^{6}Li as a three-particle system in the (p, ^{3}He) reaction at astrophysical energies

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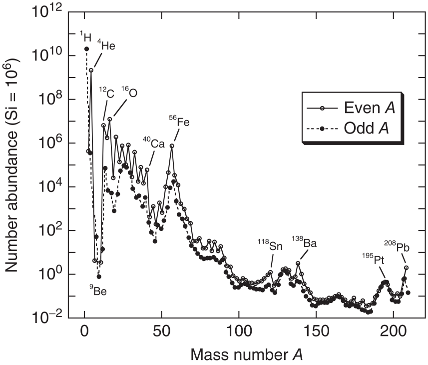
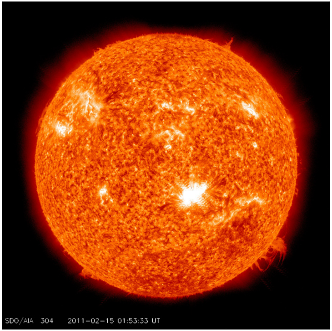
Context: reactions of astrophysical interest

Theoretical investigation on nuclear reactions between light charged particles at energies below the Coulomb barrier.

Focus on systems of astrophysical interest



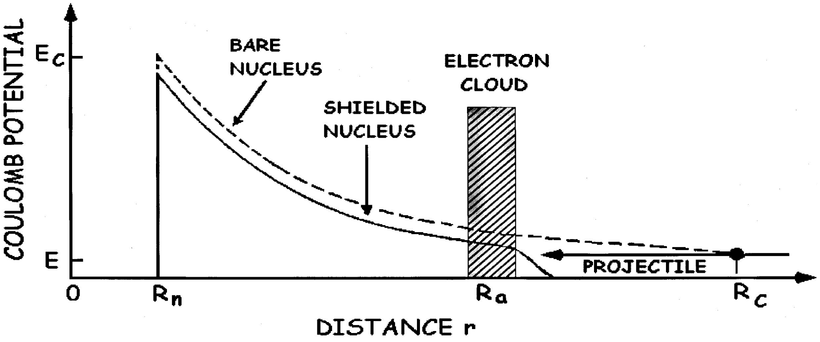
C. Iliadis. *Nuclear Physics of Stars*. Wiley-VCH, 2015, fig. 1.2



sdo.gsfc.nasa.gov/gallery

Theoretical investigation on nuclear reactions between light charged particles at energies below the Coulomb barrier.

Focus on systems of astrophysical interest



H. J. Assenbaum et al. *Zeitschrift für Physik A* 327.4 (1987)

Astrophysical factor $S(E)$

Process dominated by quantum tunnelling of the Coulomb barrier.

Astrophysical S-factor:

$$S(E) = E\, e^{2\pi\eta(E)}\sigma(E) \quad , \quad \eta(E) = \alpha_e Z_1 Z_2 \sqrt{\frac{\mu c^2}{2E}}$$

(σ angle-integrated cross-section, E center-of-mass collision energy,
Z_i reactants charge number, α_e fine-structure constant,
μ reactants reduced mass, c speed of light).

- Correlations between the reactants internal degrees of freedom can alter the sub-barrier cross sections.

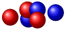
Goal and focus

Goal

Study the influence of ground-state ("static") structure on the reaction dynamics in a fully quantum framework.

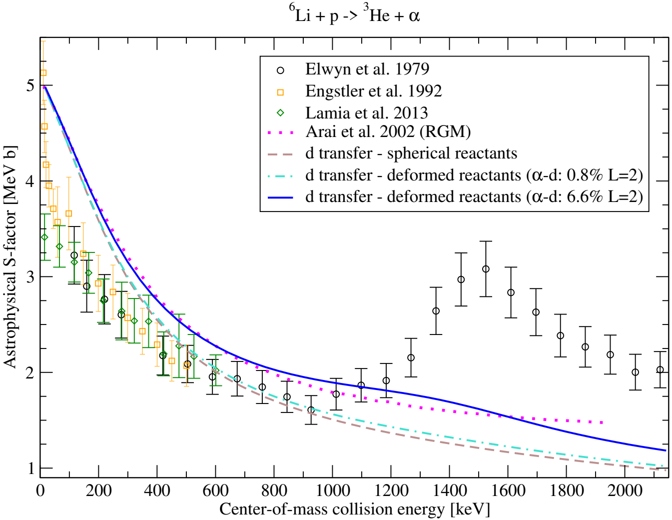
- Explicit evaluation of the cross-section in terms of the properties and interactions of reactants.
- No adjusting on reaction experimental data.

Study of $^{6}\mathrm{Li} + \mathrm{p} \rightarrow \alpha + {}^{3}\mathrm{He}$ transfer, focus on $^{6}\mathrm{Li}$ structure.

- Two-cluster models: $|{}^{6}\mathrm{Li}$  $\rangle = |\alpha\,\mathrm{d}$  $\rangle$
- Three-cluster models: $|{}^{6}\mathrm{Li}$  $\rangle = |\alpha\,\mathrm{p}\,\mathrm{n}$  $\rangle$

Contents

^{6}Li + p → ^{3}He + α: deuteron transfer

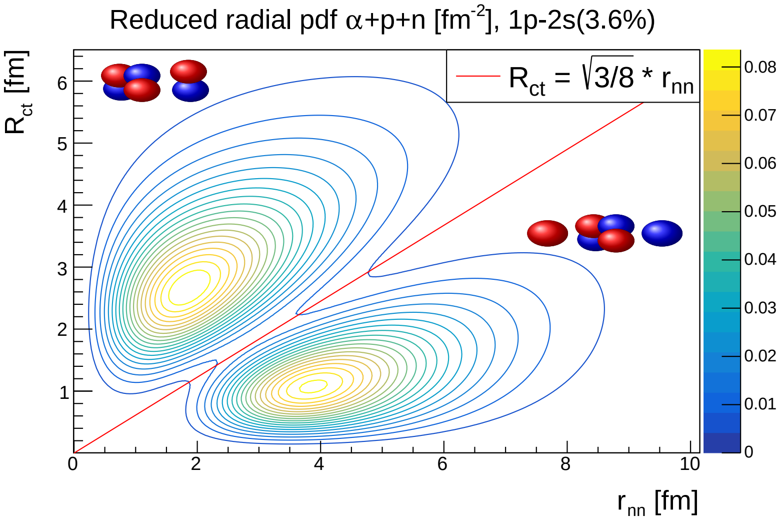


Present 1st-order DWBA

in good agreement with Resonating Group Method calculation.

Contents

α+n+p reduced probability density function



Reconstruction of J. Bang et al. *Nuclear Physics A* 313.1 (1979)

Reconstruction of 3-particle WFs

The bound state $\Phi_{\alpha\,\mathrm{p}\,\mathrm{n}}$ can be written as:

$$\Phi_{\alpha\,\mathrm{p}\,\mathrm{n}} = \sum\nolimits_{i,j} c_{i,j}\ \phi_{\alpha\,\mathrm{p},i}(\underline{r}_{\alpha\,\mathrm{p}})\phi_{\alpha\,\mathrm{n},j}(\underline{r}_{\alpha\,\mathrm{n}})$$

"Bang": Reconstruction of J. Bang et al. *Nuclear Physics A* 313.1 (1979): Faddeev, α–n Bang 1979, n–n de Tourreil-Sprung 1975.

"Casal": Reconstruction of J. Casal et al. private communication. 2021: HH, α–n Bang 1979, n–n Gogny-Pirres-Tourreil 1970.

core-n sp shell	tot S	tot L	Norm from Bang	Norm from Casal
$1p \times 1p$	1	0	$90.3\,\%$	$82.7\,\%$
$1p \times 1p$	0	1	$5.7\,\%$	$6.6\,\%$
$1p \times 1p$	1	2	$0.4\,\%$	$0.6\,\%$
$2s \times 2s$	1	0	$3.6\,\%$	$10.1\,\%$

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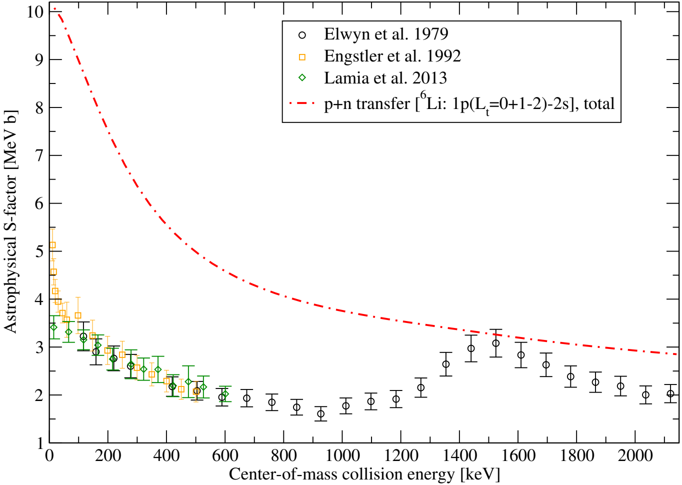
$^6Li + p \to {}^3He + \alpha$: two-particle transfer cross-section

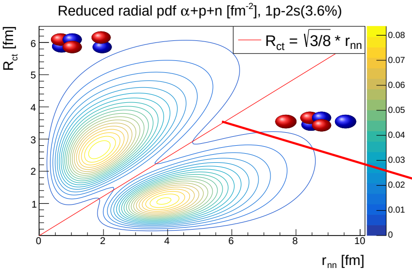


Total $p+n$ transfer, ^6Li from Faddeev.

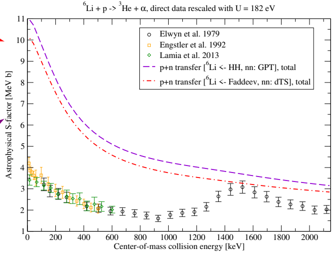
High absolute value connected to issue in reaction calculation.

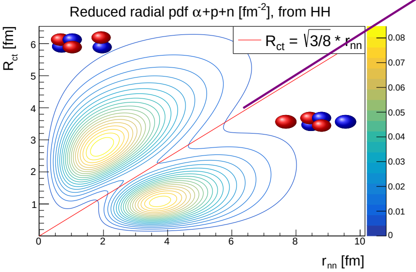
Can still study structure role by comparing theory to theory.

α+p+n Faddeev and HH comparison



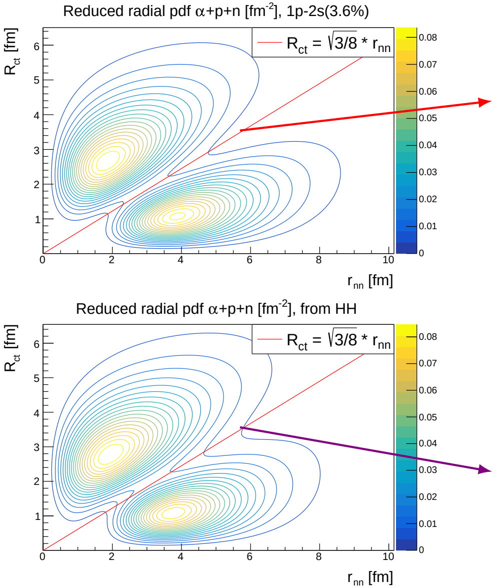
from Bang 1979: Faddeev,
α–*n* Bang 1979,
n–*n* dTS 1975.





from Casal 2021: HH,
α–*n* Bang 1979,
n–*n* GPT 1970.

$\alpha+\mathrm{p}+\mathrm{n}$ Faddeev and HH comparison

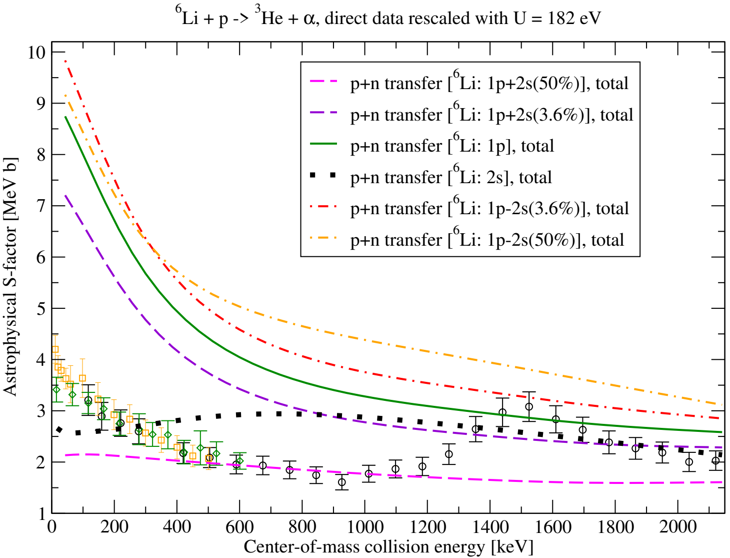


$I_{\text{clustered}} = 0.60$

$I_{\text{clustered}}$ = integral of PDF above the red line ("clustered region").

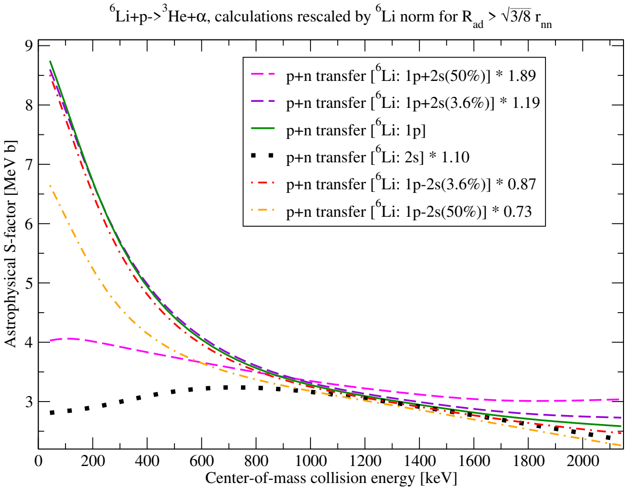
$I_{\text{clustered}} = 0.64$

^{6}Li + p → ^{3}He + α: role of ^{6}Li $(2s)^2$ contribution



Lines: different weight and sign for ^{6}Li $(2s)^2$ component.

Calculations rescaled by clustering strength



Cross-section absolute value scales with $^6\mathrm{Li}$ WF "clustered norm".

Summary

What: $^{6}\mathrm{Li} + \mathrm{p} \rightarrow {}^{3}\mathrm{He} + \alpha$ around and below the Coulomb barrier

How:
- DWBA 2-nucleon transfer
- Emphasis on the role of cluster structure.

So far:
- Cross-sections scale with the "clustering strength".
- Greater clustering in $^{6}\mathrm{Li}$ WF predicted by hypersph. harmonics with *n*-*n* Gogny-Pirres-Tourreil 1970 than Faddeev with *n*-*n* de Tourreil-Sprung 1975.

To do:
- Compare different *n*-*n* potentials in HH.
- Directly use three-body WFs in the transfer.
- Better treatment of unbound $^{5}\mathrm{Li}$ in sequential transfer.

PhD thesis `arxiv.org/abs/2307.01835`, PRC under review.

Thank you

Hamiltonian for 1-step p+n transfer

Total Hamiltonian, e.g. for initial state ($A = a + \mu$, $\mathcal{A} = a + \mu + \nu$):

$$\mathcal{H} = [(K_{a\mu} + V_{a\mu}) + (K_{A\nu} + V_{\nu a} + V_{\nu\mu})] + K_{\mathcal{A}b} + V_{ba} + V_{b\mu} + V_{b\nu}$$

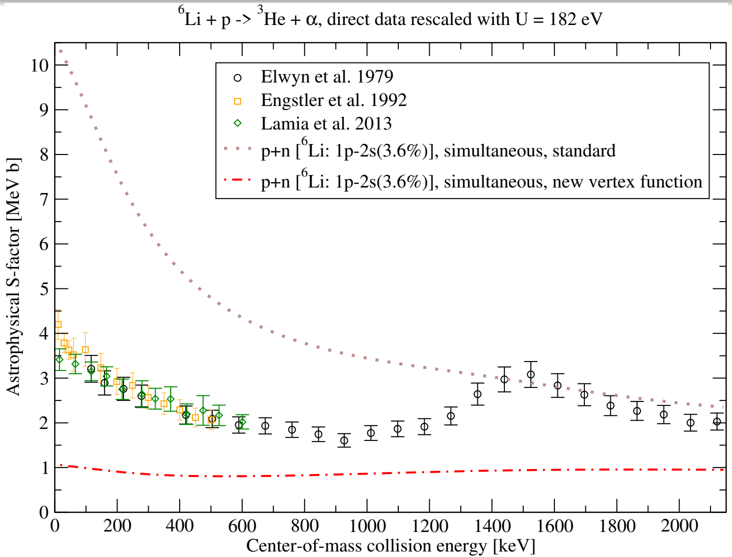
K_{ij}: kinetic energy of relative i-j motion. V_{ij}: i-j potential.

Approximation:

$$\mathcal{H} = [(K_{a\mu} + V_{a\mu}) + (K_{A\nu} + V_{A\nu})] + K_{\mathcal{A}b} + V_{ba} + V_{B\mu} + V_{b\nu}$$

- Simplifies the calculation.
- Accurate if $V_{\mu\nu}$ is comparatively small (e.g. heavy ions).
- Problematic in the present case.

^{6}Li(p, ^{3}He)α 1-step p+n transfer, ^{6}Li from Faddeev



More accurate vertex function (red dot-dashed) removes 1-step cross-section over-estimation.