Antisymmetrization of The Wave Functions Consisting of Spin-Isospin and Hyperspherical Parts

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Problem statement

- Investigating few-body systems with identical particles in a hyperspherical basis yields the problem of obtaining antisymmetrized functions from functions with arbitrary quantum numbers.
- This article introduces construction schemes for the construction of Four-Body Wave Functions with spin, isospin,and hyperspherical parts that are antisymmetric under particle permutations.

Jibuti R.I., Krupennikova N.B., Chachanidze-Margolin L.L.
Few Body Systems, 4, 151, (1988)
Margolin L.L. Journal of Physics, 343, 1 (2012)
Margolin L.L. EPJ Web of Conferences 66, 09013 (2014)

Objectives:

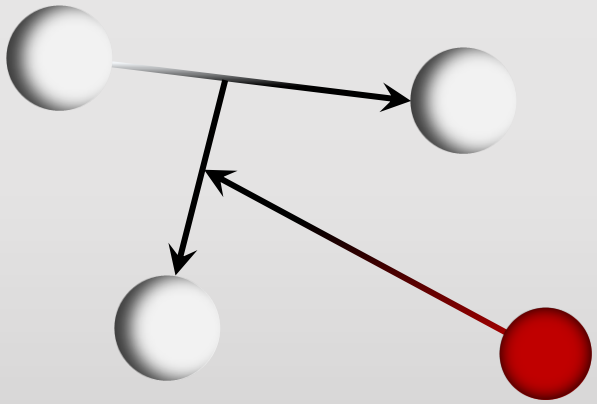
- Hyperspherical basis symmetrization for four-body systems using Parentage Scheme of Symmetrization.
- Parentage coefficients corresponding to the [4], [31], [22], [211], representations of S_4 groups, are obtained,
- Young operators, acting on Four body hyperspherical functions symmetrized with respect to three particles, are derived. The connection between the transformation coefficients for the identical particle systems and the parentage coefficients is demonstrated
- Fully antisymmetrized wave functions with the spin. Isospin and hyper spherical parts are constructed

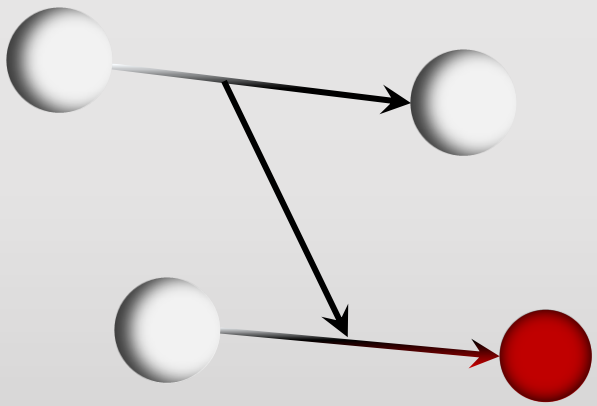
Construction Scheme of Four-Body Wave Functions in a Hyperspherical Basis

- The transformation between Hyperspherical functions corresponding to the (3+1) and (2+2) configurations is considered
- Unitary transformation coefficients under particle permutations are obtained
- Symmetrized Hyperspherical basis for the (3+1) configuration is constructed
- Constructed symmetrized Four-Body Hyperspherical basis with respect to three identical particles

Four Particle System Configurations

- (3+1) Configuration (2+2) Configuration





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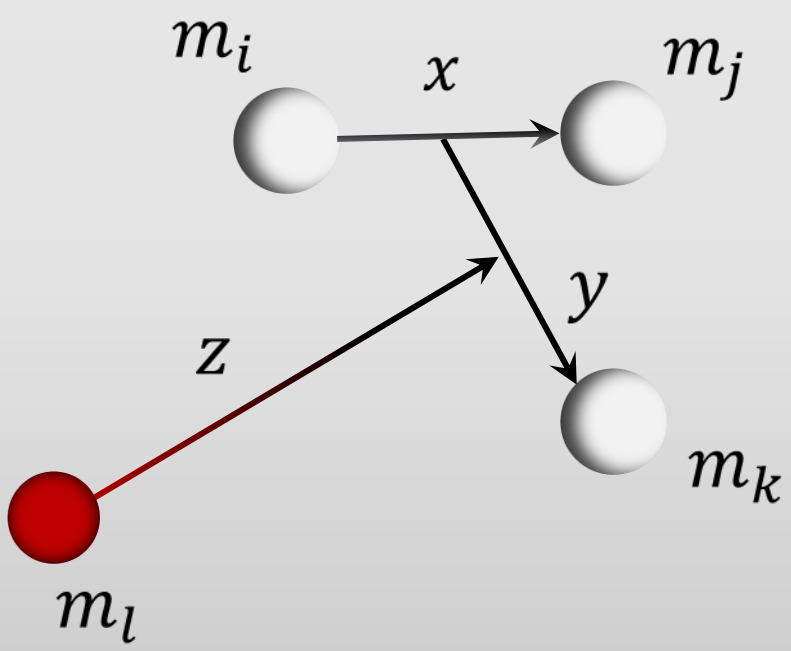
(3+1) Configuration for $\mathrm{He}_{\lambda}^{4}$



$$x = \sqrt{\frac{m_i m_j}{m_i + m_j}}\left(r_i - r_j\right)$$

$$y = \sqrt{\frac{m_k\left(m_i + m_j\right)}{m_i + m_j + m_k}}\left(-r_k + \frac{m_i r_i + m_j r_j}{m_i + m_j}\right)$$

$$z = \sqrt{\frac{\left(m_i + m_j + m_k\right)m_l}{m_i + m_j + m_k + m_l}}\left(-r_l + \frac{m_i r_i + m_j r_j + m_k r_k}{m_i + m_j + m_k}\right)$$

Transformation from Jacobi to Four Body Hyperspherical Coordinates

$$(\vec{x},\vec{y},\vec{z}) \rightarrow \begin{Bmatrix} \rho^2 = x^2 + y^2 + z^2 \\ \omega = \left(\alpha,\beta,\hat{\vec{x}},\hat{\vec{y}},\hat{\vec{z}}\right) \end{Bmatrix},$$

where

$$x = \rho\cos(\alpha)\sin(\beta)\,,$$
$$y = \rho\sin(\alpha)\sin(\beta)\,,$$
$$z = \rho\cos(\beta)\,,$$

$$d\vec{x}d\vec{y}d\vec{z} = \rho^8 d\rho d\omega$$ A

$$d\omega$$
$$= \sin^2(\alpha)\cos^2(\alpha)\sin^5(\beta)\cos^2(\beta)\, d\alpha d\beta d d\Omega_{\hat{\vec{x}}} d\Omega_{\hat{\vec{y}}} d\Omega_{\hat{\vec{z}}}$$

Four Particle HF in Nine Dimensional Space of Jacobi Vectors

$$\Psi_{k_3k_4}^{l_1l_2l_3m_1m_2m_3}(\omega) = \mathcal{N}_{k_3}^{l_1l_2}(\cos(\alpha))^{l_1}(\sin(\alpha))^{l_2}P_n^{l_2+1/2l_1+1/2}(\cos(2\alpha))$$

$$\cdot\, \mathcal{N}_{k_4+3/2}^{l_3k_3+3/2}(\cos(\beta))^{l_3}(\sin(\beta))^{k_3}P_m^{k_3+2,l_3+1/2}(\cos(2\beta))\mathcal{Y}_{l_1m_1}(\hat{\vec{x}})\mathcal{Y}_{l_2m_2}(\hat{\vec{y}})\mathcal{Y}_{l_3m_3}(\hat{\vec{z}}),$$

where

$$\mathcal{N}_a^{bc} = \sqrt{\frac{2d!\,\Gamma(d+c+b+2)(a+2)}{\Gamma(a+b+3/2)(d+c+3/2)}}; \qquad \begin{aligned} 2d &= a-b-c, \\ 2m &= k_4-k_3-l_3, \\ 2n &= k_3-l_1-l_2 \end{aligned}$$

$$K_4^2(\omega) = \frac{k_3^2(\Omega)}{\sin^2(\beta)} - \frac{\partial^2}{\partial\beta^2} - \frac{7\cos^2(\beta)-2}{\sin(\beta)\cos(\beta)}\frac{\partial}{\partial\beta} + \frac{l_z^2}{\cos^2(\beta)}$$

Four Body HF with $L=l_{12}+l_3$

$$\Psi_{k_3k_4}^{l_1l_2l_{12}l_3LM}(\omega) = \sum_{m_1m_2m_{12}m_3} (l_1l_2m_1m_2|l_{12}m_{12})(l_{12}l_3m_{12}m_3|LM)\Psi_{k_3k_4}^{l_1l_2l_3m_1m_2m_3}(\omega)$$

$$\Psi_{k_3k_4}^{l_1l_2l_{12}l_3LM}(\omega) = \mathcal{N}_{k_4+3/2}^{l_3k_3+3/2}(\cos(\beta))^{l_3}(\sin(\beta))^{k_3}P_m^{k_3+2,l_3+1/2}(\cos(2\beta))\left[\Psi_{k_3l_{12}}^{l_1l_2}(\Omega)Y_{l_3}(\hat{\vec{z}})\right]_M^L$$

$\Psi_{k_3l_{12}}^{l_1l_2}$ is three Body HF

Four body Hyperspherical Functions symmetrized with respect to three identical particles

$$\Psi_{k_4 L}^{[\bar{f}](n) l_{12} l_3 k_3} = \sum_{l_1 l_2} C_{k_3 l_{12}}^{[\bar{f}]\bar{\nu}} (l_1 l_2) \Psi_{k_3 L}^{l_1 l_2}(\Omega)$$

Where C_{Kl} are three body symmetrization coefficients,

1	**2**	**4**
3		

1	**2**
3	

Margolin L.L. Journal of Physics, 343, 1 (2012)

Transformation matrix for four particle systems with three identical particles

$$\hat{a}(P_{34}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\dfrac{M_l}{3(2M+M_l)}} & \sqrt{\dfrac{2(3M+M_l)}{3(2M+M_l)}} \\ 0 & \sqrt{\dfrac{2(3M+M_l)}{3(2M+M_l)}} & -\sqrt{\dfrac{M_l}{3(2M+M_l)}} \end{pmatrix}$$

Transformations of Four Body HF with three identical particles

$$P_{i4}\phi_{\mu L}^{[f]\lambda_{[f]}\nu_{[f]}l_{12}l_3K}(\omega) =$$

$$= \sum_{[f]\lambda_{[f]}\nu_{[f]}} \sum_{l'_{12}l'_3K'} \phi_{\mu L}^{[f']\lambda_{[f']}\nu_{[f']}l'_{12}l'_3K'} *$$

$$\left\langle [f']\lambda_{[f']}\nu_{[f']}l'_{12}l'_3K' \middle| [f]\lambda_{[f]}\nu_{[f]}l_{12}l_3K \right\rangle_{\mu L}$$

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Transformation Coefficients of four-body HF with three identical particles

$$\left\langle \left[\bar{f}'\right]\overline{m}'\,\bar{\nu}'\,l_{12}'\,l_3'\,K_3' \middle| \left[\bar{f}\right]\overline{m}\,\bar{\nu}\,l_{12}\,l_3\,K_3 \right\rangle = \sum_{l_1 l_2 l_1' l_2'} C_{K_3' l_{12}'}^{[\bar{f}']\overline{m}'\,\bar{\nu}'}\left(l_1' l_2'\right) \left\langle l_1' l_2' l_{12}' l_3' K_3' \middle| l_1 l_2 l_{12} l_3 K_3 \right\rangle_{K_4 L}^{P_{i4}} C_{K_3 l_{12}}^{[\bar{f}]\overline{m}\,\bar{\nu}}\left(l_1 l_2\right)$$

$$\left\langle l_1' l_2' l_{12}' l_3' K_3' \middle| l_1 l_2 l_{12} l_3 K_3 \right\rangle_{K_4 L} = \int \Psi_{K_3' K_4}^{l_1' l_2' l_{12}' l_3' L^*}\left(\omega'\right) \Psi_{K_{K_3} K_4}^{l_1 l_2 l_{12} l_3 L}\left(\omega\right) d\omega$$

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Table 2 **Coefficients** $\left\langle [\bar{f}]' \bar{m}' \bar{v}' l'_{12} l'_3 k'_3 \middle| [\bar{f}]' \bar{m} \bar{v} l_{12} l_3 k_3 \right\rangle$, with $K_4 = 2, L = 0$

$[\bar{f}]\bar{m}, l_{12} l_3 k_3$	$\langle[3], 000\|$	$\langle[2\mathrm{I}]\mathrm{I}, 002\|$	$\langle[2\mathrm{I}]\mathrm{I}, \mathrm{III}\|$
$\|[3], 000\rangle\rangle$	$^1/_2\,(3a_{33}^2 - 1)$	$^{\sqrt{3}}/_2\,(a_{31}^2 - a_{32}^2)$	$-\sqrt{3}a_{32}a_{33}$
$\|[2\mathrm{I}]1{,}002\rangle$	$^{\sqrt{3}}/_2\,(a_{13}^2 - a_{23}^2)$	$^1/_2\,(a_{11}^2 - a_{12}^2 - a_{21}^2 + a_{22}^2)$	$a_{22}a_{23} - a_{12}a_{13}$
$\|[2\mathrm{I}], \mathrm{III}\rangle$	$-\sqrt{3}a_{23}a_{33}$	$a_{22}a_{32} - a_{21}a_{31}$	$a_{22}a_{33} + a_{32}a_{23}$
$\|[2\mathrm{I}]2{,}002\rangle$	$\sqrt{3}a_{13}a_{23}$	$-a_{12}a_{22} + a_{11}a_{21}$	$-a_{22}a_{13} - a_{12}a_{23}$
$\|[2\mathrm{I}]2, \mathrm{III}\rangle$	$-\sqrt{3}a_{13}a_{33}$	$-a_{11}a_{31} + a_{32}a_{12}$	$a_{12}a_{33} + a_{32}a_{13}$
	$\langle[2\mathrm{I}]2{,}002\|$	$\langle[2\mathrm{I}]\mathrm{I}, \mathrm{III}\|$	
$\|[3], 000\rangle$	$\sqrt{3}a_{31}a_{32}$	$-\sqrt{3}a_{31}a_{33}$	
$\|[2\mathrm{I}], 1{,}002\rangle$	$a_{11}a_{12} - a_{21}a_{22}$	$a_{21}a_{23} - a_{11}a_{13}$	
$\|[2\mathrm{I}], 1, \mathrm{III}\rangle$	$-a_{22}a_{31} - a_{21}a_{32}$	$a_{21}a_{33} + a_{31}a_{23}$	
$\|[2\mathrm{I}], 2{,}002\rangle$	$a_{11}a_{22} + a_{12}a_{21}$	$-a_{11}a_{23} - a_{21}a_{13}$	
$\|[2\mathrm{I}], 2, \mathrm{III}\rangle$	$-a_{11}a_{32} - a_{12}a_{31}$	$a_{11}a_{33} + a_{31}a_{13}$	

$\langle[\bar{f}']\bar{m}'\bar{v}'l'_{12}l'_3k'_3 \,|\, [\bar{f}]\bar{m}\bar{v}l_{12}l_3k_3\rangle_{20}^{P_{34}}$ coefficients corresponding to $(3+1)$-configuration of four identical bodies

$[\bar{f}]\bar{m}_1 l_{12} l_3 k_3$	$\langle[3],000\|$	$\langle[21]1,002\|$	$\langle[21]1,111\|$	$\langle[21]2,002\|$	$\langle[21]2,111\|$
$\|[3],000\rangle$	$-1/3$	$-4\sqrt{3}/9$	$2\sqrt{6}/9$	0	0
$\|[21]1,002\rangle$	$-4\sqrt{3}/9$	$5/9$	$2\sqrt{2}/9$	0	0
$\|[21]1,111\rangle$	$2\sqrt{6}/9$	$2\sqrt{2}/9$	$7/9$	0	0
$\|[21]2,002\rangle$	0	0	0	$1/3$	$-2\sqrt{2}/3$
$\|[21]2,111\rangle$	0	0	0	$-2\sqrt{2}/3$	$-1/3$

$$A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/3 & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & -\frac{1}{3} \end{pmatrix}$$

Transformation Matrix for identical particles

Parentage scheme of Symmetrization

$$\omega^{[f]}_{[\bar{f}]_m}\Psi^{[\bar{f}]} = \sum_{in}\Gamma^{[f]}_{mn}\left(P_{iN}\right)P_{iN}\Psi^{[\bar{f}]}$$

N=3 we have [3],[21] and [111]

N=4 we have [4],[22],[31],[211]

N=5 we have [5],[41],[221],[311],[2111],[11111]

N=6 we have [6],[51],[42],[411],[2211],[3111],[111111]

Young Operators of Group S_3

1	2	3

$$\omega_{[2]}^{[3]}\Psi^{[2]} = \frac{1}{3}[1 + P_{13} + P_{23}]\Psi^{[2]};$$

$$\omega_{[1^2]}^{[1^3]}\Psi^{[1^2]} = \frac{1}{3}[1 - P_{13} - P_{23}]\Psi^{[1^2]}$$

1	2
3	

$$\omega_{[2]_1}^{[21]}\Psi_1^{[2]} = \frac{1}{3}[2 - P_{13} - P_{23}]\Psi_1^{[2]};$$

$$\omega_{[2]_2}^{[21]}\Psi_2^{[2]} = \frac{1}{\sqrt{3}}[-P_{13} + P_{23}]\Psi_2^{[2]}$$

1	3
2	

$$\omega_{[1^2]}^{[\widetilde{21}]}\Psi_1^{[1^2]} = \frac{1}{3}[2 + P_{13} + P_{23}]\Psi_1^{[1^2]};$$

$$\omega_{[1^2]_2}^{[\widetilde{21}]}\Psi_2^{[1^2]} = \frac{1}{\sqrt{3}}[P_{13} - P_{23}]\Psi_2^{[1^2]}$$

1
2
3

1	2
3	

Young Operators of Group S4

$\omega_{[3]}^{[4]}\Psi^{[3]}$	$= \frac{1}{4}[1 + P_{14} + P_{24} + P_{34}]\Psi^{[3]}$
$\omega_{[1^3]}^{[1^4]}\Psi^{[1^3]}$	$= \frac{1}{4}[1 - P_{14} - P_{24} - P_{34}]\Psi^{[1^3]}$
$\omega_{[21]_1}^{[22]}\Psi_1^{[21]}$	$= \frac{1}{4}\left[\Psi_1^{[21]} + P_{34}\Psi_1^{[21]} + P_{14}\left(-\frac{1}{2}\Psi_1^{[21]} + \frac{\sqrt{3}}{2}\Psi_2^{[21]}\right) + P_{24}\left(-\frac{1}{2}\Psi_1^{[21]} - \frac{\sqrt{3}}{2}\Psi_2^{[21]}\right)\right]$
$\omega_{[21]_2}^{[22]}\Psi_2^{[21]}$	$= \frac{1}{4}\left[\Psi_2^{[21]} - P_{34}\Psi_2^{[21]} + P_{14}\left(\frac{\sqrt{3}}{2}\Psi_1^{[21]} + \frac{1}{2}\Psi_2^{[21]}\right) + P_{24}\left(-\frac{\sqrt{3}}{2}\Psi_1^{[21]} + \frac{1}{2}\Psi_2^{[21]}\right)\right]$
$\omega_{[1^3]_1}^{[211]}\Psi_1^{[1^3]}$	$= \frac{\sqrt{6}}{4}[P_{14} - P_{24}]\Psi^{[1^3]}$
$\omega_{[1^3]_2}^{[211]}\Psi^{[1^3]}$	$= \frac{\sqrt{2}}{4}[2P_{34} - P_{14} - P_{24}]\Psi^{[1^3]}$
$\omega_{[1^3]_3}^{[211]}\Psi^{[1^3]}$	$= \frac{1}{4}[3 + P_{14} + P_{24} + P_{34}]\Psi^{[1^3]}$

1	2	3	4

1	2
3	4

1	3
2	4

1	2
3	
4	

1	3
2	
4	

1	4
2	
3	

Parentage Scheme of Symmentrization

$$\Psi_{k_4L}^{[f]m\nu} = \sum_{\alpha} B_{k_4L}^{[f]\nu}\left([\bar{f}]_m\alpha\right)\Psi_{k_4L}^{[\bar{f}]_m\bar{n}_m\alpha}$$

$$\sum_{[f]\nu} B^{[f]\nu}([\bar{f}]\alpha)B^{[f]\nu}([\bar{f}]\alpha') = \delta'_{\alpha\alpha}$$

$$\sum_{\alpha} B^{[f]\nu}([\bar{f}]\alpha)\,B^{[f']\nu'}([\bar{f}]\alpha) = \delta_{[f][f']}\delta_{\nu\nu'}$$

$$P_{i4} = P_{i3}P_{34}P_{i3}$$

$$\omega_{[\bar{f}]_m t}^{[f]}\Psi^{[\bar{f}]_m\alpha} = \sum_{\nu} B^{[f]\nu}\left([\bar{f}]_m\alpha\right)\Psi_t^{[f]\nu} = \sum_{\nu\alpha'} B^{[f]\nu}([f]_m\alpha)B^{[f]\nu}\left([\bar{f}]_t\alpha'\right)\Psi_{\bar{n}_t}^{[\bar{f}]_t\alpha'}$$

Parentage Coefficients for Three Body Systems

<u>Group S3</u>:

$$\sum_{\nu} B_{k_3L}^{[3]\nu}([2]\alpha)B_{k_3L}^{[3]\nu}([2]\alpha') = \frac{1}{3}\delta'_{\alpha\alpha} + \frac{2}{3}\langle[2]\alpha|[2]\alpha'\rangle_{k_3L}^{P_{23}}$$

$$\sum_{\nu} B_{k_3L}^{[1^3]\nu}([1^2]\alpha)B_{k_3L}^{[1^3]\nu}([2]\alpha') = \frac{1}{3}\delta'_{\alpha\alpha} - \frac{2}{3}\langle[1^2]\alpha|[2]\alpha'\rangle_{k_3L}^{P_{23}}$$

$$\sum_{\nu} B_{k_3L}^{[21]\nu}([2]\alpha)B_{k_3L}^{[21]\nu}([2]\alpha') = \frac{2}{3}\delta'_{\alpha\alpha} - \frac{2}{3}\langle[2]\alpha|[2]\alpha'\rangle_{k_3L}^{P_{23}}$$

$$\sum_{\nu} B_{k_3L}^{[21]\nu}([2]\alpha)B_{k_3L}^{[21]\nu}([1^2]\alpha') = \frac{2}{\sqrt{3}}\langle[2]\alpha|[1^2]\alpha'\rangle_{k_3L}^{P_{23}}$$

Parentage Coefficients for Four Body Systems

$$\sum_{\nu} B_{k_4L}^{[4]\nu}([3]\alpha)B_{k_4L}^{[4]\nu}([3]\alpha') = \frac{1}{4}\left[\delta'_{\alpha\alpha} + 3\langle[3]\alpha|[3]\alpha'\rangle_{k_4L}^{P_{34}}\right];$$

$$\sum_{\nu} B_{k_4L}^{[1^4]\nu}([1^3]\alpha)B_{k_4L}^{[1^4]\nu}([1^3]\alpha') = \frac{1}{4}\left[\delta'_{\alpha\alpha} - 3\langle[1^3]\alpha|[1^3]\alpha'\rangle_{k_4L}^{P_{34}}\right];$$

$$\sum_{\nu} B_{k_4L}^{[22]\nu}([21]\alpha)B_{k_4L}^{[22]\nu}([21]\alpha') = \frac{1}{4}\delta'_{\alpha\alpha} + \frac{3}{8}\left[\langle[21]1\alpha'|[21]1\alpha\rangle_{k_4L}^{P_{34}} - \langle[21]2\alpha'|[21]2\alpha\rangle_{k_4L}^{P_{34}}\right];$$

$$\sum_{\nu} B_{k_4L}^{[211]\nu}([1^3]\alpha)B_{k_4L}^{[211]\nu}([21]\alpha') = \frac{3\sqrt{2}}{4}\langle[1^3]\alpha'|[21]2\alpha'\rangle_{k_4L}^{P_{34}};$$

$$\sum_{\nu} B_{k_4L}^{[211]\nu}([1^3]\alpha)B_{k_4L}^{[211]\nu}([1^3]\alpha') = \frac{3}{4}\left(\delta'_{\alpha\alpha} + \langle[1^3]\alpha|[1^3]\alpha'\rangle_{k_4L}^{P_{34}}\right);$$

$$\sum_{\nu} B_{k_4L}^{[31]\nu}([3]\alpha)B_{k_4L}^{[31]\nu}([3]\alpha') = \frac{3}{4}\left(\delta'_{\alpha\alpha} - \langle[3]\alpha|[3]\alpha'\rangle_{k_4L}^{P_{34}}\right);$$

$$\sum_{\nu} B_{k_4L}^{[31]\nu}([3]\alpha)B_{k_4L}^{[31]\nu}([21]\alpha') = \frac{3\sqrt{2}}{4}\langle[3]\alpha'|[21]1\alpha'\rangle_{k_4L}^{P_{34}}$$

Table I.3: **Coefficients** $\left\langle [\bar{f}]' \bar{m}' \bar{\nu}' l'_{12} l'_3 k'_3 \middle| [\bar{f}]' \bar{m} \bar{\nu} l_{12} l_3 k_3 \right\rangle$, w/ $K_4 = 2, L = 0$

$[\bar{f}]\bar{m}, l_{12} l_3 k_3$	$\langle[3], 000\|$	$\langle[2\mathrm{I}]\mathrm{I}, 002\|$	$\langle[2\mathrm{I}]\mathrm{I}, \mathrm{III}\|$
$\|[3], 000\rangle\rangle$	$^1/_2\,(3a_{33}^2 - 1)$	$^{\sqrt{3}}/_2\,(a_{31}^2 - a_{32}^2)$	$-\sqrt{3}a_{32}a_{33}$
$\|[2\mathrm{I}]1{,}002\rangle$	$^{\sqrt{3}}/_2\,(a_{13}^2 - a_{23}^2)$	$^1/_2\,(a_{11}^2 - a_{12}^2 - a_{21}^2 + a_{22}^2)$	$a_{22}a_{23} - a_{12}a_{13}$
$\|[2\mathrm{I}], \mathrm{III}\rangle$	$-\sqrt{3}a_{23}a_{33}$	$a_{22}a_{32} - a_{21}a_{31}$	$a_{22}a_{33} + a_{32}a_{23}$
$\|[2\mathrm{I}]2{,}002\rangle$	$\sqrt{3}a_{13}a_{23}$	$-a_{12}a_{22} + a_{11}a_{21}$	$-a_{22}a_{13} - a_{12}a_{23}$
$\|[2\mathrm{I}]2, \mathrm{III}\rangle$	$-\sqrt{3}a_{13}a_{33}$	$-a_{11}a_{31} + a_{32}a_{12}$	$a_{12}a_{33} + a_{32}a_{13}$
	$\langle[2\mathrm{I}]2{,}002\|$	$\langle[2\mathrm{I}]\mathrm{I}, \mathrm{III}\|$	
$\|[3], 000\rangle$	$\sqrt{3}a_{31}a_{32}$	$-\sqrt{3}a_{31}a_{33}$	
$\|[2\mathrm{I}], 1{,}002\rangle$	$a_{11}a_{12} - a_{21}a_{22}$	$a_{21}a_{23} - a_{11}a_{13}$	
$\|[2\mathrm{I}], 1, \mathrm{III}\rangle$	$-a_{22}a_{31} - a_{21}a_{32}$	$a_{21}a_{33} + a_{31}a_{23}$	
$\|[2\mathrm{I}], 2{,}002\rangle$	$a_{11}a_{22} + a_{12}a_{21}$	$-a_{11}a_{23} - a_{21}a_{13}$	
$\|[2\mathrm{I}], 2, \mathrm{III}\rangle$	$-a_{11}a_{32} - a_{12}a_{31}$	$a_{11}a_{33} + a_{31}a_{13}$	

Constructing Four Body Symmetrized Hyperspherical Basis

$$\Psi_{k_4L}^{[\bar{f}](n)l_{12}l_3k_3} = \sum_{l_1l_2} C_{k_3l_{12}}^{[\bar{f}]\bar{\nu}}(l_1l_2)\Psi_{k_3L}^{l_1l_2}(\Omega)$$

$$\Psi_{k_4L}^{[f]m\nu} = \sum_{\alpha} B_{k_4L}^{[f]\nu}\left([\bar{f}]_m\alpha\right)\Psi_{k_4L}^{[\bar{f}]_m\bar{n}_m\alpha}$$

$$\Psi_{k_4L}^{[f](m)\nu} = \sum_{l_1l_2l_{12}l_3k_3} B_{k_4L}^{[f]_m\nu}\left([\bar{f}]_m\bar{\nu}l_{12}l_3k_3\right)C_{k_3l_{12}}^{[\bar{f}]_m}(l_1l_2)\Psi_{k_3k_4}^{l_1l_2l_{12}l_3L}(\Omega)$$

$$= \sum_{l_1l_2l_{12}l_3k_3} C_{k_4L}^{[f]m\nu}(l_1l_2l_{12}l_3k_3)\Psi_{k_4k_3}^{l_1l_2l_{12}l_3L}(\Omega)$$

$$C_{k_4L}^{[f](m)\nu}(l_1l_2l_{12}l_3k_3) = \sum_{\bar{\nu}} B_{k_4L}^{[f]m\nu}\left([\bar{f}]_m\bar{\nu}l_{12}l_3k_3\right)C_{k_4L}^{[\bar{f}]_m\bar{\nu}}(l_1l_2)$$

Four Body Symmetrization Coefficients

$C_{20}^{[f]m\nu}(l_1 l_2 l_{12} l_3 k_3)$ coefficients corresponding to $(2+2)$-configuration of four identical bodies

$[f]_m$ \ $l_1 l_2 l_{12} l_3 k_3$	00000	00002	11002	10111	01111
[31]1	$-1/2$	$\sqrt{3}/6$	0	0	$-\sqrt{6}/3$
[31]2	$-\sqrt{2}/2$	$\sqrt{6}/6$	0	0	$\sqrt{3}/3$
[31]3	0	0	1	0	0
[22]1	$1/2$	$\sqrt{3}/2$	0	0	0
[22]2	0	0	0	1	0

$$C_{k_4 L}^{[f](m)\nu}(l_1 l_2 l_{12} l_3 k_3) = \sum_{\bar{\nu}} B_{k_4 L}^{[f]m\nu}\left([\bar{f}]_m \bar{\nu} l_{12} l_3 k_3\right) C_{k_4 L}^{[\bar{f}]_m \bar{\nu}}(l_1 l_2)$$

$Parentage\ Coefficients\ B_{k_4L}^{[f]m\nu}\left([\bar{f}]_m \bar{\nu} l_{12} l_3 k_3\right)$ with K4 = 4, L = 0

$[f]_\nu[\bar{f}]$ \ $l_{12}l_3k_3$	000	002	222	111	113	004
$[4]_1[3]$	$\sqrt{26}/9$	0	$\frac{1}{9}\sqrt{11/2}$	0	$-\frac{1}{3}\sqrt{11/3}$	$\frac{1}{3}\sqrt{11/6}$
$[4]_2[3]$	0	0	$1/\sqrt{2}$	0	$1/\sqrt{3}$	$1/\sqrt{6}$
$[31]_1[3]$	$\sqrt{55}/9$	$-\frac{1}{9}\sqrt{13/5}$	0	0	$\frac{1}{3}\sqrt{26/15}$	$-\frac{1}{3}\sqrt{13/15}$
$[31]_1[\widetilde{21}]$	0	$\frac{7}{9\sqrt{5}}$	$\frac{1}{9}\sqrt{26/5}$	$-\frac{4}{9}\sqrt{6/5}$	$-\frac{1}{3}\sqrt{26/15}$	$\frac{2}{3}\sqrt{13/15}$
$[31]_2[3]$	0	0	$\sqrt{2/5}$	0	$-1/\sqrt{15}$	$-4/\sqrt{30}$
$[31]_2[\widetilde{21}]$	0	$-\frac{1}{3}\sqrt{13/10}$	$\frac{4}{3\sqrt{5}}$	$\frac{1}{6}\sqrt{39/5}$	$-\frac{1}{2}\sqrt{3/5}$	$\sqrt{2/15}$
$[211][1^3]$	0	0	0	0	1	0
$[211][21]$	0	$\frac{1}{3}\sqrt{13/6}$	$-\frac{2}{3\sqrt{3}}$	$\sqrt{13}/6$	$\frac{1}{6}$	$-\sqrt{2}/3$
$[22]_1[21]$	0	$\frac{2\sqrt{10}}{9}$	$-\frac{1}{9}\sqrt{13/5}$	$-\frac{1}{3}\sqrt{5/3}$	$\frac{1}{3}\sqrt{13/5}$	$-\frac{1}{9}\sqrt{78/5}$
$[22]_2[21]$	0	0	$\sqrt{2/5}$	0	$4/\sqrt{30}$	$1/\sqrt{15}$

Importance of The Recurrence Method

- The transformations of Hyperspherical functions become sufficiently complex when number of particles in the system increases
- For four and more particles kinematic rotations (KR) include both particle permutations and transitions from one configuration to another
- Finding (KR) coefficients for four particle systems using general formula is extremely difficult and is practically impossible for the systems with five and more particles
- Recurrent method allows to obtain KR coefficients for the systems with any number of particles

Building Spin-Isospin Functions in (3+1) Configuration

- $\chi_s = [[\chi^1(12)\chi(3)]^{3/2}\chi(4)]^2$;
- $\chi_e = [[\chi^0(12)\chi(3)]^{1/2}\chi(4)]^0$;

$$\chi_\alpha = [[\chi^0(12)\chi(3)]^{1/2}\chi(4)]^1$$

$$-\chi_\beta = [[\chi^1(12)\chi(3)]^{1/2}\chi(4)]^1$$

$$\chi_f = [[\chi^1(12)\chi(3)]^{1/2}\chi(4)]^0 ;$$

$$\chi_\gamma = [[\chi^1(12)\chi(3)]^{1/2}\chi(4)]^1$$

Where $e = (2121)$, s=(1111), $\bar{s}$=(4321), f=(2211),α=(1121), β=(1211),γ=(2111), $\bar{\alpha}$=(3211), $\bar{\beta}$=(3121), $\bar{\gamma}$=(1321).

Building Spin-Isospin Functions in (3+1) Configuration

- We can represent a product of [31] and [31] configurations as the sum of the following representations of the four-particle permutation groups:

$$[31] \times [31] = [4] + [31] + [22] + [211]$$

- The four-particle permutation group [31] includes α=(1121), β=(1211),γ=(2111),
- permutation group [211] includes $\bar{\alpha}$=(3211), $\bar{\beta}$=(3121), $\bar{\gamma}$=(1321),
- and group [22] is represented by $e = (2121)$ and f=(2211):

Spin-Isospin Functions in (3+1) Configuration

- $\alpha = \frac{1}{\sqrt{3}}\alpha_1\gamma_2 + \frac{1}{\sqrt{6}}\gamma_1\alpha_2 - \frac{1}{\sqrt{3}}\alpha_1\beta_2 - \frac{1}{\sqrt{3}}\beta_1\alpha_2$
- $\beta = -\frac{1}{\sqrt{3}}\alpha_1\alpha_2 + \frac{1}{\sqrt{3}}\beta_1\beta_2 + \frac{1}{\sqrt{6}}\beta_1\gamma_2 + \frac{1}{\sqrt{2}}\gamma_1\beta_2$
- $\gamma = \frac{1}{\sqrt{6}}\alpha_1\alpha_2 + \frac{1}{\sqrt{6}}\beta_1\beta_2 - \frac{2}{\sqrt{6}}\gamma_1\gamma_2$
- $\bar{\alpha} = \frac{1}{\sqrt{2}}\gamma_1\beta_2 - \frac{1}{\sqrt{2}}\beta_1\gamma_2$
- $\bar{\beta} = \frac{1}{\sqrt{2}}\alpha_1\gamma_2 - \frac{1}{\sqrt{2}}\gamma_1\alpha_2$
- $\bar{\gamma} = -\frac{1}{\sqrt{2}}\alpha_1\beta_2 + \frac{1}{\sqrt{2}}\beta_1\alpha_2$
- $e = {}^{1}/_{\sqrt{6}}\{-\alpha_1\beta_2 - \beta_1\alpha_2 - \sqrt{2}\alpha_1\gamma_2 - \sqrt{2}\gamma_1\alpha_2\}$

  $f = 1/\sqrt{6}\{\alpha_1\alpha_2 - \beta_1\beta_2 + \sqrt{2}\beta_1\gamma_2 + \sqrt{2}\beta_2\gamma_1\}$

Spin-Isospin Functions in (3+1) Configuration

- The product of two [211] configurations can be represented as $[211] \times [211] = [211] + [22] + [31] + [4]$.
- Where formulas for the [211], [31], and [22] configurations can easily be obtained from by replacing α =(1121), β=(1211),γ=(2111) with $\bar{\alpha}$=(3211), $\bar{\beta}$=(3121), $\bar{\gamma}$=(1321) correspondingly

Spin-Isospin Functions in (3+1) Configuration

- $\mathbf{[4] \times [4] \times [1111]}$
- $\chi_s(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_s$
- $[31] \times [31] \times [1111]$
- $\frac{1}{\sqrt{3}}[\chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_\beta(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_\gamma(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{s}}]$
- $\mathbf{[31] \times [31] \times [31]}$
- $\frac{1}{\sqrt{6}}[\chi_\gamma(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\alpha - \chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\alpha + \chi_\alpha(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\beta - \chi_\gamma(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\beta - \chi_\alpha(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\gamma + \chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\gamma]$
- $\mathbf{[31] \times [31] \times [22]}$
- $\frac{1}{2\sqrt{3}}[-\chi_\alpha(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_f - \chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_f - \sqrt{2}\chi_\alpha(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_f - \sqrt{2}\chi_\gamma(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_f - \chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_e + \chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_e + \sqrt{2}\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_e + \sqrt{2}\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_e]$

- $\mathbf{[31] \times [31] \times [211]}$
- $\frac{1}{3}[\frac{1}{\sqrt{2}}\chi_\alpha(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\alpha}} + \frac{1}{\sqrt{2}}\chi_\gamma(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\alpha}} - \chi_\alpha(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\alpha}} - \chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\alpha}} - \chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\beta}} + \chi_\beta(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\beta}} + \frac{1}{\sqrt{2}}\chi_\beta(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\beta}} + \frac{1}{\sqrt{2}}\chi_\gamma(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\beta}} + \frac{1}{\sqrt{2}}\chi_\alpha(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_{\bar{\beta}} + \frac{1}{\sqrt{2}}\chi_\beta(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_{\bar{\gamma}} - \sqrt{2}\chi_\gamma(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_{\bar{\gamma}}]$

Full Wave Functions in (3+1) Configuration

- $[\mathbf{22}] \times [\mathbf{22}] \times [\mathbf{1111}]$
- $\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_{\bar{s}}]$
- $[\mathbf{22}] \times [\mathbf{22}] \times [\mathbf{4}]$
- $\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_{\bar{s}} - \chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{s}}]$
- $[\mathbf{22}] \times [\mathbf{22}] \times [\mathbf{22}]$
- $\frac{1}{2}[\chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_f + \chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_f - \chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_l + \chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_l]$
- $[\mathbf{22}] \times [\mathbf{4}] \times [\mathbf{22}]$
- $\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_f - \chi_f(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_e]$
- $[\mathbf{22}] \times [\mathbf{31}] \times [\mathbf{31}]$
- $\frac{1}{\sqrt{3}}[\frac{1}{2}\chi_e(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\alpha + \frac{1}{\sqrt{2}}\chi_f(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\alpha + \frac{1}{2}\chi_f(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\alpha + \frac{1}{\sqrt{2}}\chi_e(\boldsymbol{\sigma})\chi_\gamma(\boldsymbol{\tau})\phi_\beta + \frac{1}{2}\chi_f(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\beta - \frac{1}{2}\chi_e(\boldsymbol{\sigma})\chi_\beta(\boldsymbol{\tau})\phi_\beta + \frac{1}{\sqrt{2}}\chi_\beta(\boldsymbol{\sigma})\chi_\alpha(\boldsymbol{\tau})\phi_\gamma]$

Full Wave Functions in (3+1) Configuration

- The remaining four combinations can be easily constructed by switching spin and isospin functions $\boldsymbol{\sigma} \leftrightarrow \boldsymbol{\tau}$. Namely:
- $[31] \times [22] \times [31] = [22] \times [31] \times [31]$ $\boldsymbol{\sigma} \leftrightarrow \boldsymbol{\tau}$
- $[31] \times [22] \times [211] = [22] \times [31] \times [211]$ $\boldsymbol{\sigma} \leftrightarrow \boldsymbol{\tau}$
- $[4] \times [22] \times [22] = [22] \times [4] \times [22]$ $\boldsymbol{\sigma} \leftrightarrow \boldsymbol{\tau}$

  $[4] \times [31] \times [211] = [31] \times [4] \times [211]$ $\boldsymbol{\sigma} \leftrightarrow \boldsymbol{\tau}$

Full Wave Functions in (2+2) Configuration

- Spin and isospin functions in (2+2) configuration are expressed as follows:
- $\chi_e = \chi^0(12)\chi^0(34)$ $\quad$ $\chi_a = \chi^0(12)\chi^1(34)$
- $\chi_f = [\chi^1(12)\chi^1(34)]^0$ $\quad$ $\chi_b = \chi^1(12)\chi^0(34)$
- $\chi_s = [\chi^1(12)\chi^1(34)]^2$ $\quad$ $\chi_c = [\chi^1(12)\chi^1(34)]^1$
- $[\mathbf{4}] \times [\mathbf{4}] \times [\mathbf{1111}]$
- $\chi_s(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_{\bar{s}}$
- $[\mathbf{31}] \times [\mathbf{31}] \times [\mathbf{1111}]$
- $\frac{1}{\sqrt{3}}[\chi_a(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_b(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_c(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_{\bar{s}}]$
- $[\mathbf{31}] \times [\mathbf{31}] \times [\mathbf{31}]$
- $\frac{1}{\sqrt{6}}[\chi_c(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_a - \chi_b(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_a + \chi_a(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_b - \chi_c(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_b + \chi_b(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_c - \chi_a(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_c]$

Full Wave Functions in (2+2) Configuration

- $[\mathbf{31}] \times [\mathbf{31}] \times [\mathbf{22}]$
- $\frac{1}{2}[\frac{1}{\sqrt{3}}\chi_a(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_e + \frac{1}{\sqrt{3}}\chi_b(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_e - \frac{2}{\sqrt{3}}\chi_c(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_e + \chi_a(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_f + \chi_b(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_f]$
- $[\mathbf{31}] \times [\mathbf{31}] \times [\mathbf{211}]$
- $1/\sqrt{6}[\chi_a(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{\alpha}} + \chi_c(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{\alpha}} - \chi_b(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_b - \chi_c(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_b + \chi_a(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{c}} + \chi_b(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{c}}]$
- $[\mathbf{22}] \times [\mathbf{22}] \times [\mathbf{1111}]$
- $\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{s}} + \chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_{\bar{s}}]$
- $[\mathbf{22}] \times [\mathbf{22}] \times [\mathbf{4}]$
- $\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_s - \chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_s]$

Full Wave Functions in (2+2) Configuration

- $[\mathbf{22}] \times [\mathbf{22}] \times [\mathbf{22}]$
- $\frac{1}{2}[\chi_e(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_e + \chi_f(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_e - \chi_e(\boldsymbol{\sigma})\chi_f(\boldsymbol{\tau})\phi_f - \chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_f]$
- $[\mathbf{22}] \times [\mathbf{31}] \times [\mathbf{31}]$
- $\frac{1}{2}[\frac{1}{\sqrt{3}}\chi_e(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_a + \chi_f(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_a + \chi_f(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_b + \frac{1}{\sqrt{3}}\chi_e(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_b - \frac{2}{\sqrt{3}}\chi_e(\boldsymbol{\sigma})\chi_c(\boldsymbol{\tau})\phi_c]$
- $[\mathbf{22}] \times [\mathbf{31}] \times [\mathbf{211}]$
- $\frac{1}{2}[-\frac{1}{\sqrt{3}}\chi_f(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{\alpha}} + \chi_e(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{\alpha}} + \chi_e(\boldsymbol{\sigma})\chi_a(\boldsymbol{\tau})\phi_{\bar{b}} - \frac{1}{\sqrt{3}}\chi_f(\boldsymbol{\sigma})\chi_b(\boldsymbol{\tau})\phi_{\bar{b}} + \frac{2}{\sqrt{3}}\chi_f(\boldsymbol{\sigma})\chi_e(\boldsymbol{\tau})\phi_{\bar{c}}]$
- $[\mathbf{22}] \times [\mathbf{4}] \times [\mathbf{22}]$
- $\frac{1}{\sqrt{2}}[\chi_e(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_f - \chi_f(\boldsymbol{\sigma})\chi_s(\boldsymbol{\tau})\phi_e]$

Conclusion

- **The problem of the construction of the wave functions that are anti-symmetric under particle interchange has been solved in both (3+1) and(2+2) configurations. Complete set of all sixteen possible combinations of spin, isospin and hyperspherical functions have been obtained. Proposed mathematical formalism can easily be generalized for the systems with five and more particles.**

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