Examples and counterexamples of renormalizability of an EFT in the nonperturbative regime.

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Importance of *explicit renormalization* of an EFT: . Example of nuclear chiral EFT.

Expansion parameter: (soft scale)/(hard scale) $Q = \frac{q}{\Lambda_b} \quad q \in \{|\vec{p}|\,, M_\pi\}\,, \quad \Lambda_b \sim M_\rho$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Contains bare parameters (LEC’s) $C_i = C_i^r + \delta C_i$

Counter terms δC_i absorb divergent and power-counting breaking contributions

Renormalization: power counting and expansion in terms of renormalized quantities C_i^r

“Perturbative” calculation of the S-matrix, spectrum, etc.

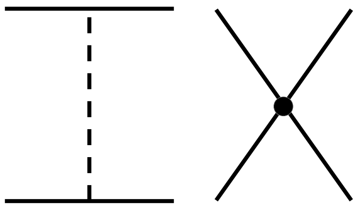
Nonperturbative effects: → ***Explicit renormalization*** of nuclear chiral EFT is complicated

Power counting for NN chiral EFT

Weinberg, S., **NPB363**, 3 (1991)

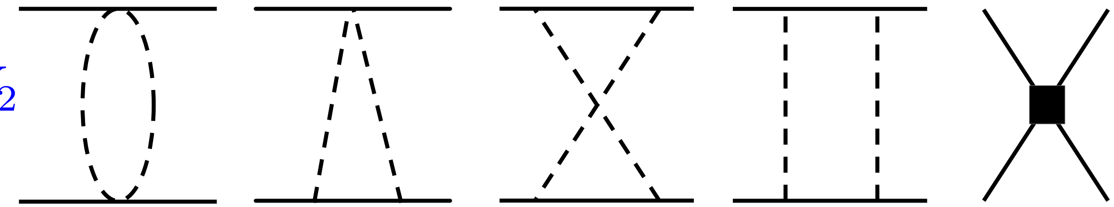
LO

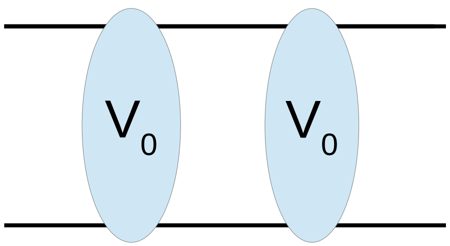
$$V_{\mathrm{LO}} = V_0$$



NLO

$$V_{\mathrm{NLO}} = V_2$$





2N-reducible diagrams are enhanced: V_0 must be iterated

$$\mathrm{LO:}\ T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$\mathrm{NLO:}\ T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

V_2 is treated perturbatively to have the expansion of the amplitude under control

Regularization

The unregularized amplitude is divergent:

→ Regulator: cutoff Λ

$$\text{LO: } T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim \Lambda^n$$

$$\text{NLO: } T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m\, V_2\, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim \Lambda^{m+n+2}$$

Positive powers of Λ violate the power counting even if we keep Λ finite

One needs an infinite number of counter terms to absorb ***all*** positive power of Λ

Power counting restoration via renormalization.
The case of a finite cutoff (of the order of the hard scale).

Power-counting violating contributions originate from large loop momenta: $p \sim \Lambda$

$$\text{LO: } T_0^{[n]} = V_0 (G V_0)^n \sim \mathcal{O}(Q^0), \qquad V_0 \sim \frac{1}{\Lambda_V}$$

$$\Lambda \approx \Lambda_V \approx \Lambda_b : \qquad G V_0 \sim \int \frac{dp}{\Lambda_V} \sim \frac{\Lambda}{\Lambda_V} \sim \frac{\Lambda_b}{\Lambda_b} \sim \mathcal{O}(Q^0)$$

$$\text{NLO: } T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \frac{\Lambda^2}{\Lambda_b^2} \left(\frac{\Lambda}{\Lambda_V} \right)^{m+n} \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting breaking contributions can be absorbed by lower order contact (counter) terms

AG, E.Epelbaum, **PRC 105**, 024001 (2022)

Renormalization condition: $\mathbb{R}(T_2)(p=0) = 0$

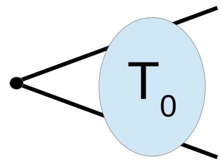
$$\mathbb{R}(T_2^{[m,n]})(p) \sim \mathcal{O}(Q^2)$$

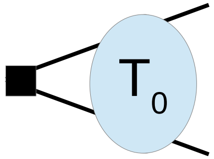
Constraints due to nonperturbative effects

AG, E.Epelbaum, **PRC107**, 044002 (2023)

NLO with 2 contact interactions:

$$\mathbb{R}\,(T_2)\,(p) = T_{2;\text{long}}(p) + C_0\psi_p(r\approx 0)^2 + C_2\psi_p(r\approx 0)\psi_p'(r\approx 0) + \ldots$$

$\psi_p(r\approx 1/\Lambda\approx 0)$ =1+ 

$\psi_p'(r\approx 0)$ =p^2+ 

C_i are fixed, e.g., by renormalization conditions at momenta p_i

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$$\det(p_1,p_2) = \begin{vmatrix} \psi_{p_1}(0) & \psi_{p_1}'(0) \\ \psi_{p_2}(0) & \psi_{p_2}'(0) \end{vmatrix} = 0 \;\Longrightarrow\; C_0,\, C_2 = \infty$$

Destroys renormalizability if $\det(p_1, p_3 \neq p_2) \neq 0$ (zero is not factorizable)

Typical situation for singular attractive interactions. Wave functions oscillate close to r=0

⇓

Renormalizability constraints on (the short-range part of) the LO potential.
The simplest formulation: LECs must be of natural size (If $\Lambda\sim\Lambda_b$).

NN scattering for at NLO. $^3P_0$ partial wave. Sending the cutoff to infinity.

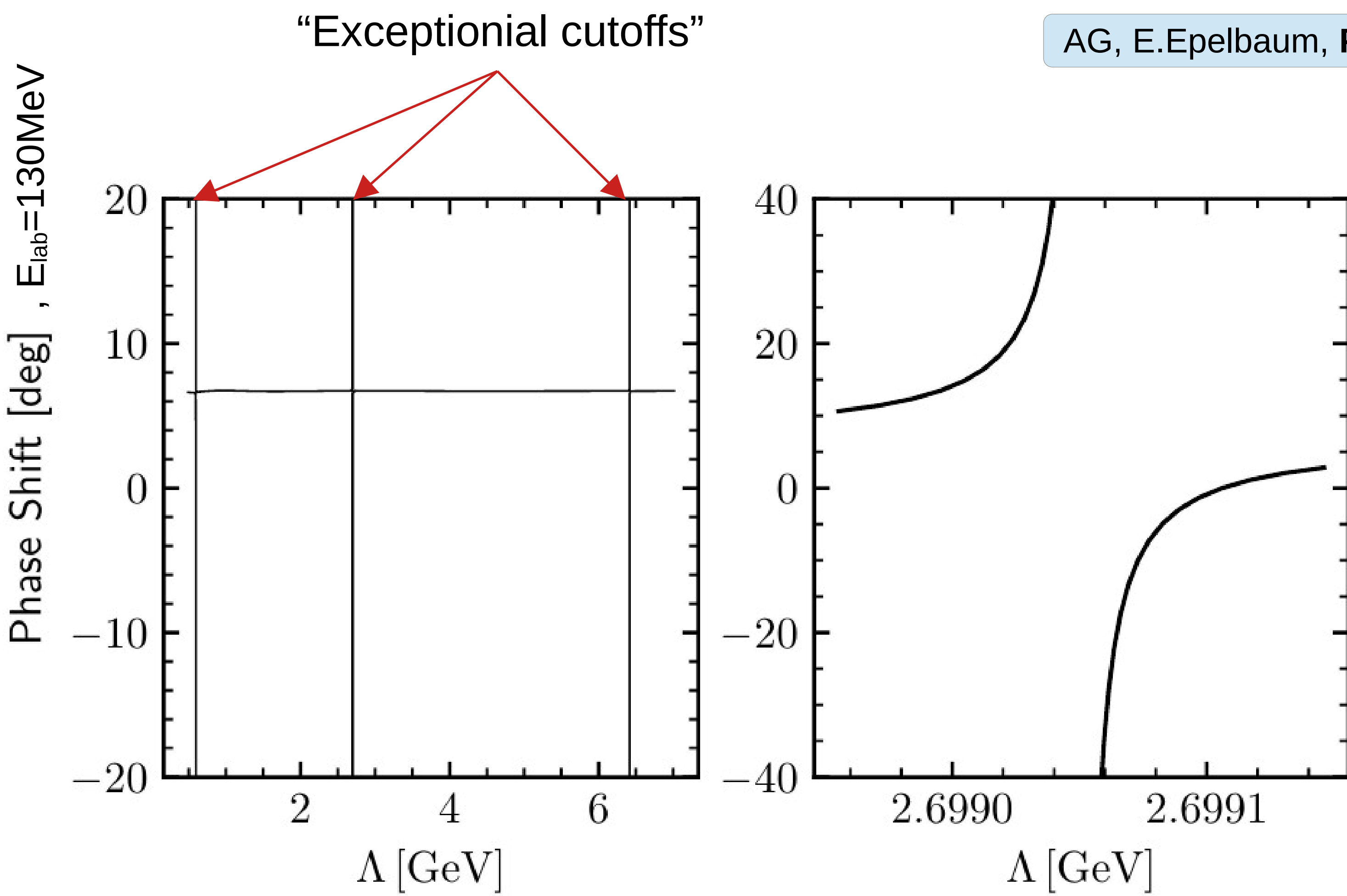
Scheme of Long and Yang:

B. Long, C. J. Yang, **PRC84**, 057001 (2011)

$$V^{(2)}(p',p) = V_{2\pi}(p',p) + C_0^{(2)}(\Lambda)p'p + C_2^{(2)}(\Lambda)p'p(p^2+p'^2)$$

Perturbative NLO: $T^{(2)} = \left[\mathbb{1} + T^{(0)}G\right]V^{(2)}\left[\mathbb{1} + GT^{(0)}\right]$ (avoiding pathologies due to repulsive singular interaction)

AG, E.Epelbaum, **PRC107**, 034001 (2023)



Generalizations

Singular ($1/r^n, n \geq 2$) LO interactions, perturbative NLO

Pionless EFT in 3N systems,...

For pure $1/r^2$ LO potential, $1/r^4$ NLO potential and the sharp cutoff regularization,
There appear no “exceptional” cutoffs (factorization)

Unique situation.
Any slight modification of the potential (short- or long-range part) or
of a regulator leads to appearance of “exceptional” cutoffs

B. Long, U. van Kolck,
Annals Phys. 323, 1304 (2008)
AG, E.Epelbaum, **PRC107**, 034001 (2023)

Non-local separable long-range interaction

AG, E.Epelbaum, N.Jacobi, in progress

$$V_0 = C_0 F_\Lambda(p') F_\Lambda(p) + \dots,$$

$$V_2 = C_2 \frac{p'^2 + p^2}{\Lambda_b^2} \frac{p'^2 p^2}{(M_\pi^2 + p'^2)(M_\pi^2 + p^2)} F_\Lambda(p') F_\Lambda(p).$$

$$F_\Lambda(p) = \frac{\Lambda^2}{(\Lambda^2 + p^2)}$$

$$V_0 G V_2 \sim \frac{\Lambda^2}{\Lambda_b^2} \frac{p^2}{(M_\pi^2 + p^2)} \sim O(Q^0)$$

$$\int dp', \qquad p' \sim \Lambda$$

→ Long-range power-counting-breaking terms

→ Nonrenormalizability (in terms of local counter terms)

Similar result when V_0 is nonperturbative

Non-local separable interaction

Interaction obtained from chiral EFT: $V(\vec{p}\,', \vec{p}) = V_{\rm short}(\vec{p}\,', \vec{p}) + V_{\rm long}(\vec{p}\,', \vec{p})$

$$V_{\rm short}(\vec{p}\,', \vec{p}) = {\rm Polynomial}(\vec{p}\,', \vec{p})\, F_\Lambda(\vec{p}\,', \vec{p})$$

$$V_{\rm long}(\vec{p}\,', \vec{p}) = V_L(\vec{q} = \vec{p}\,' - \vec{p})\, \tilde{F}_\Lambda(\vec{p}\,', \vec{p}), \qquad V_L = V_{1\pi} + V_{2\pi} + \ldots$$

Subtractions: $\left|V(p', p) - V(p', 0)\right| \le \left|\frac{p}{p'}\right| \times (\ldots)$ if $|p'| > |p|$

$$\left|V(p', p) - \sum_{i=0}^{n} \frac{\partial^i V(p', p)}{i!(\partial p)^i}\bigg|_{p=0} p^i\right| \le \left|\frac{p}{p'}\right|^{n+1} \times (\ldots) \text{ if } |p'| > |p|$$

AG, E.Epelbaum, **PRC 105**, 024001 (2022)

→ Large loop momenta are suppressed

→ Renormalizability

Fully local interaction

AG, E.Epelbaum, Y.Komissarova, in progress

All potentials (short- and long-range) are local

A.Gezerlis et al., **PRC 90**, 054323 (2014)

$$V(\vec{p}\,', \vec{p}) = V(\vec{q} = \vec{p}\,' - \vec{p}).$$

$$V(\vec{r}\,', \vec{r}) = V(r)\delta(\vec{r} - \vec{r}\,'),$$

Analysis of renormalizability is much easier

NLO, S-wave: $T_2(p) = \int r^2 dr V_2(r) \psi_p^{(+)}(r)^2 = \frac{(4\pi)^2}{f(p)^2} \int dr V_2(r) \phi_p(r)^2$

$f(p)$ -Jost function (Fredholm determinant) containts nonperturbative physics

$\phi_p(r)$-regular solution is represented by the rapidly converging series:

$$\phi_p = \sum_{n=0}^{\infty} \phi_p^{(n)}, \qquad \phi_p^{(n+1)}(r) = \frac{m_N}{p} \int_0^r dr' \sin[p(r - r')] V_0(r') \phi_p^{(n)}(r')$$

Summary

- ✔ *Explicit renormalization* of nuclear (chiral) EFT is crucial to obtain a consistent power counting for observable quantities.
- ✔ Beyond LO there appear certain constraints on the LO interaction to make a theory *renormalizable* in the nonperturbative regime
- ✔ For non-EFT interactions, *renormalizability* is not guaranteed