Nuclear structure effects in the Lamb shift of μD in pionless EFT

Vadim Lensky

VL, A. Hiller Blin, V. Pascalutsa, PRC 104, 054003 (2021)
VL, F. Hagelstein, V. Pascalutsa, PLB 835, 137500 (2022); EPJ A 58, 224 (2022)

EFB 2023
Mainz, July 31 – August 4, 2023

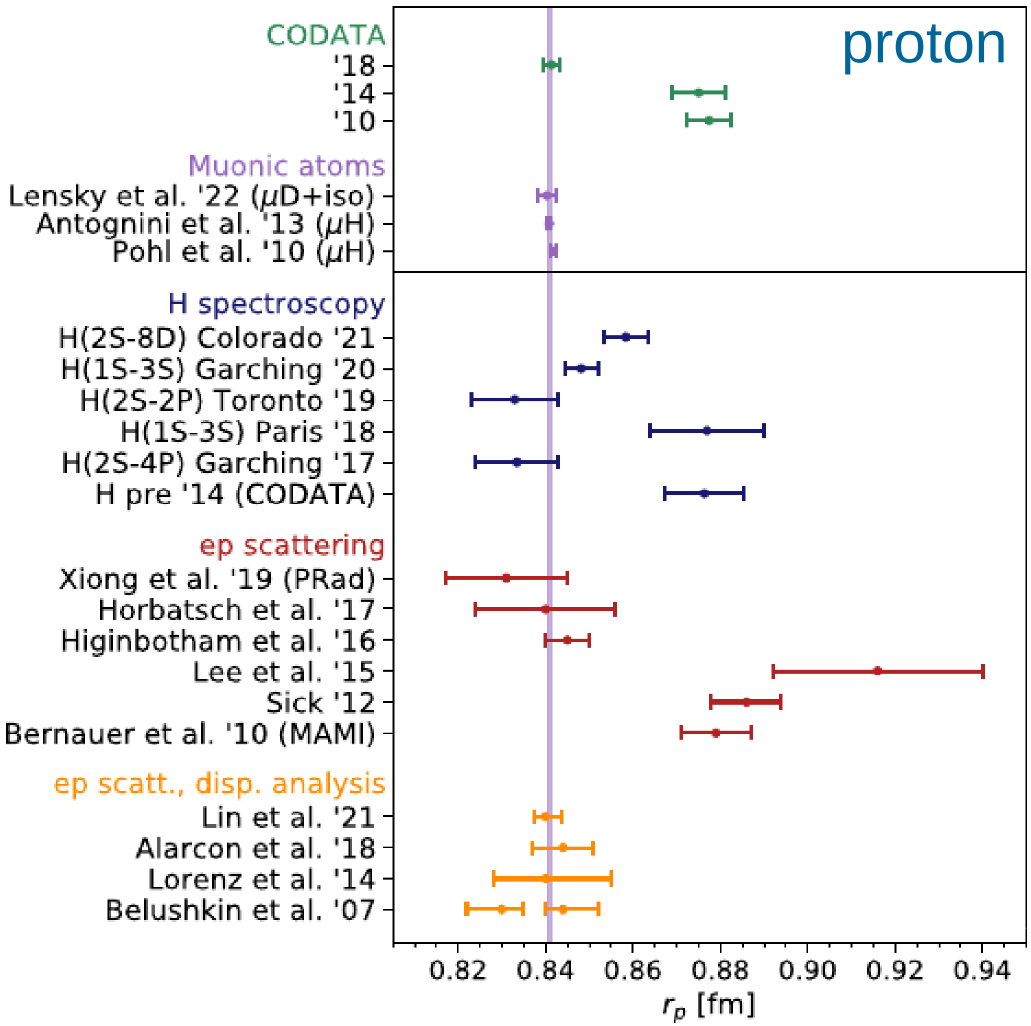
Proton and Deuteron Radii and Isotope Shift

A. Antognini's talk on Monday

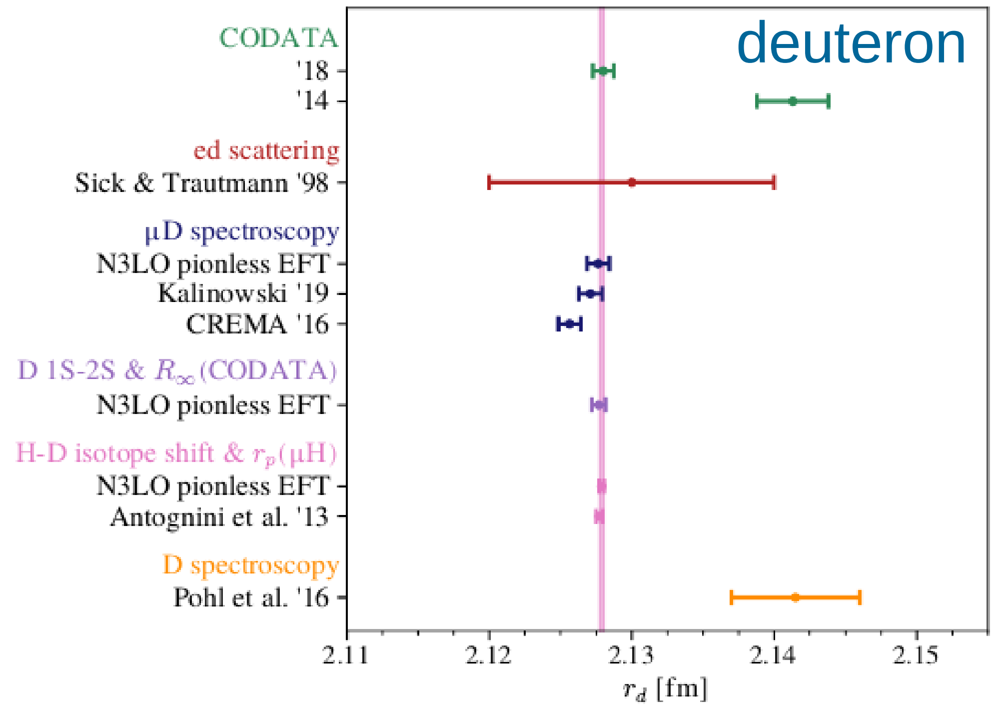
- H-D isotope shift: $E(H, 1S-2S) - E(D, 1S-2S)$

$$r_d^2 - r_p^2 = 3.820\,61(31)\ \mathrm{fm}^2$$

Jentschura et al. (2011)
VL, Hagelstein, Pascalutsa (2022)



Antognini, Hagelstein, Pascalutsa (2022)



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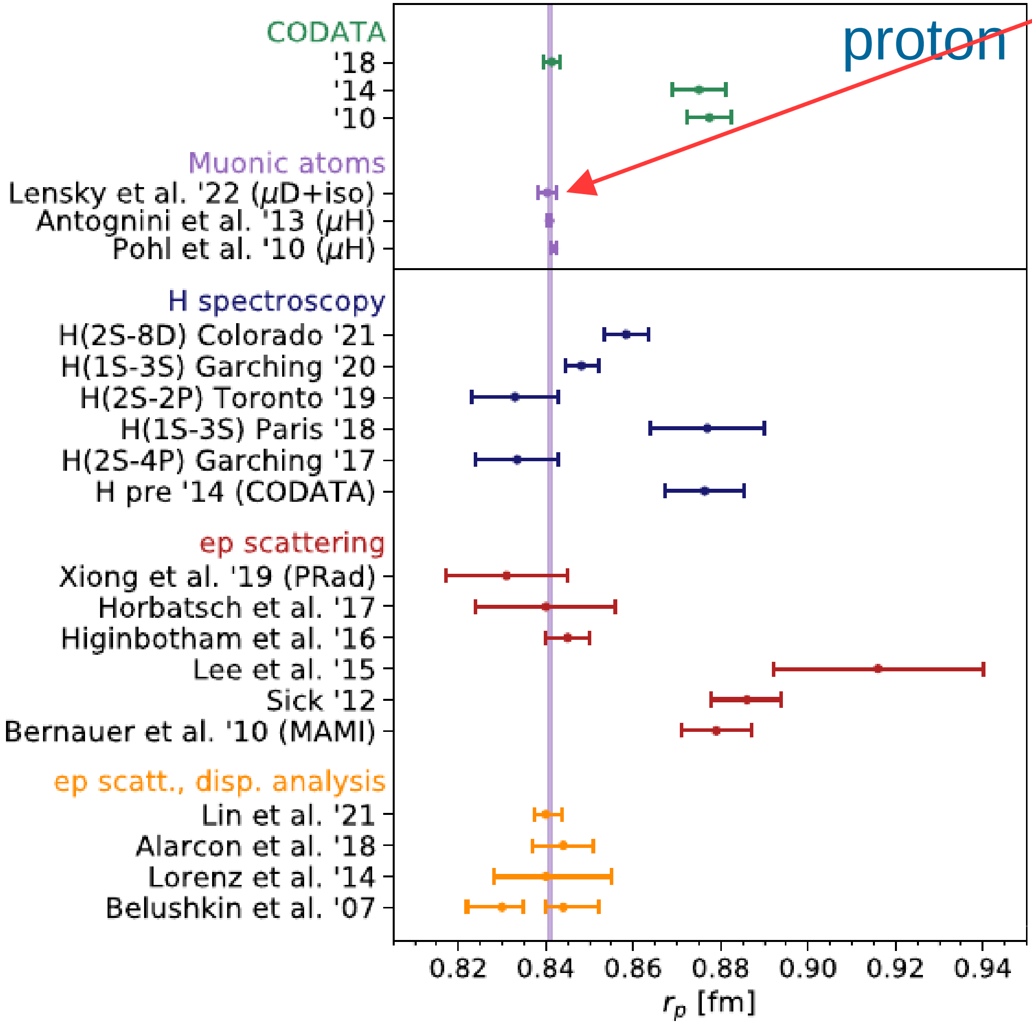

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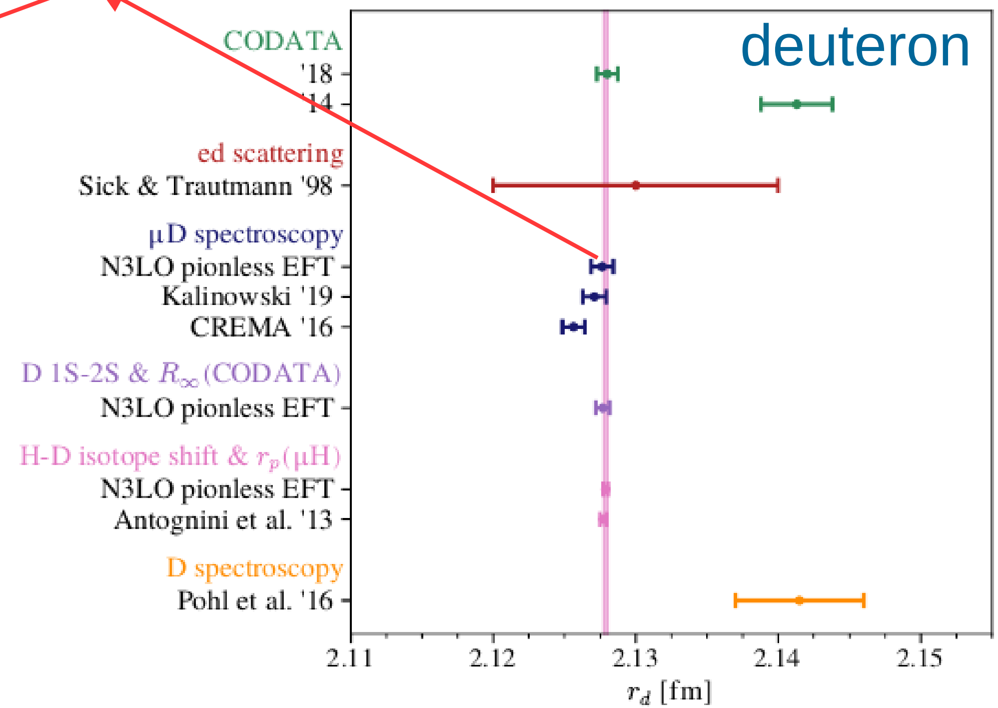


VL, Hagelstein, Pascalutsa (2022)

Two-Photon Exchange (TPE) in (Muonic) Atoms

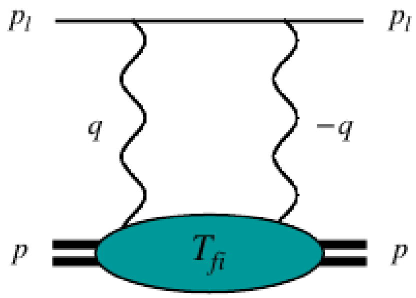
Bohr radius

$$a = (Z\alpha m_r)^{-1}$$

- Muonic atoms: greater sensitivity to charge radii
- But also greater sensitivity to subleading nuclear response

$$\propto \alpha^5$$

Lamb Shift: $$\Delta E_{nS} = \frac{2\pi Z\alpha}{3}\frac{1}{\pi(an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2}R_F^3\right] + \dots$$

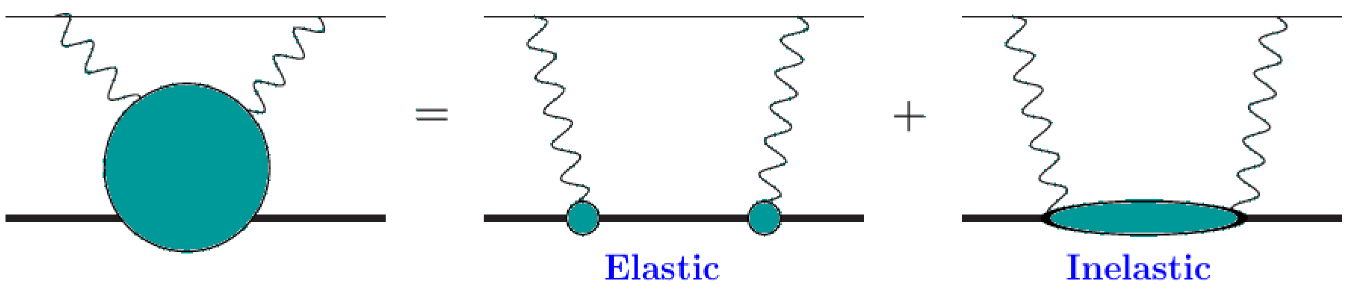


Friar radius: $$R_{\mathrm{F}}^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

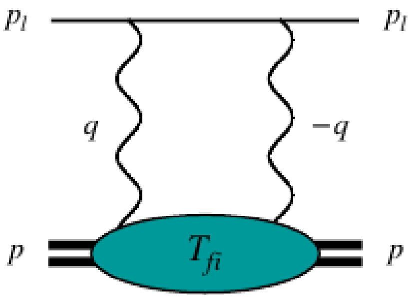
– (part of the) two-photon response

- Described in terms of (doubly virtual forward) Compton scattering: VVCS
- Elastic ($\nu = \pm Q^2/2M_{\text{target}}$, elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)

F. Hagelstein's talk today



VVCS



- Forward unpolarised VVCS amplitude

$$\alpha_{\rm em}\, M^{\mu\nu}(\nu,Q^2) = -\left\{\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right) T_2(\nu,Q^2)\right\}$$

$Q^2 = -q^2,\ \nu = p\cdot q/M_{\rm target}$ photon virtuality and lab frame energy

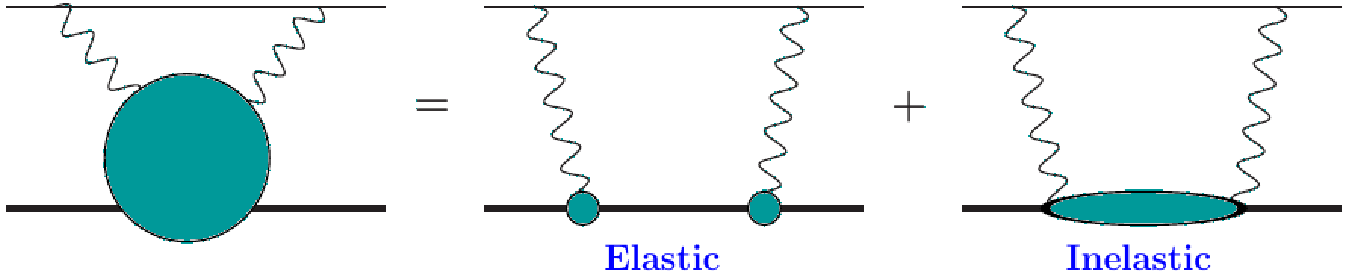
Lamb Shift: $$E^{2\gamma}_{nS} = -8i\pi\alpha m\,[\phi_n(0)]^2 \int \frac{d^4q}{(2\pi)^4}\frac{(Q^2-2\nu^2)\,T_1(\nu,Q^2) - (Q^2+\nu^2)\,T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

Point-like and finite size contributions need to be subtracted!

- Need to know both the elastic and inelastic parts of the amplitude

$$T_{1,2}(\nu,Q^2) = T^{\rm elastic}_{1,2}(\nu,Q^2) + T^{\rm inel}_{1,2}(\nu,Q^2)$$

F. Hagelstein's talk today



Pionless EFT for Deuteron VVCS

- Typical energies in (muonic) atoms are small: use effective field theories
  - pionless EFT for nuclear effects
  - expansion in powers of a small parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
  - order-by-order Bayesian uncertainty estimate

Talks of L. Contessi, M. Schäfer, M. Schindler, W. Dekens, E. Epelbaum, C. Ji, ...

- Counting: $p \sim q = O(\gamma),\ E \sim \nu = O(\gamma^2)$
- LO loops are resummed to reproduce the pole(s)
- z-parametrization

Phillips, Rupak, Savage (2000)

$$f(k) = \frac{1}{-\gamma - \mathrm{i}k + \frac{1}{2}\rho_d(k^2+\gamma^2) + w_2(k^2+\gamma^2)^2 + \dots}$$

$$= \frac{1}{(-\gamma - \mathrm{i}k)[\underbrace{1-\gamma\rho_d}_{1/Z} + \frac{1}{2}\rho_d(\mathrm{i}k+\gamma) + \dots]} = \frac{Z}{-\gamma-\mathrm{i}k}\left[1 - \frac{Z}{2}\rho_d(\mathrm{i}k+\gamma) + \dots\right]$$

- reproduces the residue at NLO, better convergence for low-energy observables

- We go beyond strict pionless and use χEFT/data driven DR to estimate higher-order individual nucleon contributions

Counting for VVCS and TPE

- Longitudinal and Transverse amplitudes

$$f_L(\nu, Q^2) = -T_1(\nu, Q^2) + \left(1 + \frac{\nu^2}{Q^2}\right) T_2(\nu, Q^2), \qquad f_T(\nu, Q^2) = T_1(\nu, Q^2)$$

Lamb Shift:

$$\Delta E_{nl} = -8i\pi m \, [\phi_{nl}(0)]^2 \int \frac{d^4q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

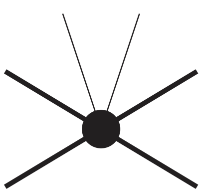
$$f_L = O(p^{-2}), \qquad f_T = O(p^0)$$ in the VVCS amplitude

$\alpha_{E1} = 0.64 \text{ fm}^3$

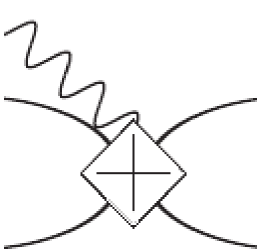
$\beta_{M1} = 0.07 \text{ fm}^3$

$$\text{longitudinal} = O(p^{-2}), \qquad \text{transverse} = O(p^2)$$ in TPE

- Transverse contribution to TPE starts only at N4LO



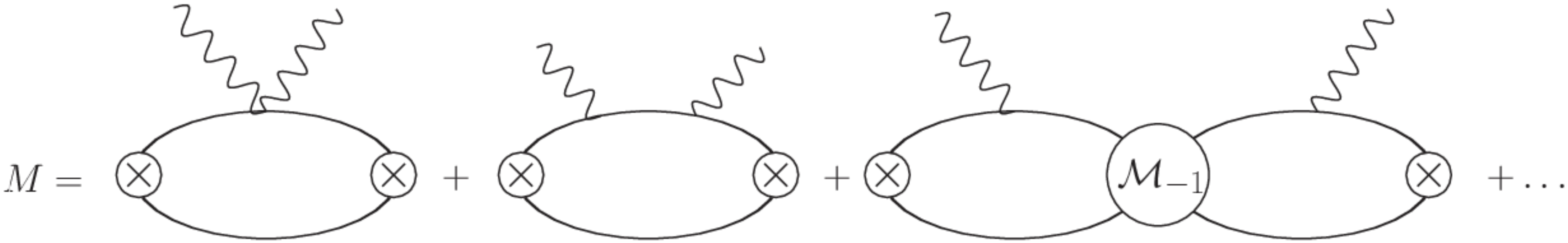
- N4LO: ΔE_{nl} needs to be regularised, an unknown lepton-NN LEC
- We go up to N3LO in f_L, and up to (relative) NLO in f_T [cross check]
- One unknown LEC at N3LO in f_L
  - important for the charge form factor
  - extracted from the H-D isotope shift and proton R_E



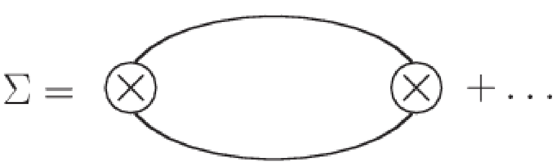
In Practice: Amplitude with Deuterons

- The reaction amplitude is given by the LSZ reduction

$$T = M \left[\frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E}\bigg|_{E=E_d} \right]^{-1}$$



– irreducible VVCS graphs (here full LO for f_L; crossed not shown)



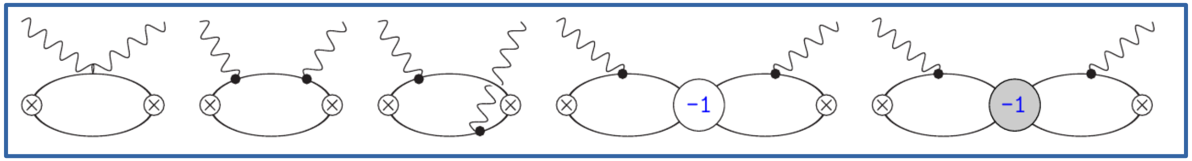
– deuteron self-energy (here at LO)

- The expression for the residue is very simple up to N3LO:

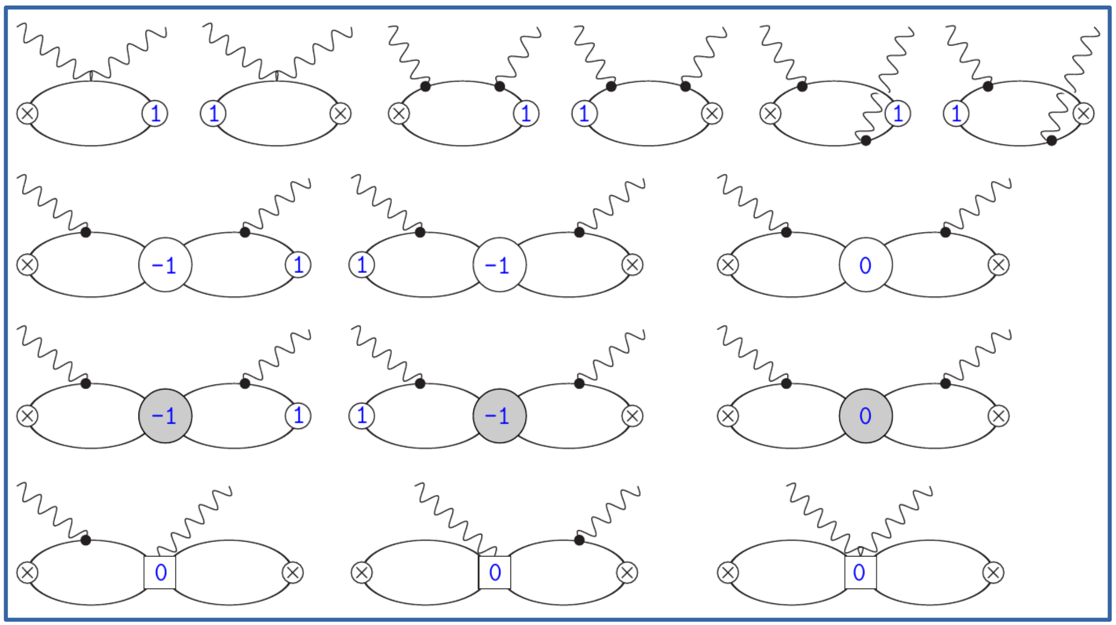
$$\left[\frac{\mathrm{d}\Sigma(E)}{\mathrm{d}E}\bigg|_{E=E_d} \right]^{-1} = \frac{8\pi\gamma}{M^2}\left[1 + (Z-1) + 0 + 0 + \ldots\right]$$

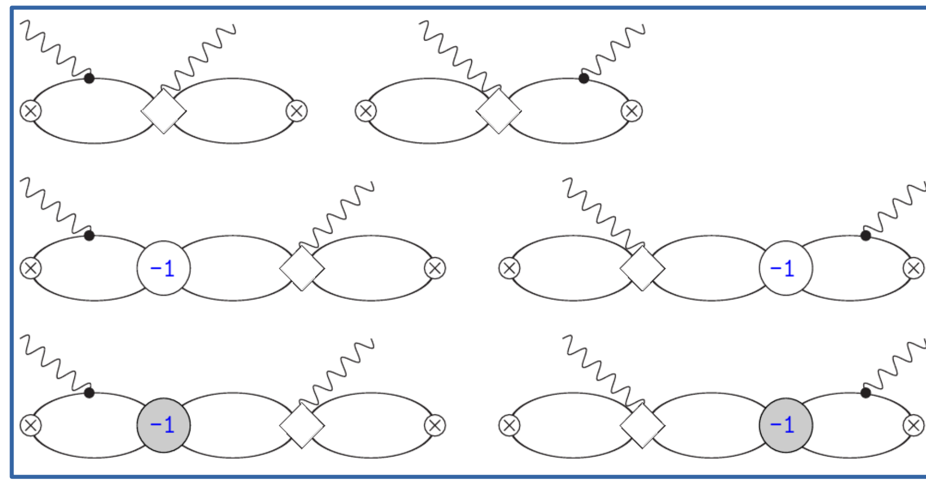
Deuteron VVCS: Feynman Graphs

LO



NLO

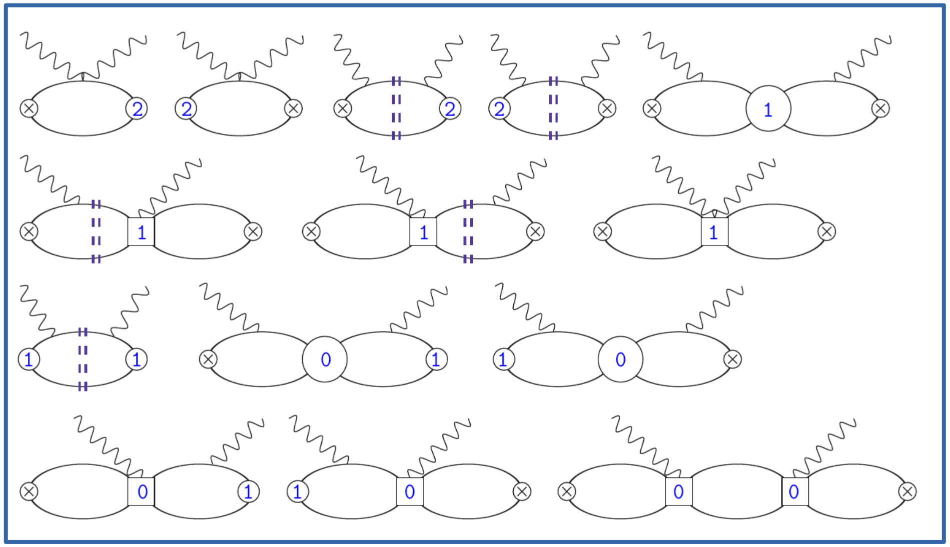


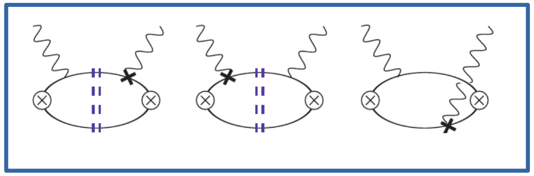


- Amplitudes are calculated analytically (dimreg+PDS) Kaplan, Savage, Wise (1998)
- Checks:
  - the sum of each subgroup (+ respective crossed graphs) is gauge invariant
  - regularisation scale dependence has to vanish

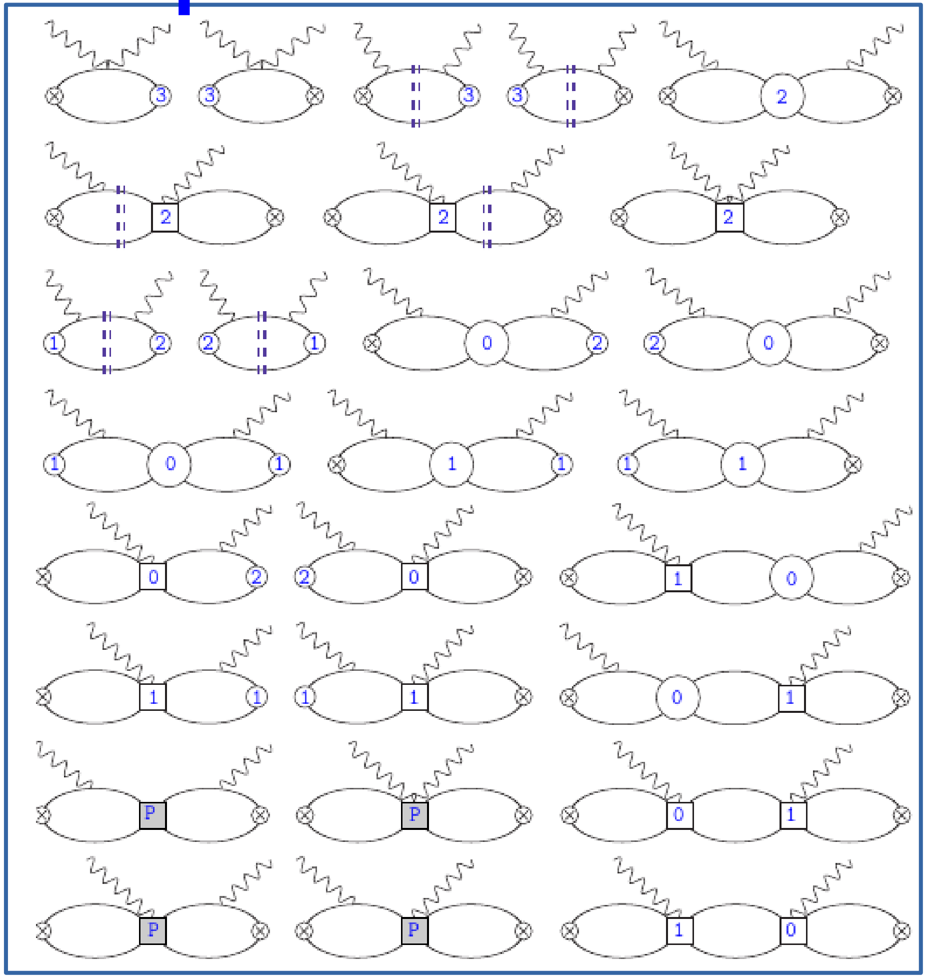
Deuteron VVCS: Feynman Graphs

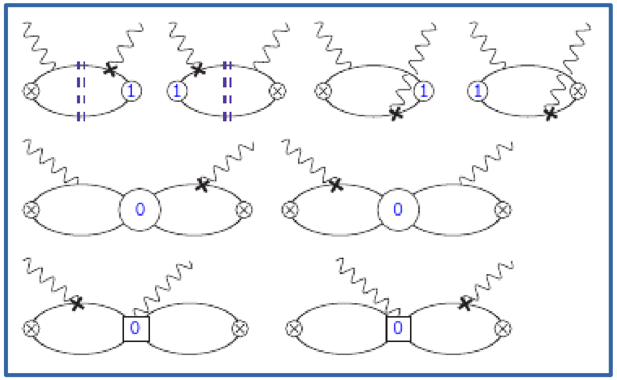
NNLO

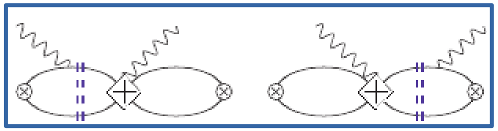




N3LO



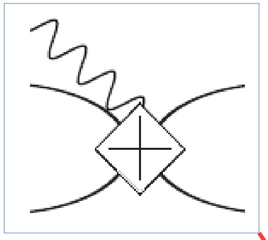


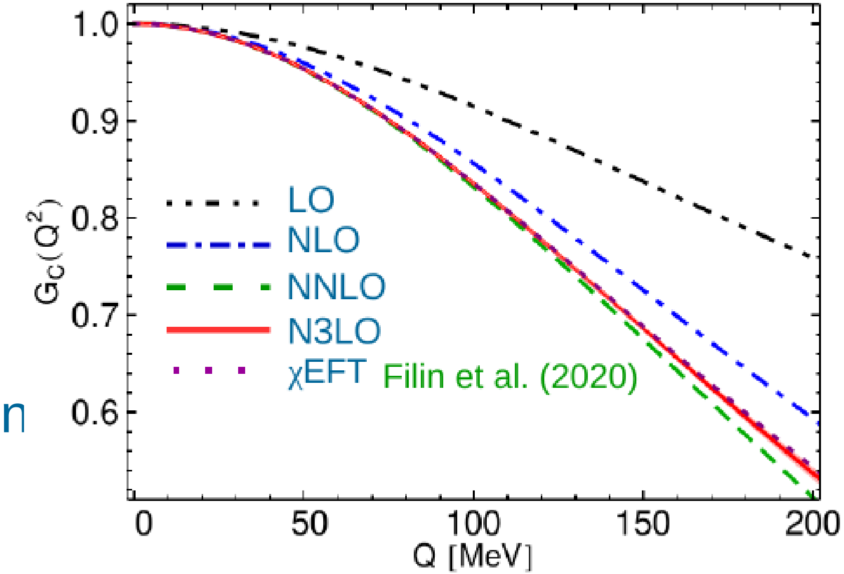


Deuteron Charge Form Factor and TPE in μD

- The deuteron charge form factor obtained from the residue of the VVCS amplitude
- The result is consistent with **χ**EFT
- Correlation between the charge and Friar radii; can be used to test FF parametrisation
- Generated by the N3LO LEC

$$R_{\mathrm{F}}^3 = \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[G_C^2(Q^2) - 1 - 2G_C'(0)\, Q^2 \right]$$





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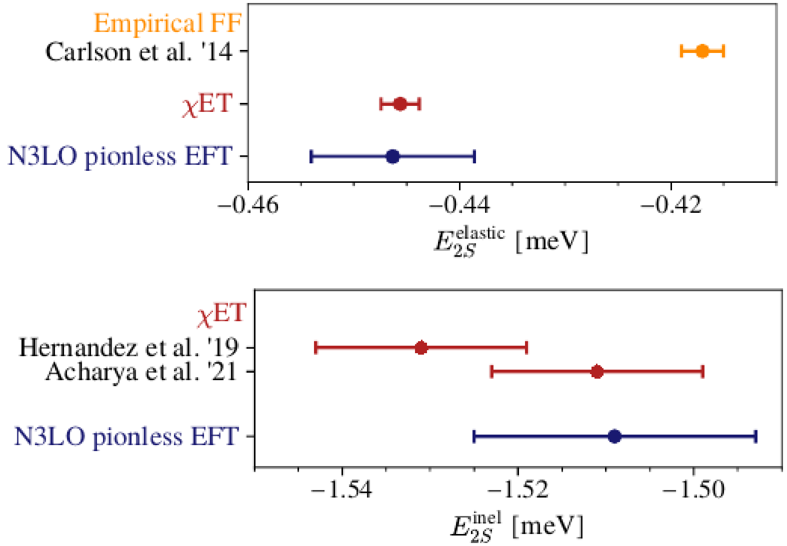

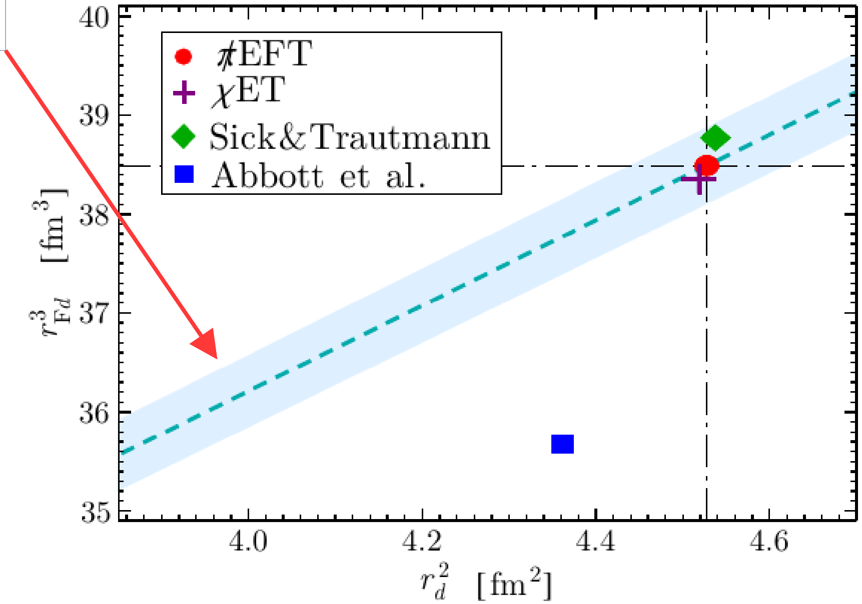

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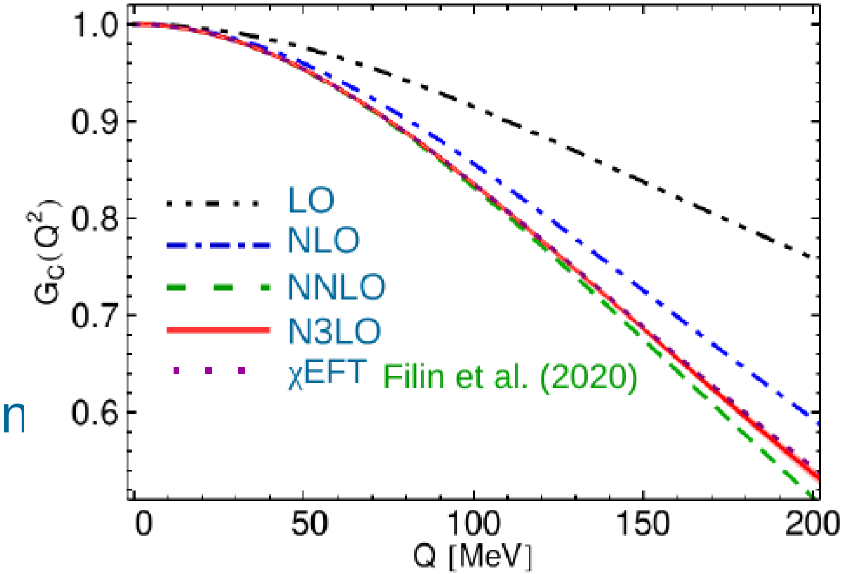

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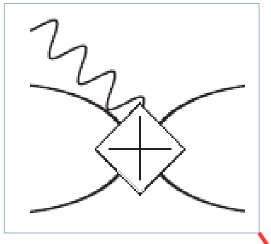
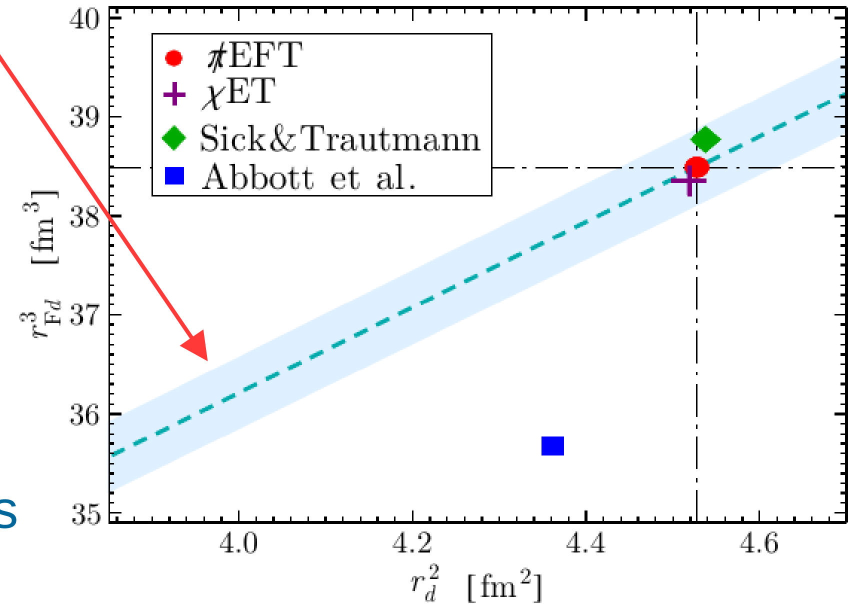


$$R_F^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

$$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$$

- Some parametrisations fail to describe the low-Q properties
- Agreement with χEFT vindicates both EFTs



VL, Hagelstein, Pascalutsa (2022)

TPE in μD: Higher-Order Corrections

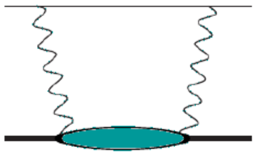
$\Delta E_{2S} = \Delta E_{2S}^{\mathrm{elastic}} + \Delta E_{2S}^{\mathrm{inel}} = -1.955(19)\mathrm{meV}$

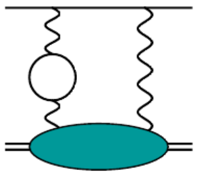
- Higher-order in α terms are important in D
  - Coulomb $\left[\mathcal{O}(\alpha^6 \log \alpha)\right]$

    taken from elsewhere $\Delta E_{2S}^{\mathrm{Coulomb}} = 0.2625(15)\ \mathrm{meV}$
  - eVP $\left[\mathcal{O}(\alpha^6)\right]$ Kalinowski (2019)

    reproduced in pionless EFT $\Delta E_{2S}^{\mathrm{eVP}} = -0.027\ \mathrm{meV}$

non-forward





- Single-nucleon terms at N4LO in pionless EFT and higher
  - insert empirical FFs in the LO+NLO VVCS amplitude
  - polarisability contribution (inelastic+subtraction)
    - inelastic: *ed* scattering data Carlson, Gorchtein, Vanderhaeghen (2013)
    - subtraction: nucleon subtraction function from χEFT

      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\mathrm{hadr}} = -0.032(6)\ \mathrm{meV}$

TPE in μD: Higher-Order Corrections

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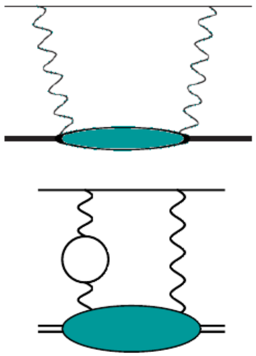

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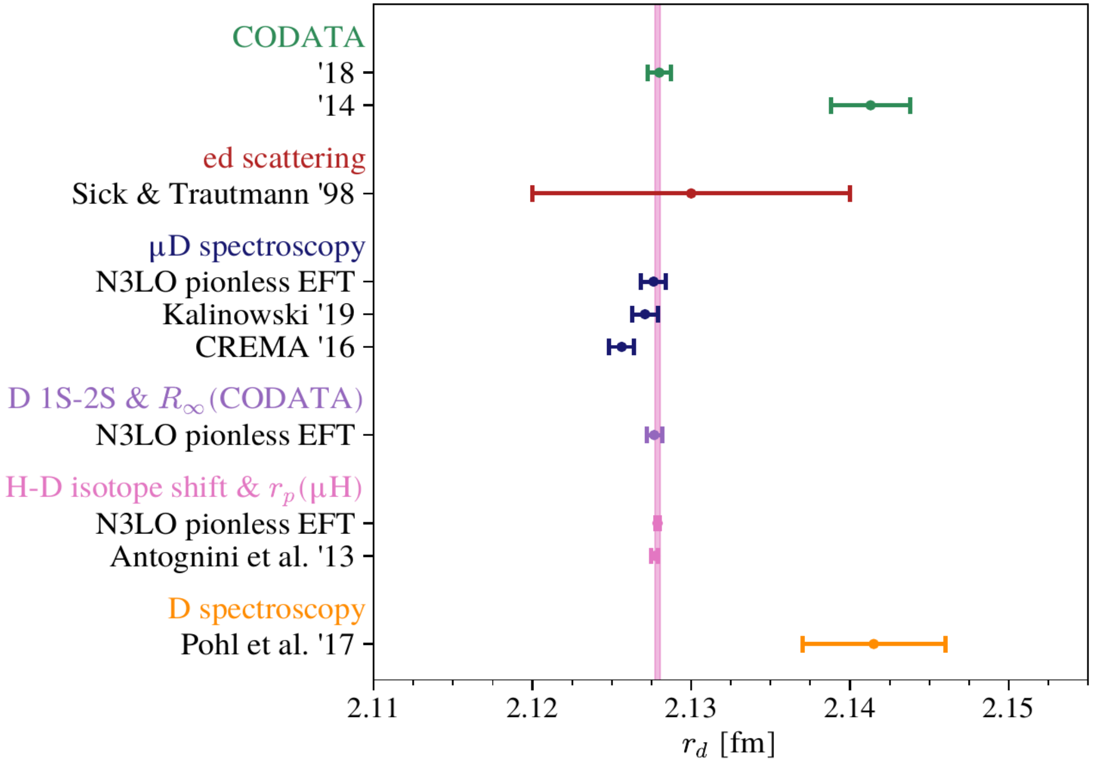
$$\Delta E_{2S}^{2\gamma} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} + \Delta E_{2S}^{\text{hadr}} + \Delta E_{2S}^{\text{eVP}} + \Delta E_{2S}^{\text{Coulomb}} = -1.752(20) \text{ meV}$$

Deuteron Charge Radius and TPE in μD

- Reassessed with pionless EFT
- μD, D, and H-D isotope shift
  all consistent with one another
- Agreement with the very precise
  empirical value of 2γ exchange

	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	−1.7096(200)
Kalinowski '19 [6, Eq. (6) + (19)]	−1.740(21)
$\not\pi$EFT (this work)	−1.752(20)
Empirical (μH + iso)	
Pohl et al. '16 [3]	−1.7638(68)
This work	−1.7585(56)



VL, Hagelstein, Pascalutsa (2022)

- Agreement between μD and μH+iso: small proton radius
- Experimental precision presents a challenge for theory

Summary and Outlook

- μD and H-D isotope shift in pionless EFT consistent with each other
  - small proton radius
- Agreement with the very precise empirical value of 2γ exchange
  - experimental precision presents a challenge for theory
- Correlation between charge and Friar radius
  - a criterion to check form factor parametrizations
- Single-nucleon effects are starting to be sizeable
  - more importaint in heavier nuclei

- HFS in μD: work in progress, more difficult (cancellations!)
- 3H, 3He: can pionless EFT shed light on discrepancies?

Thank You for Your Attention!

Slighly More Details on Pionless EFT

- Nucleons are non-relativistic → $E \simeq p^2/M = O(p^2)$
- Loop integrals $dE\, d^3p = O(p^5)$
- Nucleon propagators $(E - p^2/2M)^{-1} = O(p^{-2})$
- Typical momenta $p \sim \gamma = \sqrt{ME_d} \simeq 45$ MeV
- Expansion parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
- *NN* system has a low-lying bound/virtual state → enhance S-wave coupling constants, resum the LO *NN* S-wave scattering amplitude

- Easier to solve than χEFT (analytic results for *NN*)
- Easier to analyse (e.g., discover correlations between various quantities)
- Explicit gauge invariance and renormalisability
- Slower convergence (~larger uncertainty) and (potentially) a narrower range of applicability than χEFT

More Details on the Counting for VVCS and TPE

$$\Delta E_{nl} = -8i\pi m\, [\phi_{nl}(0)]^2 \int \frac{d^4q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

- Transverse contribution starts at N4LO in TPE

$$\alpha_{E1} = 0.64\ \text{fm}^3$$

$$\beta_{M1} = 0.07\ \text{fm}^3$$

$$f_L(\nu, Q^2) = 4\pi\alpha_{E1} Q^2 + \ldots$$

$$f_T(\nu, Q^2) = -\frac{e^2}{M_d} + 4\pi\beta_{M1} Q^2 + 4\pi(\alpha_{E1} + \beta_{M1})\nu^2 + \ldots$$

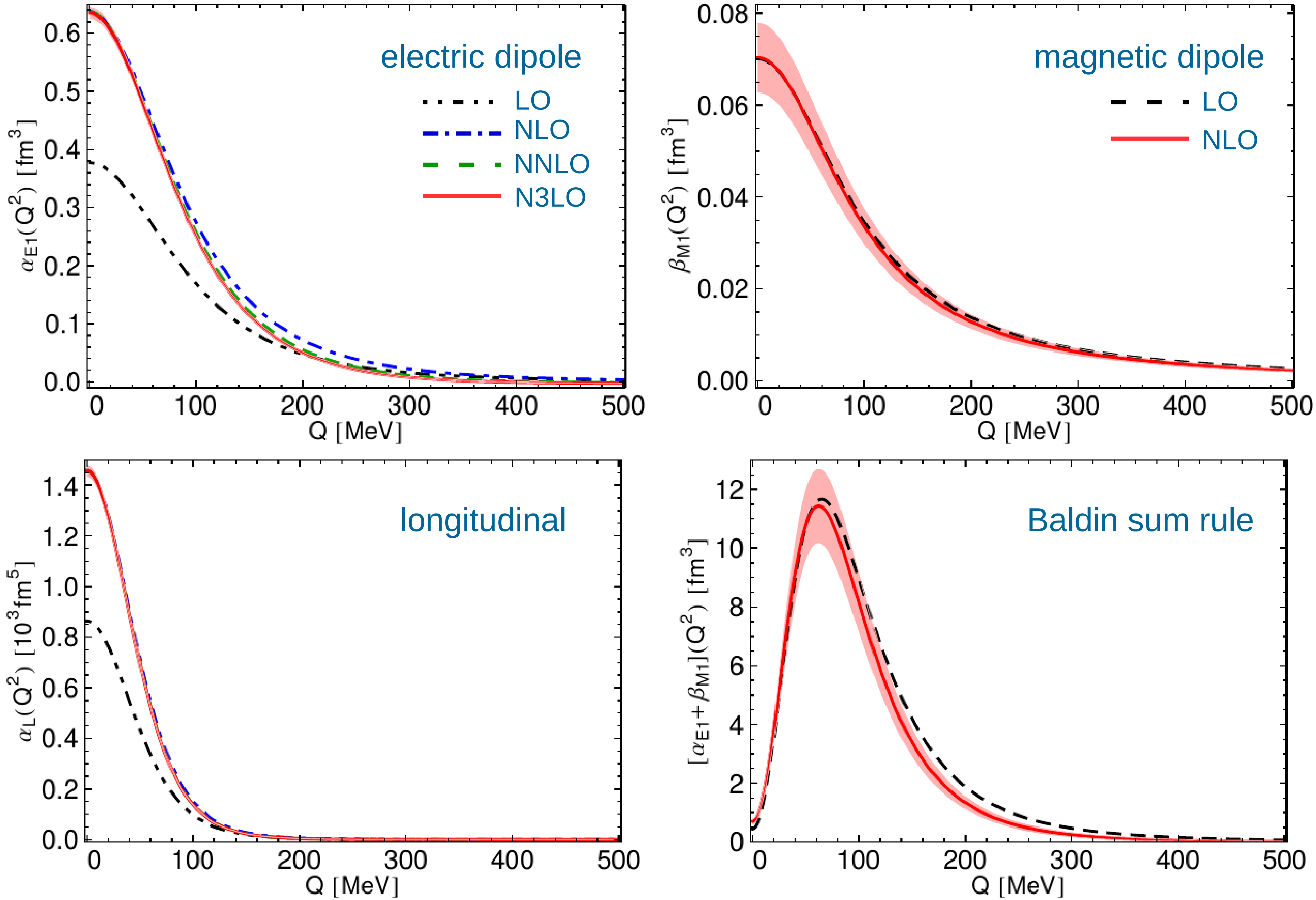
$$\alpha_{E1} = \frac{\alpha M}{32\pi\gamma^4} + \ldots$$

$$\beta_{M1} = -\frac{\alpha}{32M\gamma^2}\left[1 - \frac{16}{3}\mu_1^2 + \frac{32}{3}\mu_1^2 \frac{\gamma}{\gamma_s - \gamma}\right] + \ldots$$

$f_L = O(p^{-2}), \qquad f_T = O(p^0)$ in VVCS

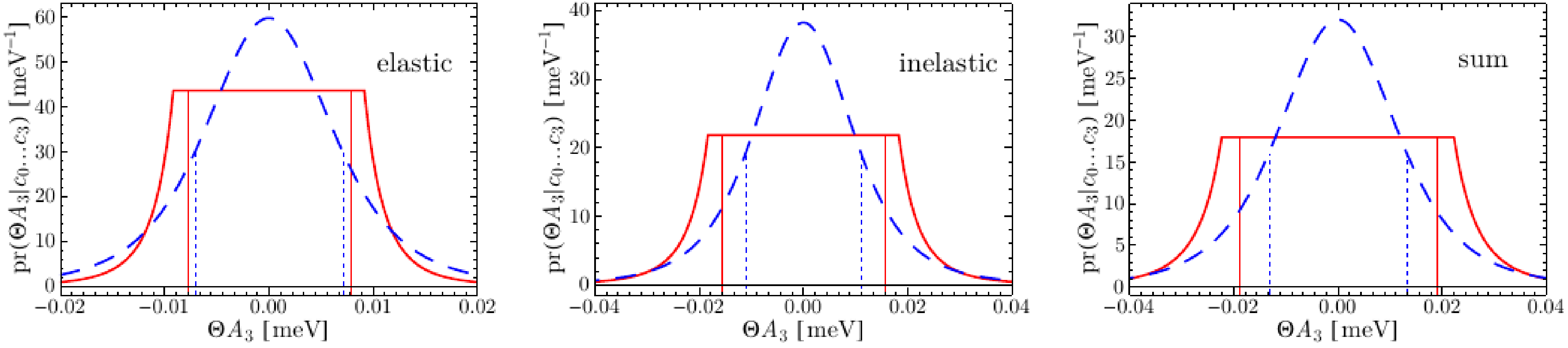
$\text{longitudinal} = O(p^{-2}), \qquad \text{transverse} = O(p^2)$ in TPE

Deuteron VVCS: Generalised Polarisabilities



Bayesian Uncertainty Estimate

- Probabilty distribution functions corresponsing to the uncertainty estimate



VL, Hagelstein, Pascalutsa (2022)
Along the lines of Furnstahl, Klco, Phillips, Wesolowski (2015), Coello Perez, Papenbrock (2015)