

$$V(G) = V(\Lambda_G) + \frac{1}{2}m_G^2 G^2 + \frac{1}{3!}\left(5\frac{m_G^2}{\Lambda_G}\right)G^3 + \frac{1}{4!}\left(11\frac{m_G^2}{\Lambda_G^2}\right)G^4 + \frac{1}{5!}\left(6\frac{m_G^2}{\Lambda_G^3}\right)G^5 + \ldots$$





Can one see that? In YM-lattice, probably yes.
In experiment? Hard, but…

Extension: **Higgsonium**!
Eur.Phys.J.C 83 (2023) 8, 713 2212.01272

Tensor glueball

PHYSICAL REVIEW D **108**, 014023 (2023)

Is $f_2(1950)$ the tensor glueball?

Arthur Vereijken,[1,*] Shahriyar Jafarzade,[1,†] Milena Piotrowska,[1,‡] and Francesco Giacosa[1,2,§]
[1]*Institute of Physics, Jan Kochanowski University, ul. Uniwersytecka 7, 25-406, Kielce, Poland*

Decay ratios

- Coupling constant is not known so we can only compute decay ratios
- Computation is done for a tensor glueball mass of 2210 MeV (later other masses)
- Vector channels are dominant, in particular $\rho\rho$ and K^*K^*

Decay Ratio	theory
$\frac{G_2(2210)\longrightarrow K\,K}{G_2(2210)\longrightarrow \pi\,\pi}$	0.4
$\frac{G_2(2210)\longrightarrow \eta\,\eta}{G_2(2210)\longrightarrow \pi\,\pi}$	0.1
$\frac{G_2(2210)\longrightarrow \eta\,\eta'}{G_2(2210)\longrightarrow \pi\,\pi}$	0.004
$\frac{G_2(2210)\longrightarrow \eta'\,\eta'}{G_2(2210)\longrightarrow \pi\,\pi}$	0.006
$\frac{G_2(2210)\longrightarrow \rho(770)\,\rho(770)}{G_2(2210)\longrightarrow \pi\,\pi}$	55
$\frac{G_2(2210)\longrightarrow K^*(892)\,K^*(892)}{G_2(2210)\longrightarrow \pi\,\pi}$	46
$\frac{G_2(2210)\longrightarrow \omega(782)\,\omega(782)}{G_2(2210)\longrightarrow \pi\,\pi}$	18
$\frac{G_2(2210)\longrightarrow \phi(1020)\,\phi(1020)}{G_2(2210)\longrightarrow \pi\,\pi}$	6
$\frac{G_2(2210)\longrightarrow a_1(1260)\,\pi}{G_2(2210)\longrightarrow \pi\,\pi}$	0.24
$\frac{G_2(2210)\longrightarrow K_{1,A}\,K}{G_2(2210)\longrightarrow \pi\,\pi}$	0.08
$\frac{G_2(2210)\longrightarrow f_1(1285)\,\eta}{G_2(2210)\longrightarrow \pi\,\pi}$	0.02
$\frac{G_2(2210)\longrightarrow f_1(1420)\,\eta}{G_2(2210)\longrightarrow \pi\,\pi}$	0.01

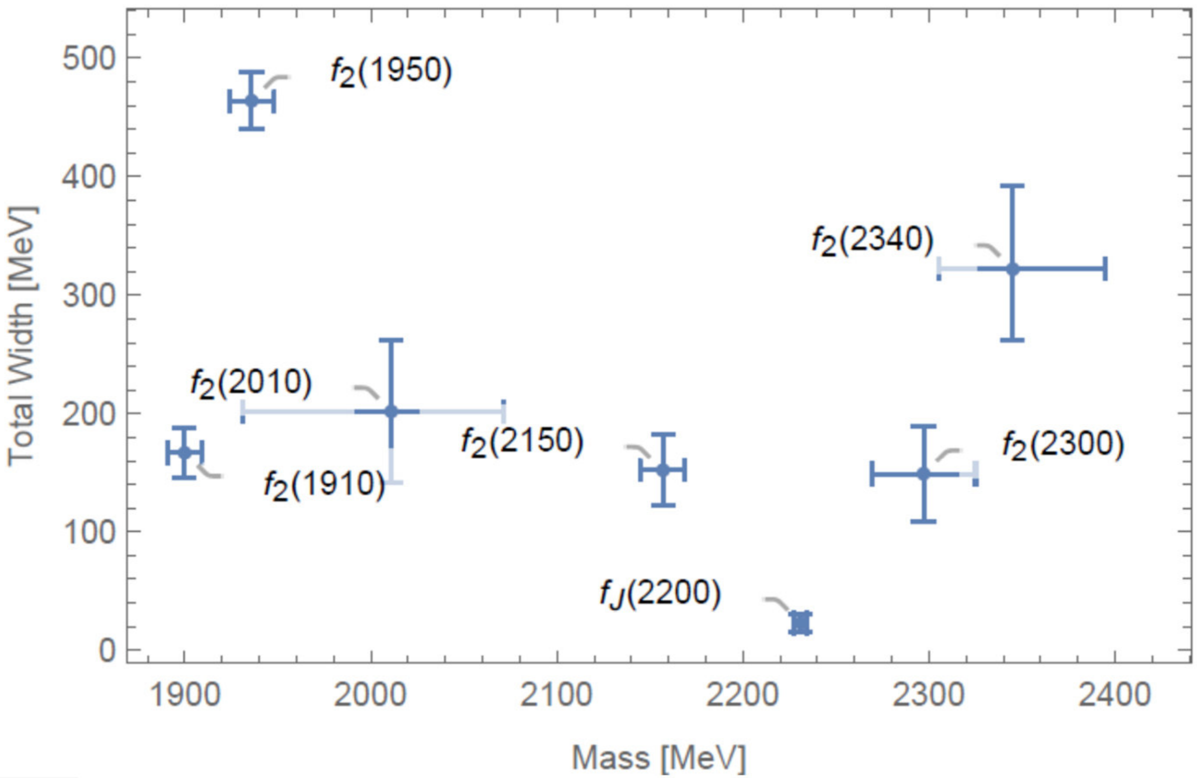
$$\mathcal{L} = \lambda G_{\mu\nu}\left(\mathrm{Tr}\left[\{L^\mu, L^\nu\}\right] + \mathrm{Tr}\left[\{R^\mu, R^\nu\}\right]\right)$$

ρ ρ dominant!

Similar result in holographic approach, see e.g.

Brünner,Parganlija Rebhan
Phys.Rev.D91,no.10,106002(2015)

Isoscalar tensor resonances: comparison



Resonance	Decay Ratio	PDG	Model Prediction
$f_2(1910)$	$\rho\rho/\omega\omega$	2.6 ± 0.4	3.1
$f_2(1910)$	$f_2(1270)\eta/a_2(1320)\pi$	0.09 ± 0.05	0.07
$f_2(1910)$	$\eta\eta/\eta\eta'$	< 0.05	~ 8
$f_2(1910)$	$\omega\omega/\eta\eta'$	2.6 ± 0.6	~ 200
$f_2(1950)$	**$\eta\eta/\pi\pi$**	**0.14 ± 0.05**	**0.081**
$f_2(1950)$	**$K\bar{K}/\pi\pi$**	**~ 0.8**	**0.32**
$f_2(1950)$	**$4\pi/\eta\eta$**	**> 200**	**> 700**
$f_2(2150)$	$f_2(1270)\eta/a_2(1320)\pi$	0.79 ± 0.11	0.1
$f_2(2150)$	$K\bar{K}/\eta\eta$	1.28 ± 0.23	~ 4
$f_2(2150)$	$\pi\pi/\eta\eta$	< 0.33	~ 10

Decay ratios for the decay channels with available data.

Pseudoscalar glueball

PHYSICAL REVIEW LETTERS **129**, 042001 (2022)

Observation of a State $X(2600)$ in the $\pi^+\pi^-\eta'$ System in the Process $J/\psi \to \gamma\pi^+\pi^-\eta'$

$\pi^+\pi^-$ invariant mass spectrum. A simultaneous fit on the $\pi^+\pi^-\eta'$ and $\pi^+\pi^-$ invariant mass spectra with the two η' decay modes indicates that the mass and width of the $X(2600)$ state are $2618.3 \pm 2.0^{+16.3}_{-1.4}$ MeV$/c^2$ and $195 \pm 5^{+26}_{-17}$ MeV, where the first uncertainties are statistical, and the second systematic.

PHYSICAL REVIEW D **87**, 054036 (2013)

Decay of the pseudoscalar glueball into scalar and pseudoscalar mesons

Walaa I. Eshraim,[1] Stanislaus Janowski,[1] Francesco Giacosa,[1] and Dirk H. Rischke[1,2]

Quantity	$M_{\tilde{G}} = 2.6$ GeV
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{\rm tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{\rm tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma^{\rm tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma^{\rm tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma^{\rm tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{\rm tot}_{\tilde{G}}$	0.47
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma^{\rm tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma^{\rm tot}_{\tilde{G}}$	0.095

Chiral-anomaly driven decays!

$$\mathcal{L}^{int}_{\tilde{G}} = ic_{\tilde{G}\Phi}\tilde{G}\left(\det\Phi - \det\Phi^\dagger\right)$$

Conclusions and outlook

By using a chiral model for QCD, the eLSM, we discussed:

Lightest hybrids

- Nonet of light hybrid states: two resonances still missing
- Postdictions/Predictions
- Search for the missing resonances promising

Lightest glueballs

- Scalar (and the glueballonium)
- **Tensor** (new results)
- Pseudoscalar

Uniwersytet Jana Kochanowskiego w Kielcach

Thanks!

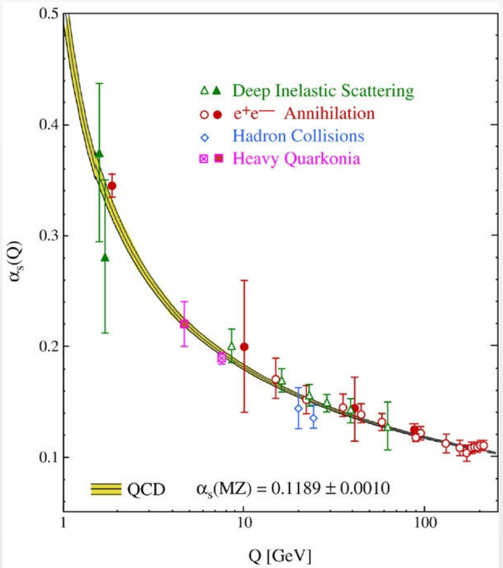
Trace anomaly:
the emergence of a dimension

Chiral limit: $m_{q} = 0$

$x^\mu \to x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250\ \text{MeV}$

$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \to m^*_{gluon} \approx 500 - 800\ \text{MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	$I=1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$I=\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	$I=0$ f'	$I=0$ f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.3	-24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	K_{1B} †	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	K_{1A} †	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)$ †	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Beyond Breit-Wigner

Uniwersytet Jana Kochanowskiego w Kielcach

Eur. Phys. J. A (2021) 57:336
https://doi.org/10.1140/epja/s10050-021-00641-2

THE EUROPEAN PHYSICAL JOURNAL A

Check for updates

Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

ArXiv: 2106.03749

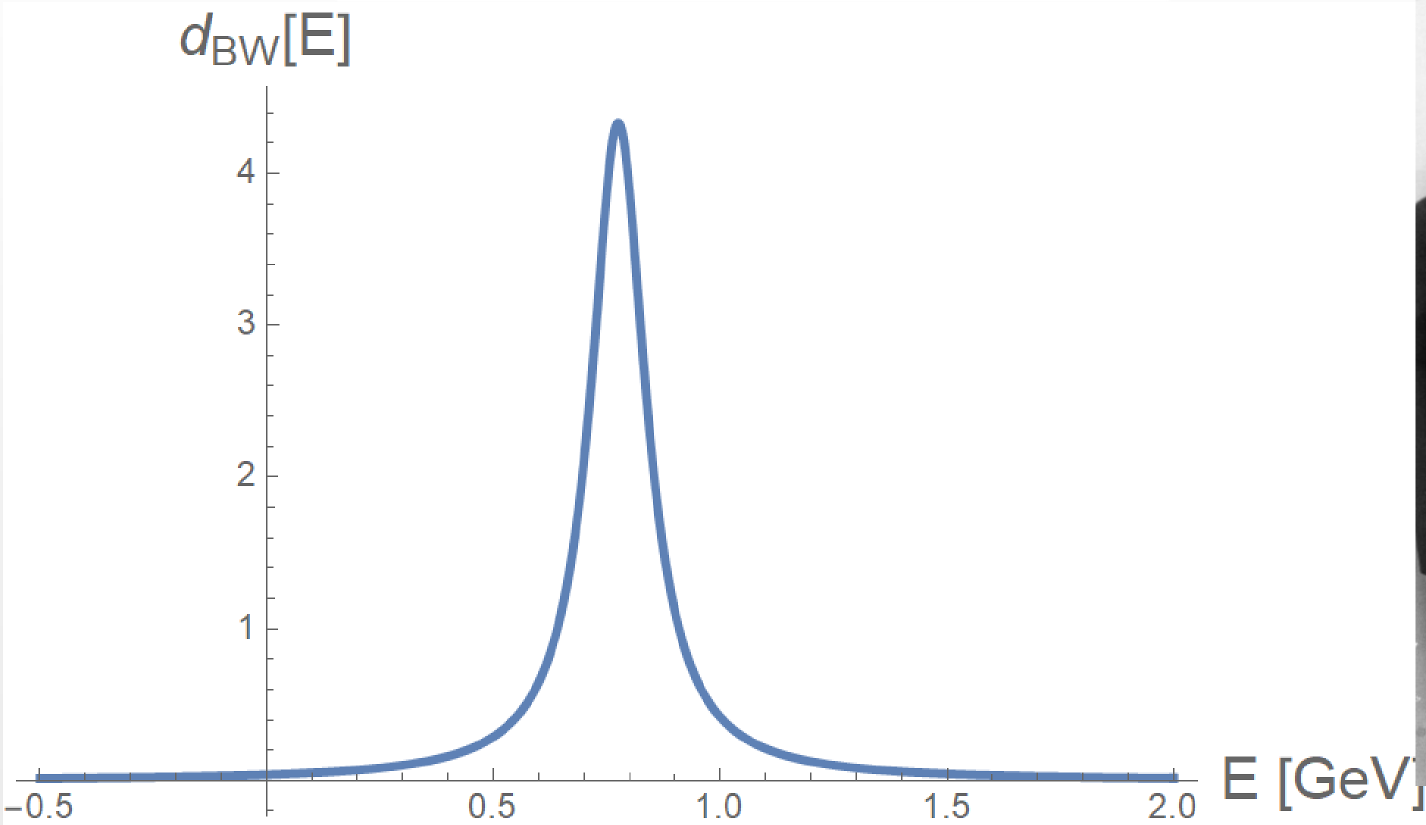
$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2-M^2)^2 + (M\Gamma)^2} \theta(E)$$

$$d_S^{\mathrm{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2-E_{th}^2}\,\tilde{\Gamma}}{(E^2-M^2)^2 + (\sqrt{E^2-E_{th}^2}\,\tilde{\Gamma})^2} \theta(E-E_{th})$$

Breit-Wigner distribution

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E - M)^2 + \frac{\Gamma^2}{4}}$$



Rho-meson as example.

BW extends from –inf to +inf. There is no left threshold.

Relativistic Breit-Wigner (rBW)

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2 - M^2)^2 + (M\Gamma)^2} \theta(E)$$

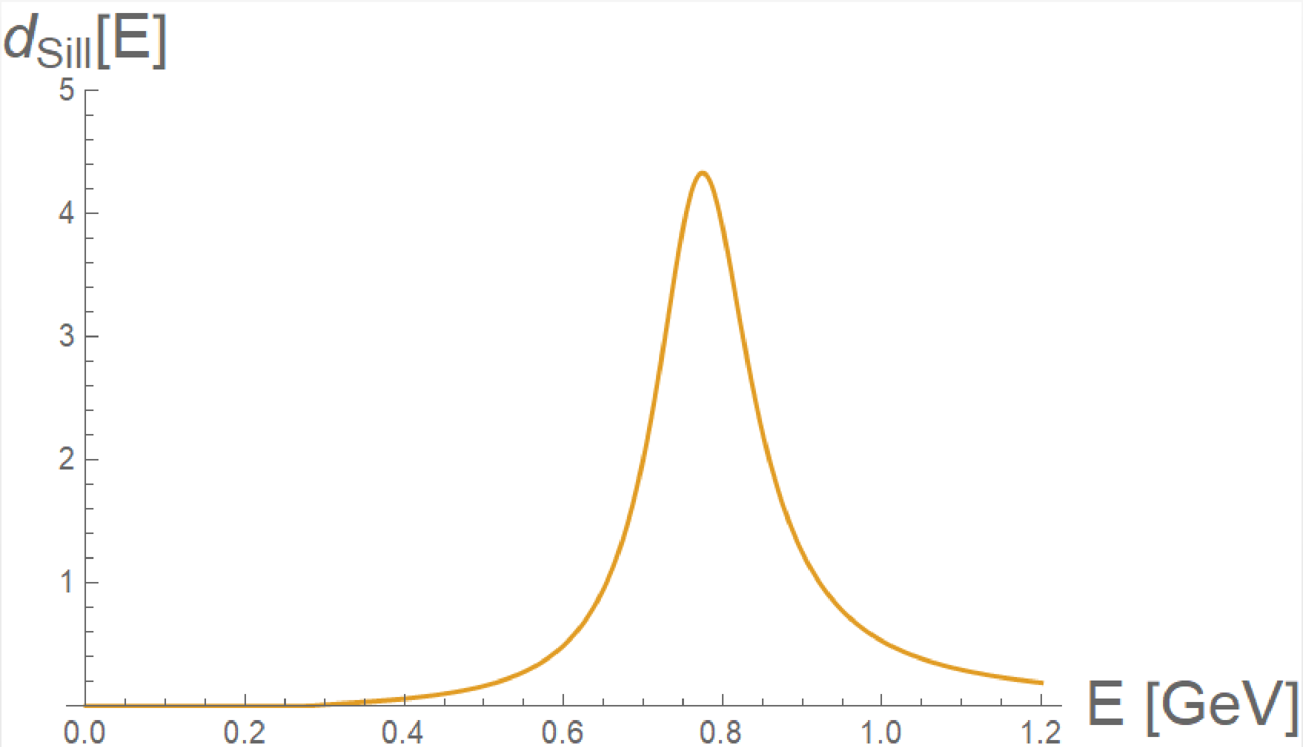
In a relativistic framework there is always a threshold! (eventually zero).

Function above not normalized as it stands.

From above often used in various applications.

‘Sill’ distribution

$$d_S(E) = d_S^{\text{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}}{(E^2 - M^2)^2 + \left(\sqrt{E^2 - E_{th}^2}\,\tilde{\Gamma}\right)^2} \theta(E - E_{th})$$



Sill for the rho-meson

Comments

$$s_{pole} = M^2 - \frac{\tilde{\Gamma}^2}{2} - i\sqrt{(M^2 - s_{th})\tilde{\Gamma}^2 + \frac{\tilde{\Gamma}^4}{4}} \, .$$

Note, for $\tilde{\Gamma}^2$ sufficiently smaller than $M^2 - s_{th}$, the pole of s can be approximated as

$$s_{pole} \simeq M^2 - i\sqrt{(M^2 - s_{th})}\tilde{\Gamma} = M^2 - iM\Gamma \, ,$$

The normalization

$$\int_{E_{th}}^{+\infty} \mathrm{dE} \, d_S^{\mathrm{Sill}}(E) = 1$$

for any E_{th}, M, and $\tilde{\Gamma}$ is a consequence of the proper treatment of the real part of the loop

Sill extension to multi-channel case

The extension to the N channels is straightforward::

$$G_S(s) = \frac{1}{s - M^2 + i\sum_{k=1}^{N} \tilde{\Gamma}_k \sqrt{s - s_{k,th}} + i\varepsilon}$$

with

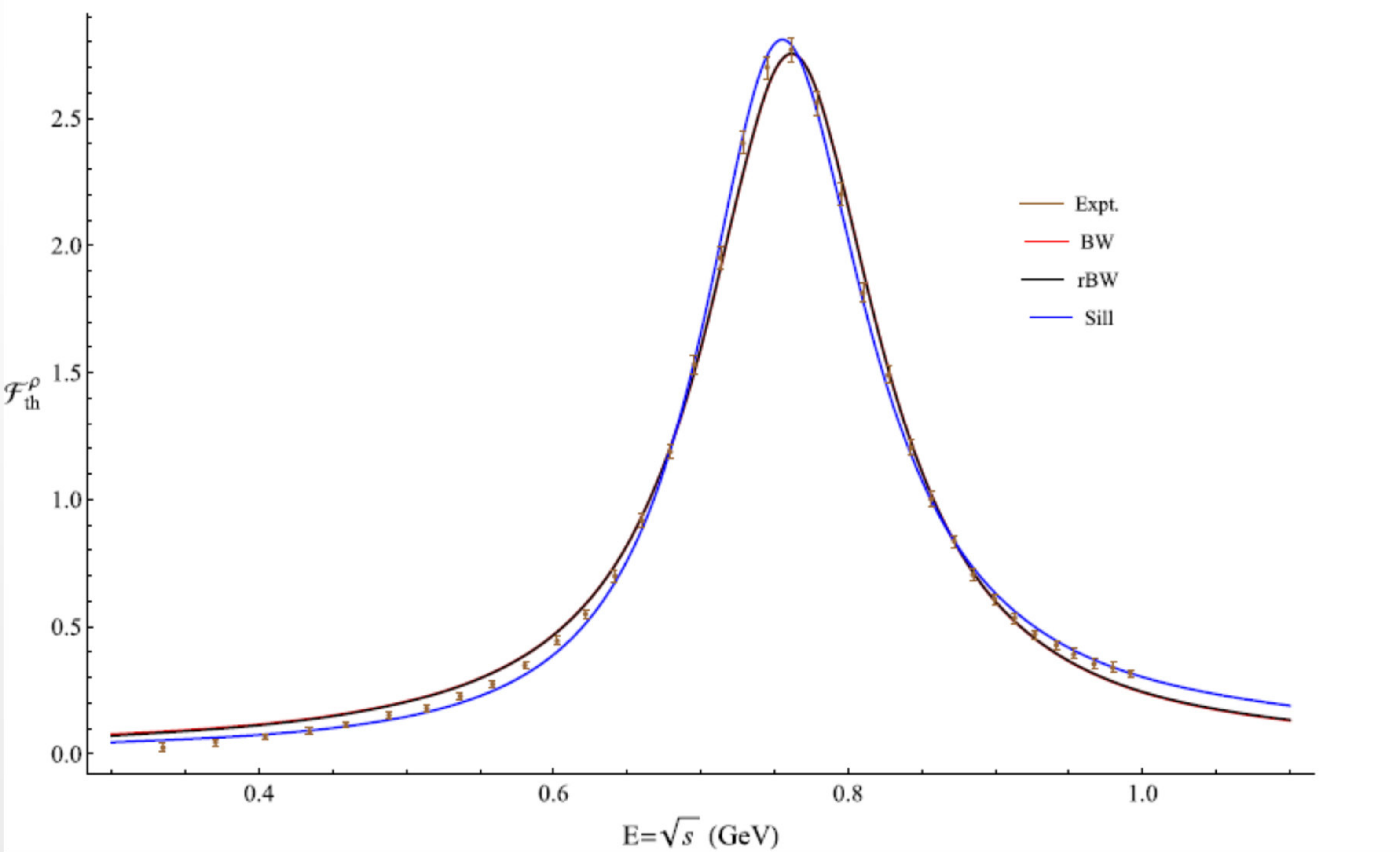
$$\tilde{\Gamma}_k = \Gamma_k \frac{M}{\sqrt{M^2 - E_{k,th}^2}} \text{ and}$$

$$s_{1,th} = E_{1,th}^2 \leq s_{2,th} \leq \ldots \leq_{N,th} = E_{N,th}^2 \,.$$

$$d_s^k(s) = \frac{1}{\pi} \frac{\sqrt{s - s_{\mathrm{th,k}}}\ \tilde{\Gamma}_k}{(s - M^2 - \sum_{i=1}^{Q} \sqrt{s_{\mathrm{th,i}} - s}\ \tilde{\Gamma}_i)^2 + \sum_{i=Q+1}^{N} (\sqrt{s - s_{\mathrm{th,i}}}\ \tilde{\Gamma}_i)^2} \theta(s - s_{\mathrm{th,k}})$$

where, $s_{\mathrm{th,k}}$ is the k^{th} threshold, and the integer Q is such that, for all $i < Q$, $s_{th,i} < s_{th,k}$

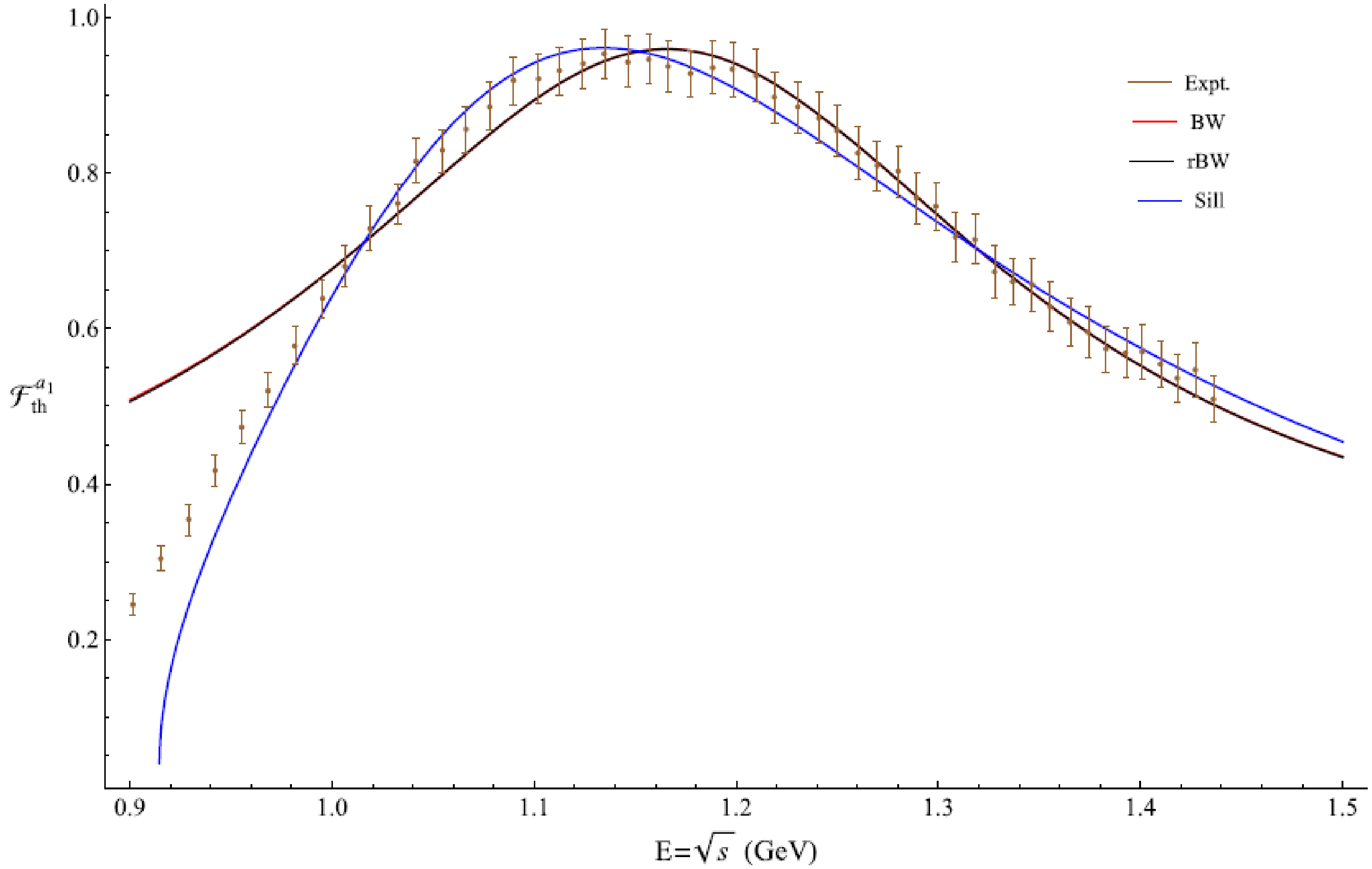
ρ meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	761.64 ± 0.32	144.6 ± 1.3	10.16	$761.6 - i\,72.3$
Relativistic BW	758.1 ± 0.33	145.2 ± 1.3	9.42	$761.5 - i\,72.3$
Sill	755.82 ± 0.33	137.3 ± 1.1	3.52	$751.7 - i\,68.6$

a1 meson



Aleph data for tau decay

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1165.6 ± 1.2	415 ± 15	4.31	$1166 - i\,208$
Relativistic BW	1146.5 ± 1.6	424 ± 16	4.25	$1165 - i\,209$
Sill	1181.3 ± 3.4	539 ± 27	3.52	$1046 - i\,250$

The Delta baryon

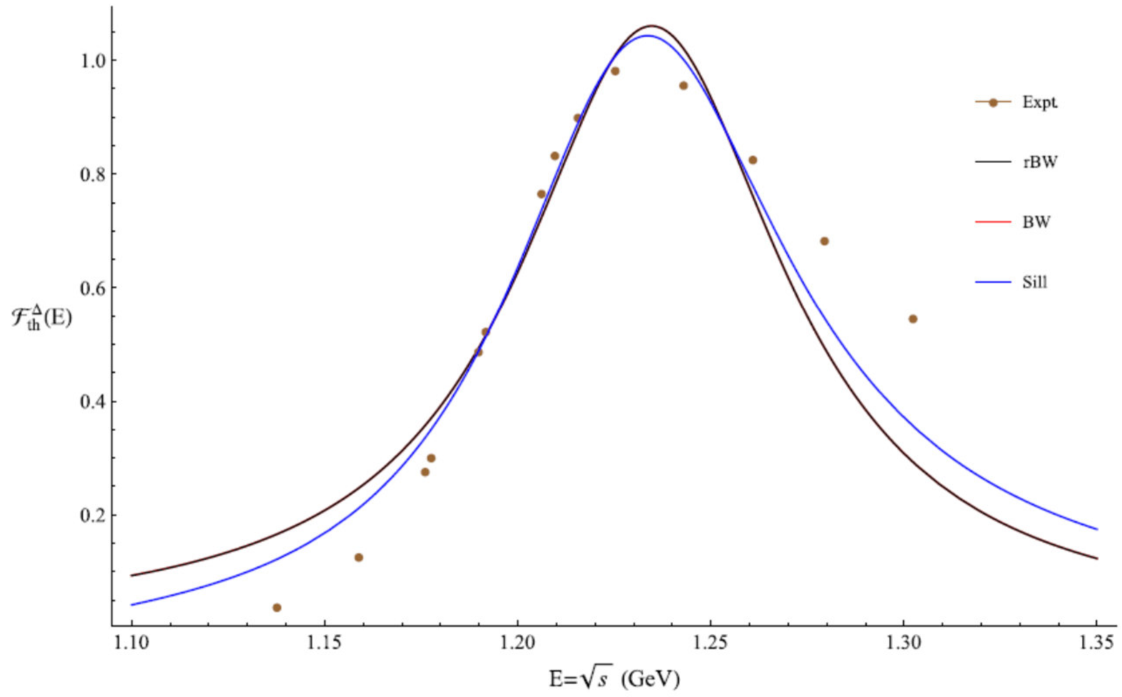


Fig. 6 The spectral function for the $\Delta(1232)$. Experimental data from [82]. It is visible that the Sill fairs marginally better than the (r)BW distributions

Table 5 Mass and width of $\Delta(1232)$ fitted using the three distributions discussed in the text, their error estimates, and the poles (as described in the text)

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1234.6 ± 1.3	83.6 ± 4.1	2.92	$1234.7 - i\,41.8$
Relativistic BW	1233.9 ± 1.2	83.7 ± 4.2	2.92	$1234.7 - i\,41.8$
Sill	1236.2 ± 1.5	90.4 ± 4.9	1.53	$1235.4 - i\,45.2$

Data from: J.R. Haskins, Am. J. Phys. **53**, 988–991 (1985)

Recent Sill application/JPAC and CLAS

Uniwersytet Jana Kochanowskiego w Kielcach

PHYSICAL REVIEW D **106**, 094009 (2022)

XYZ spectroscopy at electron-hadron facilities. II. Semi-inclusive processes with pion exchange

D. Winney,[1,2,*] A. Pilloni,[3,4,†] V. Mathieu,[5,‡] A. N. Hiller Blin,[6,7] M. Albaladejo,[8] W. A. Smith,[9,10] and A. Szczepaniak[9,10,11]

(Joint Physics Analysis Center)

description of the πp mass distribution in the Δ mass region:

$$d_{\Delta\to\pi p}(M^2) = \frac{1}{\pi}\frac{\rho(M^2)\tilde{\Gamma}_\Delta}{[M^2 - m_\Delta^2]^2 + [\rho(M^2)\tilde{\Gamma}_\Delta]^2}, \quad (39)$$

with $\rho(M^2) = \sqrt{M^2 - M_{\text{min}}^2}$ and $\tilde{\Gamma}_\Delta = \Gamma_\Delta m_\Delta/\rho(m_\Delta^2)$. Interestingly, this function is normalized across the mass

First measurement of hard exclusive $\pi^-\Delta^{++}$ electroproduction beam-spin asymmetries off the proton

(The CLAS Collaboration)

ArXiv: 2303.11762

As a second completely independent method, a bin-by-bin background subtraction was performed based on a fit of the complete distribution (signal + background) with a so-called "Sill" function, which is a Breit-Wigner distribution including threshold effects [28] plus a fifth-order polynomial background in each Q^2, x_B, -t and ϕ bin and for each helicity state. After the combined fit, the signal and background contributions were separated and the asymmetry was calculated based on the pure signal events. It was found that both methods provided consistent results for the signal asymmetry within the statistical uncertainty.

Recent Sill application/2

First measurement of hard exclusive $\pi^- \Delta^{++}$ electroproduction beam-spin asymmetries off the proton

(The CLAS Collaboration)

ArXiv: 2303.11762

As a second completely independent method, a bin-by-bin background subtraction was performed based on a fit of the complete distribution (signal + background) with a so-called "Sill" function, which is a Breit-Wigner distribution including threshold effects [28] plus a fifth-order polynomial background in each Q^2, x_B, $-t$ and ϕ bin and for each helicity state. After the combined fit, the signal and background contributions were separated and the asymmetry was calculated based on the pure signal events. It was found that both methods provided consistent results for the signal asymmetry within the statistical uncertainty.

Recent Sill application/3: Xi(1620)

ArXiv: 2305.19093

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

ALICE

CERN

CERN-EP-2023-106
30 May 2023

Accessing the strong interaction between Λ baryons and charged kaons with the femtoscopy technique at the LHC

ALICE Collaboration*

$$f(k^*) = \frac{-2\tilde{\Gamma}_{\Lambda K^-}}{E^2 - M^2 + i\tilde{\Gamma}_{\Xi\pi}\sqrt{E^2 - E^2_{\text{thr.}\Xi\pi}} + i\tilde{\Gamma}_{\Lambda K^-}\sqrt{E^2 - E^2_{\text{thr.}\Lambda K^-}}}$$

$\boxed{\Xi(1620)}$ $I(J^P) = \frac{1}{2}(?^?)$ Status: $*$

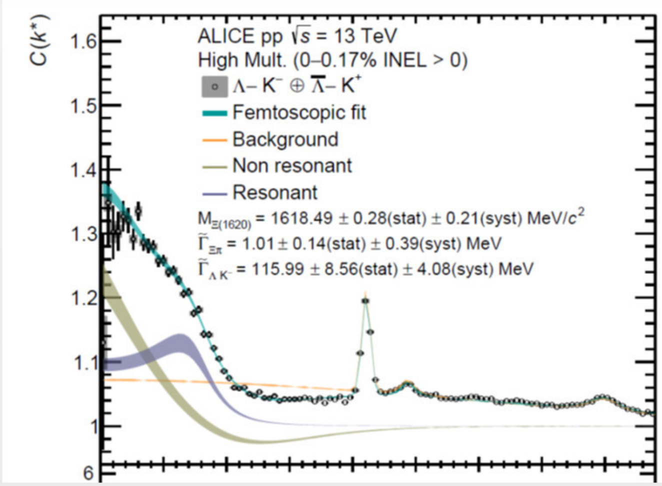
J, P need confirmation.

OMITTED FROM SUMMARY TABLE

What little evidence there is consists of weak signals in the $\Xi\pi$ channel. A number of other experiments (e.g., BORENSTEIN 72 and HASSALL 81) have looked for but not seen any effect.

Ξ(1620) MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
≈ 1620 OUR ESTIMATE				



Ξ(1620) DECAY MODES

	Mode
Γ_1	$\Xi\pi$

Comments

The Sill is Flatte-like, but not equal.

PHYSICAL REVIEW D **99,** 093007 (2019)

Isovector scalar $a_0(980)$ and $a_0(1450)$ resonances in the $B \to \psi(K\bar{K}, \pi\eta)$ decays

Zhou Rui,* Ya-Qian Li, and Jie Zhang

$$M_{a_0(980)}(\omega^2) = \frac{m_0^2}{m_0^2 - \omega^2 - i(g_{\pi\eta}^2\rho_{\pi\eta} + g_{KK}^2\rho_{KK})}$$

It does not reduce to Flatte (even not in the KK channel)

$$\rho_{\pi\eta} = \sqrt{\left[1 - \left(\frac{m_\eta - m_\pi}{\omega}\right)^2\right]\left[1 - \left(\frac{m_\eta + m_\pi}{\omega}\right)^2\right]},$$

$$\rho_{K\bar{K}} = \frac{1}{2}\sqrt{1 - \frac{4m_{K^\pm}^2}{\omega^2}} + \frac{1}{2}\sqrt{1 - \frac{4m_{K^0}^2}{\omega^2}}.$$

Comment

The Sill is Flatte-like, but not equal.

Flatté-like distributions and the $a_0(980)/f_0(980)$ mesons

V. Baru[1], J. Haidenbauer[2], C. Hanhart[2], A. Kudryavtsev[1], Ulf-G. Meißner[2,3]

Eur.Phys.J.A 23 (2005) 523-533e-Print: nuclth/0410099 [nucl-th]

$$\frac{d\sigma_i}{dm} \propto \left| \frac{m_R\sqrt{\Gamma_{\pi\eta}\Gamma_i}}{m_R^2 - m^2 - i m_R(\Gamma_{\pi\eta} + \Gamma_{K\bar{K}})} \right|^2 ,$$

with the partial widths $\Gamma_{\pi\eta} = \bar{g}_\eta q_\eta$ and

$$\Gamma_{K\bar{K}} = \bar{g}_K \sqrt{m^2/4 - m_K^2}$$

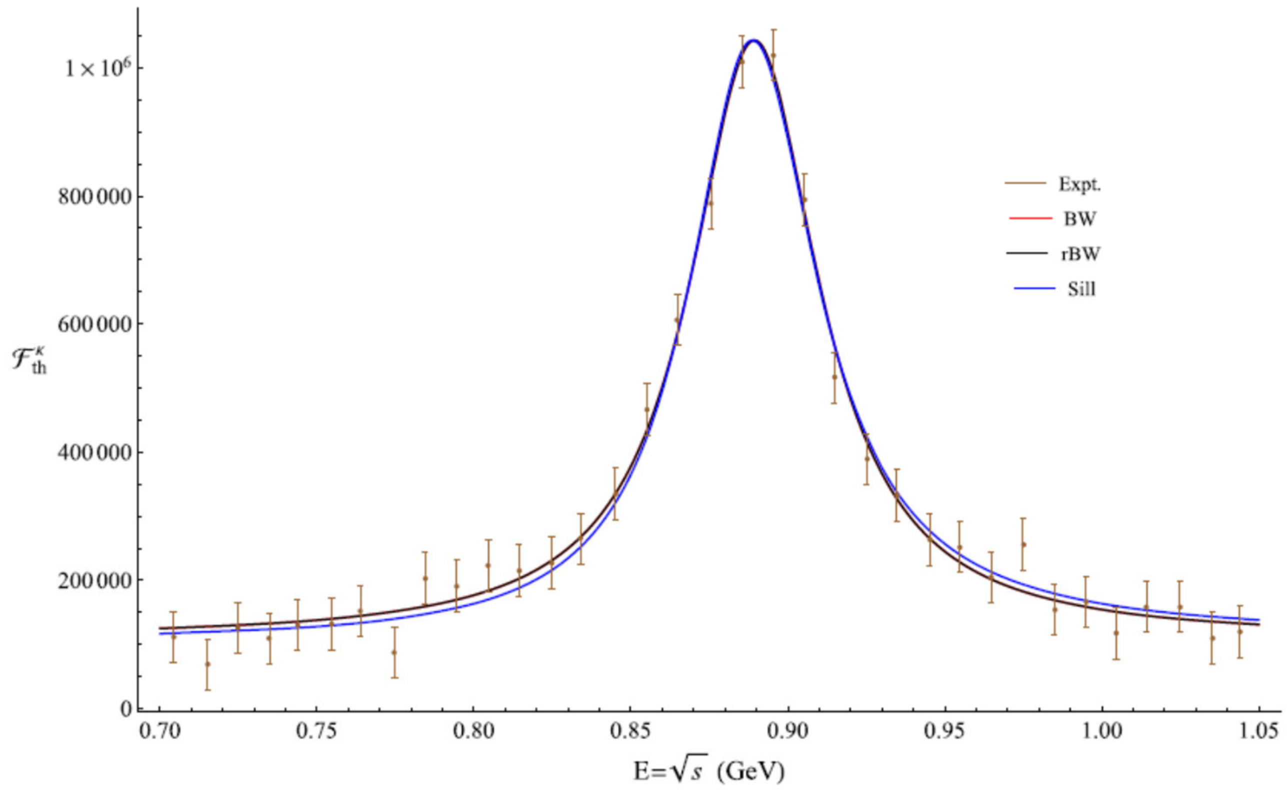
above threshold and

$$\Gamma_{K\bar{K}} = i\bar{g}_K \sqrt{m_K^2 - m^2/4}$$

The Sill is as Flatte along KK
(but not along pion-eta)

The K*(892) meson: basically no difference

Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	889.37 ± 0.43	50.1 ± 1.6	1.78	$889.4 - i\,25.0$
Relativistic BW	889.01 ± 0.43	50.1 ± 1.6	1.78	$890.1 - i\,24.9$
Sill	889.06 ± 0.43	49.9 ± 1.6	2.08	$888.0 - i\,25.0$



J. Adam *et al.* [ALICE], arXiv:1601.07868

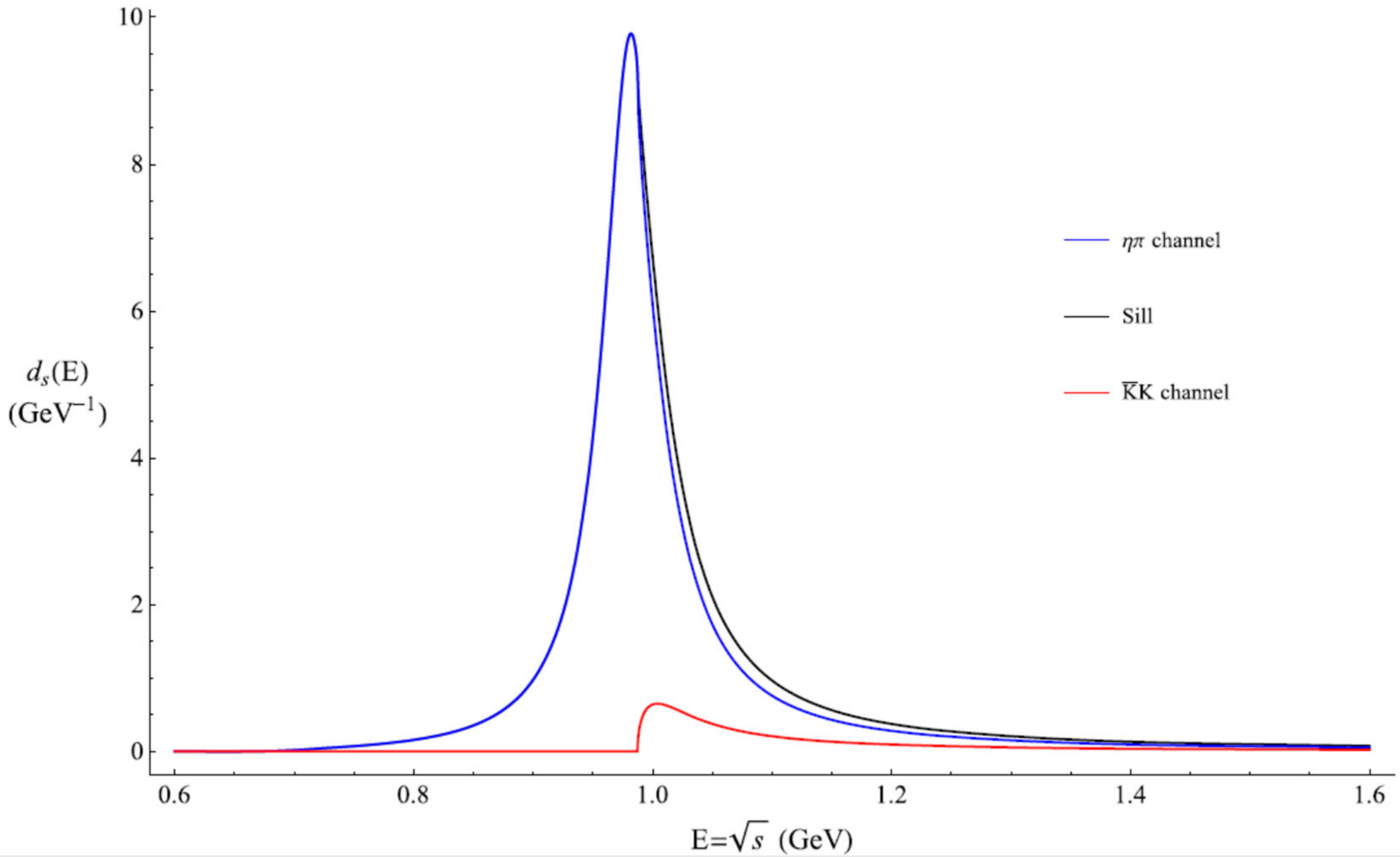
Sill: two-channel case

$$G_S(s) = \frac{1}{s - M^2 + i\tilde{\Gamma}_1\sqrt{s - s_{1,th}} + i\tilde{\Gamma}_2\sqrt{s - s_{2,th}} + i\varepsilon};$$

$$d_S(s) = -\frac{1}{\pi}\,\mathrm{Im}[G_S(s)] = \begin{cases} \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}}}{(s-M^2)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}}+\tilde{\Gamma}_2\sqrt{s-s_{2,th}})^2} & \text{for } s > s_{2,th} \\ \frac{1}{\pi}\frac{\tilde{\Gamma}_1\sqrt{s-s_{1,th}}}{\left(s-M^2-\tilde{\Gamma}_2\sqrt{s_{2,th}-s}\right)^2+(\tilde{\Gamma}_1\sqrt{s-s_{1,th}})^2} & \text{for } s_{1,th} \le s \le s_{2,th} \\ 0 & \text{for } s < s_{1,th} \end{cases}$$

a0(980) example

Fig. 8 The Sill distribution of the $a_0(980)$ and the $\eta\pi$ and $\bar{K}K$ channels. The non-BW form due to the KK threshold is evident



Multichannel decay law

Physics Letters B 831 (2022) 137200

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Physics Letters B

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Multichannel decay law

Francesco Giacosa [a,b,*]

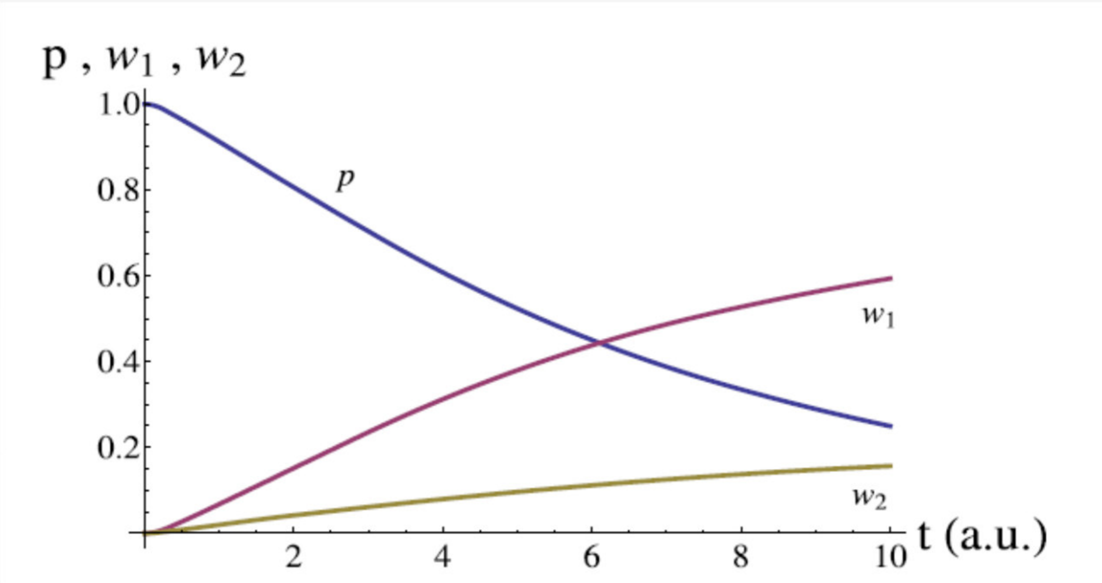

Fig. 1. The survival probability $p(t)$ of Eq. (1) and the decay probabilities $w_1(t)$ and $w_2(t)$ of Eq. (14) are plotted as function of t. The constraint $p+w_1+w_2=1$ holds. Note, t is expressed in a.u. of $[M^{-1}]$.

w1(t) is the probability that the decay has occurred in the first channel between (0,t)

$$\sum_{i=1}^{N} w_i = 1 - p(t)$$

$$w_i(t) = \int_{E_{th,i}}^{\infty} \mathrm{d}E \frac{2E^2\Gamma_i(E)}{\pi} \left| \int_{E_{th,1}}^{\infty} \mathrm{d}E' d_S(E') \frac{e^{-iE't} - e^{-iEt}}{E'^2 - E^2} \right|^2$$

w1/w2 is not a constant