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Nucleon-nucleon interaction in manifestly Lorentz-invariant ChEFT

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In collaboration with:

Evgeny Epelbaum (RUB), Jambul Gegelia(RUB), and Vadim Baru (RUB)

OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

Nuclear forces — Weinberg's seminal work

- Apply **chiral perturbation theory (ChPT),** as the low-energy EFT of QCD, to derive nuclear forces

  S. Weinberg, PLB251(1990)288-292; NPB363(1991)3-18

  - **Self-consistently** include many-body forces

$$V = V_{2N} + V_{3N} + V_{4N} + \cdots$$

  - **Systematically improve** order by order (heavy baryon ChPT)

$$V_{iN} = V_{iN}^{\text{LO}} + V_{iN}^{\text{NLO}} + V_{iN}^{\text{NNLO}} + \cdots$$

  - Scattering amplitude: **Schrödinger equation** (or integral equations in momentum space, e.g. Lippmann-Schwinger eq.)

$$\left[\left(\sum_{i=1}^{A} -\frac{\nabla_i^2}{2m_N}\right) + V_{2N} + V_{3N} + V_{4N} + \ldots\right] |\Psi\rangle = E|\Psi\rangle$$

- Provide <u>**a systematic and solid theoretical approach**</u> to study the few-nucleon scattering

Renormalization issue of chiral NF

- Iteration of the chiral NN potential within LSE

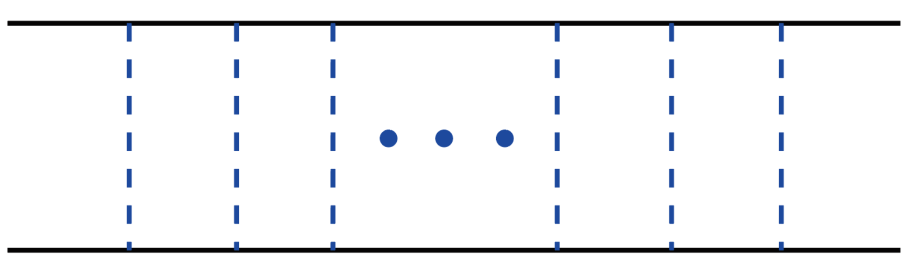
$$T\left(\boldsymbol{p}',\boldsymbol{p}\right)=V\left(\boldsymbol{p}',\boldsymbol{p}\right)+\int\frac{d^3k}{(2\pi)^3}V\left(\boldsymbol{p}',\boldsymbol{k}\right)\frac{m_N}{\boldsymbol{p}^2-\boldsymbol{k}^2+i\epsilon}T(\boldsymbol{k},\boldsymbol{p})$$

➡ UV divergencies cannot be absorbed by contact terms!

- Leading order NN potential

$$V_{\mathrm{LO}}=C_S+C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+m_\pi^2}$$

- Iterated one-pion exchange potential (ladder diagrams)



M. Savage, arXiv:nucl-th/9804034

$k\to\infty$ → Spin-triplet

Logarithmic Divergence

$\sim(Qm_N)^n$

cannot be absorbed by C_S, C_T

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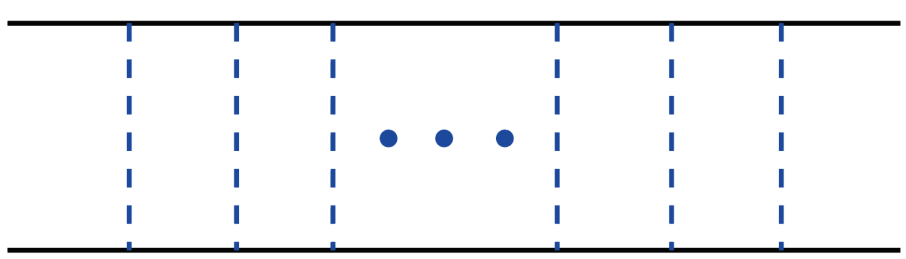

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WPC is inconsistent with renormalization, even at LO!

Deal with the renormalization issue

- Possible solutions (still controversial…)
  - **Keep cutoff lower than hard scale:** $\Lambda < \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$
    - ✓ WPC is consistent
      G.P. Lepage, nucl-th/9706029; E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161
      A.M. Gasparyan, E. Epelbaum PRC105(2022)024001; 107 (2023) 044002
    - ✓ Achieve great successes
      E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
      R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1
  - **Kaplan, Savage, and Wise (KSW) power counting**
    - ✓ Treat the exchange of pions perturbatively *D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390*
    - ✓ Fail to converge in certain spin-triplet channels *S. Fleming, et al., Nucl.Phys. A677 (2000) 313*
      - Deepen examine: only lowest spin-triplet partial waves *D.B. Kaplan, PRC102(2020)034004*
  - **Modified WPC with renormalization group invariance (RGI)**
    - ✓ Rearrange the higher order contact terms to the lower chiral order
      A. Nogga, et al., PRC72(2005)054006 M. C. Birse, PRC74(2006)014003 M. Pavon Valderrama,PRC72(2005) 054002.
      B. Long and C.-J. Yang, PRC84(2011)057001 … ***U. van Kolck, Front. in Phys. 8 (2020) 79***

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  - **Lorentz invariant framework to reformulate chiral force**
    - ✓ Fundamental symmetry of our nature

Chiral forces in Lorentz invariant framework

- Initial idea: modified Weinberg approach — E. Epelbaum and J. Gegelia, PLB716(2012)338-344
  - Use Weinberg power counting to expand the NN potential
    - ✓ Relativistic corrections are perturbatively included

$$V(p', p) = \bar{u}_1 \bar{u}_2 \, \mathcal{A} \, u_1 u_2, \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

  - Use the Kadyshevsky equation to calculate T-matrix

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{2(\boldsymbol{k}^2 + m_N^2)} \frac{1}{\sqrt{\boldsymbol{p}^2 + m_N^2} - \sqrt{\boldsymbol{k}^2 + m_N^2} + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

  - **LO study: a renormalizable framework** (except $^3P_0$ channel)

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  - **LO study: a renormalizable framework** (except $^3P_0$ channel)

- Based on this idea, we proposed **a systematic framework** within the time-ordered perturbation theory (TOPT) using the covariant chiral Lagrangians
  - Formulate the NN interaction up to next-to-next-to-leading order

    V. Baru, E. Epelbaum, J. Gegelia, XLR, Phys. Lett. B 798, 134987 (2019)
    XLR, E.Epelbaum, J.Gegelia, Phys. Rev. C 101, 034001 (2020)
    XLR, PoS(CD2021)007
    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022) / in preparation

Chiral Lagrangian up to NNLO

Lorentz-invariant effective Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

- Purely pionic sector *J.Gasser, H. Leutwyler, Ann.Phys.(1984)*

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu + \chi_+\rangle.$$

- One-nucleon sector *J. Gasser, M. E. Sainio, and A. Svarc, NPB(1988)*

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1\langle\chi+\rangle - \frac{c_2}{4m_N^2}\langle u^\mu u^\nu\rangle \left(D_\mu D_\nu + \text{ h.c. }\right) + \frac{c_3}{2}\langle u^\mu u_\mu\rangle - \frac{c_4}{4}\gamma^\mu\gamma^\nu\left[u_\mu, u_\nu\right] \right\} \Psi_N$$

✓ $f_\pi = 92.4$ MeV, $g_A = 1.267$, $c_{1,2,3,4}$ determined by πN scattering data

- Two-nucleon sector (with unknown LECs)

$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2}\left[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N) + C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right) + C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right) + C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right) + C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\right]$$

N.Fettes, U.-G. Meißner, S. Steininger, NPA(1998)

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N\bar{\Psi}_N\mathcal{O}_i\Psi_N\Psi_N$$

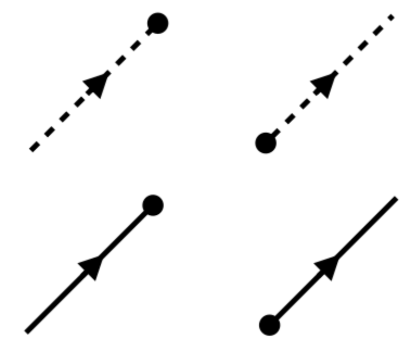
L.Girlanda, S. Pastore, R. Schiavilla, M. Viviani, PRC(2010)
Yang Xiao, Li-Sheng Geng, XLR, PRC(2019)
E. Filandri, L. Girlanda, PLB (2023)

Diagrammatic rules in TOPT

XLR, PoS(CD2021)007

- **External lines**

Spin 0 boson (in, out)



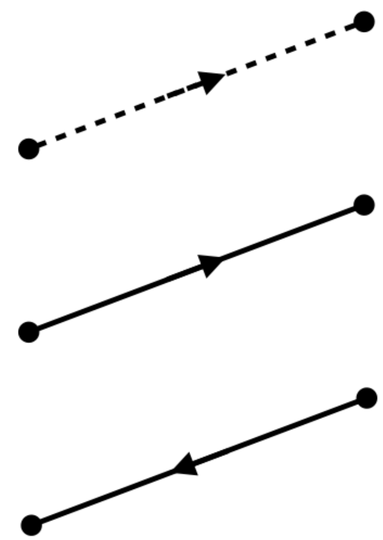
$$\mathbf{1}$$

Spin 1/2 fermion (in, out)

$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

- **Internal lines**

Spin 0 (anti-)boson



$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

Spin 1/2 fermion

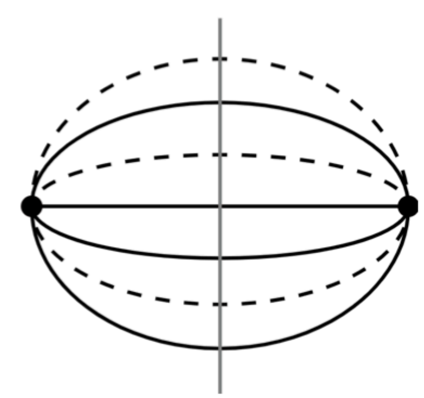
$$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

anti-fermion

$$\frac{m}{\omega_p}\sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

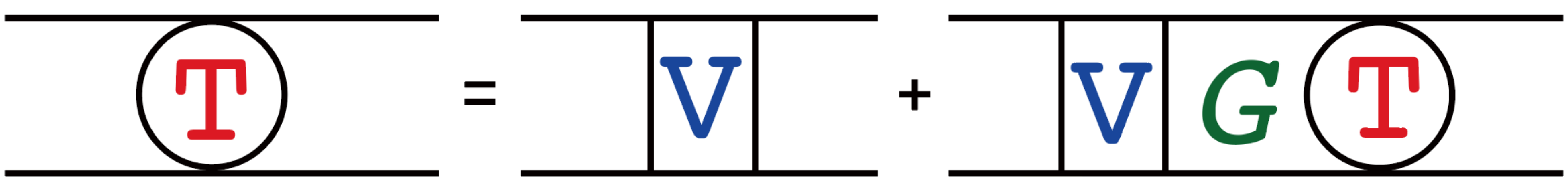
- **Interaction vertices:** the standard Feynman rules
  - Zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Nucleon–nucleon scattering in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the **two-nucleon irreducible** time-ordered diagrams
  - **Weinberg power counting:** systematic ordering of all graphs
- Scattering equation



- **Two-nucleon Green function** $G(E,k) = \dfrac{m_N^2}{k^2+m_N^2}\dfrac{1}{E-2\sqrt{k^2+m_N^2}+i\epsilon}$
- **Uniquely determined** the scattering equation

$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{k}) \frac{m_N^2}{k^2+m_N^2} \frac{1}{E-2\sqrt{k^2+m_N^2}+i\epsilon} T(\boldsymbol{k},\boldsymbol{p})$$

V. Kadyshevsky, NPB (1968)

✓ SELF-CONSISTENTLY obtained in our TOPT framework

✓ Milder UV behaviour than the Lippmann-Schwinger equation

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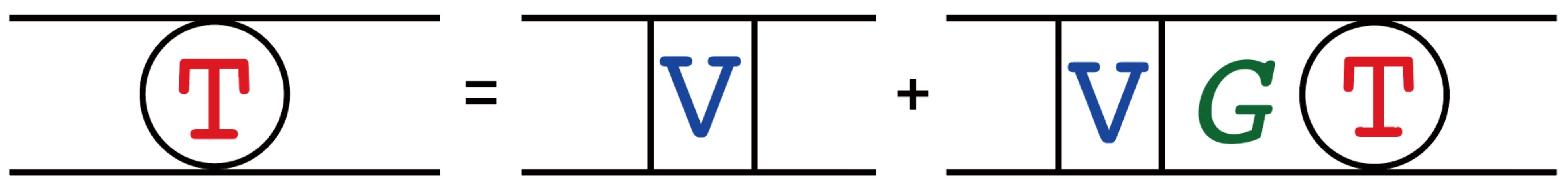

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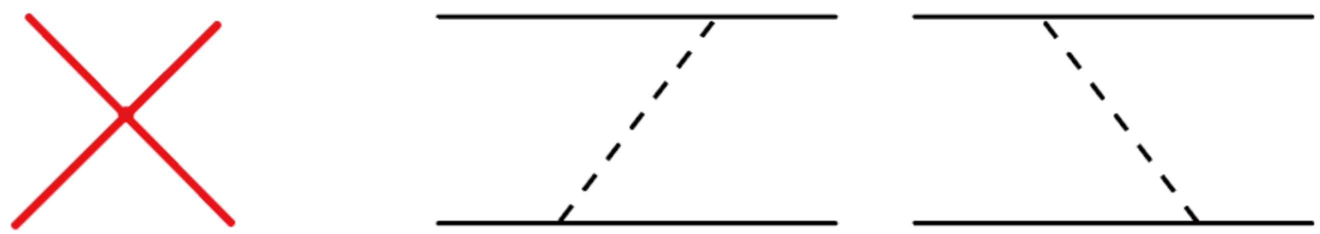
Potential and scattering equation are obtained on an equal footing!

Results and discussion

Leading order potentials

- Follow TOPT rules



- Perform the expansion for the nucleon energies (Weinberg P.C.)

$$V_{LO,C} = (C_S + C_V) - (C_{AV} - 2C_T)\,\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

S. Weinberg, PLB251(1990)288-292

- Consistent with the non-relativistic contact terms

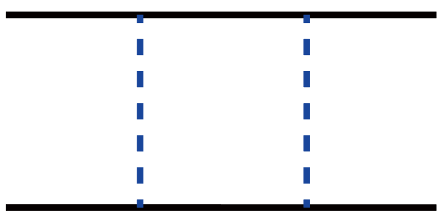
$$V_{\rm OPE} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \frac{4m_N^2}{\omega(q,M_\pi)\,(m_N+\omega(p,m_N))\,(m_N+\omega(p',m_N))} \times \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}$$

- Milder UV behaviour than that of the non-relativisitc OPEP

$V_{\rm OPE}(p',k) \xrightarrow{k\longrightarrow\infty}$ Our $\frac{1}{k}$ vs. Non-Rel. $\frac{1}{1}$

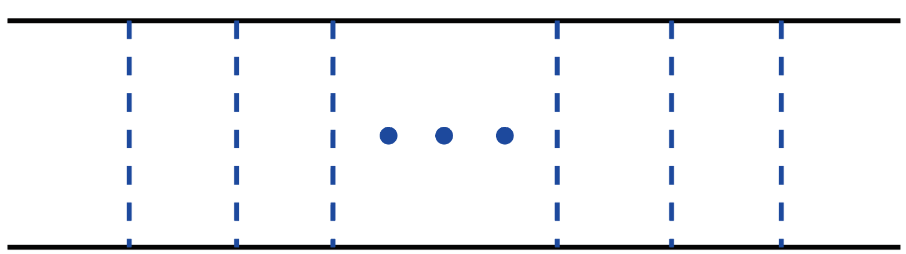
UV behavior of the OPE potential

- Once-iterated OPEP: $V\,G\,V$ XLR, PoS(CD2021)007



$$\begin{cases} I_{VGV}^{\mathrm{Our}} \to \int dk^3 \frac{1}{k}\frac{1}{k^3}\frac{1}{k} = \int dk^3 \frac{1}{k^5} & \textbf{UV convergent} \\ I_{VGV}^{\mathrm{NR}} \to \int dk^3\, 1\, \frac{1}{k^2}\, 1 = \int dk^3 \frac{1}{k^2} & \textbf{UV divergent} \end{cases}$$

- Iteration of our OPEP



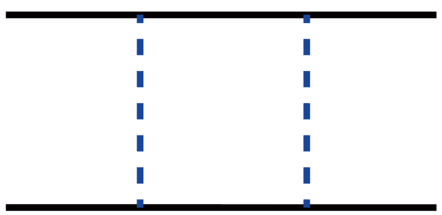
$\xrightarrow{k \to \infty}$ Finite

- Scattering amplitude from OPEP is **cutoff independent**

$$T_{\mathrm{OPE}} = V_{\mathrm{OPE}} + V_{\mathrm{OPE}}\, G\, T_{\mathrm{OPE}}$$

UV behavior of the OPE potential

- Once-iterated OPEP: $V\,G\,V$ *XLR, PoS(CD2021)007*



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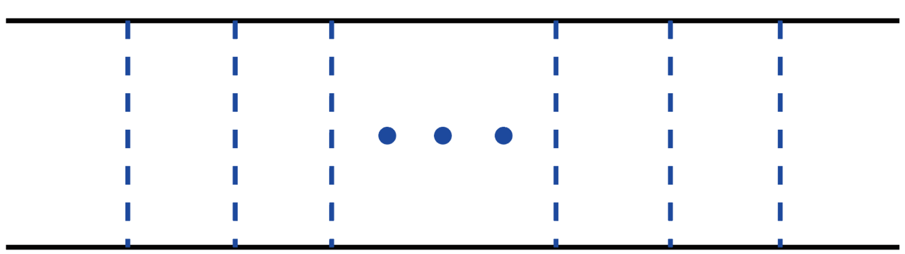


$\xrightarrow{k \to \infty}$ Finite

- Scattering amplitude from OPEP is **cutoff independent**

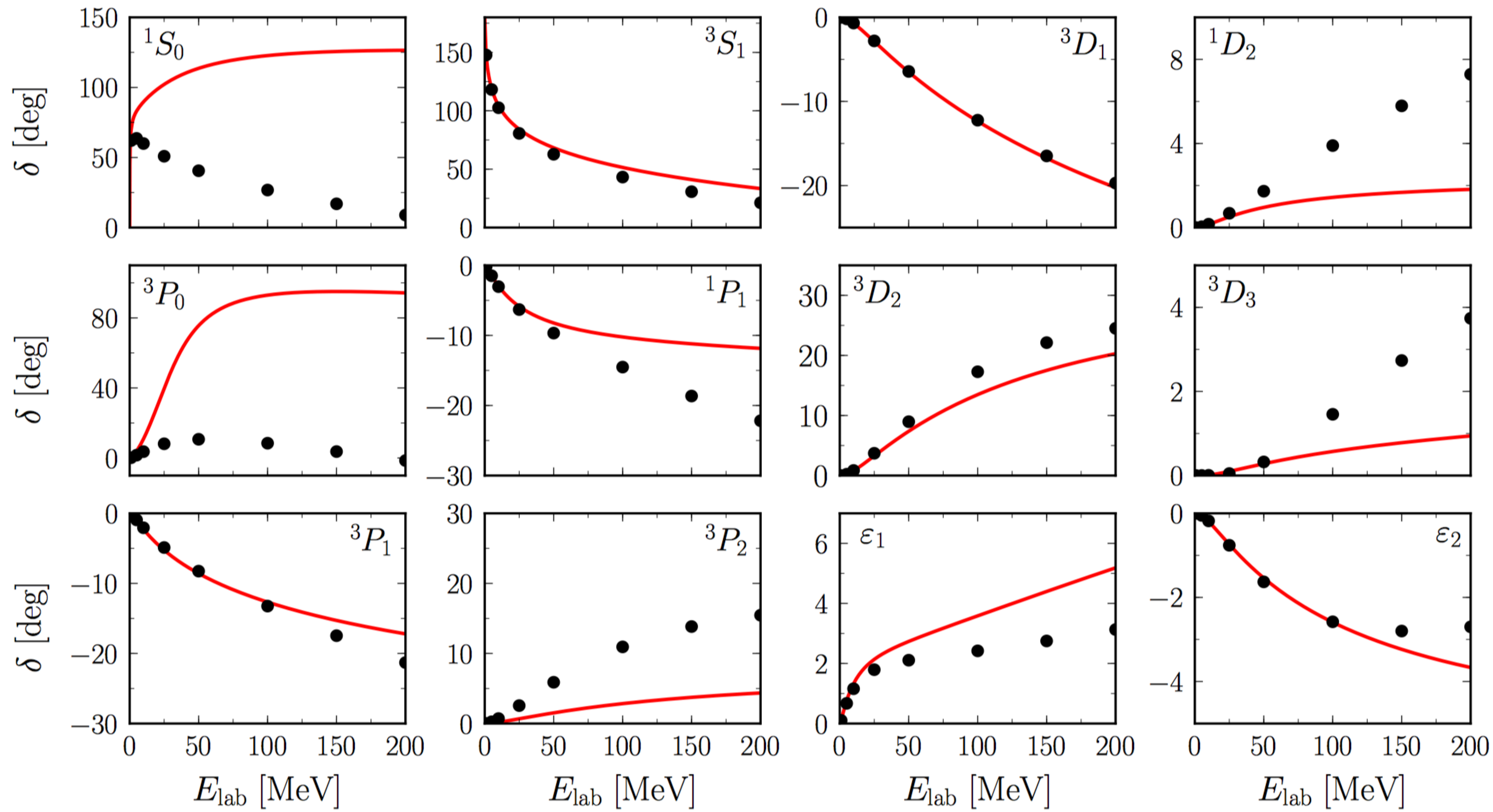
$$T_{\rm OPE} = V_{\rm OPE} + V_{\rm OPE}\, G\, T_{\rm OPE}$$

→Our LO potential is renormalizable!

- Unique solutions for all partial waves, no limit-cycle behavior
- Avoid finite-cutoff artefacts inherent to the conventional NR framework

Phase shifts at LO

- Two LECs: fixed by scattering lengths of $^1S_0$ and $^3S_1$ ($\Lambda = 20$ GeV)



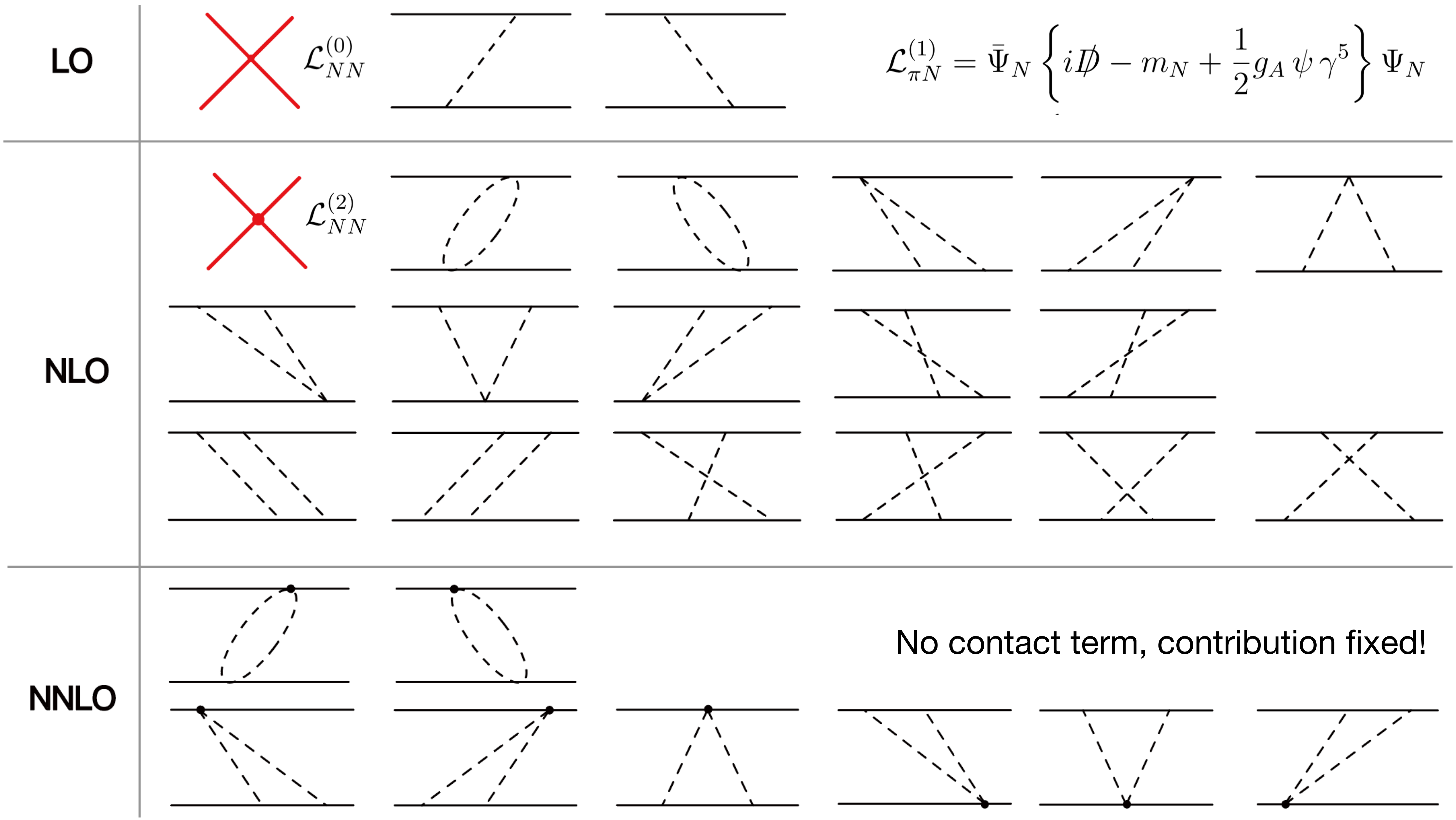
- Provides a reasonable description of the empirical phase shifts
  - ✓ $^1S_0$ and $^3P_0$: Large deviation
  - ✓ Part of the subleading corrections must be treated non-perturbatively

Beyond LO

V. Baru, E. Epelbaum, J. Gegelia, XLR*, Phys. Lett. B 798, 134987 (2019)

NNLO potential in TOPT

- Time ordered diagrams up to NNLO



$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}^{(2)}_{\pi N} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

$c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \mathrm{GeV}^{-1}$

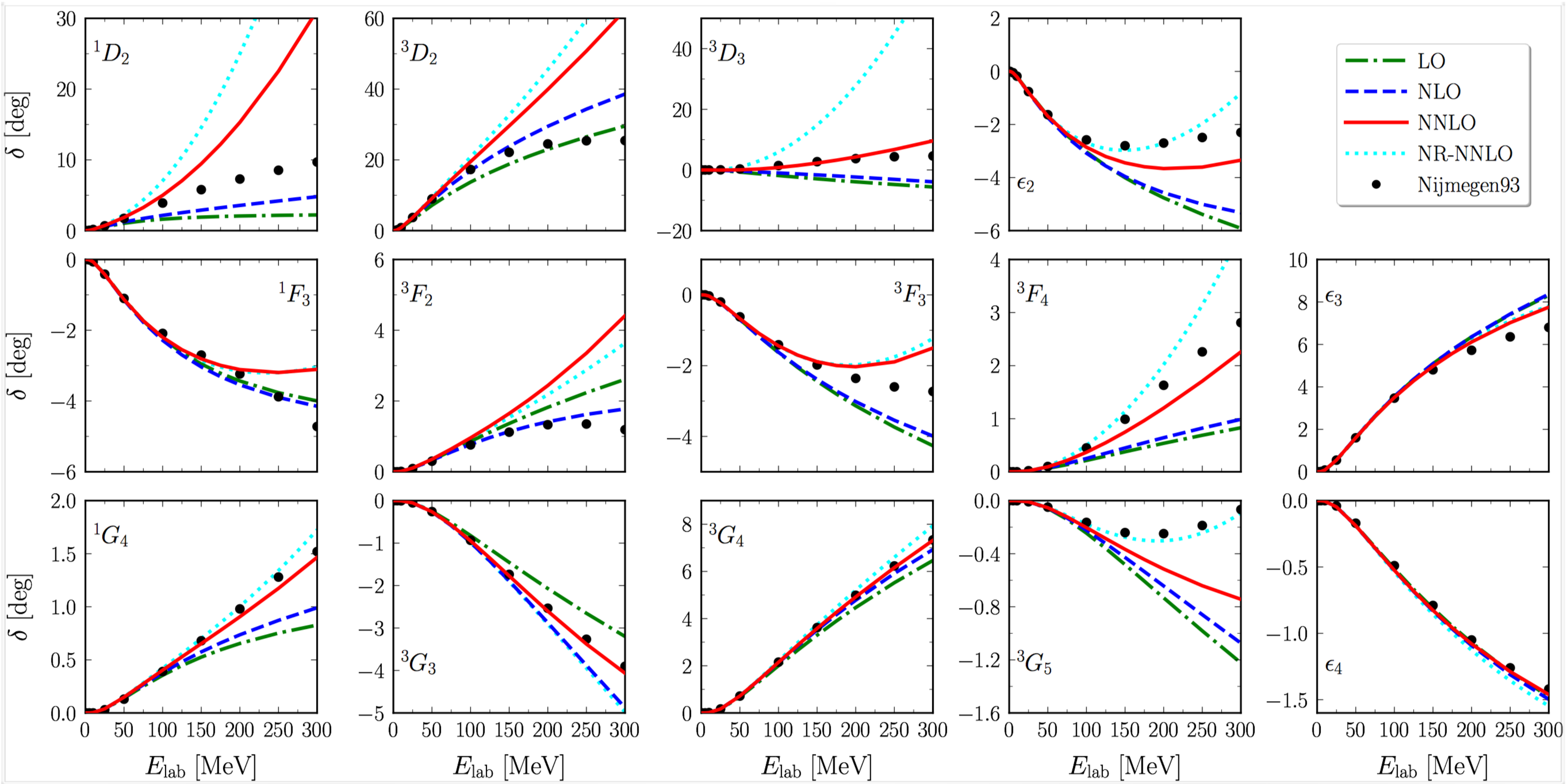
D. Siemens, et al.,PLB 770 (2017) 27-34

Pion-exchange contribution

- On-shell T-matrix under the Born approximation

$$T(p',p) = V_{\rm OPE}(p',p) + V^{(2)}_{2\pi,irr}(p',p) + V^{(3)}_{2\pi,irr}(p',p) + V_{\rm OPE}\, G\, V_{\rm OPE}$$

- **Prediction: phase shifts of D, F, G waves**



✓ Improve the description of D waves; globally similar results for F, G waves

- $^3G_5$: non-rel. result is accidental, c_i/m_N effect (N^4LO) is large *D. Entem, et al., 1411.5335*

XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)

S, P partial wave phases

◻ Partial wave T-matrix

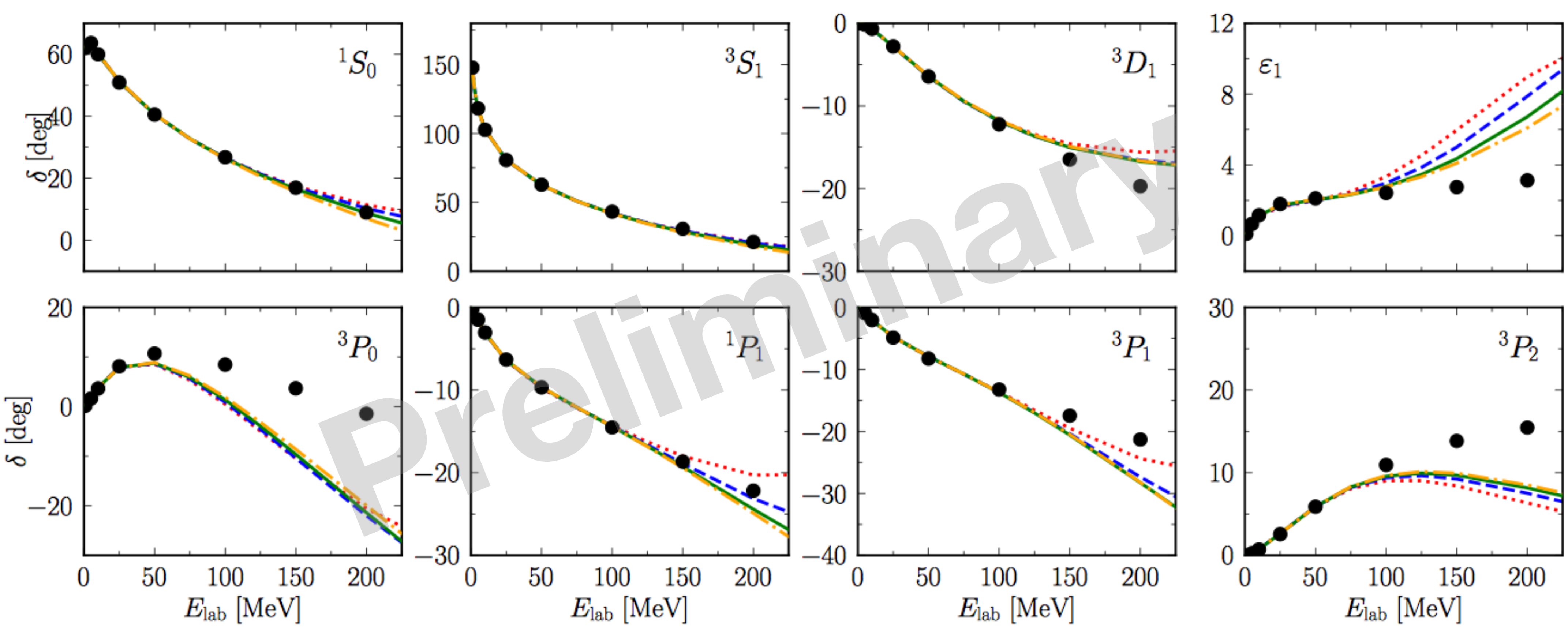
- V_{NNLO} non-perturbatively iterated in the Kadyshevsky equation

$$T^{sj}_{ll'}(p',p) = V^{sj}_{ll'}(p',p) + \sum_{l''}\int \frac{d^3k}{(2\pi)^3} V^{sj}_{ll''}(p',k)\,\frac{m_N^2}{2(k^2+m_N^2)}\,\frac{1}{\sqrt{p^2+m_N^2}-\sqrt{k^2+m_N^2}+i\epsilon}\,T^{sj}_{l''l'}(k,p)$$

- Pion-loop potential: cutoff regularization with $k_{\text{max.}} = 500$ MeV
  - ✓ Effective remove the short range contribution
- Exponential regulator: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$, with $n = 2$, $\Lambda = 400 \sim 550$ MeV

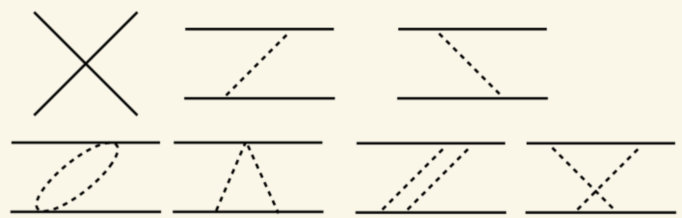
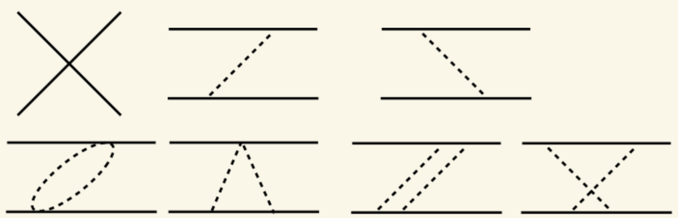
◻ Phase shifts

- Fit **NPWA ($E_{\text{lab}} \leq 100$ MeV)**: ***np*** scattering Nijmegen 93



Summary

- Proposed a systematic framework to formulate chiral forces

	Non-relativistic (Heavy-baryon)	**Manifestly Lorentz invariant**
Chiral Lagrangians	$N^\dagger\left[i(v\cdot D)+g_A(S\cdot u)\right]N$ $-\frac{1}{2}C_S\left(N^\dagger N\right)\left(N^\dagger N\right)-\frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\left(N^\dagger\vec{\sigma}N\right)+\cdots$	$\bar{\Psi}_N\left\{i\gamma_\mu D^\mu-m_N+\frac{1}{2}g_A\psi\gamma^5\right\}\Psi_N$ $+\frac{1}{2}\Big[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N)+C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right)$ $+C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right)$ $+C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\Big]+\ldots$
Potential TOPT diagrams		
Scattering equations (T = V + VGT)	Lippmann-Schwinger eq.	Kadyshevsky eq.
Power counting	Weinberg p.c.	Weinberg p.c.

- Obtained **the non-singular LO potential**, achieve the cutoff independence
- Formulated the chiral potential up to **NNLO**
  - Calculated the complicated two-pion-exchange potential at one-loop level
  - Achieved a rather reasonable description of phase shifts

Summary

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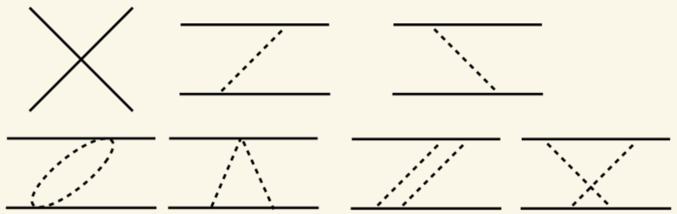
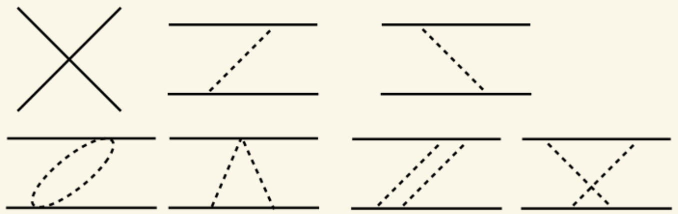
Potential TOPT diagrams		
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Thank you for your attention!

Additional slides

D, F, G partial wave phases

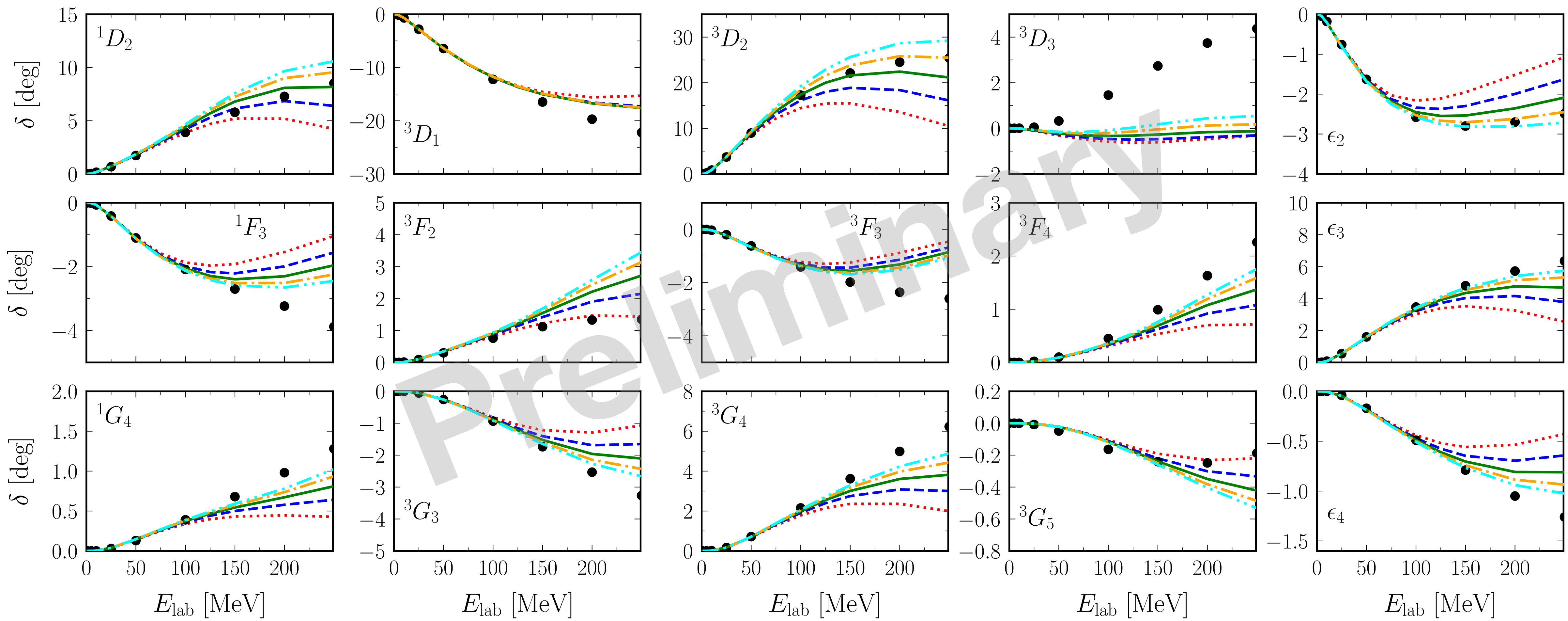
Partial wave T-matrix

- V_{NNLO} non-perturbatively iterated in the Kadyshevsky equation

$$T_{ll'}^{sj}(p',p) = V_{ll'}^{sj}(p',p) + \sum_{l''} \int \frac{d^3k}{(2\pi)^3} V_{ll''}^{sj}(p',k) \frac{m_N^2}{2(k^2+m_N^2)} \frac{1}{\sqrt{p^2+m_N^2} - \sqrt{k^2+m_N^2} + i\epsilon} T_{l''l'}^{sj}(k,p)$$

- Pion-loop potential: cutoff regularization: $k_{\max.} = 500$ MeV
  - ✓ Effective remove the short range contribution
- Exponential regulator: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$, with $n = 2$, $\Lambda = 400 \sim 600$ MeV

Prediction of phase shifts



NN potential concept

- Often-thought as a non-relativistic quantity
  - Appear in the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(t,\boldsymbol{r}) + V(\boldsymbol{r})\Psi(t,\boldsymbol{r}) = i\hbar\frac{\partial}{\partial t}\Psi(t,\boldsymbol{r}).$$

  - (or) Appears in the Lippmann-Schwinger equation

$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d\boldsymbol{k}}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{k}) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k},\boldsymbol{p}).$$

- Generalize the definition of potential
  - An interaction quantity appearing in a three-dimensional scattering equation can be referred as a **NN potential**.
  - If one employs **a relativistic scattering equation**, one can define a **relativistic NN potential**.

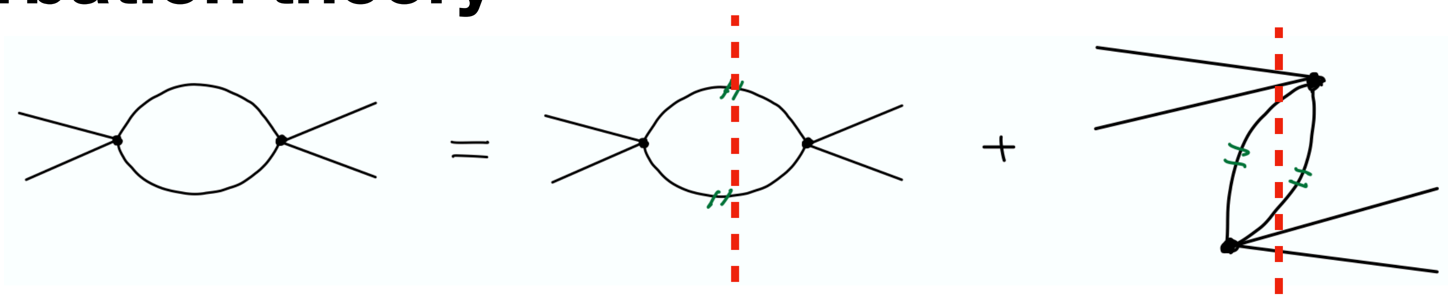
M.H. Partovi, E.L. Lomon, PRD2 (1970) 1999
K. Erkelenz, Phys.Rept. 13C(1974) 191

Time-ordered perturbation theory

- Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

  - Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
    - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
  - **TOPT or old-fashioned perturbation theory**



- Advantages
  - Explicitly show the unitarity
  - Easily to tell the contributions of a particular diagram

- Obtain the rules for time-ordered diagrams
  - Perform Feynman integrations over the zeroth components of the loop momenta
  - Decompose Feynman diagram into sums of time-ordered diagrams
  - Match to the rules of time-ordered diagrams