First Computation of ^{4}He Compton Scattering: the Transition-Density Formalism

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1. Two-Photon Response Explores System Dynamics
2. Per Aspera Ad Astra with the Transition-Density Formalism
3. Confronting Reality: Compton Scattering on ^{4}He
4. Concluding Questions

How do constituents of the nucleon react to external fields?

How to reliably extract proton, neutron, spin polarisabilities?

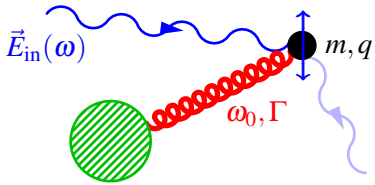
How to plan effective experiments & test theory?

Exp-Th Compton Roadmap in "Next-Gen γ Source": IJMPG49 (2022) 010502
transition density formalism and ^{3}He: hg/JMcG/AN/DRP: Few-Body Syst. **61** (2020) 61
^{4}He $\mathcal{O}(e^2\delta^3)$: Liao/hg/JMcG/AN/DRP: in preparation

1. Two-Photon Response Explores System Dynamics

(a) Polarisabilities: Stiffness of Charged Constituents in El.- Mag. Fields

Example: induced electric dipole radiation from harmonically bound charge, damping Γ Lorentz/Drude 1900/1905



$$\vec{d}_{\rm ind}(\omega) = \underbrace{\frac{q^2}{m}\,\frac{1}{\omega_0^2-\omega^2-{\rm i}\Gamma\omega}}_{=:\,4\pi\,\alpha_{E1}(\omega)}\,\vec{E}_{\rm in}(\omega)$$

"displaced volume" $[10^{-4}\ {\rm fm}^3]$

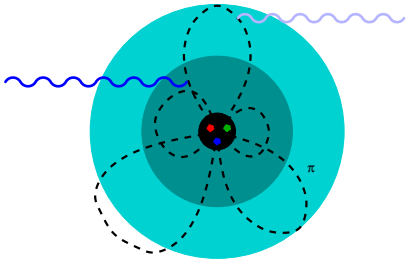
Energy- (ω)-dep. multipoles for ***interaction scales, symmetries & mechanisms*** **with & among constituents.**

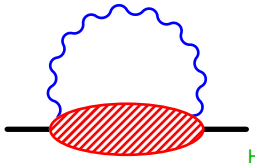
Clean, perturbative probe: χiral symmetry of pion-cloud & its breaking, $\Delta(1232)$, spin-constituents.

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2\rangle\ldots$ PDG

$$\mathcal{L}_{\rm pol} = 2\pi\left[\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2 + \ldots\right]$$

α_{E1}: **electric**
β_{M1}: **magnetic**
scalar dipole polarisability

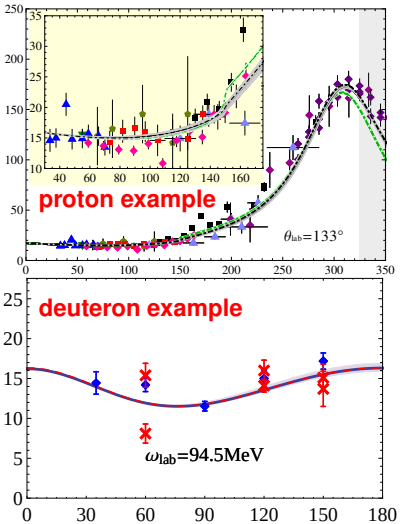




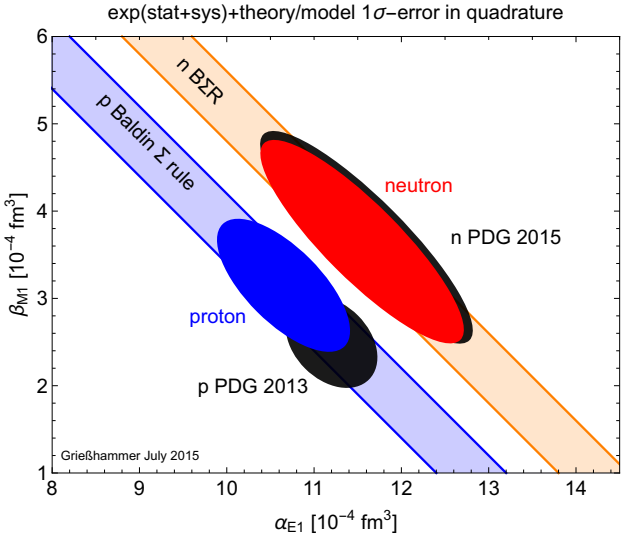
elmag. self-energy part
of nucleon mass splitting
$M_\gamma^{\rm p} - M_\gamma^{\rm n} \approx [1.1\pm0.5]\,{\rm MeV}$
with $\alpha_{E1}^{p-n} = -1.7\pm0.4_{\rm tot}$
Hoferichter/Gasser/Leutwyler/Rusetzky 2015

(b) Scalar Polarisabilities from Consistent p & d Databases

database: JMcG/DRP/hg/
Feldman PPNP 2012



$\Longleftarrow \Delta(1232)$



	α_{E1} [10^{-4} fm^3]	β_{M1} [10^{-4} fm^3]	χ^2/d.o.f.
proton (Baldin, N^{2}LO) McGovern/Phillips/hg EPJA 2013	$10.65 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}}$	$3.15 \mp 0.35_{\text{stat}} \pm 0.2_{\Sigma} \mp 0.3_{\text{theory}}$	$113.2/135$
neutron (Baldin, NLO) COMPTON@MAX-lab PRL 2014	$11.55 \pm 1.25_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.8_{\text{theory}}$	$3.65 \mp 1.25_{\text{stat}} \pm 0.2_{\Sigma} \mp 0.8_{\text{theory}}$	$45.2/44$

$\Longrightarrow$ **neutron $\approx$ proton polarisabilities:** $\alpha_{E1}^{p-n} = -0.9 \pm 1.6_{\text{tot}}$ – **exp. & neutron errors dominate**

$-0.6 \pm 1.2_{\text{tot}}$ PDG 2022

CottinghamΣR explains $M_\gamma^p - M_\gamma^n$ with $\alpha_{E1}^{p-n} = -1.7 \pm 0.4_{\text{tot}}$ **Gasser/Hoferichter/Leutwyler/Rusetzky 1506.06747**

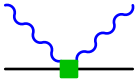
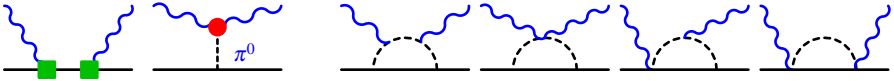
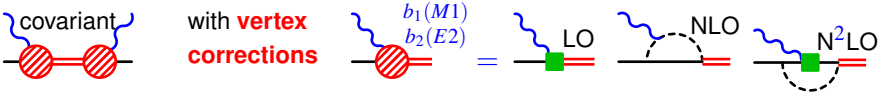
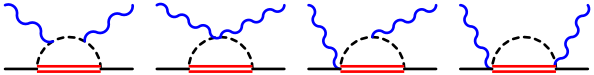
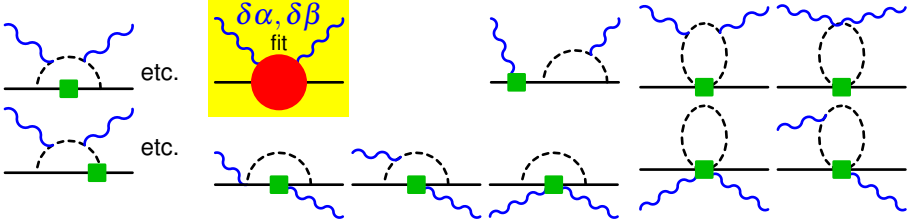
(c) All 1N Contributions to N^{4}LO

Bernard/Kaiser/Meißner 1992-4, Butler/Savage/Springer 1992-3, Hemmert/...1998
McGovern 2001, hg/Hemmert/Hildebrandt/Pasquini 2003
McGovern/Phillips/hg 2013

Unified Amplitude: accuracy decreases with ω:

in low régime $\omega \lesssim m_\pi$ **at least N^{4}LO** $(e^2\delta^4)$: **accuracy** $\delta^5 \lesssim 2\%$;

or **in high régime** $\omega \sim M_\Delta - M_N$ **at least NLO** $(e^2\delta^0)$: **accuracy** $\delta^2 \lesssim 20\%$.

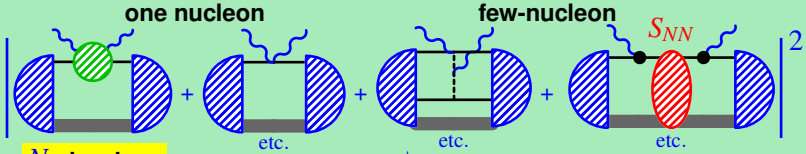
	$\omega \lesssim m_\pi$	$\sim M_\Delta - M_N \approx 300$ MeV
 Thomson term: $-\frac{Z^2\alpha_{EM}}{M}$	$e^2\delta^0$ ***LO***	$e^2\delta^0 \searrow$ NLO
	$e^2\delta^2$ N^{2}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^3$ N^{3}LO	$e^2\delta^{-1} \nearrow$ ***LO***
	$e^2\delta^3$ N^{3}LO	$e^2\delta^1$ N^{2}LO
	$e^2\delta^4$ N^{4}LO	$e^2\delta^2$ N^{3}LO

Unknowns: short-distance $\delta\alpha, \delta\beta \Longleftrightarrow$ **static** α_{E1}, β_{M1} **(offset)** $\Longrightarrow$ ω**-dependence *predicted*.**

2. Per Aspera Ad Astra with the Transition-Density Formalism

(a) How to Get to the Neutron?

deuteron: hg/. . . /+Phillips/+McGovern 2004-
MECs: Beane/. . . 1999-2005
^{3}**He:** Shukla/. . . 2009 +Strandberg/Margaryan/hg/. . . 1804.01206



Parameter-free, except N polarisabilities.

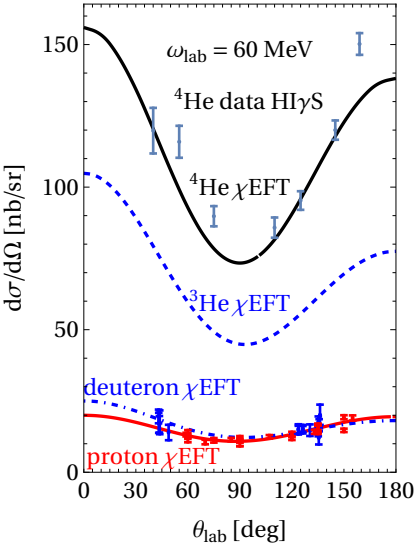
Experiment: More charge & MECs $\Longrightarrow$ more counts $\Longrightarrow$ *heavier nuclei*

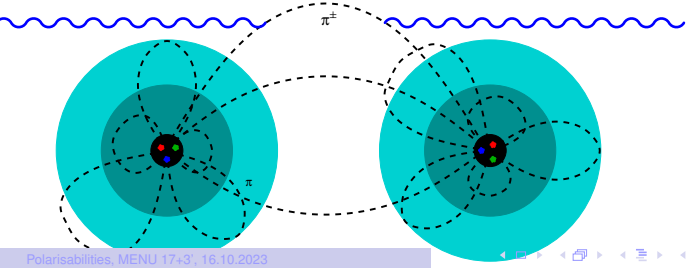
Theory: Reliable only if nuclear binding & levels accurate $\Longrightarrow$ *lighter nuclei*

Find *sweet-spot* between competing forces: deuteron, ^{3}He, ^{4}He, ^{6}Li?

Deuteron, ^{4}He: sensitive to $\alpha^p_{E1} + \alpha^n_{E1}, \quad \beta^p_{M1} + \beta^n_{M1} \Longrightarrow$ **neutron pols**

^{3}**He**: sensitive to $2\alpha^p_{E1} + \alpha^n_{E1}, 2\beta^p_{M1} + \beta^n_{M1} \Longrightarrow$ **neutron pols**





Model-independently subtract binding effects.

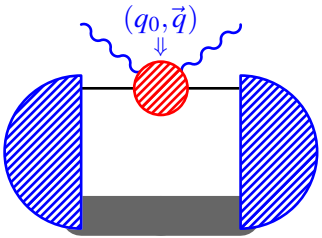
$\Longrightarrow$ χEFT: reliably quantify uncertainties.

Chirally consistent 1N & few-N: potentials, wave functions, currents, π-exchange.

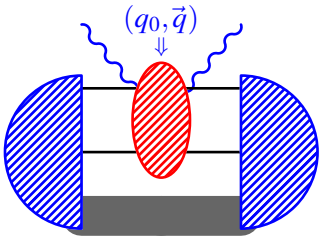
Test charged-pion component of NN force.

(b) A New Hope: The Transition-Density Formalism

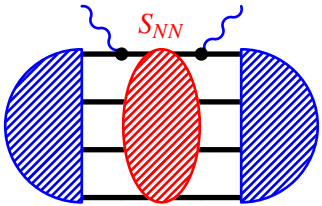
hg/McGovern/Phillips/Nogga
FewB Sys 61 (2020) 61 arXiv:2005.12207
Alex Long: PhD Project 2022-?



$A-1$ **spectator nucleons**
1 **Active Nucleon:**
one-body density



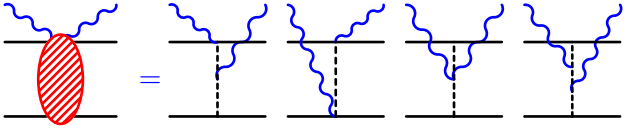
$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density



no spectators
All Nucleons active:
rescattering
important as $\omega \lesssim \frac{1}{R}$

only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
Compton



Beane/. . . 1999-2005

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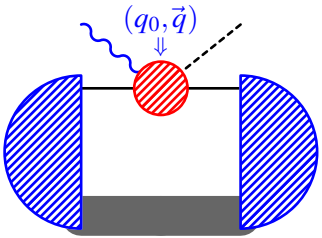

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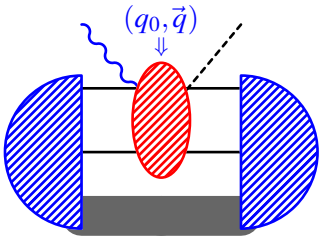

$A-2$ **spectator nucleons**
2 **Active Nucleons:**
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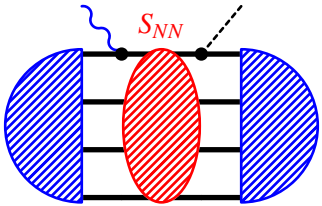

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rescattering
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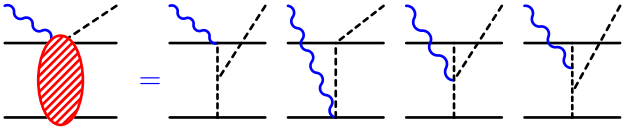
only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
π **production**



Beane/. . . 1995-97

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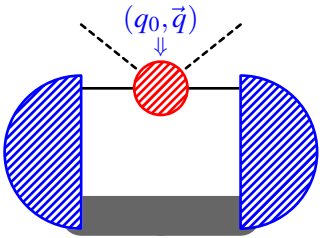

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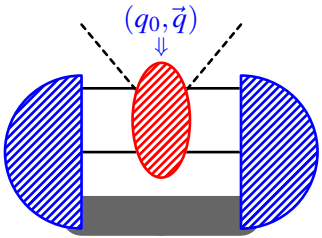
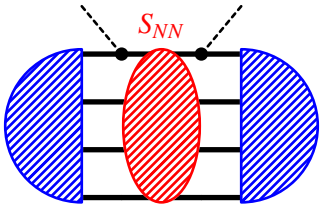


$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density



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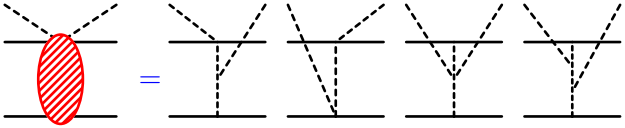
$\mathcal{O}(e^2\delta^2)$ 2N kernel:
π scattering



Beane/. . . 1998

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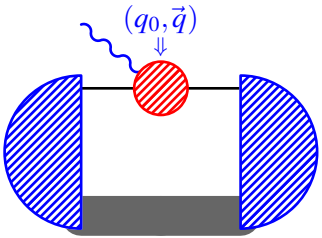

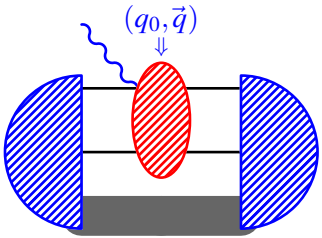

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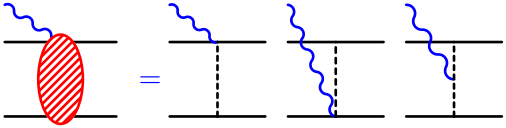
$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density

only depends on quantum numbers of actives and mom. transfer

$\mathcal{O}(e^2\delta^2)$ 2N kernel:
form factor



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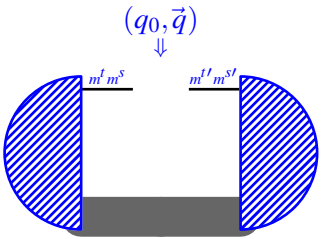

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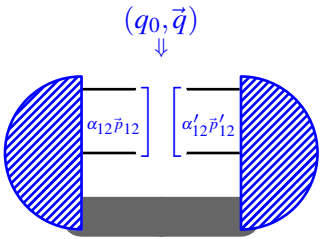
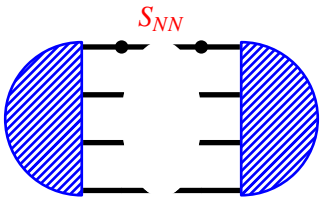


$A-2$ **spectator nucleons**
2 **Active Nucleons:**
two-body density



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Idea: Split calculation into

kernel: interaction with n active nucleons	**structure: $A-n$ spectators**
recycle same reaction for different nuclei	**recycle same nucleus for different reactions**
Compton on ^{3}He, ^{3}H, ^{4}He, ^{6}Li, ...	**^{4}He Compton, π prod., FFs, dark matter,...**

χEFT hierarchy of few-body interactions: onebody, twobody $\gg$ threebody $\gg$ fourbody...

n-body transition density amplitude: n nucleons with intrinsic momenta and specific quantum numbers α absorb momentum transfer $(q_0, \vec{q})$, re-arrange quantum numbers to α', get absorbed back into nucleus.

Computationally highly efficient: well-developed, sophisticated numerical few-body techniques.

Density repository for ^{3}He, ^{4}He at `datapub.fz-juelich.de/anogga` – more (^{6}Li) to come.

^{3}He (arXiv:2005.12207): CPU time reduced from days to hours; extensive checks; same result as traditional.

$\Longrightarrow$ Compute to higher numerical accuracy (integration mesh, j_{12},...): $\approx 1\%$ change)

Old and New: Pion Exchange Currents (Two-Body) in ^{3}He

hg/McGovern/Phillips/Nogga
FewB Sys 61 (2020) 61 arXiv:2005.12207

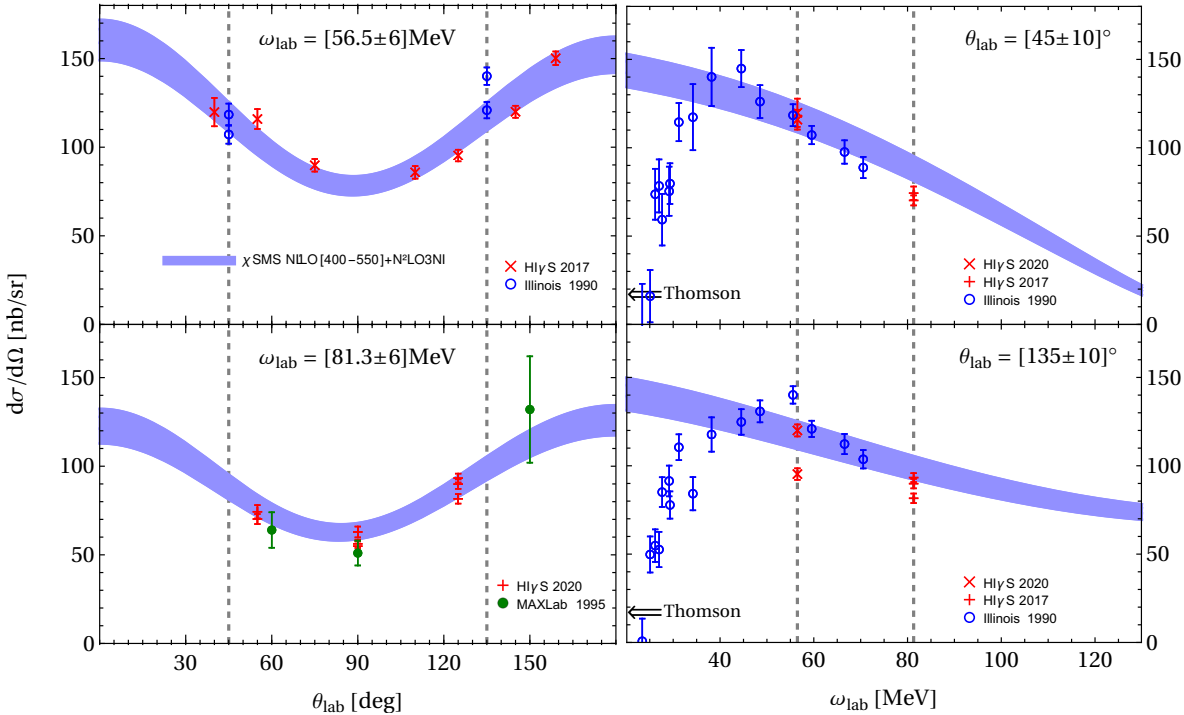
	$\omega = 50$ MeV, $\theta = 30°$				$\omega = 120$ MeV, $\theta = 165°$			
	Idaho N^{3}LO+3NFb		AV18+UIX		Idaho N^{3}LO+3NFb		AV18+UIX	
$\{M', M; \lambda', \lambda\}$	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.	value [fm^3]	rel.dev.
$\{\frac{1}{2}, \frac{1}{2}; 1, 1\}$	$-.07132$	0.1%	$-.09343$	0.2%	$-.00149$	0.0%	$-.00188$	0.2%
$\{\frac{1}{2}, \frac{1}{2}; -1, 1\}$	$-.00543$	0.3%	$-.00702$	0.3%	$-.10220$	0.8%	$-.12570$	0.8%
$\{\frac{1}{2}, \frac{1}{2}; 1, -1\}$	$-.00543$	0.3%	$-.00702$	0.3%	$-.10220$	0.8%	$-.12570$	0.8%
$\{\frac{1}{2}, \frac{1}{2}; -1, -1\}$	$-.07132$	0.1%	$-.09343$	0.2%	$-.00149$	0.0%	$-.00188$	0.2%

Table 7: Comparison of two-body matrix elements in the "density" approach and the "traditional" approach for potentials Idaho N^{3}LO+3NFb and AV18+UIX with $j_{12} \leq 2$ at $\omega = 50$ MeV, $\theta = 30°$ (where mostly diagonal matrix elements are probed) and $\omega = 120$ MeV, $\theta = 165°$ (where off-diagonal matrix elements are probed more strongly). See also text and captions to tables 5 and 3 for further details.

(a) Perfect Scalar-Isoscalar: Sensitive *Only* to α^{p+n}, β^{p+n} – Not To Spinpols γ_i

$\Longrightarrow$ In principle very clean if binding effects under control: $\mathcal{O}(e^2\delta^3)$ with established α_{E1}, β_{M1}.

Promising but wave-function dependence; some data issues. $\Longrightarrow$ No polarisability extraction (yet).



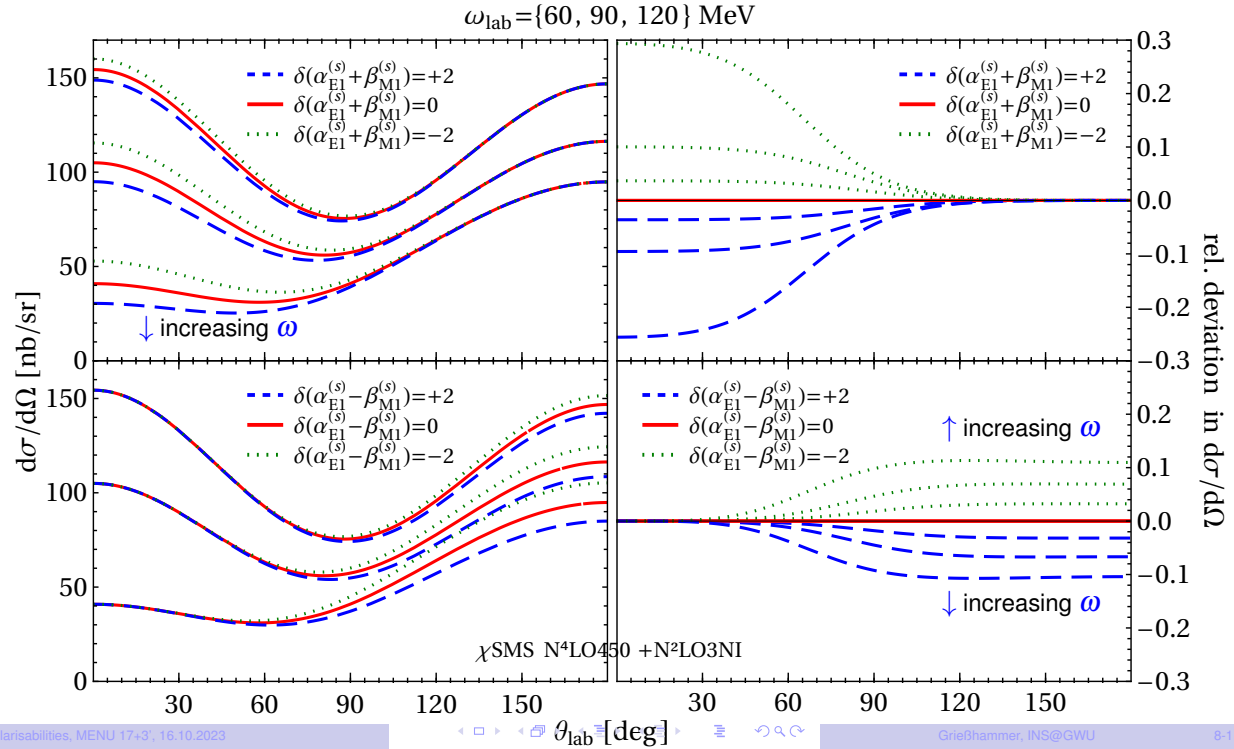
(b) Dependence on Scalar Iso-Scalar Polarisabilities

hg/J. Liao/JAMcG/A. Nogga/DRP
in preparation incl. math.nb

^{4}He: perfect scalar-isoscalar $\Longrightarrow$ sensitive *only* to α^{p+n}, β^{p+n}, not to spin-polarisabilities γ_i.

$\mathcal{O}(e^2\delta^3)$ with χSMSN^{4}LO 450+N^{2}LO 3NI (other potentials: $\lesssim \pm 8\%$). **Suffices for *planning*, not for *analysis*!**

Increasing ω gives bigger relative signals, but smaller rates.



(c) The "Only" Other Observable: Beam Asymmetry $\Sigma^{\rm lin}$

hg/J. Liao/JAMcG/A. Nogga/DRP

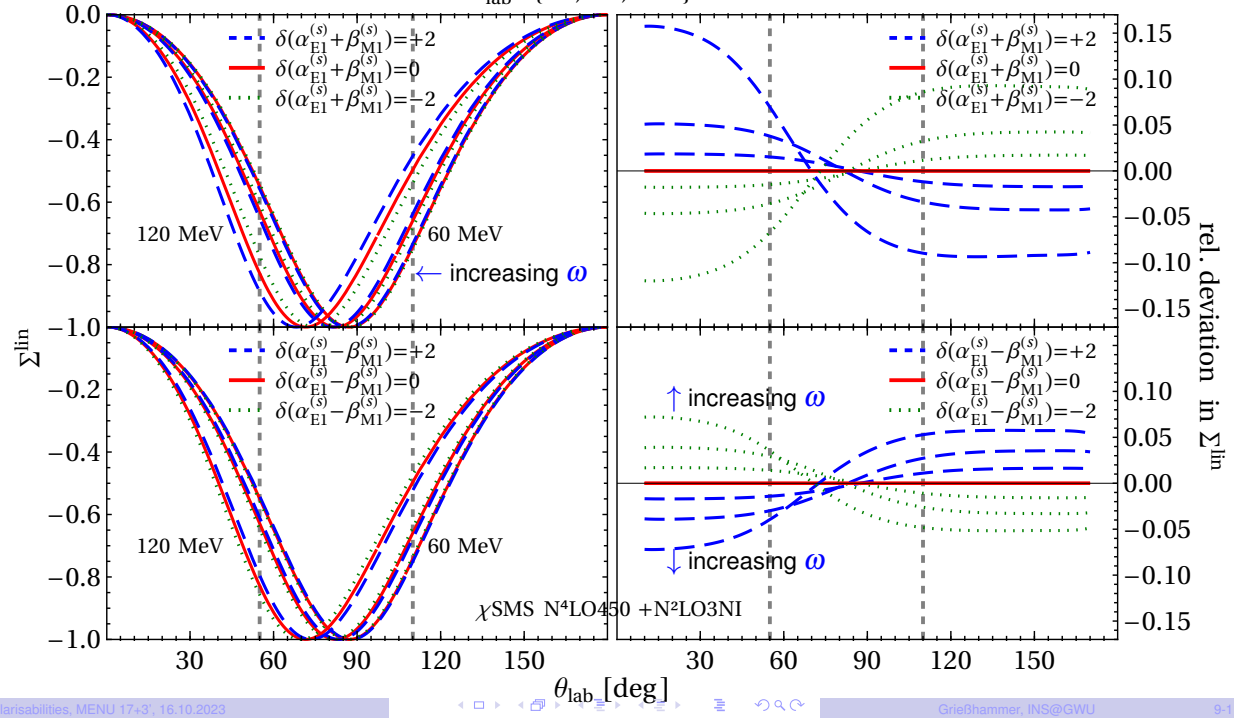
in preparation incl. math.nb

^{4}He: perfect scalar-isoscalar $\Longrightarrow$ sensitive *only* to α^{p+n}, β^{p+n}, not to spin-polarisabilities γ_i.

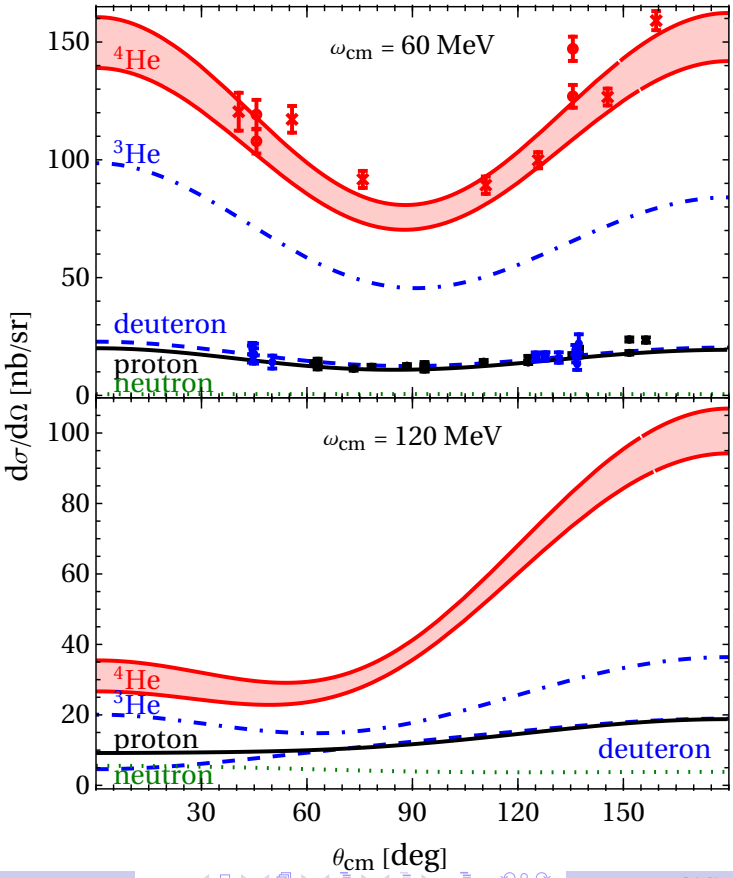
$\mathcal{O}(e^2\delta^3)$ with χSMSN^{4}LO 450+N^{2}LO 3NI (other potentials: $\lesssim \pm 4\%$). **Suffices for *planning*, not for *analysis*!**

Increasing ω gives bigger relative signals, but only around centres of rise/fall.

$\omega_{\rm lab}=\{60,\ 90,\ 120\}$ MeV



(d) Comparing ^{4}He, ^{3}He, Deuteron, Proton, Neutron



4. Concluding Questions

Polarisabilities: **ω-dependence maps out scales, symmetries & mechanisms of interactions:**
χiral symmetry of pion-cloud, $\Delta(1232)$ properties. Impact on $M_p - M_n$, p-radius,...
Spin Polarisabilities: **Stiffness of Spin Constituents; Nuclear Faraday Effect.**
χEFT: systematic, parameter-free predictions with uncertainties; lattice QCD catching up.

Target	Opportunities	Theory Status for All Observables
proton & neutron	nucleon spin polarisabilities	"done": N^{4}LO $\omega \lesssim 230\,\mathrm{MeV}$ for pols. math.nb jupyter.py
deuteron	sensitive to $p+n$ average polarised, d-wave interference: mixed spin pols $\gamma_{E1M2}, \gamma_{M1E2}$	$\omega < m_\pi$ N^{3}LO done, N^{4}LO this year math.nb $\omega \gtrsim m_\pi$ needs resources
^{3}He: increased rates	unpolarised: sensitive to $2p+n$ polarised: "n-spin" $\Longrightarrow \gamma_i^n$ only	densities method arXiv:2005.12207 ^{3}He:math.nb $\omega \in [50\,\mathrm{MeV}; m_\pi]$ N^{3}LO✓, N^{4}LO like d $\omega \to 0$ under way — $\omega \gtrsim m_\pi$ needs resources
^{4}He: increased rates	sensitive to $p+n$ average, not γ_i's	
$\gamma X \to NY\gamma$ quasifree	tag n or p directly – both at once?	$\gamma d \to np\gamma$ N^{4}LO done; more needs resources

We Need Data: **elastic & inelastic cross-sections & asymmetries – *reliable systematics!***
Low-ω for scalar, high-ω for spin-polarisabilities, **but always $\omega \lesssim 230\,\mathrm{MeV}$.**
Only *combination of dedicated experiments* meaningful! (Not "one datum for one answer".)
$\Longrightarrow$ ***Synergy of Experiment, Low-Energy Theory & Lattice QCD, competitive uncertainties!***

$\Longrightarrow$ **Compton Community programme outlined in White Paper for a**
Next Generation Laser Compton Gamma-ray Beam Facility **arXiv:2012.10843 and DOE.**

The efficient person gets the job done right. The effective person gets the right job done.