Vector mesons
and
chiral perturbation theory

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Before chiral perturbation theory (ChPT)

- Vector mesons (VM) were one of the most important building blocks of nuclear physics ($N, \sigma, \pi, \rho, \omega, \cdots$)
  - Tensor force ($\pi + \rho$)
  - Repulsive forces that prevent nucleon/nuclei from collapsing
  - Vector meson dominance (VMD) and Electromagnetic form factors

Since the emergence of ChPT

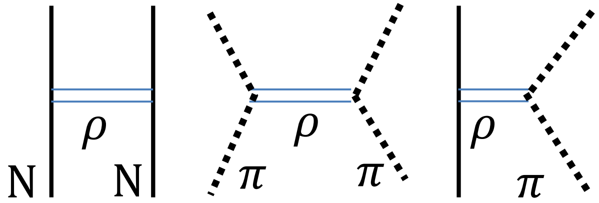
- Heavy particles have been integrated out: $(N, \sigma, \pi, \rho, \omega, \cdots) \rightarrow (N, \pi)$
- Then, vector mesons have been forgotten
- We are also forgetting pions and even nucleons…
  - Pionless EFT: $(N, \pi) \rightarrow (N)$
  - Cluster EFT: $(N) \rightarrow (\quad)$
- Less dofs: More convenient, many insights, but less predictive (more information are put into LECs)
- Effective field theory (EFT) with $(N, \sigma, \pi, \rho, \omega, \cdots)$ for more powers?
  - Things should not be (too much) complicated
  - Things should be systematic
  - How? Symmetries and power-counting

Contents

- Construction of ChPT with vector mesons
- Form factors of pion & nucleon
- NN potential up to N^2LO
- Results on peripheral NN scatterings
- Discussions

Chiral perturbation theory (ChPT)

- Expansion in ascending power of Q/Λ_χ
  - Q: small scale (q, m_π)
  - Λ_χ: large (or chiral) scale ($\sim 4\pi f_\pi \sim$ 1GeV)
- How to count p and m_ρ of vector mesons ?
  - $p^\mu \sim m_\rho \sim \Lambda_\chi$ in $\gamma N \to \rho N,\ \rho \to \pi\pi, \ldots$
  - $p^\mu \sim$ Q $\ll m_\rho$ in **low-energy processes of nucleons and pions**



- In our scheme, we count $p^\mu \sim$ Q $\ll m_\rho$. Then
  Rho-meson propagators $\frac{1}{m_\rho^2 - p^2} \sim Q^0$

Power counting of chiral perturbation theory (ChPT)

- A Feynman diagram $\sim Q^{\nu}$ with

$$\nu = \sum_i d_i - 2\, I_\pi - I_N + 4L$$

  - Using $2\, I_N + E_N = \sum_i n_i$ and $L + V = I + 1$ with $I = I_\pi + I_N$, $V = \sum_i 1$, we get Weinberg's power counting

$$\nu = 2 - \left(\frac{E_N}{2} + E_h\right) + 2L + \sum_i \left(d_i + \frac{n_i}{2} + h_i - 2\right)$$

- Leading order Lagrangian is just mass term (kinetic term appears only at NNLO)

$$L_0 = m_\rho^2 \left\langle \left(\rho_\mu - \frac{i\Gamma_\mu}{g} \right)^2 \right\rangle + \mathrm{f}_\pi^2 \langle i\Delta_\mu \, i\Delta^\mu \rangle + \frac{1}{4} f_\pi^2 \langle \chi_+ \rangle,$$

Leading order propagator

$$\frac{g^{\mu\nu}}{m_\rho^2}$$

- At LO, rho-meson can be integrated out: trivial
- At higher order, resummation of self-energy makes things non-trivial.

If we count: $p^{\mu} \sim m_{\rho}$

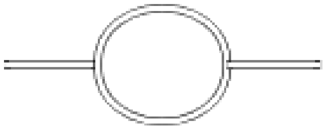
- Leading order Lagrangian (mass-term + kinetic term)

$$L_0 = m_\rho^2 \left\langle \left(\rho_\mu - \frac{i\Gamma_\mu}{g}\right)^2 \right\rangle - \frac{1}{2}\left\langle \rho_{\mu\nu}\rho^{\mu\nu}\right\rangle$$

- Leading order propagator (unitary gauge)

$$\frac{1}{m_\rho^2 - k^2 - i0^+}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_\rho^2}\right)$$

- Good for tree order calculations.
- Bad divergence



$$\delta^{ab}\frac{g_\rho^2}{16\pi^2}\frac{1}{\varepsilon}\left[-\frac{9}{2}m_\rho^2 g^{\mu\nu} + \left(-\frac{k^4}{12m_\rho^4} - \frac{7k^2}{6m_\rho^2} + 7\right)(k^\mu k^\nu - g^{\mu\nu}k^2)\right] + \cdots$$

- Required counter term: $\left\langle \nabla^2\rho_{\mu\nu}\nabla^2\rho^{\mu\nu}\right\rangle$, too high!
- We need not these complications…

$$L_2 = -\frac{1}{2} C_{1:1}\, g^2 \langle \rho_{\mu\nu}\rho^{\mu\nu} \rangle + \frac{1}{2} C_{1:2} \langle \Gamma_{\mu\nu}\Gamma^{\mu\nu} \rangle - i\, C_{1:3} \langle \rho_{\mu\nu}\Gamma^{\mu\nu} \rangle$$

$$-\mathrm{i}\ \mathrm{C}_{2:1}\mathrm{g}\, \langle \rho_{\mu\nu}[\Delta^\mu, \Delta^\nu] \rangle + \mathrm{C}_{2:2}\, \langle \Gamma_{\mu\nu}[\Delta^\mu, \Delta^\nu] \rangle$$

$$-C_3 \langle \chi_+ \rangle \langle \Delta^\mu \Delta_\mu \rangle + C_3 \langle \chi_+ \rangle \left\langle \left(g\rho_\mu - i\, \Gamma_\mu \right)^2 \right\rangle + C_5 \langle \chi_+ \rangle^2 + \cdots$$

$$C_i = \eta_i \left[\lambda + \frac{1}{32\pi^2} \left(\bar{C}_i + \ln \frac{m_\pi^2}{\mu^2} \right) \right].$$

$$\lambda = -\frac{\mu^{d-4}}{32\pi^2} \left(\frac{2}{4-d} + \ln 4\pi + \Gamma'(1) + 1 \right)$$

$$\eta_{1:1} = \frac{a^2}{6}, \qquad \eta_{1:2} = \frac{2}{3}\left(1 - \frac{a}{2}\right)^2, \qquad \eta_{1:3} = \frac{a}{3}\left(1 - \frac{a}{2}\right),$$

$$\eta_{2:1} = 0, \qquad \eta_{2:2} = 0,$$

$$\eta_3 = 1, \qquad \eta_4 = \frac{a^2}{4}, \qquad \eta_5 = \frac{3}{32}$$

$$a = 2$$

Renormalization

- Low-energy constants: $a, g, C_{1:1}, C_{1:2}, C_{1:3}, C_{1:2} - C_{2:2}$
- We fix LECs by
  - $a = 2$: KSRF
  - g & $C_{1:1}$: $m_\rho = 775.26(23)$ MeV
  - $C_{2:1}$: $\Gamma(\rho \to \pi^+\pi^-) = 147.8(9)$ MeV
  - $C_{1:3}$: $\Gamma(\rho \to e^+e^-) = 7.04(6)$ keV
  - $C_{1:2} - C_{2:2}$: $\langle r_\pi^2 \rangle = 0.434(5)\ \text{fm}^2$

FIG. 2: 1PI diagrams for $\pi_a(k) \to \pi_b(k)$. Dashed lines denote pions.

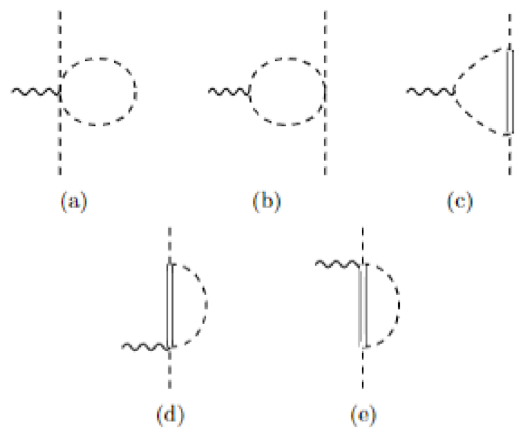


FIG. 3: 1PI diagrams for $\mathcal{V}_e^\mu(k) \to \pi_a(q_a) + \pi_b(q_b)$. Wiggled lines denote external vector fields.

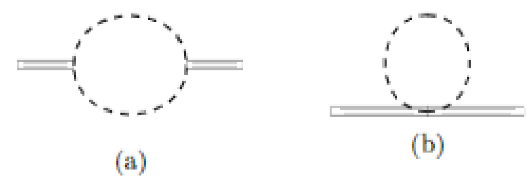


FIG. 5: 1PI diagrams for $\rho_a^\mu(k) \to \rho_b^\nu(k)$.

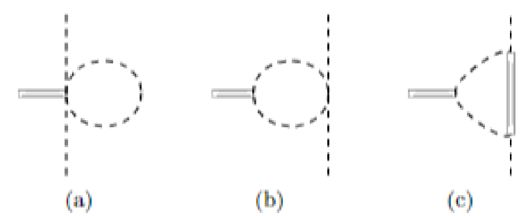
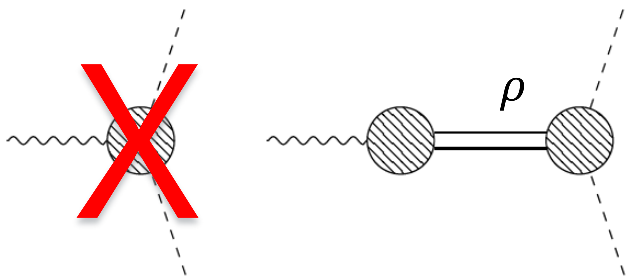


FIG. 4: 1PI diagrams for $\rho_e^\mu(k) \to \pi_a(q_a) + \pi_b(q_b)$

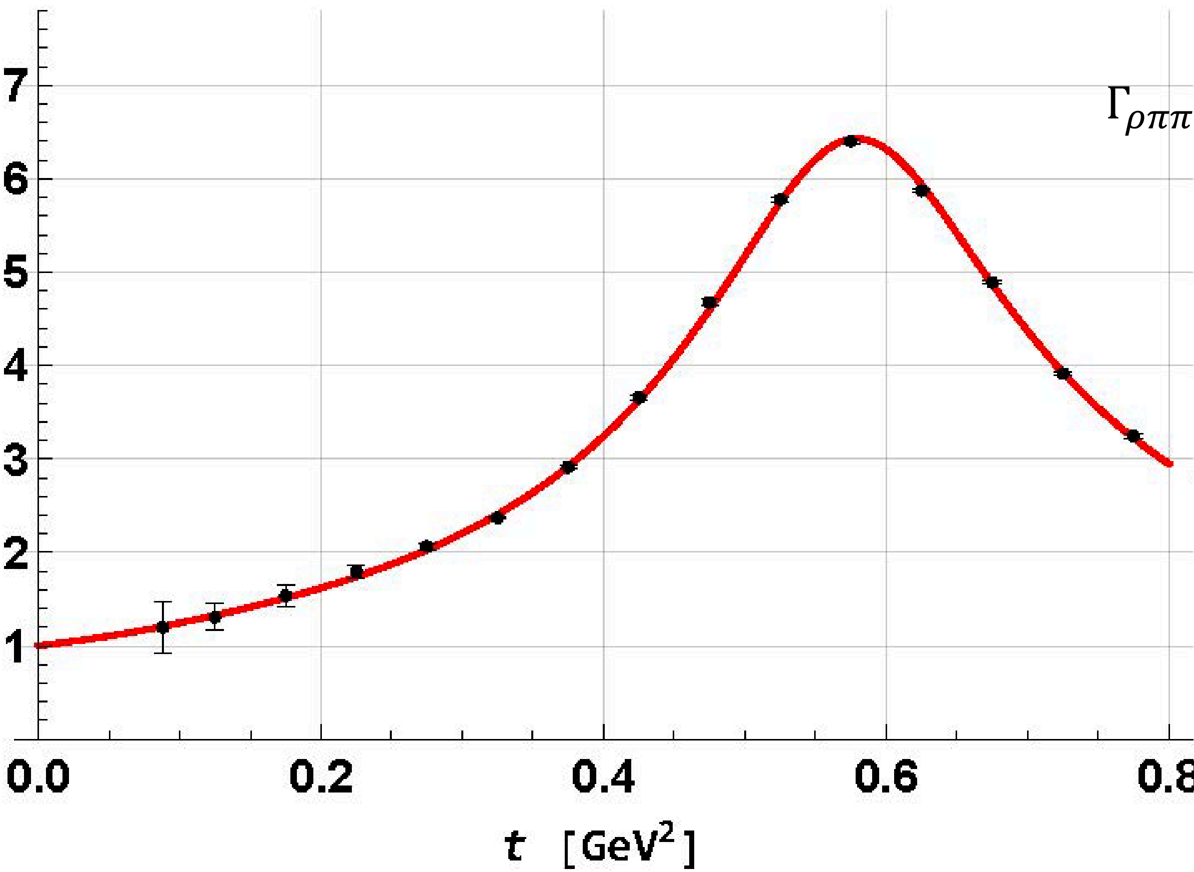
- Interestingly, **vector-meson dominance (VMD) is found to be persistent up to one-loop level**
- $\Gamma_{\gamma\pi\pi}(t) = 1 - \frac{a}{2} + \frac{t}{2f_\pi^2}\left[2\left(1-\frac{a}{2}\right)^2 f_3(t) + C_{1:2} - C_{2:2}\right] \xrightarrow{a\to 2} 0,$
- $\frac{1}{2f_\pi^2}(C_{1:2} - C_{2:2}) = (0.000 \pm 0.005)\ \mathrm{fm}^2$ (determined from $\langle r_\pi^2 \rangle = 0.434(5)\ \mathrm{fm}^2$

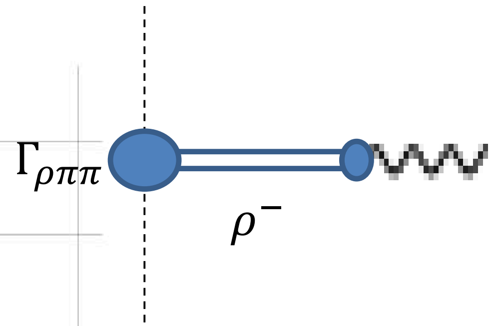


(a) (b)

Pion form factor, $|F_\pi(t)|$

BELLE data ($\tau^- \to \pi^- \pi^0 \nu_\tau$)

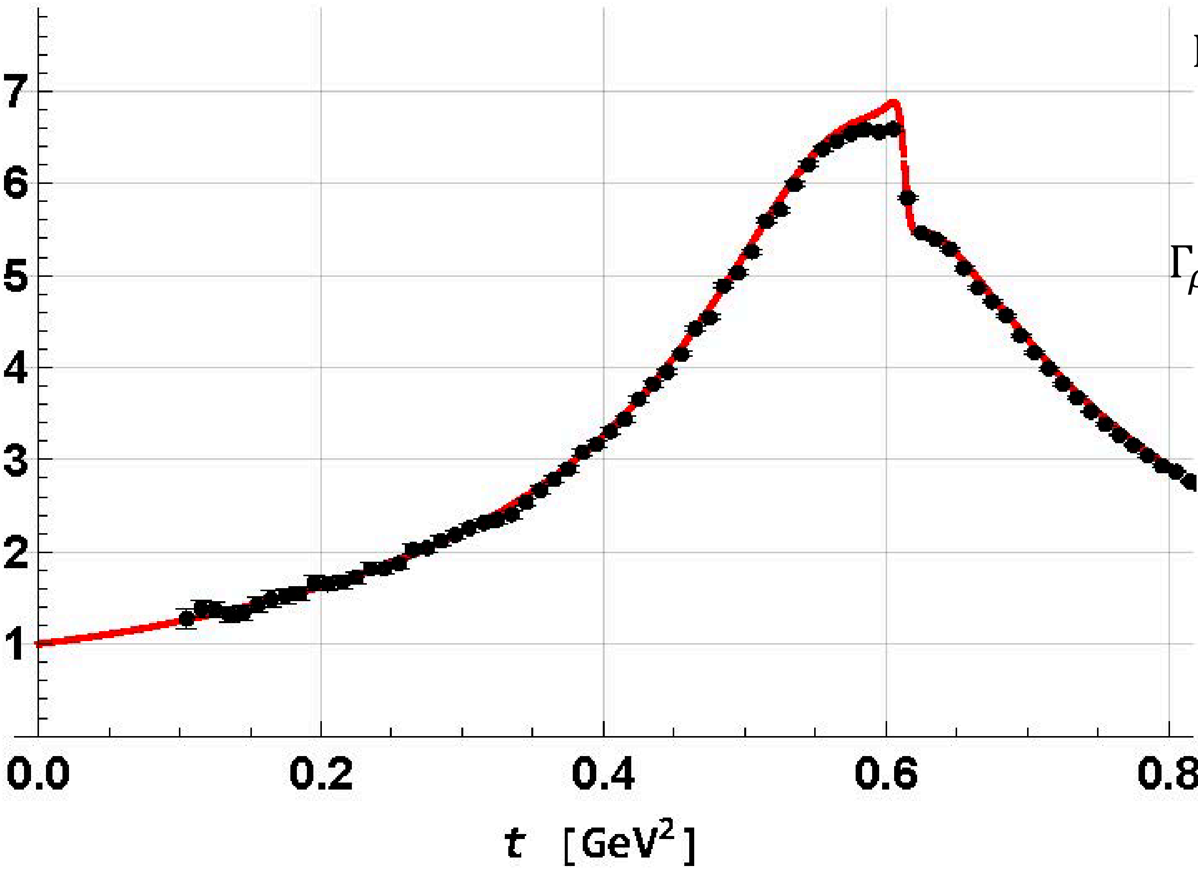


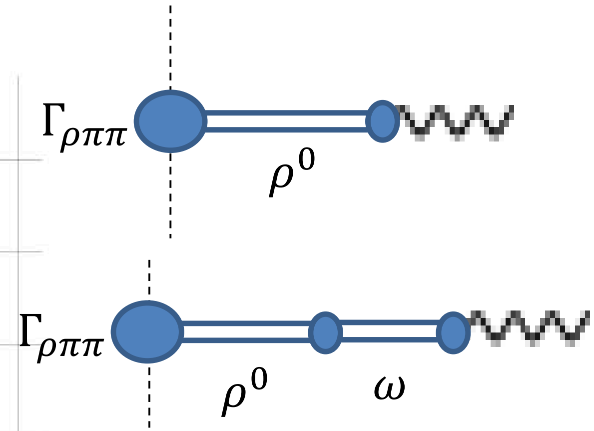


0 parameters !

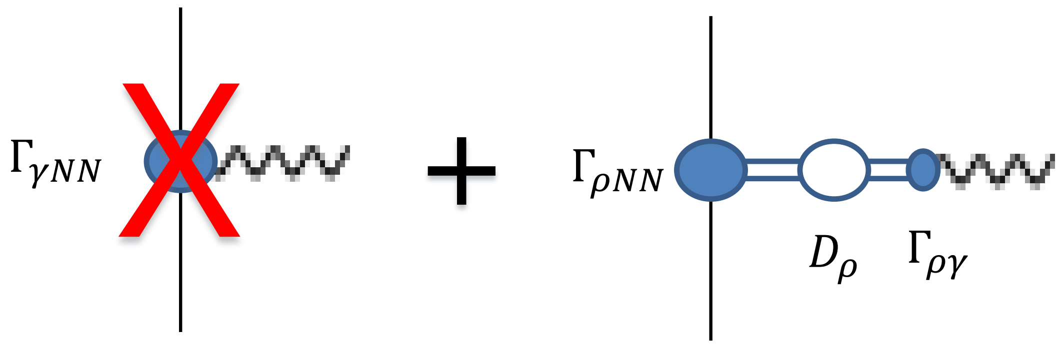
Pion form factor, $|F_\pi(t)|$

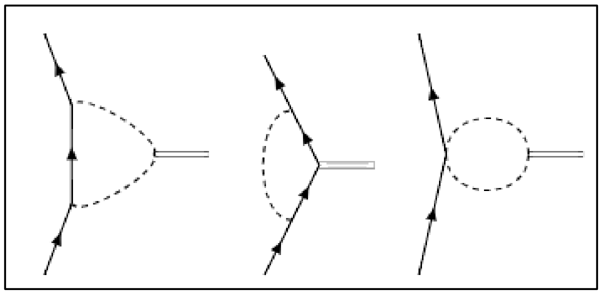
KLOE data ($e^+e^- \to \pi^+\pi^-$)

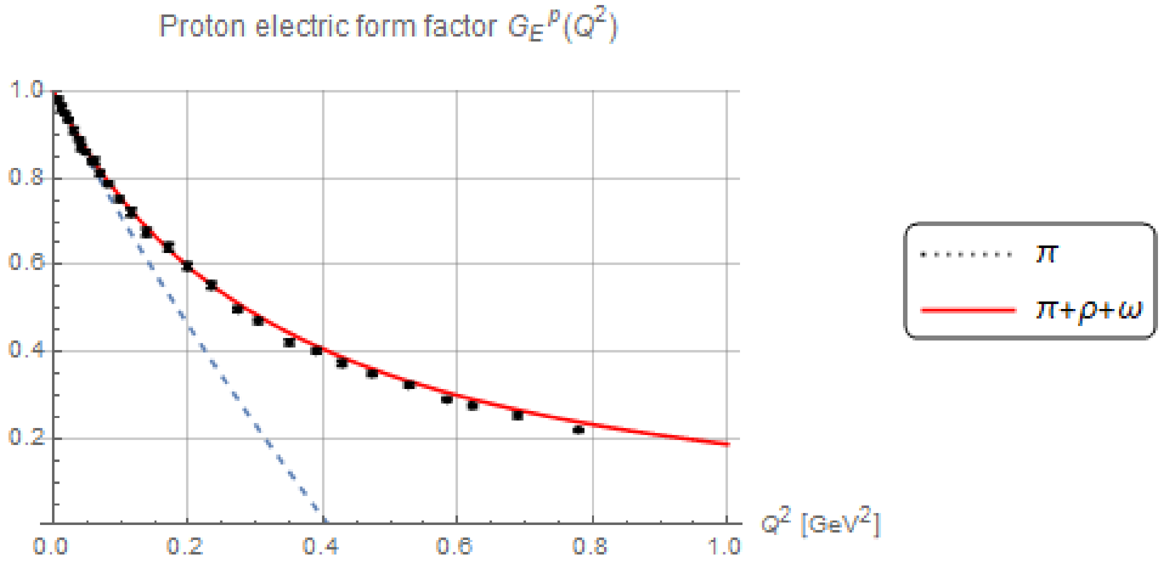




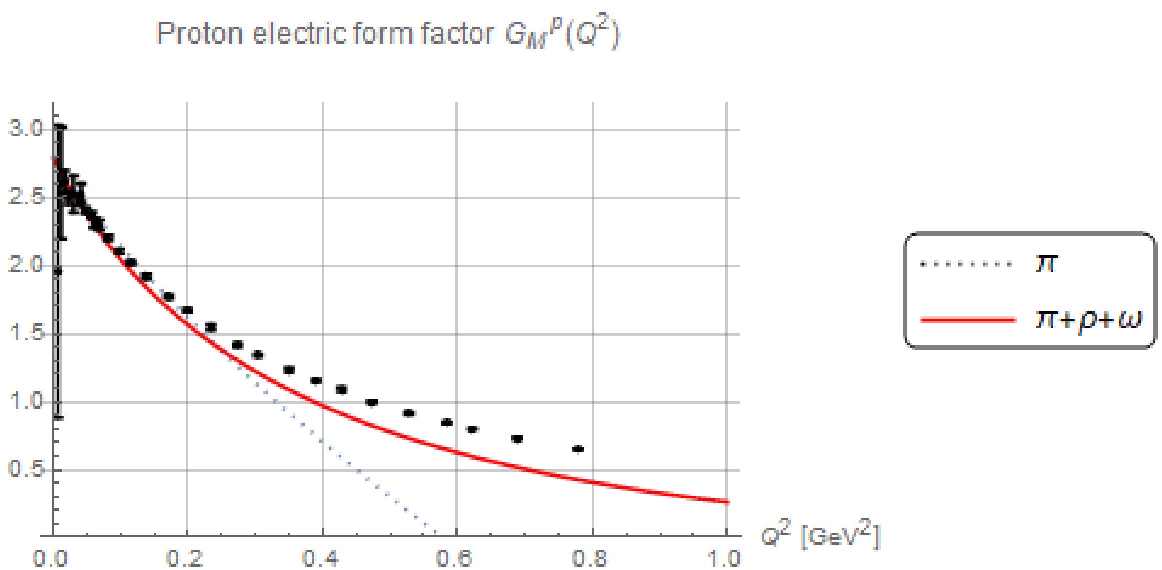
EM form factors of nucleon



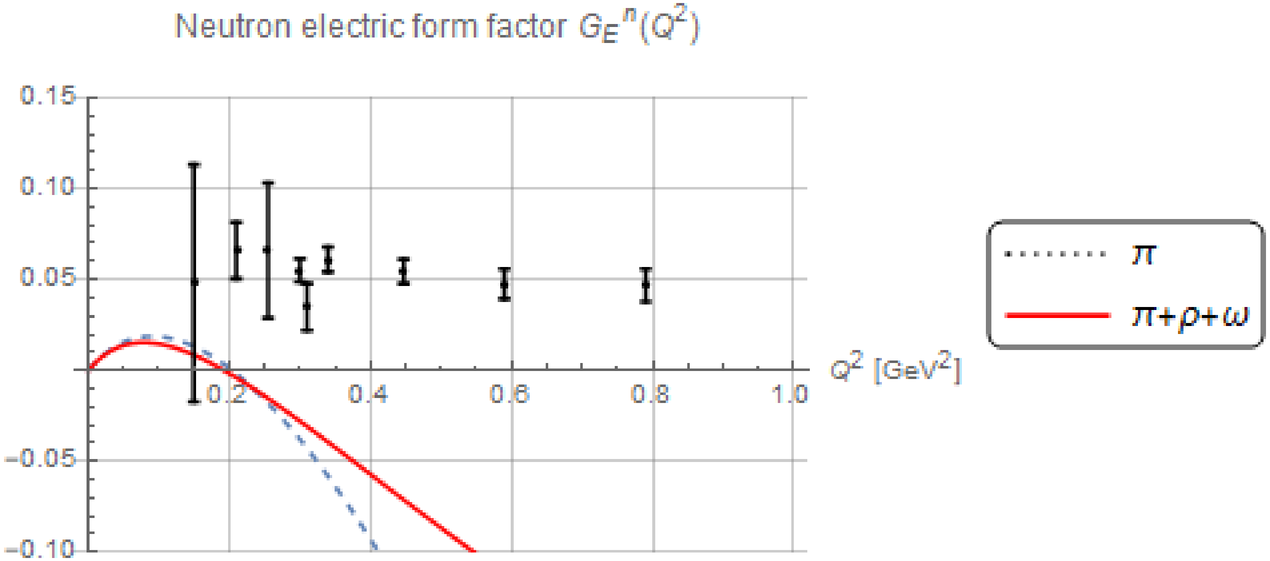


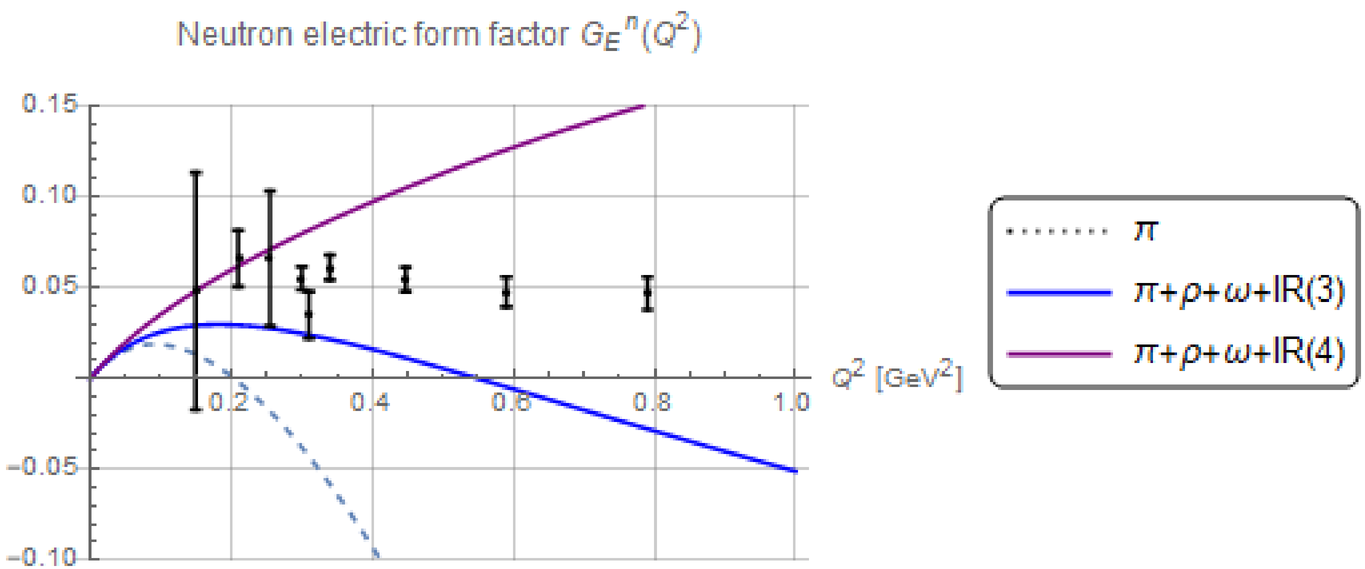


Proton electric form factor $G_E{}^p(Q^2)$
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
Q^2 [GeV^2]
π
$\pi+\rho+\omega$



Proton electric form factor $G_M{}^p(Q^2)$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
Q^2 [GeV^2]
π
$\pi+\rho+\omega$

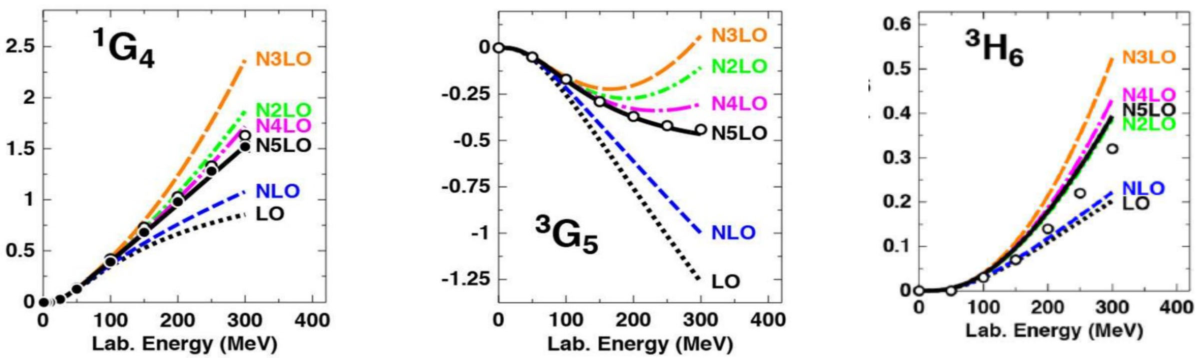




For more discussion on IR, see
Kubis & Meissner, NPA 679, 698
Schindler, Gegelia, Schrer, EPJA 26,1

Nuclear forces with ChPT ($\pi + N$)

- Quite impressive!
- Momentum cutoff is a bit too low: $\Lambda = (450 \sim 550)$ MeV
  - **Are we missing something at around ($\mathbf{450 \sim 550}$) MeV** ?
  - $\sigma + \rho + \omega$?
- Convergence (Largeness of the values of LECs are responsible, resonance saturation assumption)

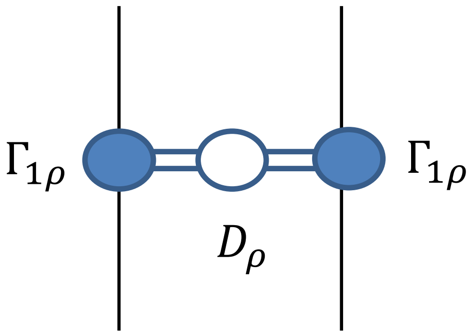


Entem, Machleidt, and Nosyk, Front. in Phys. 8, 57 (2020).

Nuclear forces

$$V_{NN}^{(\rho)} = \frac{\tau_1 \cdot \tau_2}{8 f_\pi^2} \frac{1}{1 - A_\rho(t) t / m_\rho^2} [\Gamma_{1\rho}(t)]^2$$

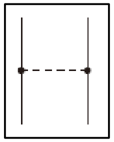
$$= \frac{\tau_1 \cdot \tau_2}{8 f_\pi^2} \left[1 - \frac{t}{m_\rho^2} - \frac{t}{f_\pi^2} \bar{f}_3(t)\right]^{-1} [\Gamma_{1\rho}(t)]^2$$



$$V_{NN}^{(\omega)} = \frac{g_\omega^2}{4 m_\omega^2} \left[1 - \frac{t}{m_\omega^2}\right]^{-1} [\Gamma_{1\omega}(t)]^2$$

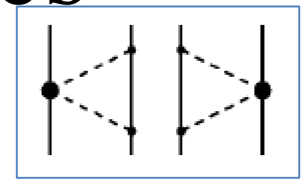
Long-range part of forces

- LO (Q^0)



  - Contact + 1-π-exchange
  - 1-ρ-exchange is just a contact term

- N^{2}LO (Q^3)

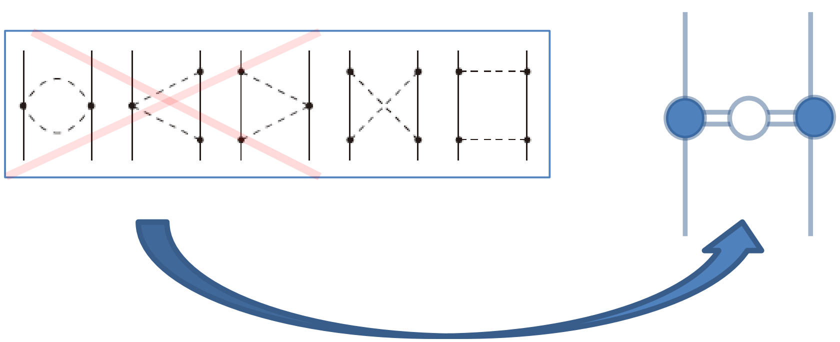


  - One-loop with NLO Lagr.

$$L_1 = \bar{N}\left[-\frac{2\,c_1}{f_\pi^2}\,m_\pi^2\,\vec{\pi}^2 + \cdots\right]N$$

  - Proportional to low-energy constants: $\{c_1, c_3, c_4\}$

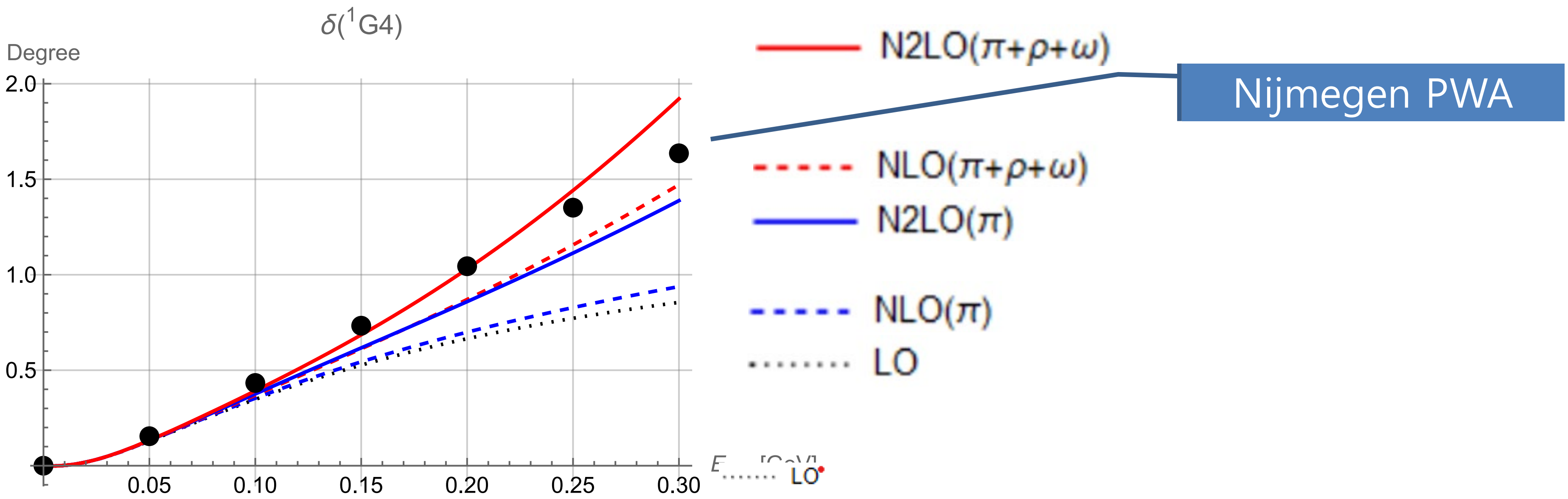
- NLO (Q^2)
  - One-loop with LO Lagr.
  - 1-ρ-exchange with loop corrections

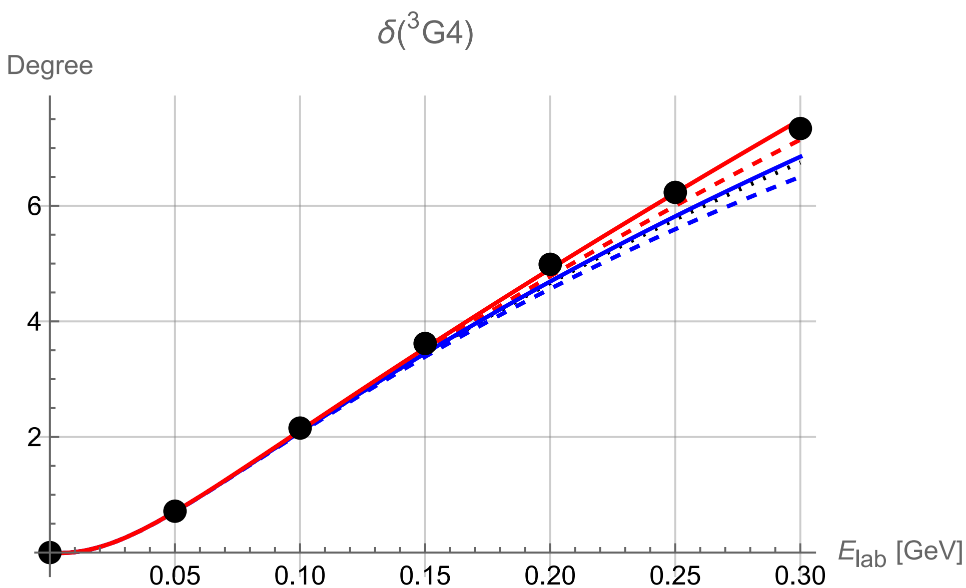


Results: peripheral NN scatterings

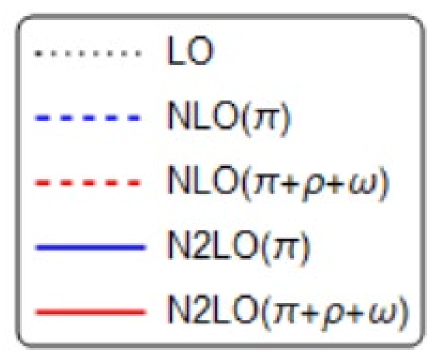
- For $L \geq 3$,
  - insensitive on low-energy constants for short-range physics
  - Interaction is weak, and the 1^{st} order Born approximation is okay (one-pion exchange is treated up to second order).

$J = 4$ phase shifts (preliminary)

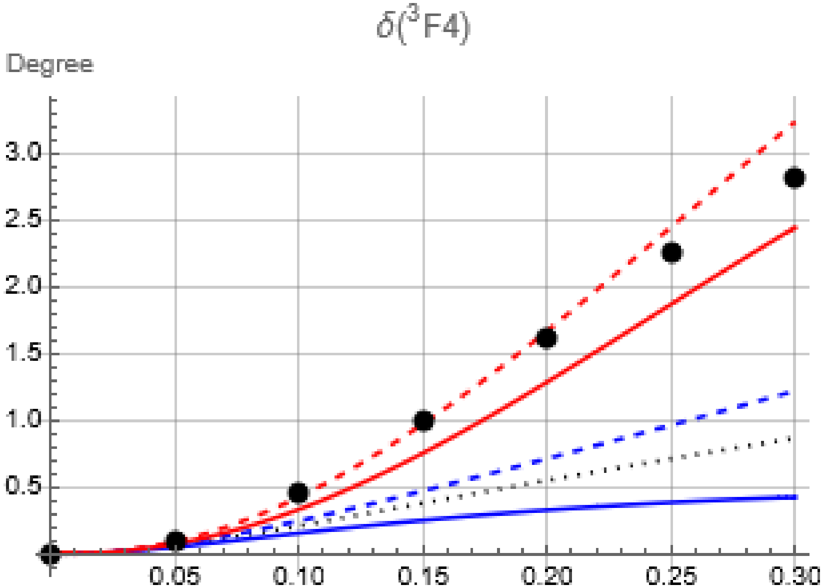


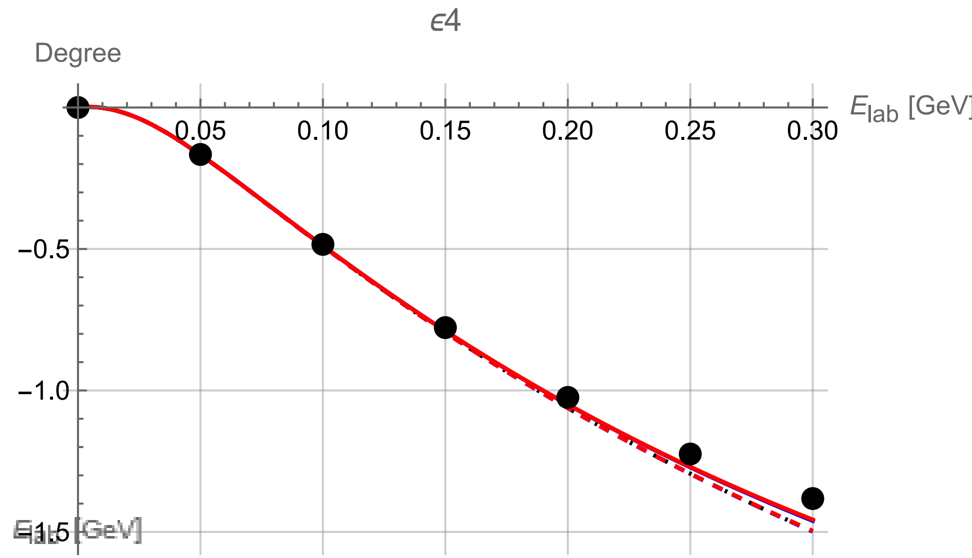


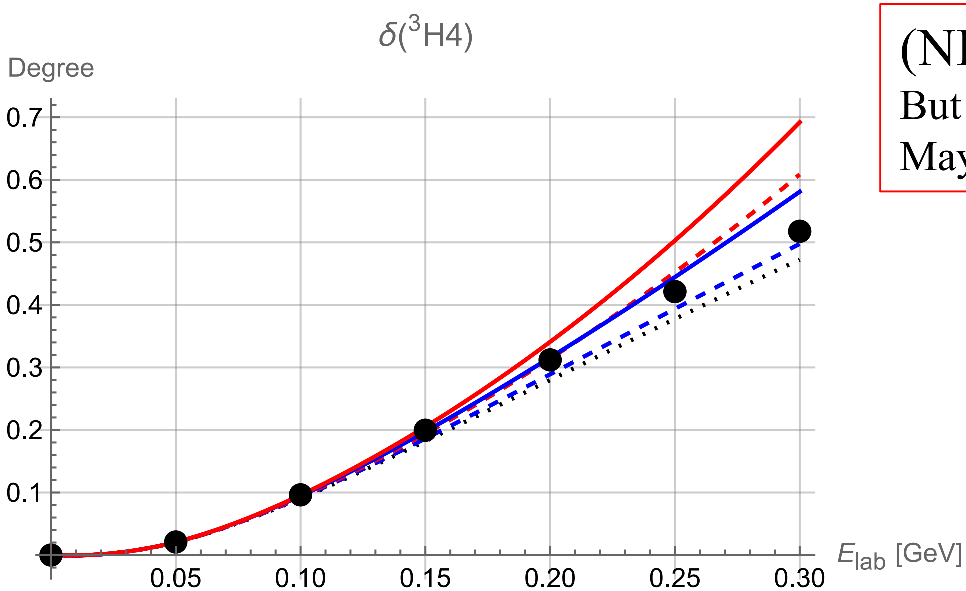
(NLO with VM) ≃ (N^2LO-ChPT) !



$J = 4$ phase shifts (cont'd, preliminary)



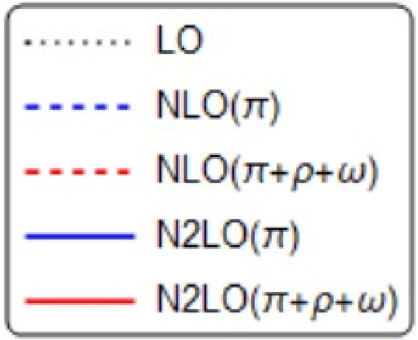




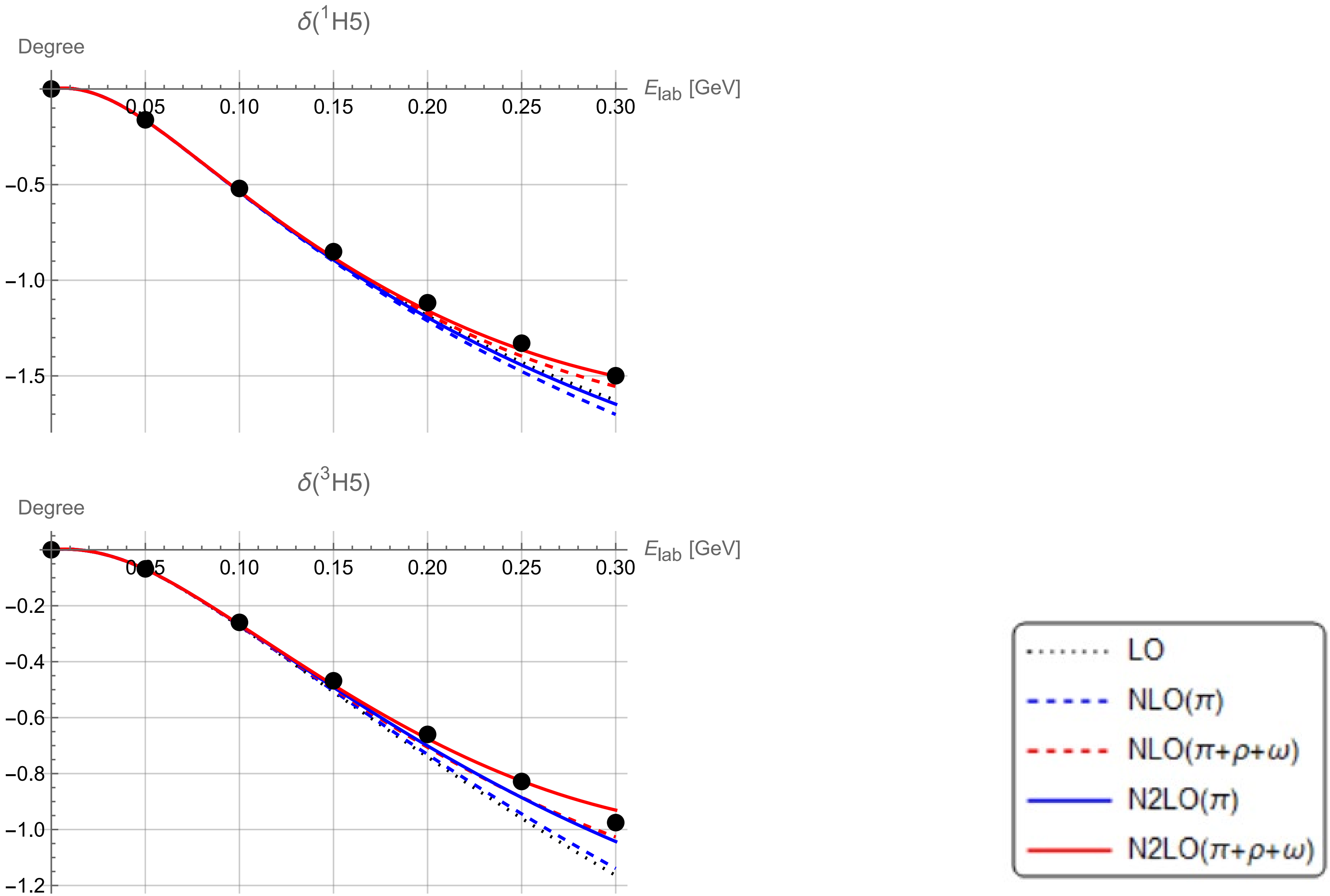
(NLO with VM) $\gg$ (N^{2}LO-ChPT),

But our N^{2}LO is not so impressive.

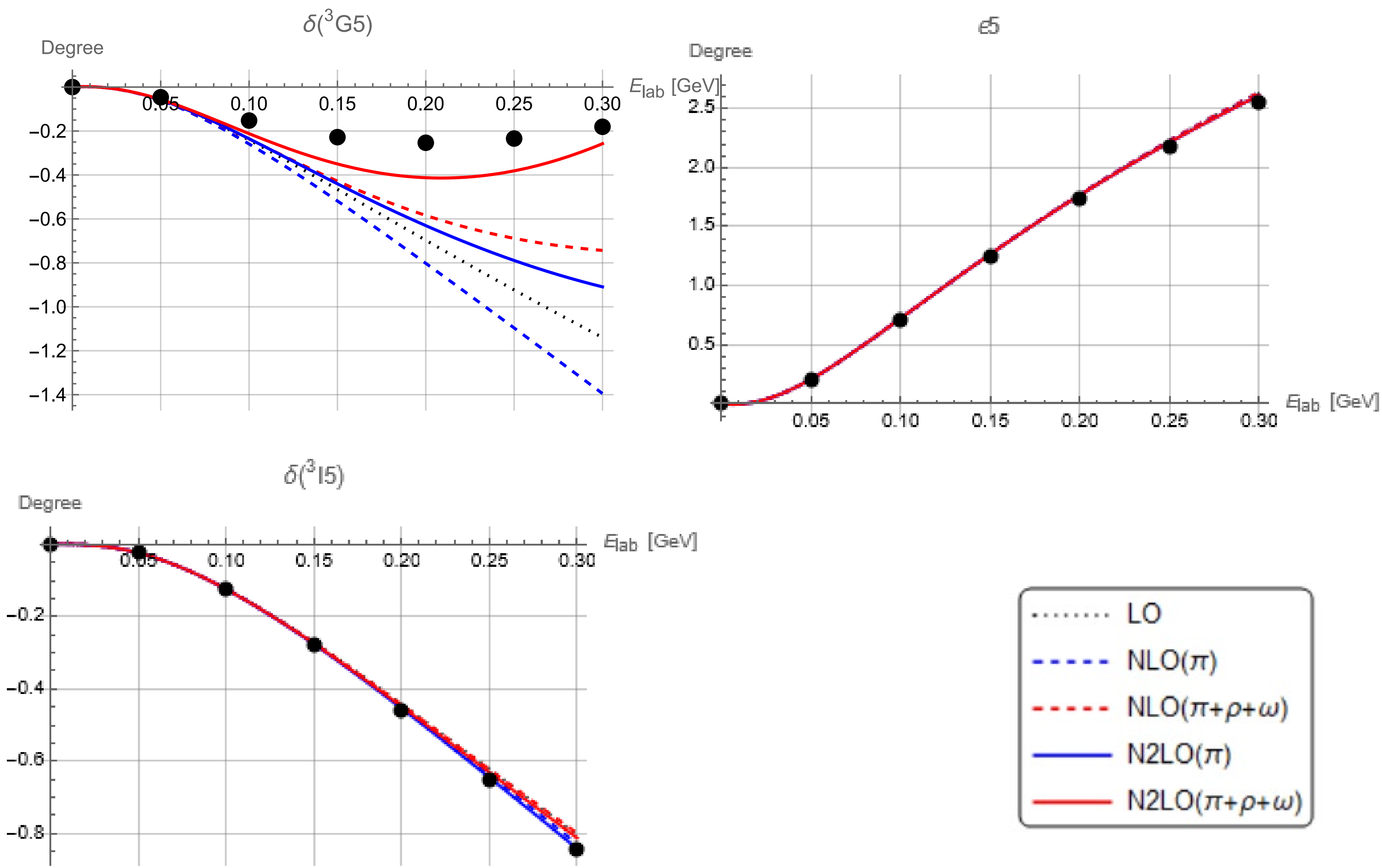
Maybe we need refit LECs with VM.



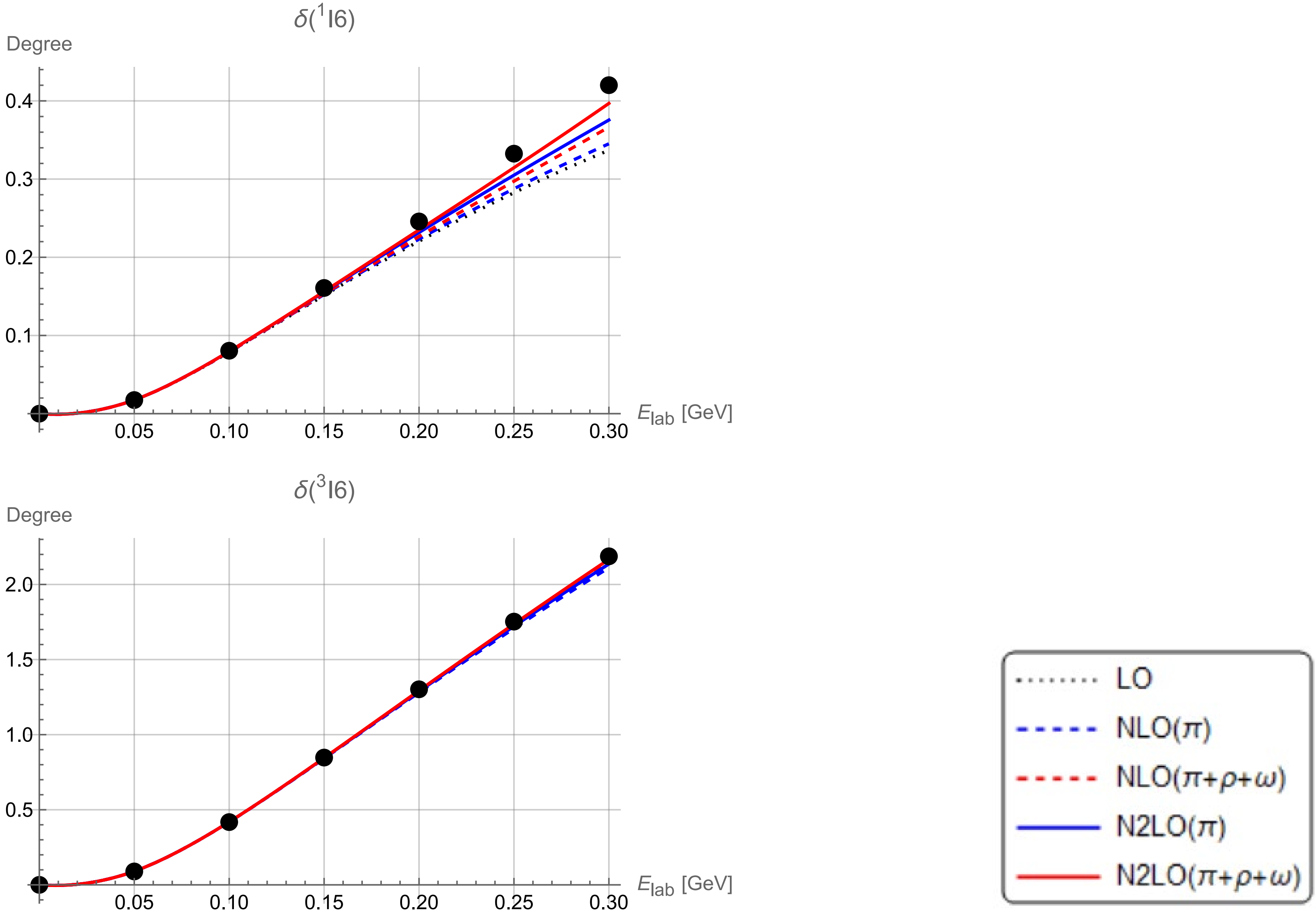
$J = 5$ phase shifts (preliminary)



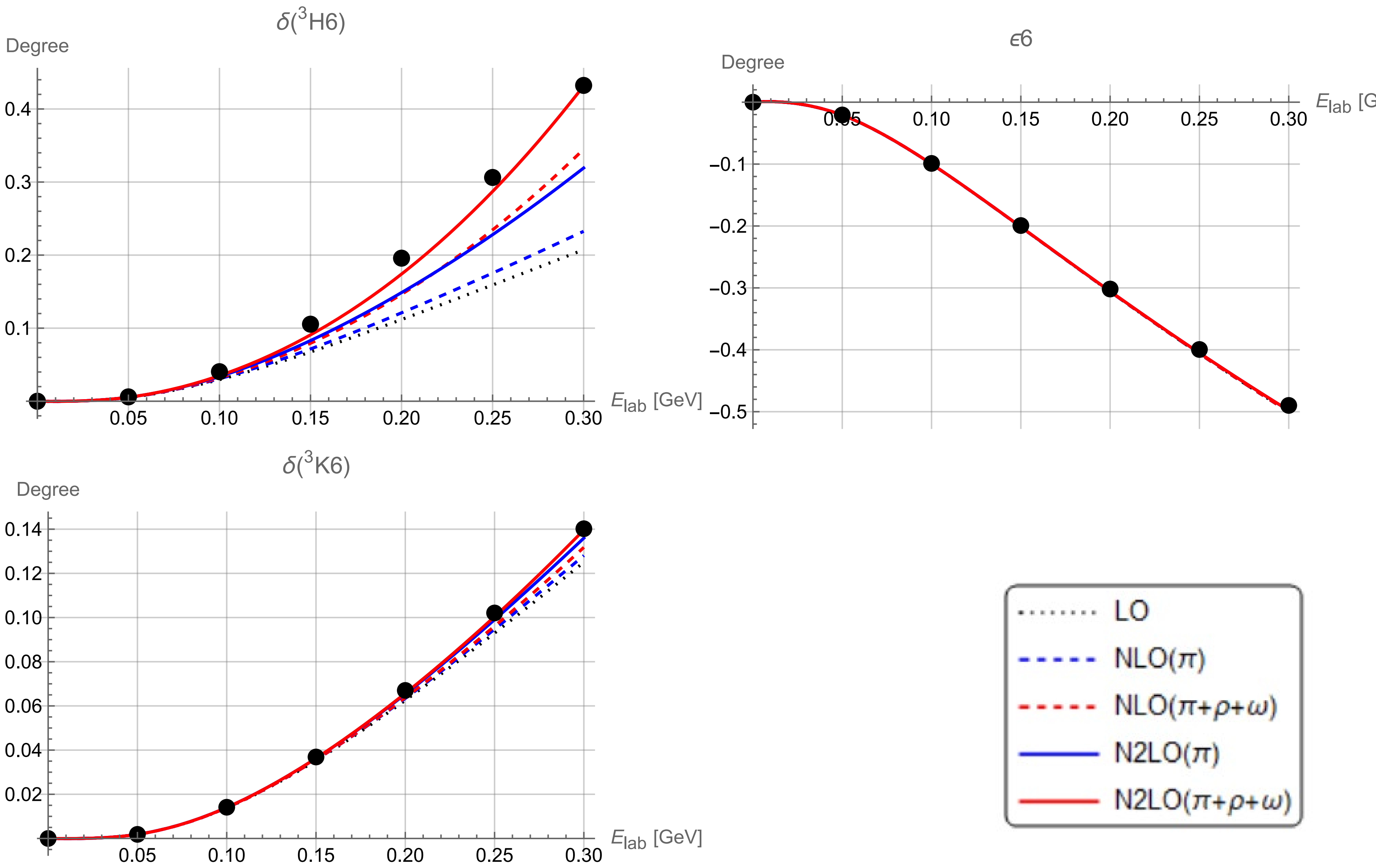
$J = 5$ phase shifts (cont'd, preliminary)



$J = 6$ phase shifts (preliminary)



$J = 6$ phase shifts (cont'd, preliminary)



Discussions

- ChPT with vector mesons has been developed.
  - Vector meson masses are treated as “heavy”, while treating their momenta as light.
  - A consistent 1-loop order renormalization could be done.
- With vector mesons, pion’s and proton’s EM form factor are described well
- Nuclear force up to N^2LO has been constructed
  - Our $N^2LO >$ Our NLO $\simeq$ ChPT N^2LO in terms of accuracy, for most peripheral partial waves
- Going beyond N^2LO is under progress

Thank you for your attention !