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THE HEBREW UNIVERSITY OF JERUSALEM

The asymptotic behaviour of the many-body wave-function

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Short Range Correlations in a many-body systems

Kenya (2016).

Generalized Contact Formalism (GCF)

Nuclear 2-body Short Range Correlations (SRCs) are successfully described by the **GCF**[1]

The GCF is based on the factorization ansatz

$$\lim_{\boldsymbol{r}_{ij}\to 0} \Psi = \sum_c \varphi_{ij}^c\left(\boldsymbol{r}_{ij}\right) A_{ij}^c\left(\boldsymbol{R}_{ij}^{\text{C.M.}}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \qquad ij \in pp,\ np,\ nn$$

- φ is a universal *zero-energy* two-body wave-function, $\hat{H}\varphi = 0$
- A is the residual part, the "wave-function" of the spectator subsystem

The contact is defined by $\quad C_{ij}^{cc'} = \frac{N_{ij}}{2J+1}\sum_m \langle A_{ij}^c | A_{ij}^{c'} \rangle$

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Short range observable - The Contact

Tan, Braaten & Platter,...

The 2-body system

$$\psi(\boldsymbol{r}) \xrightarrow[r \to 0]{} \varphi_0(\boldsymbol{r})$$

The N-body system

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N) \xrightarrow[r_{12} \to 0]{} \varphi_0(\boldsymbol{r}_{12}) A(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_N)$$

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$$O_{12} \approx \delta(\boldsymbol{r}_{12}) \Longrightarrow \langle\psi|O_{12}|\psi\rangle \approx C_2\langle\varphi_0|O_{12}|\varphi_0\rangle$$

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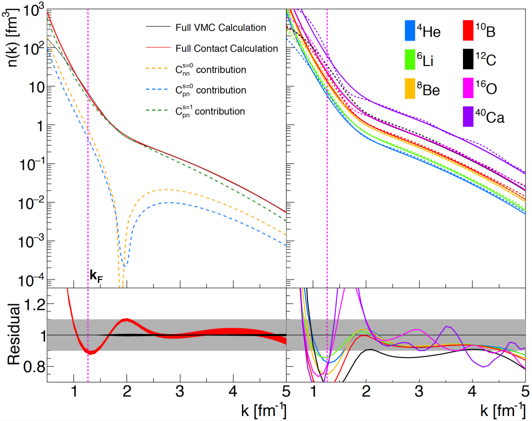
$$\langle \Psi | \sum O_{ij} | \Psi \rangle \approx \underbrace{\frac{N(N-1)}{2} \langle A | A \rangle}_{C_N} \langle \varphi_0 | O_{12} | \varphi_0 \rangle$$

The nuclear momentum distributions

The **asymptotic** 1-body momentum distribution

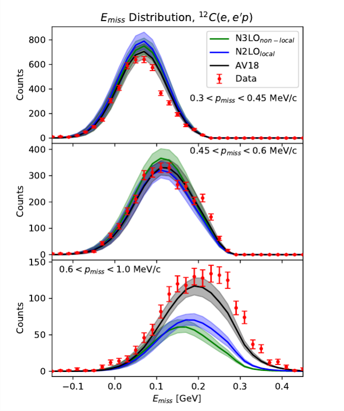
$$n_n(\boldsymbol{k}) \longrightarrow C_{np}^{s=0}|\tilde{\varphi}_{np}^{s=0}(\boldsymbol{k})|^2 + C_{np}^{s=1}|\tilde{\varphi}_{np}^{s=1}(\boldsymbol{k})|^2 + 2C_{nn}^{s=0}|\tilde{\varphi}_{nn}^{s=0}(\boldsymbol{k})|^2$$

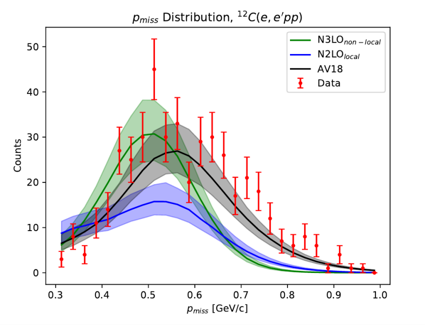
Comparing with VMC calculations:



Surprisingly, the agreement holds for $k_F \le k \le 6\ \mathrm{fm}^{-1}$

Experiments[2] at $1.4 < x_B \leq 2$





Contacts taken from **ab-initio** calculations

2 A. Schmidt; et al. (CLAS Collaboration). In: *Nature* 578 (2020).

The Generalized Contact Formalism

We look for a systematic approach to:

- Obtain the factorization ansatz
- Understand the rule of Higher-body SRCs
- Derive the universal function φ_0
- Calculate the contacts

To achieve these goals we use the coupled-cluster (CC) method

The CC method describes correlations naturally

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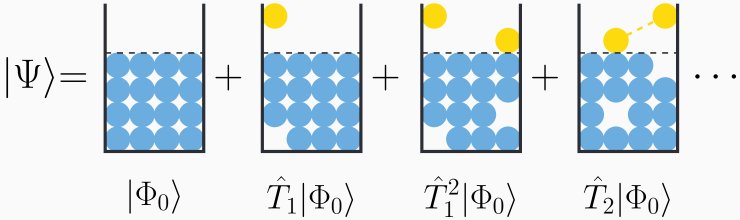
Coupled cluster[3]

The wave-function is expanded in **clusters**

$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle \qquad \hat{T} = \hat{T}_1 + \hat{T}_2 + \hat{T}_3 + \cdots$$

$\hat{T}_n$ operator excites n particles from the Slater determinant Φ_0



[3] Rodney J. Bartlett and Monika Musiał. In: *Rev. Mod. Phys.* 79 (1 2007).

Few comments

- We consider nuclear matter
- Ignore 3-body interactions (and yes, also 4,5,...)
- $\hat{T}_1 = 0$ due to momentum conservation

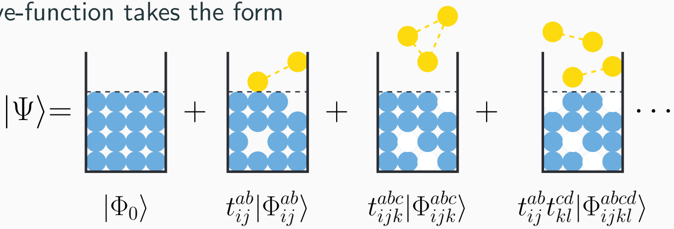
- i, j, k, l - hole states, with momentum $\boldsymbol{k}$
- a, b, c, d - particle states, with momentum $\boldsymbol{p}$ $(p > k_F)$
- An $npnh$ state - $|\Phi_{ij\cdots}^{ab\cdots}\rangle \equiv \hat{\boldsymbol{a}}^\dagger \hat{\boldsymbol{b}}^\dagger \cdots \hat{\boldsymbol{j}} \hat{\boldsymbol{i}} |\Phi_0\rangle$

The complete 2- and 3-body CC equations

$\hat{T}_n$ is the *npnh* operator

$$T_n = \frac{1}{n!^2} \sum t_{i_1 \ldots i_n}^{a_1 \ldots a_n} \boldsymbol{a}_1^\dagger \boldsymbol{a}_2^\dagger \cdots \boldsymbol{i}_2 \boldsymbol{i}_1$$

The wave-function takes the form

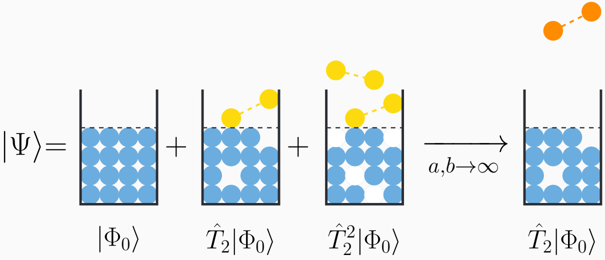


The Coupled-Cluster equations **2-body:**

$$0 = \langle \Phi_{ij}^{ab} | \hat{V} + [\hat{H}_0, \hat{T}_2] + [\hat{V}, \hat{T}_2] + \frac{1}{2}[[\hat{V}, \hat{T}_2], \hat{T}_2] + [\hat{V}, \hat{T}_3] + [\hat{V}, \hat{T}_4] | \Phi \rangle$$

The CC equations are coupled and non-linear

The Nuclear Contact Ansatz   Beck, Weiss, Barnea



Zabolitsky[4] derived the T_2 equation in the **low density** $k_F \to 0$, **high momentum** $\boldsymbol{k} \to \infty$ limit.

$$T_2(\boldsymbol{k}) = \frac{-1}{k^2}\left[V(\boldsymbol{k}) + \int \frac{d\boldsymbol{k}'}{(2\pi)^3} V(\boldsymbol{k}-\boldsymbol{k}')T_2(\boldsymbol{k}')\right]$$

where

$$T_2(\boldsymbol{k}) \equiv t_{\mathbf{0},\mathbf{0}}^{\boldsymbol{k},-\boldsymbol{k}} = \langle \boldsymbol{k}, -\boldsymbol{k}|T_2|\mathbf{0},\mathbf{0}\rangle$$

[4] J.G. Zabolitzky and W. Ey. In: *Physics Letters B* 76.5 (1978).

Substituting

$$\varphi_2(\boldsymbol{k}) = T_2(\boldsymbol{k}) + (2\pi)^3\delta(\boldsymbol{k})$$

We get

$$\boxed{\varphi_2(\boldsymbol{k}) = (2\pi)^3\delta(\boldsymbol{k}) - \frac{1}{k^2}\int\frac{d\boldsymbol{k}'}{(2\pi)^3}V(\boldsymbol{k}-\boldsymbol{k}')\varphi_2(\boldsymbol{k}')}$$

Which is just the **zero-energy Lipmann-Schwinger** equation!.

The resulting momentum distribution

$$n(\boldsymbol{k}) = \langle\Phi_0|e^{T^\dagger}a^\dagger_{\boldsymbol{k}}a_{\boldsymbol{k}}e^{T}|\Phi_0\rangle \longrightarrow \langle\Phi_0|T_2^\dagger a^\dagger_{\boldsymbol{k}}a_{\boldsymbol{k}}T_2|\Phi_0\rangle \propto |T_2(\boldsymbol{k})|^2$$

High-momentum approximations $a, b \to \infty$

The approximations follow these principals

- Ψ is normalized, hence $\hat{T}_n \to 0$
- $\hat{T}_n \to 0$ faster than the potential.
- Keep terms where a, b contract with V.
- Keep the $\hat{H}_0$ term
- Use momentum conservation, e.g. $t_{ij}^{ab} \propto \delta^3 \left(\boldsymbol{p}_a + \boldsymbol{p}_b - \boldsymbol{k}_i - \boldsymbol{k}_j \right)$

$$0 = \langle \Phi_{ij}^{ab} | \hat{V} + [\hat{H}_0, \hat{T}_2] + [\hat{V}, \hat{T}_2] + \frac{1}{2}[[\hat{V}, \hat{T}_2], \hat{T}_2] + [\hat{V}, \hat{T}_3] + [\hat{V}, \hat{T}_4] | \Phi \rangle$$

The effects on the 2-body eq. $(p_a \gg k_{\text{f}},\ E_{ij}^{ab} \equiv \epsilon_a + \epsilon_b - \epsilon_i - \epsilon_j)$

- $E_{ij}^{ab} t_{ij}^{ab} \gg V_{id}^{ak} t_{jk}^{bd} \quad \Rightarrow \quad [\hat{V}, \hat{T}_2] \to \frac{1}{2} V_{de}^{ab} t_{ij}^{de}$
- $E_{ij}^{ab} t_{ij}^{ab} \gg V_{de}^{kl} t_{jl}^{de} t_{ik}^{ab} \quad \Rightarrow \quad [\hat{H}_0, \hat{T}_2] \gg \frac{1}{2}[[\hat{V}, \hat{T}_2], \hat{T}_2]$
- $t_{ikl}^{abd}, t_{ijkl}^{abde} \approx t_{00}^{ab} \quad \Rightarrow \quad [\hat{H}_0, \hat{T}_2] \gg [\hat{V}, \hat{T}_3],\ [\hat{V}, \hat{T}_4]$
- $E^{ab} \gg E^{ij} \quad \Rightarrow \quad E_{ij}^{ab} \to E^{ab}$

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The asymptotic equation

The 2-body amplitude

$$0 = \langle \Phi_{ij}^{ab} | \hat{V} + [\hat{H}_0, \hat{T}_2] + [\hat{V}, \hat{T}_2] + \frac{1}{2}[[\hat{V}, \hat{T}_2], \hat{T}_2] + [\hat{V}, \hat{T}_3] + [\hat{V}, \hat{T}_4] | \Phi \rangle$$

$$\downarrow$$

$$\boxed{0 = E^{ab} t_{ij}^{ab} + V_{ij}^{ab} + \frac{1}{2} V_{de}^{ab} t_{ij}^{de}}$$

In the CC jargon this equation is called

"The particle-particle ladder approximation"

It is equivalent to Zabolitsky's result.

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$$\boxed{0 = E^{ab} t_{ij}^{ab} + V_{ij}^{ab} + \frac{1}{2} V_{de}^{ab} t_{ij}^{de}}$$

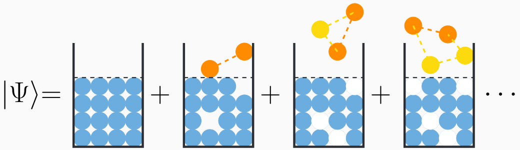
The solution, $\hat{T}_2^{\infty}$, of the asymptotic equation takes the form

$$\hat{T}_2^{\infty} = \frac{1}{1 - \hat{Q}_2 \hat{G}_0 \hat{V}} \hat{Q}_2 \hat{G}_0 \hat{V} \hat{P}_2 \qquad (t^{\infty})_{ij}^{ab} \approx t_{ij}^{ab}$$

- $\hat{G}_0$ is the zero-energy Green's function $\hat{G}_0 = \dfrac{1}{i\varepsilon - \hat{H}_0}$
- $\hat{Q}$ is the projection operator into the *particle subspace*
- $\hat{P}$ is the projection operator into the *hole subspace*

High-momentum approximation to $\hat{T}_n$

$$|\Psi\rangle = \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} \cdots$$



$|\Phi_0\rangle$ $\quad \hat{T}_2|\Phi_0\rangle$ $\quad \hat{T}_3|\Phi_0\rangle$ $\quad \hat{T}_4|\Phi_0\rangle$

The n-body equation is complicated

$$0 = \langle \Phi^{ab\cdots}_{i_1 i_2\cdots}| \left[\hat{H},\hat{T}\right] + \frac{1}{2!}\left[\left[\hat{H},\hat{T}\right],\hat{T}\right] + \\ + \frac{1}{3!}\left[\left[\left[\hat{H},\hat{T}\right],\hat{T}\right],\hat{T}\right] + \frac{1}{4!}\left[\left[\left[\left[\hat{H},\hat{T}\right],\hat{T}\right],\hat{T}\right],\hat{T}\right]|\Phi_0\rangle$$

Due to normalization of Ψ, $\hat{T}_n \to 0$ at high energy excitations (a,b).

$$\Downarrow$$

We keep $E^{ab\cdots}_{i_1 i_2\cdots} t^{ab\cdots}_{i_1 i_2\cdots}$, and the terms where the high energy excitations, a, b are contracted with the potential e.g. $V^{ab}_{c_1 c_2} t^{c_1 c_2\cdots}_{i_1 i_2\cdots}$.

The 2-body asymptotics

The leading terms must be of the form $\propto V^{ab}_{..}$

- The n-folded commutator give terms with n contractions over $\hat{V}$

  $\Rightarrow \frac{1}{3!}\left[\left[\left[\hat{H},\hat{T}\right],\hat{T}\right],\hat{T}\right], \frac{1}{4!}\left[\left[\left[\left[\hat{H},\hat{T}\right],\hat{T}\right],\hat{T}\right],\hat{T}\right]$

- $\left[\left[\hat{H},\hat{T}\right],\hat{T}\right] \quad \Rightarrow \quad \sum_{l=2}^{n-2} V^{ab}_{..}\hat{T}_l\hat{T}_{n-l}$

- $\left[\hat{H},\hat{T}\right] \quad \Rightarrow \quad V^{ab}_{..}\hat{T}_{n-1},\ V^{ab}_{..}\hat{T}_n$

Collect $\hat{T}_{n-1}, \hat{T}_l\hat{T}_{n-l}$ into one operator $\left(\hat{L}_n\right)^{r_1 r_2}$

$$\left(\hat{L}_n\right)^{r_1 r_2} = \left(\hat{\boldsymbol{r}}_2\hat{T}_{n-1}\hat{\boldsymbol{r}}_1 - \hat{\boldsymbol{r}}_1 \leftrightarrow \hat{\boldsymbol{r}}_2\right) + \left(\sum_{l=2}^{n-2}\hat{\boldsymbol{r}}_2\hat{T}_l\hat{\boldsymbol{r}}_1\hat{T}_{n-l}\right)$$

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2-body asymptotics $a_1, a_2 \to \infty$ of $\hat{T}_n$

Assuming also $E^{a_1 a_2 \cdots}_{i \cdots} \approx E^{a_1 a_2}$, the asymptotic equation becomes

$$t^{a_1 a_2 d_3 \cdots d_n}_{i_1 \cdots i_n} + \frac{V^{a_1 a_2}_{d_1 d_2}}{2E^{a_1 a_2}} t^{d_1 \cdots d_n}_{i_1 \cdots i_n} = -\frac{V^{a_1 a_2}_{r_1 r_2}}{2E^{a_1 a_2}} L^{r_1 r_2; d_3 \cdots d_n}_{i_1 \cdots i_n}$$

Making the guess that asymptotically

$$t^{a_1 a_2 d_3 \cdots d_n}_{i_1 \cdots i_n} = \frac{1}{2} \tau^{a_1 a_2}_{r_1 r_2} L^{r_1 r_2; d_3 \cdots d_n}_{i_1 \cdots i_n}$$

gives

$$\frac{1}{2} \underbrace{\left(\tau^{a_1 a_2}_{r_1 r_2} + \frac{V^{a_1 a_2}_{d_1 d_2}}{2E^{a_1 a_2}} \tau^{d_1 d_2}_{r_1 r_2} + \frac{V^{a_1 a_2}_{r_1 r_2}}{E^{a_1 a_2}} \right)}_{\text{same as the } \hat{T}_2 \text{ equation}} L^{r_1 r_2; d_3 \cdots d_n}_{i_1 \cdots i_n} = 0$$

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2-body asymptotic equation for $\hat{T}_n$

It suffices to demand

$$\frac{1}{2}\left(\tau^{a_1a_2}_{r_1r_2} + \frac{V^{a_1a_2}_{d_1d_2}}{2E^{a_1a_2}}\tau^{d_1d_2}_{r_1r_2} + \frac{V^{a_1a_2}_{r_1r_2}}{E^{a_1a_2}}\right) = 0$$

or in matrix form

$$\hat{\tau}_2 = \frac{1}{1-\hat{Q}_2\hat{G}_0\hat{V}}\hat{Q}_2\hat{G}_0\hat{V}$$

I.e. the general Coupled-Cluster amplitude factorizes to

$$\boxed{\hat{T}_n \to \hat{\tau}_2 \cdot \hat{L}^{(2)}_n}$$

Conclusion:

$\hat{T}_n$ gets the same asymptotics as $\hat{T}_2$, up to a "low energy" matrix L

Same follows also for the **3-body** asymptotics

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Summary

- Used the Coupled Cluster method to analyze the asymptotics of the nuclear matter wave-function.
- Relation to the zero-energy Lipmann-Schwinger, and Schrödinger equation has been shown.
- The **2-body** asymptotics: $\boxed{\hat{T}_n \to \hat{\tau}_2 \cdot \hat{L}_n^{(2)}}$
- The **3-body** asymptotics: $\boxed{\hat{T}_n \to \hat{\tau}_3 \cdot \hat{L}_n^{(3)}}$
- Subleading corrections affects only the L matrices.