Unitary interaction subsets

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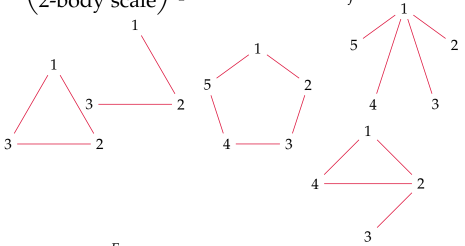
Contessi, Valderrama, and JK ('23)

The problem: What $\begin{pmatrix}\text{few-body}\\ \text{non-relativistic}\\ \text{quantum}\end{pmatrix}$ - complexity can a

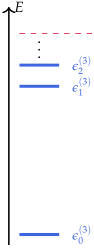
"The mind is localized and extends forever to infinity. The body is extended and remains localized." (R. Descartes)

universal/most versatile/elemental $\hat{=}$ $\begin{pmatrix}\text{no}\\ \text{2-body scale}\end{pmatrix}$ pair interaction v_{ij} evoke?

$$H = \sum_{i=1}^{A} t_i + \sum_{\{i,j\}\in\mathcal{I}} v_{ij}$$



$$v_{ij} = \begin{cases} c_0 \cdot \delta(|\boldsymbol{r}_{\rm ij}|) \\ c_0 \cdot e^{-c_1 \boldsymbol{r}_{\rm ij}^2} \\ c_0 \cdot |\boldsymbol{r}_{\rm ij}|^{-1} \cdot e^{-c_1|\boldsymbol{r}_{\rm ij}|} - c_3 \cdot |\boldsymbol{r}_{\rm ij}|^{-6} \\ c_0 \cdot |\boldsymbol{r}_{\rm ij}|^{-12} + c_1 \cdot |\boldsymbol{r}_{\rm ij}|^{-6} \\ \vdots \end{cases} \Rightarrow$$

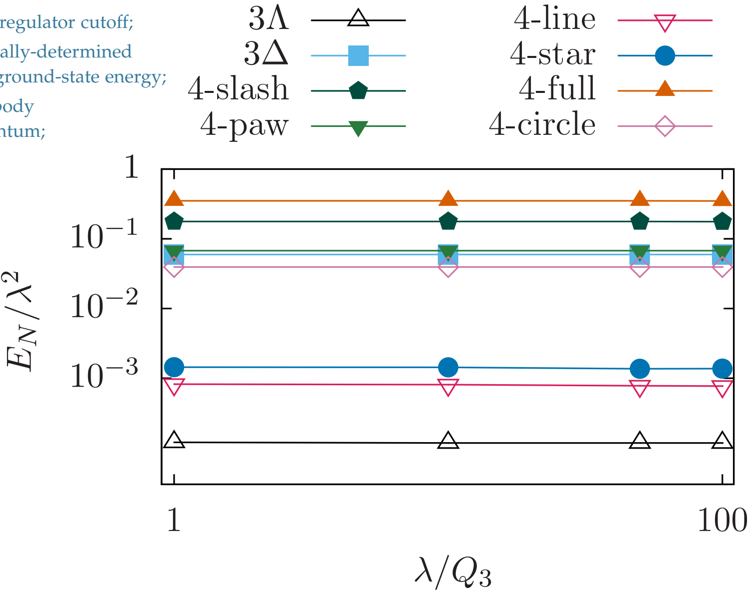


$$\lim_{n\to\infty} \frac{\epsilon_n^{(3)}}{\epsilon_{n+1}^{(3)}} = \text{constant}(\mathcal{I})$$ *(V. Efimov, '70)*

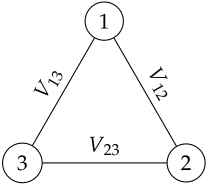
$$\lim_{\text{range}\to 0} \epsilon_0^{(3)} = \infty$$ *(L.H. Thomas, '35)*

Graph-independent collapse pattern

- $\lambda \hat{=}$ Gaussian regulator cutoff;
- $E_N \hat{=}$ numerically-determined (**SVM, RGM**) ground-state energy;
- $Q_3 \hat{=}$ fixed 3-body binding momentum;



Stabilization ("renormalization") of the universal interaction



divergence-free all-resonant 3-body system via:

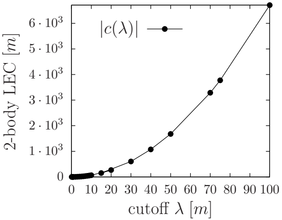
- boundary condition on 3-body wave function ↔ `radial parameter`;
- additional interaction term

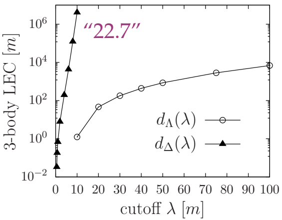
$$v_{ijk} \overset{e.g.}{=} d \cdot \delta(|\boldsymbol{r}_{\rm ij}|)\delta(|\boldsymbol{r}_{\rm ik}|) \quad ;$$

numerical constraint: functional representation

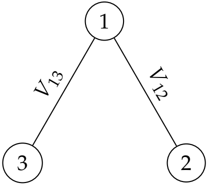
$$d \cdot \delta(r) = \lim_{\lambda\to\infty} d_\Delta(\lambda) \cdot e^{-\lambda r^2} \quad ;$$

$$d_\Delta \leftrightarrow \epsilon_0^{(3)} \quad \Rightarrow$$





Stabilization ("renormalization") of the universal interaction



divergence-free partially-resonant 3-body system via:

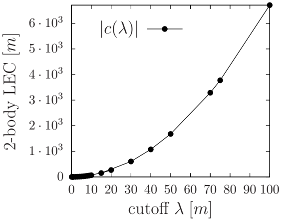
- boundary condition on 3-body wave function $\leftrightarrow$ `radial parameter`;
- additional interaction term

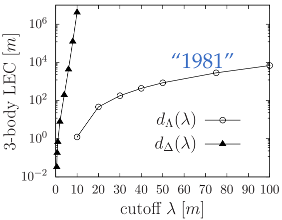
$$v_{ijk} \overset{e.g.}{=} d \cdot \delta(|\boldsymbol{r}_{\mathrm{ij}}|)\delta(|\boldsymbol{r}_{\mathrm{ik}}|) \quad ;$$

numerical constraint: functional representation

$$d \cdot \delta(r) = \lim_{\lambda \to \infty} d_{\Lambda}(\lambda) \cdot e^{-\lambda r^2} \quad ;$$

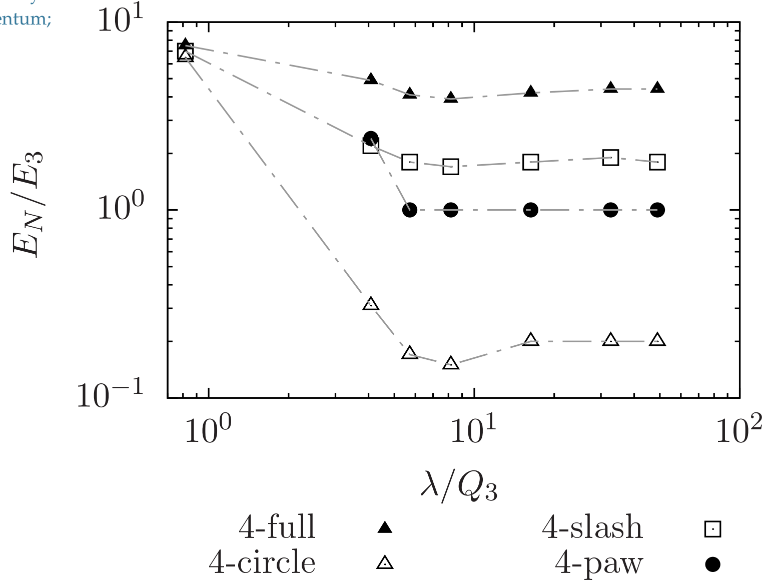
$$d_{\Lambda} \leftrightarrow \epsilon_0^{(3)} \quad \Rightarrow$$





- $E_3 \hat{=}$ 3-body-Δ ground-state energy;
- $E_N \hat{=}$ numerically-determined (**SVM, RGM**) ground-state energy;
- $Q_3 \hat{=}$ fixed 3-body binding momentum;

Loop GRAPHS



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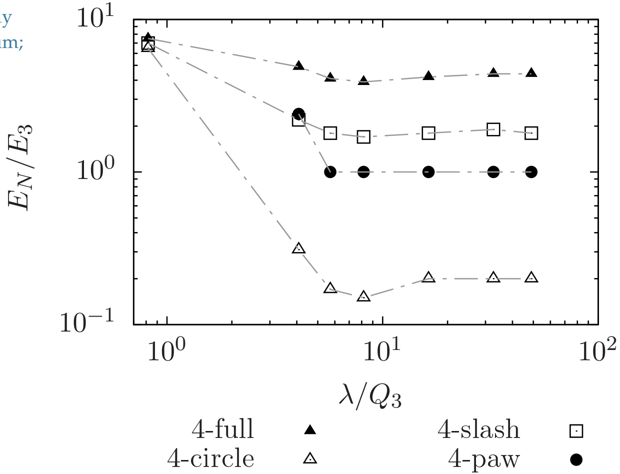
Loop GRAPHS



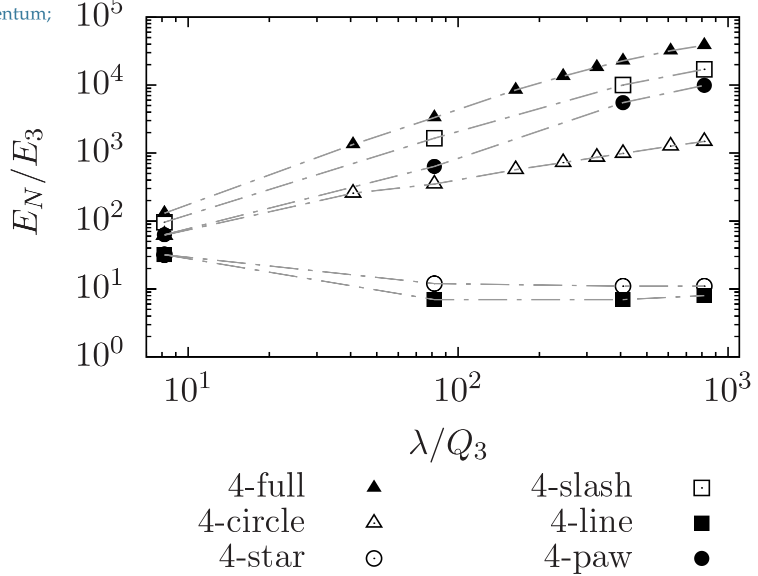
corr $[B_0(3), B_0(4)]$ insensitive to short-distance structure of v_{ij}

if

$B_0(3)$graph $\in B_0(4)$graph ;

- $E_3 \hat{=}$ 3-body-Λ ground-state energy;
- $E_N \hat{=}$ numerically-determined (**SVM, RGM**) ground-state energy;
- $Q_3 \hat{=}$ fixed 3-body binding momentum;

Open line GRAPHS



- $E_3 \hat{=}$ 3-body-Λ ground-state energy;
- $E_N \hat{=}$ numerically-determined (**SVM, RGM**) ground-state energy;
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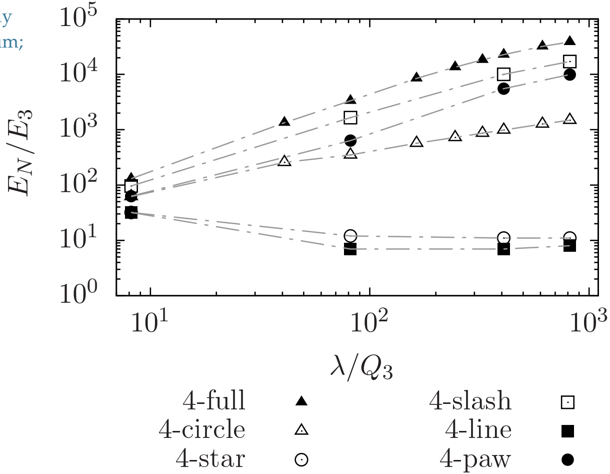
Open line GRAPHS



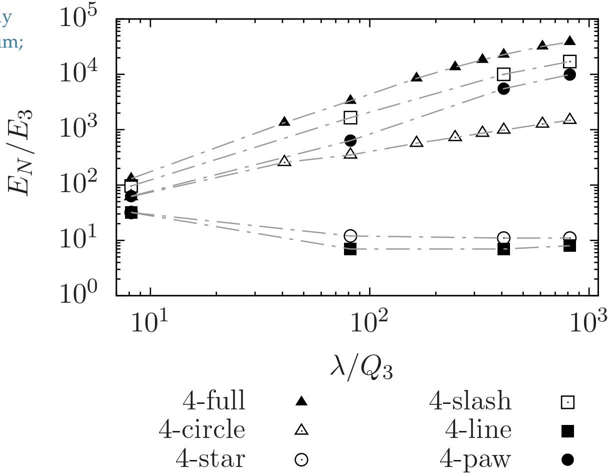
corr $[B_0(3), B_0(4)]$ insensitive to short-distance structure of v_{ij}

if

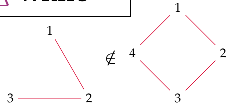
$B_0(3)$graph $\in B_0(4)$graph except...

- $E_3 \hat{=}$ 3-body-Λ ground-state energy;
- $E_N \hat{=}$ numerically-determined (**SVM, RGM**) ground-state energy;
- $Q_3 \hat{=}$ fixed 3-body binding momentum;

Open line GRAPHS



circle/loop anomaly: $B(4, \text{circle})$ stabilized with d_Δ while



4-body S-wave Bethe-Peierls boundary condition

$$\lim_{r_{ij}\to 0}\left(-\frac{1}{r_{ij}\Psi}\frac{\partial}{\partial r_{ij}}\left[r_{ij}\Psi\right]\right)=\frac{1}{a_{ij}}\quad \forall \text{ contact-interacting pairs } (ij).$$

Idea (MPV): Independent sub-constraints on particular Faddeev-Yakubovsky components resembling Efimov conditions.

$$\begin{array}{lclcl}
\left[\frac{\partial}{\partial r}\left(r\,\phi_K(r,\rho,\sigma)\right)\right]_{r\to 0} & + & \left[\frac{\partial}{\partial r}\left(r\,\phi_K(r,\rho',\sigma')\right)\right]_{r\to 0} & + & B\left(\phi_K(\vec{\xi}\,')+\phi_K(\vec{\xi}\,'')+\phi_K(\vec{\xi}\,''')\right) \quad + \\
A\,\phi_K(a\rho,b\rho,\sigma) & + & A\,\phi_K(a\rho',b\rho',\sigma') & + & \\
\left[\frac{\partial}{\partial r}\left(r\,\phi_H(r,\tilde{\rho},\tilde{\sigma})\right)\right]_{r\to 0} & + & \left[\frac{\partial}{\partial r}\left(r\,\phi_H(\tilde{\sigma},\tilde{\rho},r)\right)\right]_{r\to 0} & + & \\
A\,\phi_H(c\tilde{\rho}+d\tilde{\sigma},e\tilde{\sigma},f\tilde{\rho}+g\tilde{\sigma}) & + & A\,\phi_H(f\tilde{\rho}+g\tilde{\sigma},e\tilde{\sigma},c\tilde{\rho}+d\tilde{\sigma}) & & \stackrel{!}{=} 0
\end{array}$$

4-body S-wave Bethe-Peierls boundary condition

$$\lim_{r_{ij}\to 0}\left(-\frac{1}{r_{ij}\Psi}\frac{\partial}{\partial r_{ij}}\left[r_{ij}\Psi\right]\right)=\frac{1}{a_{ij}}\quad \forall \text{ contact-interacting pairs } (ij).$$

Idea (MPV): Independent sub-constraints on particular Faddeev-Yakubovsky components resembling Efimov conditions.

$$\begin{array}{ccccc}
\left[\frac{\partial}{\partial r}\left(r\,\phi_K(r,\rho,\sigma)\right)\right]_{r\to 0} & + & \left[\frac{\partial}{\partial r}\left(r\,\phi_K(r,\rho',\sigma')\right)\right]_{r\to 0} & + & B\left(\phi_K(\vec{\xi}\,')+\phi_K(\vec{\xi}\,'')+\phi_K(\vec{\xi}\,''')\right) \quad + \\
A\,\phi_K(a\rho,b\rho,\sigma) & + & A\,\phi_K(a\rho',b\rho',\sigma') & + & \\
\left[\frac{\partial}{\partial r}\left(r\,\phi_H(r,\tilde{\rho},\tilde{\sigma})\right)\right]_{r\to 0} & + & \left[\frac{\partial}{\partial r}\left(r\,\phi_H(\tilde{\sigma},\tilde{\rho},r)\right)\right]_{r\to 0} & + & \\
A\,\phi_H(c\tilde{\rho}+d\tilde{\sigma},e\tilde{\sigma},f\tilde{\rho}+g\tilde{\sigma}) & + & A\,\phi_H(f\tilde{\rho}+g\tilde{\sigma},e\tilde{\sigma},c\tilde{\rho}+d\tilde{\sigma}) & & \stackrel{!}{=} 0
\end{array}$$

circle/loop anomaly: A value classifies circle as Λ like.

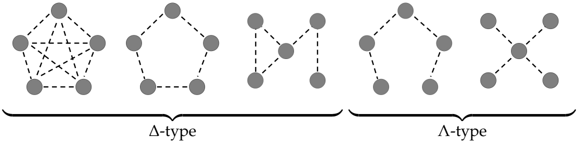
Epilogue

- corr $[B_0(3), B_0(3+n)]$
  (existence of a universal, stable cluster for any $N > 3$)

- Any connected, unitary N-body system exhibits discrete scale invariance with 22.7 and/or 1986.1 ;

- No new scale can be introduced solely with additional, unitary particle species/flavours/types.

- numerical verification for 5-body shapes:



- c.f. mass-imbalanced, partially-resonant systems & their universal properties, *e.g.* ,
  B. Bazak and D. S. Petrov ('17) , *P. Naidon ('18)*